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Abstract

Asymmetric information regarding firm prospects causes dilution, leading to adverse selection and inefficiencies in the market for new investments. However, if the market obtains information about the firm over time, issuing callable convertible securities with restrictive call provisions is optimal. Even when the market's information is noisy, such securities can be designed to make the payoff to new claimholders independent of the private information of the manager. The restrictive call provision serves as a commitment device, enabling the manager to call only when the stock price rises in the future. This solves the dilution and adverse selection problem costlessly. The same efficient outcome can also be implemented by issuing optimally designed floating price convertibles. This role of convertibles is similar to that of warranties in a lemons market for a durable good.

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1 Introduction

We investigate the classic problem of inefficient under–investment when a manager has superior information about the firm's prospects relative to the market, as in Myers and Majluf (1984). Due to the asymmetry of information, any security issued by the firm is priced in competitive markets at its expected value. This leads to dilution in the claims of the existing owners of the firm, when the manager's information is better than average. Such dilution may lead the manager, acting in the interest of the existing owners, to take the socially inefficient decision of not investing in positive net present value projects.

In this paper we start from the premise that the initial asymmetry of information about the firm's assets in place and investment opportunities is likely to be resolved over time, even though at each date the manager's information is superior to that held in the market. Analyst announcements, future earnings, R&D outcomes, M&A announcements or decisions by regulators are few of the events that may reveal valuable information to the public over time. Our main goal is to use the future imperfect resolution of the initial asymmetry of information and design a security whose value is *independent of the initial private information* of the manager. In equilibrium, the price obtained for such a security in competitive markets will be a 'fair' price from the perspective of the manager, regardless of his private information. As a result, the symmetric information outcome of no dissipation or dilution will be implemented, solving the under-investment problem costlessly.

We show that suitably designed convertible securities can perform such a role, thereby providing a rationale for their common features and practices. In a simple binary state model, we find a callable convertible debt (or preferred stock) contract, with fixed conversion prices and restrictive call provisions, that mitigates adverse selection completely. The convertibility feature allows investors to choose which kind of security they would like to hold ex-post — the senior debt claim or the junior common stock claim. The callability feature allows the manager to force conversion into common stock following good news. Convertibility combined with callability ensures that different types of securities are held, depending on the nature of information that is publicly disclosed.[1]

The optimally designed callable convertible security has the property that the market value of the debt claim is higher than the market value of the common stock claim. Consequently, the manager seeks to force conversion to common stock whenever he is able to. In order to keep the manager honest, the security also has a restrictive call provision that does not allow the manager to call and force conversion unless good information has been revealed to the market and the share price is high

[1] In order to focus on inefficiencies arising out of asymmetric information we abstract away from considerations of tax or clientele effects as well as bankruptcy and financial distress costs. As a result, debt is equivalent to preferred stock in the context of our model. For simplicity we will refer to the senior claim as debt.

enough. This restriction acts as a commitment device, raising the value of the security for new claim-holders, ultimately benefitting the firm and existing claim-holders.

The better the initial information of the manager the higher is the chance that good information will eventually reach the market, raising the stock price and enabling the manager to force conversion. However, if and when such information does not reach the market, the manager is unable to force conversion. Since the likelihood of being able to force conversion is positively related to the manager's initial information, he can choose the conversion ratio and face value such that the expected value of the claims sold is constant across all private signals that he receives. Put differently, a callable convertible security can be thought of as equity, plus a put option (or guarantee), less a call provision that allows the manager to extinguish the put via a forcing call. Thus, if the manager thinks that the equity part of the convertible is undervalued by any given amount, he can ensure that the value of the guarantee adjusted for the call provision is overvalued by the identical amount, provided the call provision is suitably restrictive.

The symmetric information outcome can also be implemented with floating price convertibles (i.e., convertibles with conversion ratios that depend on publicly observed market values, including mandatory convertibles that are automatically converted into equity). We show this in the context of a general model with an arbitrary number of states where, in addition, the public disclosure of information is endogenous and provided by a self-motivated analyst. Once again, the optimal security has the property that, in equilibrium, the market value of the claims sold is lower the more favorable is the public information in the market, with its expected value independent of the manager's information. Such a security exists as long as the resolution of the initial asymmetry of information occurs with enough fidelity, enabling the manager to costlessly solve the underinvestment problem.

The seminal work of Myers and Majluf (1984) has been followed by a large literature attempting to identify securities that mitigate the dilution and associated underinvestment problem. The paper by Brennan (1986) is perhaps the closest in spirit to our work. Brennan points out that a floating-priced convertible security can avoid the adverse selection problem if the conversion price depends on the market price. Such a security is automatically converted into n shares, where n is the inverse of market price at the time of conversion. Thus, it pays a fixed dollar amount independent of the market price (i.e., the *public* information in the economy). If the private information of the manager is perfectly reflected in the market price at the time of conversion, then the adverse selection problem can be costlessly solved with such a security. When the manager's private information is imperfectly incorporated into the market price however, issuing such a security leads to dilution and may cause underinvestment. We show that in such cases efficiency can be regained by using commonly used securities such as a callable convertible preferred/bond with fixed a conversion ratio and restrictive call provisions, or floating price convertibles whose payoff depends non–trivially on the market price, as long as the expected payoff from such a security is independent of the *private* information of the manager.

A significant portion of the literature following Myers and Majluf focusses on modes of financing that allow the management to separate by signaling its type and thus solve the under-investment problem. Since separation by signaling quality is typically costly, it creates another source of inefficiency and dissipation in value that might even exceed the dissipation in value caused by dilution.[2] In fact, Nachman and Noe (1994) show that non-dissipative signaling is not possible if the firm is limited to issuing securities with payoffs that are weakly increasing in the underlying cash flows. Complementing this result, Gibson and Singh (2001) show that costless separation can be achieved when the firm is allowed to issue put warrants, whose payoffs are non-increasing in cash flows. Our work shows that the first-best outcome can in fact be implemented in equilibrium, without any dissipation in value, using securities with payoffs that are non-decreasing in cash flows. The difference with Nachman and Noe is that the manager is able to use the future imperfect resolution of the current asymmetry of information.

A number of papers restrict their attention to securities with non-decreasing payoffs and yet manage to attain separation without dissipation in value. Their results are achieved by using signaling devices that are costly to mimic for the bad type but not costly in equilibrium for the good type. Among these, Stein (1992) demonstrates the value of callable convertible securities in avoiding costs of financial distress, in a model where the initial asymmetry of information is completely resolved over time. He shows that callable convertible debt can be used by good firms in order to signal their types and separate from bad firms. The bad firm does not mimic the good firm whenever the expected cost of financial distress from doing so is high enough to overcome the benefits of selling an overvalued claim. Good firms are necessarily able to call the bonds and force conversion in Stein's model, thereby avoiding the costs of financial distress. In contrast, the value of the optimal security that we characterize is independent of the private information of the manager. As a result, there is no scope for mispricing whether or not the bad firm mimics the good firm, and even when no type of the manager can guarantee that he will be able to force conversion. Furthermore, we do not require the initial asymmetry of information to be completely resolved at any point in time. Our results thus complement those of Stein by pointing out that the "back-door equity" value of convertible securities arise purely from considerations of asymmetric information, and not from considerations of financial distress costs.

Nyborg (1995) considers the optimality of convertibles in a model where new private information arrives at each date to a risk averse manager. In his model there is not only information content in

[2]We will discuss only the most related papers here. The reader is referred to Harris and Raviv (1991) for a more thorough survey of the earlier signaling literature. In additon to costly signaling, there are papers analyzing how costly information acquisition might be used to mitigate adverse selection. See, e.g., Fulghieri and Lukin (2001).

the type of security initially issued, but also in the decision to call a previously issued convertible. He shows that risk-averse managers signal their quality by not calling immediately, whereas bad managers whose equity is expected to decline in the near future are forced to call.[3] Therefore, he concludes that voluntary and forced conversions are accompanied with different price reactions. This distinction between forced versus voluntary conversions is not present in Stein (1992), since information becomes public exogenously, prior to the call period, and no new private information arrives. Furthermore, the management always calls to force conversion, not allowing a voluntary conversion in equilibrium. In our model, we allow asymmetric information at the time of call, although the decision to force conversion is not informative in equilibrium. Furthermore, the manager is able to force conversion of the optimal callable convertible security only after good news comes to the market. Therefore, voluntary conversion is a feasible action for investors following bad news, but they never choose to exercise it.

Constantinides and Grundy (1989) show that securities similar to (noncallable) convertible bonds can costlessly solve the adverse selection problem by signaling information, provided the firm is also allowed to buy back shares. On the other hand, Brennan and Kraus (1987) show that the good type may separate from the bad type by retiring existing debt, which is too costly for the bad type to mimic. We also allow the manager to buy back previously issued securities in our model, but such strategies are not utilized in equilibrium. Brennan and Schwartz (1987) (and Brennan and Kraus) show that convertible bonds can solve the problem caused by asymmetric information about volatility. In our model, the asymmetric information is about the distribution of cash flows and so covers the possibility of at least some types of asymmetric information about volatility.

Other explanations have been offered for the use of convertible securities. Cornelli and Yosha (2003) analyze a problem in which a manager can manipulate the interim signal about the quality of a project. If the investment occurs in multiple stages, this possibility of "window dressing" results in a conflict of interest and thus inefficient investment. Convertible debt can be used to solve this problem. Convertibility features may also mitigate incentive problems. Green (1984) shows that the incentive problem caused by conflicts of interest among claim holders can be mitigated by convertible debt rather than straight debt. A growing literature is also providing explanations for the use of convertible securities solving moral hazard problem within staged venture capital financing (see, e.g., Repullo and Suarez (1998)) while a large and earlier literature provides insights on the pricing of convertible securities, the optimal exercise of call options, and on stock returns at the announcement of convertible debt calls (e.g., Ingersoll (1977a and 1977b), Brennan and Schwartz (1977), Brennan and Schwartz (1980)).

[3] This is similar to Harris and Raviv (1985), except that in Nyborg the decision to issue a convertible is also endogenous.

The considerable attention that convertibles have received in the literature reflects the importance of the convertibles market. Such securities are quite common. The global convertible market is worth approximately $610 billion and growing. In 2001, for example, there were around 400 new issues in the U.S. convertible market that raised a total of $106.8 billion.[4] About 95% of convertible securities are callable (see, e.g., Lewis, Rogalski and Seward, 1998) and the available evidence suggests that an overwhelming majority of these have restrictive call provisions similar to those we specify (i.e., the convertible can be called early only if the stock price exceeds a pre–specified trigger price).[5] To take one recent case, on December 8, 1999, Human Genome Science (HGS) raised $200 million by issuing subordinated convertible notes that were due in 2006. Under the terms of the issue, HGS could not call the bond before December 2002 unless the stock price crossed a trigger price (equal to $107.44, or 150% of the conversion price) and stayed there for 20 out of 30 consecutive trading days prior to the call date. In retrospect, HGS could not have chosen a better time for the issue, as the market was energized by prospects of genomics led medical discovery. The genomics/biotech sector turned out to be the best performing equity sector for that period, gaining 60% during the first three months of 2000. As a result, HGS was able to satisfy its call restrictions and call the bonds to force conversion on March 2, 2000, just 85 days after the original issue, its own stock having gained 96.1% in the meantime. Buoyed by the enthusiastic response of the market to genomics research, in those heady days at the turn of a new millennium, HGS undertook a second convertible issue on March 6, 2000. The bonds in the issue were due in 2007, and they also had a three year restrictive call provision with a trigger price set at $164.25 (again, equal to 150% of the conversion price). The issue is still outstanding although the call restrictions have since expired and the bonds are freely callable. The market has treated HGS somewhat less favorably than originally anticipated, with its stock price currently around $12 and a triple–C rating on the bond.

Our results have several other empirical implications with respect to commonly observed features of callable convertible securities. The optimally designed such security has the property that the market value of the debt claim (with embedded options) is significantly higher than the market value of the common stock claim. This seems to be in line with the evidence especially given the fact that options with long maturities are typically worth significantly more alive than dead.[6] Our results also imply that

[4]Source: Securities Data Company, Inc.. See also www.convertbond.com, a division of Morgan Stanley Dean Witter, and Francis, Toy and Whittaker (2000). Other details of the discussion in the rest of this paragraph are available from the same sources.

[5]For example, 44 out of the 49 securities in the sample analyzed in the Morgan Stanley Dean Witter U.S. Convertible Research Report (Iyer et al., 2000), covering the period April 1999 to March 2000, have such call restrictions, referred to usually as 'soft'or 'provisional' call restrictions in the industry.

[6]In addition to such high option values, the yield on the bond is typically significantly higher than the dividend yield. In the sample analyzed by Lewis, Rogalski and Seward (1998) for example, the difference is 702 basis points on average.

calling firms should experience high earnings or some other good news prior to call announcements, again in line with the empirical evidence (see, e.g., Campbell, Ederington and Vankudre (1991)).

The main focus of the existing empirical literature has been on the price impact of convertible debt issues as well as that of calls of these securities. Mikkelson (1981) pointed out that announcement of convertible debt calls is followed by a decline in the stock price. The subsequent empirical literature (Mazzeo and Moore (1992), Byrd and Moore (1996), Ederington and Goh (2001)) finds out that such a decline is typically short–lived and is more likely to be related to liquidity effects arising out of an increase in the number of available shares rather than due to asymmetric information effects. We abstract away from considerations of liquidity and our results do not predict any significant decline or increase in the stock price after the call announcement. In contrast to the empirical evidence on the announcement effect of calls, the empirical evidence on the announcement effect of the issue of convertible securities suggests that there is a negative effect on the stock price (e.g., Dann and Mikkelson (1984), Field and Mais (1991)). In the pooling equilibrium that we focus on however, there should not be any significant announcement effect of the issue. Therefore, our analysis suggests that the negative announcement effect must be due to reasons other than adverse selection.[7] Finally, our results imply that the manager should call whenever he is able to, at least in the absence of secondary effects arising out of tax shields or short–term movements in the stock price during the call notice period. This is in line with results obtained by Asquith (1995) among others.[8]

Using future public information to alleviate a current adverse selection problem of course has applications beyond project financing. In the durable good context, starting with the work of Spence (1977) and Grossman (1981), there has been a large literature investigating the role of warranties in solving the lemons problem of Akerlof (1970). Grossman, in particular, shows that pooling with an optimally designed warranty contract is optimal, in a setting where the future performance of the good is public information that perfectly reveals its value. We extend this insight in the context of a general model, with a possibly payoff–relevant noisy public signal, and discuss the similarities and dissimilarities of the results obtained in the durable goods framework with those obtained in the context of the project financing problem.

The rest of the paper is organized as follows. In Section 2, we set up our basic model. In Section 3, we first consider the benchmark case where the asymmetry of information is perfectly resolved over time and then the case where the asymmetry of information is never fully resolved. In Section 4, we

[7]It is well documented that hedge funds buy significant portions of convertible securities and simultaneously hedge their positions, mostly by short selling the underlying stock at the time of the convertible issue, thus creating a negative pressure on stock prices. See, e.g., L'habitant (2002).

[8]In contrast with results obtained by Ingersoll (1977b) and the literature on call delay that followed. In Section 3.2 we discuss in greater detail how our results on the role of soft or restrictive call provisions may reconcile these two contrasting themes in the literature.

present our general model while in Section 5 we provide an application in a durable goods context. Section 6 concludes and the Appendix contains the details of some of the proofs.

2 The Basic Model

The basic structure of our model is essentially identical to that of Myers and Majluf (1984). We consider a firm that has both assets in place and a new investment opportunity. The values of both the new investment opportunity and assets in place are uncertain. The uncertainty is captured by the "type" of the firm $\theta \in \{\theta_1, \theta_2\}$.[9] Let $\Pr[\theta = \theta_i] = \lambda_i \in (0,1)$ with $\sum_i \lambda_i = 1$. We assume that the manager privately knows θ. Both the assets in place of the firm and the cash flows from the new investment opportunity depend on the type θ. Initially the firm is all equity with the number of shares outstanding given by $M = 1$.[10] The firm does not have sufficient internal funds to invest in the new project and has to raise capital by selling additional securities. The manager makes his decisions to maximize the welfare of the existing shareholders, the riskless rate is normalized to 0, and all agents are risk–neutral.

Let A_i stand for the *expected* value of the cash flows from the assets in place given $\theta = \theta_i$. This is the expected value of the firm for type θ_i of the manager if he does not invest in the new project. In order to invest in the new project, the manager has to raise an amount $C = \$1$ from outside investors. The new investment and assets in place *combined* produce a random cash flow of X, taking values in some set $\mathbf{X}$, a subset of the non–negative real numbers. Let $G(.|\,\theta)$ denote the cumulative distribution function of X given θ. We assume that project cash flows for type $\theta = \theta_2$ first order stochastically dominate those for type $\theta = \theta_1$:

$$\text{For all } x \in \mathbf{X},\ G(x|\theta_i) \leq G(x|\theta_j) \text{ for } i > j. \tag{1}$$

Define the expected value of the total cash flows for type θ_i of the firm, given that it invests, to be $V_i = E[X|\theta_i]$. Let $\overline{V} = \sum_i \lambda_i V_i$ be the ex–ante expected value of V_i. From (1), V_i must be non–decreasing in i. To make matters interesting, we assume henceforth that

$$V_i > V_j \text{ if } i > j. \tag{2}$$

Furthermore, we also assume that projects have positive NPV regardless of θ:

$$V_i - A_i > 1 \text{ for all } \theta_i. \tag{3}$$

[9] In Section 4 we consider the N type case for $N > 2$. The restriction to finite types is for convenience only.

[10] The restriction to an all–equity firm simplifies the exposition. All results extend in a straightforward manner to the case where the firm has existing senior debt outstanding, provided we interpret all cash flows as net of prior obligations.

With symmetric information, and in competitive markets, all types of the manager will undertake the socially efficient outcome of investing in the project. The expected payoff of the existing shareholders from doing so will equal $V_i - 1$, the net present value of the firm of type θ_i. As will be seen below however, the manager might inefficiently under-invest and forego a positive NPV project in the presence of informational asymmetries between him and the market.[11]

If the cash flows from the project together with the assets in place of the firm are greater than or equal to the cost of the project with probability one, then the firm can always issue riskless secured debt at zero cost and the problem would be uninteresting. To rule out the possibility of riskless debt we assume that

$$G(1|\theta_1) > 0. \tag{4}$$

Thus, when $\theta = \theta_1$, with strictly positive probability the total cash flows X will fail to cover the required outlay of \$1.

Let $S(x)$ be the payoff from a security S when realized cash flows are x. We will restrict attention to securities with payoffs $S(x)$ that are non–decreasing in x and that satisfy limited liability, i.e.,

$$0 \leq S(x) \leq x \text{ for all } x. \tag{5}$$

Let $\mathbf{S}$ be the set of admissible securities. Note from (1) that for any admissible security $S \in \mathbf{S}$, $S_i \equiv E[S(X)|\theta_i]$ is non–decreasing in i.

Equity and debt are admissible securities. An equity share will be denoted by $\alpha \in (0,1)$, with the expected value of the cash flows from a share α, given $\theta = \theta_i$, equal to αV_i. For any bond with face value $F \geq 0$ let $D_i(F)$ be the expected value of the cash flows from the bond given $\theta = \theta_i$:

$$D_i(F) = E[\min(X, F)\,|\,\theta_i]. \tag{6}$$

$D_i(F)$ is continuous in F and $D_i(F) \leq V_i$ for each i. In case of preferred stock, F should be thought of as the sum of promised dividends and liquidation value.

Our model has two (groups of) players, the manager (who maximizes the welfare of old shareholders) and the potential investors. The manager knows θ when he makes his investment and financing decisions. In contrast, we will assume that initially investors are uninformed about θ, though later they will obtain information about the firm type. Furthermore, these investors are competitive and efficient, so that at each date they value all securities at their expected value given publicly available information. We will refer to the set of potential investors collectively as the market. We specify our

[11] As is well–known, for adverse selection to cause inefficient under–investment, it is necessary that the assets in place A_i, as well as the total cash flows V_i, be both increasing in i. We allow for this possibility, as we do the possibility considered by Myers and Majluf where the difference $V_i - A_i$ is also increasing in i, but do not restrict attention to them.

timing structure, strategies and available securities, and the resolution of informational asymmetries over time in more detail now.

Our model has three dates 0, 1 and 2. At date 0, given his private information, the manager decides whether or not to invest and what securities to issue to finance the investment. The market is uninformed about θ at date 0 and competitively values the securities issued by the manager. The manager invests at date 0 if the issue succeeds.[12]

At date 1, some of the asymmetric information present at date 0 is resolved. Specifically, the market publicly observes a signal $m \in \{m_1, m_2\}$ of θ, with

$$\Pr[m = m_i \,|\, \theta = \theta_i] = \beta \in (\frac{1}{2}, 1] \text{ for all } i. \tag{7}$$

The parameter β is a proxy for the degree to which the initial asymmetry of information between the manager and the market is ultimately resolved.[13] The case $\beta = 1$ corresponds to the case of perfect resolution. On the other hand, the case $\beta = \frac{1}{2}$ corresponds to the case where none of the asymmetry is ever resolved before project cash flows are realized. Though our results do not depend on a specific interpretation of the signal m, it might help the reader to think of it as an analyst announcement or the outcome of a patent application which may or may not be approved by the government. For the present moment we will assume that the signal m is exogenous. In Section 4 we provide a simple story to endogenize the signal so that its distribution will depend on the date 0 financing strategy of the manager.

We allow the manager to issue securities at date 0 whose payoffs depend on the endogenous date 1 *response* of the market (e.g., the market value of equity or the stock price) to the realized public signal m. For example, the manager is allowed to issue a callable convertible bond that can be called only if the stock price exceeds a threshold value (or a floating price convertible whose conversion rate depends on the stock price). If such a security is used in equilibrium, the threshold value may be such that the bond can be called only when the public signal is equals m_2 and the ensuing stock price is higher than the threshold. Even though the outcome will be in some sense identical, we do not allow the manager to set a call restriction or a conversion provision *directly* in terms of the public signal m. Such a restriction is desirable as in practice, securities whose payoffs depend on the endogenous stock price in some manner are quite common, whereas securities whose payoffs depend directly on some public signal such as an analyst announcement or an earnings announcement are not so.[14]

[12] Though this possibility never arises in equilibrium, we assume that if the manager fails to raise the required outlay for the project, he invests the amount raised in a riskless asset. On the other hand if he raises more than the required outlay he immediately distributes the excess as dividends.

[13] The symmetry in the conditional distribution of the signal is not needed for our results but simplifies the exposition.

[14] Presumably because such public signals are frequently quite amorphous and contracts directly contingent on them are not enforceable in a court.

To complete our description of the timing structure, we assume that at date 2 the project cash flows are realized and distributed. The information and timing structure above implicitly defines a dynamic game of incomplete information between the manager and the market. Our notion of equilibrium will correspond to the perfect Bayesian equilibria of this game.

Before we present our results, a few further remarks with respect to the model and the timing structure are in order. First, since the dilution costs of adverse selection arise via transfers from the old claim–holders to the new claim–holders, it is necessary for our results that the manager attach a 'welfare' weight to old claimholders that is strictly greater than what he attaches to the new group. For simplicity, we assume that the manager cares only about the old shareholders. Such an assumption is common in the literature and may be innocuous in our context, since in practice younger firms (that are more susceptible to adverse selection in the first place) do seem to display a higher degree of managerial ownership, presumably aligning managerial interests with those of the existing claimholders.[15] Second, we implicitly assume that the manager cannot postpone his investment decision to a later date, possibly because actions from competitors will erode the value of the project if he does so. Similarly, it is also not possible for the manager to anticipate a future need for cash and raise the required cash early, at a date before date 0, under conditions of symmetric information.[16]

In order to put our results in perspective we conclude this section with a numerical example that recalls the dilution effect of issuing a simple security like common stock or straight debt.

Example 1

Suppose that $V_1 = 4$ and $V_2 = 8$ with $\lambda_1 = \lambda_2 = 1/2$. In a symmetric information world, each type θ_i of the manager will invest, for example by issuing an equity share equal to $1/V_i$, for an expected payoff to the existing claim–holders equal to $V_i - 1$. Consider now the dilution effect of issuing a share α of common stock, under conditions of asymmetric information.

Suppose that $A_2 = 6.5$. It is not difficult to see then that there exists an equilibrium where only type θ_1 of the manager issues equity, while type θ_2 foregoes the investment opportunity.[17] In this equilibrium, type θ_1 has to offer a share $\alpha = 1/V_1 = 1/4$ in order to raise the required cash from a

[15] See, e.g., Graham and Harvey (2001). It is not clear however that this is an optimal provision of managerial incentives from the perspective of the shareholders. Ex–ante, uninformed shareholders may prefer that the manager maximize the total value of the firm and so not under–invest. (See Dybvig and Zender (1991) for a similar point). However, such a contract may not be renegotiation proof at the interim stage. A complete characterization of the optimal provision of managerial incentives is beyond the scope of this paper.

[16] Perhaps because there is no such date— the firm may be 'born' under conditions of asymmetric information. Note in this respect that we allow the expected value of the assets in place A_i to depend on the manager's type. A strategy of raising a lot of cash early may also create agency problems between the manager and shareholders, arising out of inefficient use of 'free' cash, and so may be prevented by the latter group.

[17] Such an equilibrium is supported by degenerate beliefs on type θ_1 following any deviation.

market that infers from the issue decision that the manager's type is θ_1. Given this, the payoff from undertaking the investment for type θ_2 of the manager is $(1-\alpha)V_2 = 6$, less than $A_2 = 6.5$, the payoff from foregoing the investment. In this under–investment equilibrium, type θ_2 of the manager prefers to miss out on a positive NPV investment opportunity rather than sell comparatively larger claims against the cash flows from the assets in place. A simple security like equity may thus lead to socially inefficient under–investment.

Even when equity does not cause under–investment, it will lead to dilution. In the example above, there is a pooling equilibrium where both types of the manager issue the same equity share $\alpha' = 1/\overline{V} = 1/6$, and the market responds neutrally to the equity issue. In this equilibrium, type θ_2 of the manager still sells undervalued claims given his information and his expected payoff equals $(1-\alpha')V_2 = 20/3$, less than the payoff of $V_2 - 1 = 7$ that he obtains in the symmetric information world, with the difference, equal to 1/3, being the dilution effect of issuing equity.[18] As we will show below, with the optimally designed convertible security such dilution (that is a necessary condition for under–investment to occur) will be avoided by all types of the manager. Before we do so, observe that the dilution and possible under–investment that is created by a simple security like equity will also exist for other securities like debt, as long as riskless debt is infeasible for at least one type.

3 The Optimality of Callable Convertible Securities

In this section, we convey the basic idea of the paper using the simple binary model described above. To gain intuition we start by analyzing the benchmark case where the date 0 asymmetry of information is perfectly resolved at date 1. This corresponds to the case where $\beta = 1$. In Subsection 3.2, we will consider the case where $\beta < 1$, so that the date 0 asymmetry of information is never perfectly resolved.

3.1 Perfect Resolution of Asymmetric Information

Denote a callable convertible security by the tuple (F, α, k, T), where T is the (common) maturity date of the call option and the convertibility option, k is the call price, α is the share of the firm the bondholders will have, if they decide to convert into common stock and F denotes the face value of the bond. In line with common practice, we assume that if the firm calls the convertible security then the holders still retain the right to convert into equity and do not have to surrender the security as long as they convert.[19]

[18] Since type θ_1 of the manager sells an overvalued claim, he strictly prefers to invest in either equilibrium, regardless of A_1.

[19] Notice that we do not specify any call restrictions on our security. In the next section, we will see that such restrictions are needed only when $\beta < 1$.

Consider the following callable convertible security $(F^*, \alpha^*, k^*, T^*)$. Suppose F^* is equal to the face value that would be chosen in a world where it is common knowledge that the manager has bad news, i.e., $\theta = \theta_1$. That is, F^* satisfies

$$D_1(F^*) = 1. \tag{8}$$

Hence, the debt part of the convertible is a fairly valued security for the low type. Suppose that the equity part is a fairly valued security for the high type. That is, α^* solves

$$\alpha^* V_2 = 1. \tag{9}$$

Suppose further that $T^* = 1$, so that all the options on the bond expire on date 1, when there is no asymmetry of information. We show now that the call price k^* can be chosen suitably to finance the project regardless of θ, at zero cost (i.e., with no dilution) for the existing shareholders. To do this we proceed backwards in time and analyze the optimality of the manager's decision to call the bonds and the optimality of the investors' decisions to convert.

Suppose that we are in date 1 and $m = m_2$, so that it is common knowledge that $\theta = \theta_2$. We want the call value k^* to be such that the manager wants to call the bonds in this case. The optimality of the call decision depends in turn on the optimal conversion decision of the bondholders, both in the case where the bonds are called and in the case where the bonds are not. If the bonds are not called, the bondholders will not want to convert, as their payoff from not converting is at least as high as their payoff from converting:

$$D_2(F^*) \geq D_1(F^*) = 1 = \alpha^* V_2. \tag{10}$$

If the bond is called, then the bondholders will want to convert if their payoff from converting is at least as high as the payoff from holding the bond, i.e., if

$$k^* \leq \alpha^* V_2 = 1. \tag{11}$$

Suppose that k^* is such that (11) holds. Then, when $\theta = \theta_2$, the manager will want to call the bonds and force conversion, as the payoff of the old shareholders from doing so, $(1-\alpha^*)V_2$, is at least as high as $V_2 - D_2(F^*)$, the payoff from not forcing conversion.

Suppose next that we are in date 1 and $m = m_1$ so that it is common knowledge that $\theta = \theta_1$. We want the call value k^* to be such that the manager does not want to call the bonds and the bondholders do not want to convert the bonds if they are not called. If the bonds are not called, the bondholders do not want to convert as

$$\alpha^* V_1 < \alpha^* V_2 = 1 = D_1(F^*). \tag{12}$$

Therefore the manager will not want to call the bonds if

$$k^* \geq D_1(F^*) = 1. \tag{13}$$

From (11) and (13), if

$$k^* = 1, \tag{14}$$

then the manager will call the bond to force conversion if $\theta = \theta_2$ and will not call the bond if $\theta = \theta_1$. In the latter case, bondholders will not convert. For such a bond and sequentially optimal call and conversion decisions, the payoff to the bondholders will be equal to 1 regardless of the private information of the manager, from (8) and (9). As a result, when the bond is issued at date 0, investors will not face any adverse selection and will be willing to provide \$1, the expected value of the issue. Furthermore, the expected payoff for the old shareholders in type θ_i of the firm at date 0 will equal $V_i - 1$, the first best value of the firm given $\theta = \theta_i$. As a result, the manager will always invest. Finally, we have to specify beliefs off the equilibrium path at date 0 to complete the characterization of this perfect Bayesian equilibrium. We suppose that the uniformed investors believe that $\theta = \theta_1$ whenever the manager issues any other security at date 0. Thus, neither type of the manager has an incentive to deviate.

Proposition 1 *Suppose $\beta = 1$. Then it is an equilibrium for both types of the manager to invest by issuing the callable convertible security $(F^*, \alpha^*, k^*, T^*)$ characterized by (8), (9) and (14). The manager will call to force conversion only when m_2 is observed. The security will not be called or converted when m_1 is observed. In this equilibrium, the expected value of the claims sold to the new claim holders will equal 1 regardless of θ, and the expected payoff to old shareholders of type θ_i of the firm will equal $V_i - 1$, the first best value of the firm given $\theta = \theta_i$.*

Proof. Follows from the discussion above. □

The value of the optimal security that we characterize above is independent of the private information of the manager. Thus, the security is correctly valued even though the bad type mimics the good type. This property of the optimal security will be seen to carry over to the case where the resolution of the asymmetry of information is imperfect.

Note that the outcome implemented above can also be implemented simply by using short–term debt maturing in period 1 and then refinancing when there is no asymmetry of information.[20] Specifically, at date 0, the manager can issue short–term risk–free debt, issuing any other security to retire the debt at zero cost in the symmetric information environment of date 1. This equivalence between a callable convertible security and short term debt breaks down when the date 0 asymmetry of information is never perfectly resolved. The simple scenario of this section nevertheless serves to bring out the intuition why callable convertible securities mitigate adverse selection problems.

[20] At least as long as we ignore issue costs. We assume away such issue costs throughout the paper.

3.2 Imperfect Resolution of Asymmetric Information

We now turn to the case where the date 0 asymmetry of information is only imperfectly resolved at date 1, i.e., $\beta \in (\frac{1}{2}, 1)$. As we show below, as long as β is high enough, there exists a callable convertible security such that if the manager finances the investment by issuing this security then the first best outcome will still be implemented in equilibrium. The structure of the efficient equilibrium as well as the optimal security will be very similar to that characterized in the previous section. In particular, the manager will call to force conversion into equity only when good news is disclosed at date 1 and the security will not be called or converted when bad news is disclosed. The important difference from the previous case is that in the case of bad news, the manager will be prevented by a restriction on the call provision from calling the bond and forcing conversion. This restriction on the call provision will take the form that the security can be called only when the share price of the firm exceeds a certain threshold or trigger value. By specifying this restriction at date 0, the manager will be able to commit to not calling the bond and using his privileged information in the future at the expense of the new investors, ultimately benefitting the existing claim holders.

Formally, a callable convertible security with a restrictive call provision consists of a tuple (F, α, k, p, T) where F is the face value, α is the fraction of equity obtained upon conversion, k is the call price and T is the (common) maturity date of the call and convertibility options, as before. The only difference from the security of the previous section is that the bond can be called only if the share price at date 1 exceeds a trigger price p. For brevity, we will refer to this trigger price as the call restriction. We first characterize the optimal such security keeping in mind the properties it must have in order to implement the efficient outcome in equilibrium. Then we show that it is an equilibrium for the manager to issue such a security regardless of his private information.

Let the maturity date T on our optimal callable convertible security (F, α, k, p, T) be set for date 1.[21] Suppose that the both types of the manager issue the security at date 0 and expect the security to be converted (due to a forcing call) to equity at date 1, but if and only if good news is disclosed (i.e., $m = m_2$) at that date. Suppose further that, given this expectation, each type $\theta \in \{\theta_1, \theta_2\}$ of the manager estimates that the expected value of the claims sold equals \$1, the cost of the project. Then F and α must satisfy the following two equations:

$$\beta D_1(F) + (1-\beta)\alpha V_1 = 1, \tag{15}$$

$$(1-\beta) D_2(F) + \beta\alpha V_2 = 1. \tag{16}$$

[21] This is for simplicity. We also assume that the debt part of the security is a zero–coupon bond. These assumptions keep the analysis as simple as possible, but are not necessary for our results. See also the discussion at the end of this section.

The first equation states that the required outlay of 1 dollar is equal to the expected value of the security, conditional on $\theta = \theta_1$ *and* conditional on the fact that the security will be converted to equity when $m = m_2$ but not when $m = m_1$. The second equation has the same interpretation, but for $\theta = \theta_2$. We show in the Appendix that (15) and (16) possess a solution $\alpha \in (0, 1)$ and $F > 0$ with $D_1(F) < V_1$ when β is high enough. For now assume that such a solution exists. If the market conjectures at date 0 that the security will be converted to equity at date 1 when $m = m_2$ and good news about the firm is disclosed, but not when $m = m_1$, it follows from (15) and (16) that the expected value of the security will be equal to \$1 at date 0. In competitive markets, such a security will trade at that price at date 0.

We show next that if the security is converted at date 1, it must be the outcome of a conversion forcing call. More precisely, at date 1 the expected value of the equity claim, conditional on the *market's* information m, will be strictly less than the expected value of the debt claim, for all $m \in \{m_1, m_2\}$, implying that the holders of the bond will never voluntarily convert to equity. In fact, we show in the Appendix that, at a solution to (15) and (16) the following must hold:

$$\alpha V_i < D_i(F) \text{ for } i = 1, 2. \tag{17}$$

That is, the expected value of the equity claim must be less than the expected value of the debt claim conditional, in fact, on the *manager's* information θ, for each $\theta \in \{\theta_1, \theta_2\}$. Inequality (17) is central to our analysis and it states that from the manager's perspective, conversion to equity lowers the expected value of the claims sold to new claim-holders. This is the "back–door equity" value of the convertible security in the context of our model — when converted, the equity share of the new claim-holders will be lower than what they would obtain under symmetric information. To compensate the new claim-holders, the value of the debt claim is "sweetened" and is higher than what they would obtain under symmetric information.

Since the expected value of the equity claim is less than that for the debt claim regardless of the manager's private information, the same relationship must hold given the market's information. This implies that the new claimholders will not convert unless forced to do so. It also implies that the manager would like to force conversion whenever he is able to. However, if the manager always forces conversion, the expected value of the security to the new claim-holders will fall below \$1 and they will be unwilling to provide the funds for the project, hurting the existing claim-holders. Consequently, a restriction on the call provision is necessary as it enables the manager to commit to not calling the bond unless good news is disclosed. In turn this will make the investors willing to pay \$1 for the issue at date 0. We now determine the call price and the trigger price that are needed for this to hold.

Let $\mu_i^1(m)$ be the posterior probability at date 1 attached by the market to the event that $\theta = \theta_i$ after observing m. Note that since $\beta > \frac{1}{2}$ we must have $\mu_2^1(m_2) > \mu_2^1(m_1)$. Choose any call price k

that is less than the expected value of the equity claim given $m = m_2$,

$$k < \sum_{i=1}^{2} \mu_i^1(m_2)\alpha V_i. \tag{18}$$

In equilibrium, when $m = m_2$, the right–hand side of (18) will be the date 1 market value of the security. As a result, the bondholders will convert when it is called.[22] Choose the trigger price p (that defines the restrictive call provision) to be in between the market value of old shareholders' claims when $m = m_1$ and when $m = m_2$, i.e.,

$$\sum_{i=1}^{2} \mu_i^1(m_1)[V_i - D_i(F)] < p < \sum_{i=1}^{2} \mu_i^1(m_2)(1-\alpha)V_i. \tag{19}$$

Such an interval for p exists from (17). In equilibrium, the date 1 stock price will equal the right–hand side of (19) when $m = m_2$ so that the manager will be able to force conversion by calling; while when $m = m_1$, the stock price will equal the left–hand side of (19), so that the bond cannot be called and will not be converted.

This completes our characterization of the optimal security. In the Appendix, we show that it is an equilibrium for all types of the manager to issue this security at date 0 and that in this equilibrium there will be no dilution. Even though the asymmetry of information is never exactly resolved, the adverse selection problem is exactly solved when β is high enough.

Proposition 2 *There exists $\beta^* \in (\frac{1}{2}, 1)$ such that for $\beta > \beta^*$, it is an equilibrium for the manager to invest by issuing a callable convertible security (F, α, k, p, T) with F and α satisfying (15) and (16), k and p satisfying (18) and (19) and with $T = 1$, regardless of his private information θ. In this pooling equilibrium, the manager calls to force conversion only when $m = m_2$ is observed, regardless of θ. The security cannot be called and is not converted when $m = m_1$ is observed. The date 0 expected value of the claims sold to the new claim holders equals 1 and that for the old shareholders of type θ_i of the firm equals $V_i - 1$, the first best value of the firm.*

Proof. In the Appendix. ■

The convertible bond characterized above has payoffs that are non–decreasing in underlying cash flows. Nevertheless, to make the expected value of the security independent of the private information of the manager and prevent dilution, the value of the equity claim must be lower than the value of the debt claim. The manager will call the bond to force conversion to equity when good information is disclosed and the restrictions on the call provision are met. The better is the initial information

[22]For the sake of completeness, we suppose that the manager raises the required money by issuing equity, in the off–the–path of play event that bondholders surrender the bond upon a call.

of the manager the higher is the chance that this occurs. However, the manager may not always be able to call and force conversion. The worse the initial information of the manager the greater is the chance that he will be unable to force conversion, so that the new claimholders will be left holding the more valuable debt. The date 0 expected value of the claims for the new claimholders is however independent of the private information of the manager.

As we remarked before, to keep the analysis as simple as possible, we have assumed that the convertibility option expires with the callability option at $T = 1$ and also that the bond has no coupons. This has the unrealistic consequence that the call price k is less than the face value of debt F. It is not difficult to see however that our results do not depend on this. In a richer model where both these simplifying assumptions are relaxed, the expression $D_i(F)$ should be interpreted as the expected value of the debt claim (including coupons) together with the option to convert to equity at a later date. Since such an option is valuable, the face value F can be chosen to be lower than the what was obtained above, and our results will have the more realistic implication that the call price is less than the market value of this more valuable debt plus embedded option at the time the bond is called.

Next, recall that for $\beta = 1$, financing with short term debt and refinancing at date 1 also implements the same outcome as the optimal callable convertible security. However, this is no longer true when $\beta < 1$ and the initial asymmetry of information is never perfectly resolved. If the manager issues short-term debt at date 0 that matures in date 1, then he has to raise cash to honor his debt obligations at that date by issuing some admissible security like equity or debt. Since there is still residual asymmetric information at date 1, the high type of the manager will still suffer from dilution at that date, regardless of whether $m = m_1$ or m_2, for reasons similar to those discussed in the example at the end of Section 2. The date 0 expected value of this date 1 dilution will be positive for the high type and may even make him unwilling to invest in the project in the first place.[23]

In Proposition 2 we provide an example of one security and one equilibrium that implements the symmetric information outcome.[24] There exist other equilibria, involving similar securities, that also achieve the same outcome. For example, there exists a separating equilibrium where the high type issues a callable, convertible security similar to the one characterized above (differing only in the call provisions k and p), and the low type issues any other security, say equity. In such a separating equilibrium, given that the convertible has been issued at date 0, the future signal m does not convey any information about the expected value of total cash flows to the market, which is known to be

[23]We illustrate this via a numerical example as well, in the next section.

[24]The lower bound β^* that we obtain in the proof of the proposition is a sufficient condition. It can however easily be seen that some lower bound on β is necessary for the solution to satisfy the admissibility conditions in the definition of $\mathbf{S}$.

equal to V_2. Nevertheless, if the market conjectures that the bond will be called in order to force conversion when $m = m_2$, then the share price will equal $(1-\alpha)V_2$ at that date and state. On the other hand, if the market conjectures that the bond cannot be called or converted when $m = m_1$, then the share price will equal $V_2 - D_2(F)$. From (17), we see that $(1-\alpha)V_2$ is greater than $V_2 - D_2(F)$. As a result, if the manager sets a call price $k < \alpha V_2$ and a trigger price p satisfying $p < (1-\alpha)V_2$ but $p > V_2 - D_2(F)$, type θ_2 of the manager will be able to force conversion when $m = m_2$ but not when $m = m_1$, thereby implementing the symmetric information outcome. Type θ_1 of the manager will be indifferent between mimicking the high type or issuing fairly priced equity given his information and so, in the candidate equilibrium, will not mimic.[25] Without going into issues of refining the set of equilibria, in this paper we simply make the point that even when the initial asymmetry of information is never perfectly resolved, there exists at least one efficient equilibrium that exactly achieves the symmetric information outcome.[26]

The experience of HGS discussed in the Introduction suggest that issuing convertibles is not without its risks. In this respect, the experience of MCI Communications Corporation was more successful, occurring as it did during a less "exuberant" period for the stock market, 1978–83. We conclude this section by briefly recalling its salient features.[27] In 1978 (largely as a result of a lifting of a court order that previously restricted its operations), MCI started on a period of dramatic growth, with total assets growing from \$161 million in March 1978 to \$2.071 billion in March 1983. This growth needed frequent infusions of external capital and MCI often decided to use convertible securities in order to raise the funds. An issue of convertible preferred stock in December 1978 raised \$28 million, followed by a second offering in September 1979 that raised \$67.5 million and a third offering in October 1980 raising \$49.5 million. All these securities were callable, with restrictive call provisions. The restriction allowed the firm to call provided that the market price of MCI stock exceeded the conversion price by a pre-specified margin of around 25% for 30 consecutive trading days around the call date. As events turned out, MCI's stock price rose enough for it to be able to force conversion on all three issues by November 1981. As MCI continued on its growth path, it raised \$100 million in an August

[25] In reality, the efficient pooling outcome characterized in Proposition 2 may be more robust than the efficient separating one if the (low type) of the manager has some interest in maintaining a high date 0 stock price.

[26] There may also exist inefficient equilibria similar to the one characterized in the example of Section 2. Furthermore, extensions of standard forward induction refinements such as the Intuitive Criterion of Cho and Kreps (1987), or Divinity (Banks and Sobel (1987)), will fail to refine the equilibrium set. However, suitable extensions of 'mistaken theory' refinements of the sort proposed by Van Damme (1989) and Hillas (1994) will imply that every equilibrium will implement the symmetric information outcome when β is high enough. On the other hand, if we allow securities whose payoffs are *directly* contingent on m, then every perfect Bayesian equilibrium must be efficient for β high enough. In this paper we do not allow such securities or consider refinements. See however the discussion in Section 5 for more on this.

[27] See Greenwald (1984) and Stein (1992) for details of this well–known case.

1981 convertible debt issue and $250 million in a May 1982 issue. A rising stock price enabled MCI to force conversion on both these issues as well by February 1983. A sixth convertible debt issue raised $400 million in March 1983 while in July of that same year MCI raised $1 billion with a 'synthetic' convertible consisting of a package of bonds and detachable warrants. As luck would have it, MCI fared poorly in product market competition with AT&T and its stock price went into sharp decline at this time. The call restrictions made it impossible for MCI to force conversion on these last two issues and it was left with a heavy debt burden that it had difficulty servicing (although it managed to do so ultimately).

The circumstantial evidence contained in the MCI and HGS cases promotes the view that growing firms that do use convertibles in a fashion suggested by our results, seek to force conversion as early as possible. This contrasts with a literature, starting with Ingersoll (1977b), that seems to show that managers delay call decisions. Ingersoll draws his conclusion from observing a large call premium (i.e., the conversion value exceeding the call price by an average of 44%), a fact suggesting that the callability option was in the money for quite a while before the actual call decision. In contrast, our results suggest that a manager may not be able to call in spite of a large call premium, due to the presence of soft call restrictions. Indeed, Asquith (1995) and Asquith and Mullins (1991) show that a large portion of the observed call delay can be explained once 'hard' call restrictions are taken into account.[28] We hope that future research will be able to empirically establish further distinctions between early conversion forcing calls, and call and conversion decisions that occur later in the bond's life once hard and soft call restrictions have expired.

4 The General Case with an Endogenous Signal

We now extend the basic model in Section 2 by letting the manager's private information take more than two values. Accordingly, let θ take values in the set $\{\theta_1, ..., \theta_N\}$, $N \geq 2$, with $\Pr[\theta = \theta_i] = \lambda_i$. We assume that (1)—(4) hold for all $i = 1, ..., N$.

When there are more than two types, one convertible bond with a fixed conversion ratio will not be able to implement the symmetric information outcome for all types. One solution is to allow the manager to issue multiple convertibles with differing face values, conversion ratios and call restrictions. For example, with $N = 3$, issuing two callable convertible securities with properties similar to those obtained above can be shown to be efficient. An alternative approach is to consider floating-price

[28] Such 'hard' call restrictions take the form of call protection periods. These papers also show that tax shield effects are a significant factor in explaining call delays, after call protection periods and notice period effects are taken into account, forces absent in our model. To the best of our knowledge, there has been no documentation of the role of soft call provisions, preventing calls unless there is a significant premium over the conversion price, in explaining call delays.

convertibles with conversion ratios that depend on date 1 endogenous variables like the market value of equity or the stock price, including mandatory convertibles that are automatically converted to equity.[29] We will take the latter approach in what follows, in order to demonstrate in closed form the existence of an equilibrium that implements the symmetric information outcome.

We will also extend the model by providing a simple story to endogenize the date 1 public signal m. Specifically, we suppose that at date 1, there is an analyst who is either an expert (i.e., informed) with probability γ, or a charlatan (i.e., uninformed) with probability $1-\gamma$, with $\gamma \in (0,1]$. The analyst's type is private information and he makes a public announcement $m \in \{m_1, ..., m_N\}$ given his type on date 1, after observing the date 0 decisions of the manager. The message m_i is to be interpreted as a statement by the analyst that the state of the world is θ_i. We assume that when the analyst is an expert he discloses the true state, i.e., sends message m_i when the state is θ_i. When the analyst is a charlatan, he tries to maintain his reputation for being an expert, i.e., chooses his disclosure strategy in order to maximize the market's posterior probability given the message that he is informed.[30]

As before, we will look for perfect Bayesian equilibria of this game. Let $\mu_i^0(S)$ denote the uninformed analyst's (as well as the market's) date 0 beliefs that the type of the manager is θ_i given that a security $S \in \mathbf{S}$ has been issued by the manager. Let $\sigma_i(S)$ be the probability with which the uninformed analyst sends message m_i at date 1 given that a security S has been issued at date 0.[31] Let $\mu_i^1(m,S)$ denote date 1 beliefs of the market that $\theta = \theta_i$ given a message m sent by the analyst and given S has been issued at date 0. Let $\nu^1(m,S)$ denote the date 1 beliefs of the market that the analyst is an expert given that the date 0 security is S and that he has sent a message m. Finally, let

$$\overline{V}(m,S) = E[X|m,S] = \sum_{i=1}^{N} \mu_i^1(m,S) V_i \tag{20}$$

be the date 1 market value of the expected cash flows of the firm given that the analyst's message is m and that the security issued is S.

We will look for a pooling equilibrium where each type of the manager issues the same floating price mandatory convertible bond at date 0. Such a security, denoted by $S^* = (\alpha^*, V^*)$, consists of a *vector* of equity shares $\alpha^* = (\alpha_1^*, ..., \alpha_N^*)$ together with a vector $V^* = (V_1^*, ..., V_N^*)$ of cut-off levels for

[29] In 2001, such securities constituted around a quarter of the convertible market. In a recent paper, Chemmanur, Nandy and Yan (2004) consider signaling equilibria involving mandatory convertibles, in a model similar to that of Stein (1992). The first–best is not achieved in these equilibria.

[30] Our results on the existence of an efficient equilibrium do not depend on the analyst being either perfectly informed or perfectly uninformed, or on the precise specification of the uninformed analyst's preferences, as long as γ is high enough.

[31] Note that in the two type model of the previous section, where the public signal was exogenous, we essentially fixed $\sigma_i = \frac{1}{2}$ for all $i = 1, 2$ and set $\beta = \gamma + (1-\gamma)\frac{1}{2}$.

the date 1 market value of the firm. The interpretation is that the security is converted to α_i^* shares when the date 1 market value of the firm is V_i^*.[32]

In order to state our result, it will be convenient to define

$$\widehat{V} = \left[\sum_{i=1}^{N} \lambda_i \frac{1}{V_i} \right]^{-1}. \tag{21}$$

$\widehat{V}$ is the inverse of the average equity shares sold in the symmetric information world. Let

$$\gamma^* = \max \left[1 - \frac{\widehat{V}}{V_N}, \frac{\widehat{V} - V_1}{V_1(\widehat{V} - 1)} \right]. \tag{22}$$

Proposition 3 *For all $\gamma > \gamma^*$ there exists a pooling equilibrium where all types of the manager issue the same mandatory convertible $S^* = (\alpha^*, V^*)$ satisfying*

$$V_i^* = \gamma V_i + (1 - \gamma)\overline{V}, \tag{23}$$

and

$$\alpha_i^* = \frac{1}{\gamma} \left[\frac{1}{V_i} - (1 - \gamma) \frac{1}{\widehat{V}} \right] \in (0, 1), \tag{24}$$

for all $i = 1, ..., N$. On the equilibrium path, $\sigma_i(S^*) = \lambda_i$ and $\overline{V}(m_i, S^*) = V_i^*$ for all i. The date 0 expected value of the claims sold to the new claim holders is equal to 1 *and that for the old shareholders of type θ_i of the firm is equal to $V_i - 1$, the first best value of the firm.*

Proof. In the Appendix. ■

In the pooling equilibrium neither the market nor the uninformed analyst will infer anything about θ from the date 0 choice of securities. Since the informed analyst always tells the truth, the uninformed analyst, in order to maximize the market's posterior probability of his expertise, announces m_i with probability $\sigma_i = \lambda_i$, the probability he attaches to the informed analyst sending message m_i. As a result, the market will attach probability γ to the analyst being informed after any message m_i and so the market value of the firm $\overline{V}(m_i, S^*)$ will be equal to V_i^* for each m_i. The new claimholders will obtain a share α_i^* when the date 1 market value of the firm equals V_i^*.

[32] We can equally let the conversion ratios depend on the date 1 stock price of the firm, instead of the total market value, without affecting anything. Mandatory conversion allows us to ignore the debt part of the claim. All results will carry over if we instead use a non–mandatory floating–price convertible. Since the conversion ratio in such a security is floating with the stock price, we can choose a sufficiently low face value for the debt part of such a security, in order to guarantee voluntary conversion. This also allows us to eliminate the need for call provisions. Call provisions forcing conversion may still be attached however, in order to make sure that conversion happens. A call price that is less than the market value of the equity claim will ensure this.

Given this equilibrium behavior, the conversion ratios α^* will be chosen in such a way that the expected value of the claims sold will equal \$1 regardless of the private information of the manager. Since the manager of type θ_i attaches probability $\gamma + (1-\gamma)\lambda_i$ to the message m_i and a probability $(1-\gamma)\lambda_j$ to a message $m_j \neq m_i$, we must have that α^* solves

$$[\gamma + (1-\gamma)\lambda_i]\alpha_i^* V_i + (1-\gamma)\sum_{j\neq i}\lambda_j \alpha_j^* V_i = 1,$$

or, equivalently,

$$\gamma\alpha_i^* + (1-\gamma)\sum_{j=1}^{N}\lambda_j\alpha_j^* = \frac{1}{V_i} \tag{25}$$

for all $i = 1, ..., N$. Equation (25) has a simple interpretation—the expected equity share sold by type θ_i must equal the share $\frac{1}{V_i}$ that would be sold by this type in the first–best world. The solution to the system (25) is given by (24). When γ is greater than its threshold value γ^*, the solution is admissible, i.e., $\alpha_i^* \in (0,1)$ for all i. To support the pooling equilibrium, we assume that if any other security is issued at date 0 everyone attaches probability 1 to type θ_1.

Note from (24) that

$$\alpha_i^* - \alpha_j^* = \frac{1}{\gamma}\left[\frac{1}{V_i} - \frac{1}{V_j}\right] \tag{26}$$

Thus, α_i^* is decreasing in i—the more optimistic is the market the lower is the share sold. Furthermore, using (23), it is easily checked that the market value $\alpha_i^* V_i^*$ of the claims sold when $m = m_i$ is also decreasing in i. Intuitively, the higher the type of the manager the greater is the chance that a favorable m will be disclosed in date 1. To keep the expected value of the claims sold constant across manager types, the market value of the claims sold must be decreasing in the date 1 market value of the company. This property of the floating price convertible is identical to the corresponding property of the callable convertible security characterized in Section 3.2, namely that the expected value of the equity claim must be less than the expected value of the debt claim (see (17)). In the latter context, this necessitated making the bond callable as well as specifying a suitable call restriction. For the security of this section however, the fact that conversion is mandatory allows us to ignore the details of the debt part of the claim.

Note moreover that for $i > j$ the difference $\alpha_i^* - \alpha_j^*$ (as well as $\alpha_i^* V_i^* - \alpha_j^* V_j^*$) is decreasing in γ. The less the probability that the analyst is informed, the more sensitive must be the (market value of) shares sold to the analyst's message, in order to keep the expected value constant. Finally, since the firm initially has one share outstanding, after the conversion the share price p_i^* will be given by $(1-\alpha_i^*)V_i^*$ which is increasing in i. The more optimistic is the market at date 1 the higher will be V_i^*, the total value of the firm. Furthermore, the lower will be α_i^* the number of shares sold and so the total number of shares outstanding. For both these reasons the stock price will be higher the more optimistic the market. To conclude this section, we turn to a numerical example that illustrates our results.

Example 2

Suppose that $N = 3$, i.e., there are three types. The following table provides the rest of the primitives.

Parameters	$\mathbf{V}_i$	$\boldsymbol{\lambda}_i$	$\mathbf{A}_i$
$\boldsymbol{\theta} = \boldsymbol{\theta}_1$	4	$\frac{1}{3}$	2.7
$\boldsymbol{\theta} = \boldsymbol{\theta}_2$	6	$\frac{1}{3}$	4.6
$\boldsymbol{\theta} = \boldsymbol{\theta}_3$	8	$\frac{1}{3}$	6.5

In the symmetric information world the expected payoffs for the existing claim–holders are equal to 3, 5 and 7 for types θ_1, θ_2 and θ_3 which can be achieved by issuing equity shares equal to 1/4, 1/6 and 1/8 respectively. As with Example 1, it is not difficult to see that with asymmetric information, there exists an equilibrium where only type θ_1 invests by issuing equity and types θ_2 and θ_3 do not invest at all.

Consider now a floating price mandatory convertible of the type considered in Proposition 3. For the parameter values chosen,

$$\gamma^* = \max\left[1 - \frac{\widehat{V}}{V_3}, \frac{\widehat{V} - V_1}{V_1(\widehat{V} - 1)}\right] = \frac{4}{13}$$

For γ greater than this cut–off, there exists one pooling equilibrium characterized by Proposition 3, in which all types invest and achieve their symmetric information first best payoff. Using (23) and (24), we characterize the properties of the optimal security in the table below, for the case $\gamma = 1/2$.

$[\gamma = \frac{1}{2}]$	$\boldsymbol{\alpha}_i^*$	$\mathbf{V}_i^*$
$\mathbf{m} = \mathbf{m}_1$	$\frac{23}{72}$	5
$\mathbf{m} = \mathbf{m}_2$	$\frac{11}{72}$	6
$\mathbf{m} = \mathbf{m}_3$	$\frac{5}{72}$	7

To see that such a security works, consider type θ_3 of the manager and the probability he assigns to different date 1 scenarios. Since the analyst is informed with probability 1/2 and since the uninformed analyst sends each message with equal probability in equilibrium, such a manager assigns a probability of $\frac{1}{2} + \frac{1}{2}(\frac{1}{3}) = \frac{2}{3}$ chance to a date 1 message of m_3, when the market value of the company equals $V_3^* = 7$, with a $\frac{1}{6}$ chance to each of the other two possibilities. Given the security above, the manager expects to sell an equity share equal to

$$\frac{1}{6}\left(\frac{23}{72}\right) + \frac{1}{6}\left(\frac{11}{72}\right) + \frac{2}{3}\left(\frac{5}{72}\right) = \frac{1}{8},$$

equal to the share he would sell in the symmetric information world. As a result, he suffers no dilution and is willing to invest. Similarly, type θ_2 of the manager assigns a 2/3 chance to the date 1 message being m_2, when the market value of the company equals $V_2^* = 6$, with a 1/6 chance to each of the other two possibilities. Given the security above, the manager then expects to sell an equity share equal to

$$\frac{1}{6}\left(\frac{23}{72}\right) + \frac{2}{3}\left(\frac{11}{72}\right) + \frac{1}{6}\left(\frac{5}{72}\right) = \frac{1}{6},$$

equal to the share he would sell in the symmetric information world. As a result, he suffers no dilution and is willing to invest; and similarly for type θ_1 of the manager. From the perspective of the uninformed investors at date 0, they assign a 1/3 probability to each of the three date 1 scenarios. As a result, they expected value of the claims sold at date 0 equals

$$\sum_{i=1}^{3} \lambda_i \alpha_i^* V_i^* = \frac{1}{3}\frac{23}{72}(5) + \frac{1}{3}\frac{11}{72}(6) + \frac{1}{3}\frac{5}{72}(7) = 1,$$

and they are willing to subscribe to the issue.

As remarked earlier, with imperfect resolution of asymmetric information ($\gamma < 1$), the strategy of postponing the adverse selection problem by issuing short–term debt at date 0 and refinancing at date 1 (by issuing equity, say), does not implement the symmetric information outcome. We conclude this section by demonstrating this in the context of the present example. Consider an equilibrium where all types of the manager pool by issuing riskless short–term debt at date 0, and pool again at each of the three possible date 1 scenarios by issuing equity shares and retiring the debt. In such a pooling equilibrium, the behavior of the analyst will be identical to that characterized by Proposition 3. This implies that the manager has to sell a equity share equal to $1/V_i^*$ at date 1, when the analyst's message is m_i and the market value of the firm is V_i^*. At date 0, type θ_3 of the manager attaches a 2/3 chance to the date 1 message m_3 and a 1/6 chance to each of the other two possibilities. Thus, he expects to sell an equity share equal to

$$\frac{1}{6}\left(\frac{1}{5}\right) + \frac{1}{6}\left(\frac{1}{6}\right) + \frac{2}{3}\left(\frac{1}{7}\right) > \frac{1}{8},$$

the equity share he sells in the symmetric information world. Thus, there is dilution in this equilibrium.[33] It can be easily verified that there is no underinvesment in this equilibrium— the date 0 expected payoff of all types of the manager from undertaking the investment is higher than the payoff from foregoing

[33] Observe that in this equilibrium, the manager sells shares that have a date 1 market value equal to $1, in each possible date 1 state. Thus, it implements the same outcome as would be implemented by a Brennan type floating price convertible discussed in the Introduction. Moreover, it is easily seen that the dilution incurred at date 1 does not depend on the date 1 security being equity and will exist even if some other admissible security such as debt is issued at that date.

the investment. It is easy to show however that, for different parameter values, such a refinancing strategy may not be able to avoid underinvestment (e.g., if A_3 is greater than 6.75 in the present example). In order to avoid dilution and underinvestment, under imperfect resolution of asymmetric information, the manager needs to adopt a financing strategy that makes the date 0 expected value of the claims sold independent of his private information.

5 Durable Goods and Pooling with Warranties

We have shown so far that informational costs arising out of an adverse selection problem can be avoided if the current asymmetry of information is resolved imperfectly in the future. As such, this has applications beyond the project financing problem considered above, in particular to market for durable goods such as used cars that was first considered by Akerlof (1970). We discuss such an application now and relate it to our previous results.

Consider a market for a durable good (e.g., used cars) in which a seller has private information about the quality of the car that is being sold to competitive uninformed buyers. Suppose that the market for this good forms at date 0 with the private information of the seller represented by his type $\theta \in \{\theta_1, ..., \theta_N\}$, $N \geq 2$ with the prior $\Pr[\ \theta = \theta_j] = \lambda_j$. At date 1, a public noisy signal $m \in \{m_1, ..., m_N\}$ is observed by all market participants, that is correlated with θ. Let $\Pr[m = m_i|\theta = \theta_j] = h_{ij}$ be the conditional probability of the i–th signal given the j–th type, $i, j = 1, ...N$, and let $H = [h_{ij}]$ represent the $N \times N$ matrix of these conditional probabilities.[34] At date 2, the durable good is finally consumed.

The value of the car to a buyer, given a type $\theta = \theta_j$ and a signal $m = m_i$ is denoted by v_{ij}, a non–negative number that is assumed to be non-decreasing in i and j (and increasing in at least one of i or j). The reservation value of the car to the seller of type θ_j is denoted by r_j, assumed to be increasing in j.[35] We assume that preferences are additively separable in value and "money", i.e., the payoff of the buyer is $v_{ij} - P$ from obtaining the good (and 0 otherwise) and the payoff to type θ_j of the seller from selling the car is P and from not selling it is r_j, where P is a price that the buyer pays. Furthermore, we assume that $E[v_{ij}|\theta = \theta_j] > r_j$ for all j, so that it is socially efficient for trade to occur at date 0 regardless of the actual quality of the car. Indeed, in the symmetric information world, this is the competitive outcome, whereas with asymmetric information when buyers do not know the

[34]The assumption of finite type and signal spaces is made for technical convenience, and to maintain continuity with previous results. The assumption that the dimension of the signal space is equal to the dimension of the type space is also made for the sake of simplicity. What is needed is the dimension of m be at least as large as that of θ, i.e., the public signal be rich enough.

[35]One can also think of r_j as the cost of production of type θ_j of the good, without altering anything to follow.

quality of the car, there may be inefficient levels of trade.

At date 0, we allow the seller to attach a warranty policy to the good being sold. Such a policy consists of a function $w(m)$ that is a monetary payment to the buyer as a function of the future public signal m. Let $w_i = w(m_i)$, $i = 1, ..., N$. While we use the term warranty for the function w it can also be interpreted as a payment plan, as will be made clear below. Thus, w_i is allowed to be negative.

We are interested in the existence of a (perfect Bayesian) equilibrium where all types of the seller attach the identical warranty policy w to the good being sold such that the socially efficient outcome is implemented in date 0. To do this, for any constant real number P, consider the system of N linear equations in N variables $w_1, ..., w_N$:

$$\sum_{i=1}^{N} h_{ij}[v_{ij} + w_i] = P \text{ for all } j = 1, ..., N. \tag{27}$$

The interpretation is the following. Conditional on each possible type of the seller θ_j, the date 0 expected value of the good *plus* the warranty policy, is equal to the constant P and so independent of the seller's type. If such a policy exists, then, in competitive markets, buyers will be willing to pay a price P for the good together with the warranty. Furthermore, for each type θ_j of the seller, the expected payoff, equal to the price P less the expected warranty obligation, is given by

$$P - \sum_{i=1}^{N} h_{ij} w_i = \sum_{i=1}^{N} h_{ij} v_{ij} > r_j, \tag{28}$$

so that the seller will be willing to sell the good and the warranty at a price P. Thus, the efficient outcome is implemented in such an equilibrium. Finally, it is immediate that a warranty policy w satisfying (27) exists if the matrix H of conditional probabilities has full rank, i.e., as long as the signal m is correlated with the type θ. We are ready to state our next result.

Proposition 4 *If H has full rank, then there is a warranty policy w and a pooling equilibrium, where the seller attaches the same warranty w to the good regardless of θ, that implements the symmetric information outcome. Furthermore, this symmetric information outcome is implemented in every equilibrium.*

Proof. The existence of one efficient pooling equilibrium follows from the discussion above. With respect to uniqueness of the equilibrium outcome, suppose to the contrary that there exists an equilibrium that is inefficient, i.e., where some type θ_j of the seller gets a payoff strictly less than $E[v_{ij}|\theta = \theta_j]$, his symmetric equilibrium payoff. If such a type deviates and sells the good with the attached warranty policy w characterized by (27) then, regardless of the response of the market to the deviation, he receives a payoff equal to his symmetric information payoff, by construction, as (28) illustrates. The deviation is thus profitable, completing the proof. ∎

In the rest of this section, we relate Proposition 4 to the results obtained earlier, as well as to the literature on warranties for durable goods. To do so, we consider a specific version of the matrix H that also allows us also to solve for w in closed form. Suppose that given $\theta = \theta_j$ the signal $m = m_j$ with probability $\gamma > 0$. With probability $1 - \gamma$, the signal is uncorrelated with θ and takes value m_i with probability h_i that does not depend on θ. In such a case, the system (27) can be written as

$$\overline{v}_j + \gamma w_j + (1-\gamma) \sum_{i=1}^{N} h_i w_i = P \text{ for all } \; j = 1, ..., N. \tag{29}$$

where $\overline{v}_j = E[v_{ij} | \theta = \theta_j]$. Multiplying the j–th equation in (29) by h_j, summing and substituting back into (29) we obtain

$$w_i = P - \frac{1}{\gamma}[\overline{v}_i - (1-\gamma)\overline{v}^h] \tag{30}$$

where $\overline{v}^h \equiv \sum_{j=1}^{N} h_j \overline{v}_j$. Note that w_i is decreasing in i so that the expected value of the warranty conditional on θ_j is decreasing in j. The higher is the value of the good, the lower is the expected value of the warranty, keeping the total value of the package constant, thus avoiding any inefficiencies arising out of adverse selection.

Grossman (1981) first demonstrated the existence of such an efficient pooling equilibrium in a special case of the model above with $N = 2$ and in which the value of the good v_{ij} depends only on the signal m_i and is independent of the seller type θ_j. The signal m_1 is interpreted as the event of the good breaking down (within the first year, say) and the signal m_2 as the complementary event. Letting $v_{i1} \equiv v_{i2} = v_i$, the difference $v_2 - v_1$ is then the utility loss to the buyer upon break–down.[36] One solution to the associated adverse selection problem (that was proposed by Grossman) is to choose $P = \frac{1}{\gamma}[\overline{v}_2 - (1-\gamma)\overline{v}^h] > 0$ yielding from (30), $w_2 = 0$ and $w_1 = \frac{1}{\gamma}[\overline{v}_2 - \overline{v}_1] = v_2 - v_1$. The interpretation is of a warranty policy that compensates the buyer fully for a loss, in the event of a breakdown. Proposition 4 generalizes this insight to more than two types and imperfect resolution of asymmetric information.

Notice that there is more than one possible interpretation for the pair (P, w). For example, if P is chosen such that $w_1 = 0$, then $w_2 = -(v_2 - v_1)$. In such a case, w can be interpreted as a deferred payment plan, with payment due only if the good does not break down. Note that in such a case the price P may be negative, which can be interpreted as a cash–back guarantee. More generally, the pair (P, w) can be interpreted as a combination of up-front payments, cash back guarantees, deferred payment plans and a warranty policies, with specific interpretations being more or less suited to particular applications.

[36]Since all payoff relevant information is revealed at date 1, this model corresponds to the project financing problem with perfect resolution of the initial asymmetry of information.

There are close similarities between the results obtained in this section and those obtained for the project financing problem considered previously, as well as some important differences. The similarities are most immediately seen for the case where the future resolution of information is perfect (see Section 3.1). In that case, if one thinks of the 'good' being sold as equal to the equity share α^* (i.e., $v_{ij} = \alpha^* V_j$) then the corresponding 'warranty policy' involves no payment when good news is disclosed, and a payment equal to $D_1(F^*) - \alpha^* V_1 = \alpha^* V_2 - \alpha^* V_1$, the fall in the expected value of the security, given bad news. Given such a good and warranty, the investors are willing to pay the required outlay of \$1 at date 0. Alternatively, one may think of the 'good' as the debt with face value F^* (i.e., $v_{ij} = D_j(F^*)$) then the corresponding policy involves no payment when bad news is disclosed, and a 'payment' *from* the buyers equal to $D_2(F^*) - \alpha^* V_2 = D_2(F^*) - D_1(F^*)$, the rise in the expected value of the debt given good news. The extension of this intuition to the case of imperfect resolution of information is straightforward.

One immediate difference between the two models is that in the durable good model the seller sells a pre–specified good and his gains from trade are summarized by the price (as well as his reservation value). In contrast, in the project financing model considered previously, the old claimholders on the firm also *directly* gain when trade is consummated, i.e., when the issue is successful and the investment undertaken.[37]

There are at least two other important structural differences between the two models that lead to differences in the results. First, for the efficiency result of this section it is sufficient that the public signal be only slightly informative. For example, (29) has a solution as long as $\gamma > 0$. In contrast, for Proposition 2 (or, 3) we need the public signal to be sufficiently informative, i.e., β (or, γ) to be higher than a cutoff value. This weaker result in the project financing model arises out of the additional limited liability constraints (5) on the set of admissible securities $\mathbf{S}$. In fact, in the context of Proposition 3, notice that the system (25) has a solution for all $\gamma > 0$. We need γ to be large enough for the solution to satisfy limited liability, i.e., for α_i^* to belong to $(0, 1)$ for all i. Such constraints are absent in the warranty application.[38]

A second important difference in results is that in the project financing context we made no uniqueness claim corresponding to that made in Proposition 4. This difference arises from the somewhat

[37] In the form of their share of the cash flow from the new project. It is not too difficult, although tedious, to construct a general model which contains both models as special cases. Since such an unification complicates nothing else other than the notation, we do not pursue it in this paper.

[38] One may also want to impose limited liability or budget balance constraints on both the buyers and the sellers in the durable good context considered in this section, for example by insisting that the warranty payments w_i be non–negative and that the price P the seller receives be at least as high as not only the expected warranty payment that he has to pay out, but in fact the maximum warranty payment that he may have to pay out. It is straightforward to verify upon inspecting (30) that for γ high enough, a warranty policy satisfying such constraints exists.

subtle fact that in the durable goods model the warranty policy is directly contingent on the public signal m, while the equilibrium call and conversion decisions in Section 3 as well as the conversion ratios in Section 4 depended instead on the endogenous market price p. Since, in equilibrium, and on–the–path of play, there is a one–to–one relation between the price p and the signal m, this difference has no substantive effect on the existence of an efficient equilibrium. However, it prevents us from obtaining the uniqueness result, as the relationship between p and m is unrestricted off–the–path of play, barring, as noted above (see footnote 26), strong refinements of the equilibrium notion. If in a candidate inefficient equilibrium, the manager deviates with a suitable (callable) convertible security, then the manager's expected payoff depends on the endogenous future market price p. Such a price is not belief independent. If the market responds adversely to the deviation and the future market price is low enough, then the manager will end up selling claims that are undervalued given his private information, and the deviation will not be profitable.

Finally, note that the main insight gained from the two models may in practice be more relevant for the project financing application. In the durable goods context, the efficient warranty policy characterized above may create in turn new sources of inefficiencies. Since for γ low enough, the warranty policy involves payments that are quite large, it follows that buyers of the good may have an incentive to (undetectably) damage the good in order to obtain a large warranty payment (see, e.g., Lutz (1989)). Such manipulation of the public signal by the buyers of the convertible security (and the associated moral hazard problem) is less likely to be an issue in the project financing context where the buyers do not actually own the assets being traded but only claims written on those assets.[39]

6 Conclusion

We show that when the asymmetry of information is imperfectly resolved over time, commonly used securities such as callable convertible preferred stock or debt can perfectly solve the adverse selection problem. By conditioning call and conversion decisions on the future public resolution of the manager's current private information such securities make the value of the claim insensitive to the private information of the manager. The manager prefers to force conversion whenever he is able to, but may not be able to force conversion due to the presence of call restrictions. Complete mitigation of adverse selection can also be achieved by a floating price and mandatory convertibles, and even when the future information disclosure is endogenous. A similar insight on the value of warranties is also obtained in the context of a market for lemons for a durable good.

[39]Similarly, when γ is low enough, the payment from the warranty policy has a large variance, which might be unpleasant for risk–averse buyers. In the project financing context, the availability of well–developed markets for hedging instruments makes such an objection less problematic.

Regarding extensions, note that in our model the manager never obtains additional information over the course of time. However, it is easily seen that as long as the manager's current information is equal to the expected value of his future information, our results are robust to this possibility. A more interesting extension is to consider the possibility that the manager may be able to influence the distribution of the future signal. The model of Section 4 allows for this in a limited form, where the manager can influence the announcement strategy of the uninformed analyst through his date 0 investment and financing choices. A fuller investigation of such 'signal jamming' possibilities, including the possibility of directly manipulating the information content of the signal by engaging in earnings management for example, is left for future research.

7 Appendix

Proof of Proposition 2

1. Existence of a Solution to (15) **and** (16)

We will use the Intermediate Value Theorem to demonstrate the existence of a solution to (15) and (16) when β is high enough and show further that the solution must satisfy (17). From (16) we can write α in terms of F as

$$\alpha = \frac{1-(1-\beta)D_2(F)}{\beta V_2} \tag{31}$$

Using this in (15) we obtain that F must satisfy

$$\beta D_1(F) + \frac{1-(1-\beta)D_2(F)}{\beta V_2}(1-\beta)V_1 = 1 \tag{32}$$

Since $D_2(F) \leq V_2$, the second term left–hand side of (32) is positive if $\beta > 1-\frac{1}{V_2}$. Since $D_1(F) \leq V_1$, the left–hand side of (32) is strictly greater than 1 if we replace $D_1(F)$ by V_1 in that expression, provided we also have $\beta > \frac{1}{V_1}$. On the other hand, for $F = 0$, $D_1(F) = D_2(F) = 0$ and so the left–hand side of (32) is equal to $\frac{(1-\beta)V_1}{\beta V_2} < 1$. Further, $D_i(F)$ is continuous in F for all θ_i. Thus, if $\beta > \max[1-\frac{1}{V_2}, \frac{1}{V_1}]$, then by the Intermediate Value Theorem, there exists a solution $F > 0$ with $D_1(F) < V_1$ to (32).[40] For such an F, the solution α to (31) is strictly positive, given $\beta > \max[1-\frac{1}{V_2}, \frac{1}{V_1}]$. Further, α is less than 1 iff

$$1 < (1-\beta)D_2(F) + \beta V_2$$

But this follows from the fact that F solves (32) so that

$$(1-\beta)D_2(F) + \beta\alpha V_2 = 1.$$

[40]One can also show the monotonicity of the left–hand side side of (32) in F demonstrating the uniqueness of the solution.

This shows that for $\beta > \max[1 - \frac{1}{V_2}, \frac{1}{V_1}]$ there exists a solution $\alpha \in (0, 1)$ and $F > 0$ with $D_1(F) < V_1$ to (15) and (16).

The proof of (17) is immediate. Suppose it does not hold for $i = 2$ so that $\alpha V_2 \geq D_2(F)$. Then we must have

$$(1-\beta)D_2(F) + \beta\alpha V_2 \geq \beta D_2(F) + (1-\beta)\alpha V_2 > \beta D_1(F) + (1-\beta)\alpha V_1. \quad (33)$$

The first inequality follows from the fact that $\beta > 1 - \beta$. The second inequality follows from the fact that $D_2(F) \geq D_1(F)$ and the fact that $V_2 > V_1$. But (33) contradicts the fact that α and F satisfy (15) and (16). Analogously one can show that (17) holds for $i = 1$.

2. Existence of a pooling equilibrium

To show that pooling with such a security is indeed an equilibrium, we proceed backwards in time.

Date 1, $m = m_1$

In this case if the market conjectures that the bond will not be converted, then the share price will be given by the left–hand side of (19). As a result the manager will not be able to call the bond and so, from (17) it follows that it will not be converted. We also allow the manager to buy back the security in the market by issuing some other security. For any security S that is issued to buy back the debt, we assume that the market puts probability 1 on the type θ_i for whom $D_i(F) - S_i$ is the maximum, where $S_i \equiv E[S(X)|\theta = \theta_i]$. Given such beliefs, it is straightforward to check that both types of the manager will either not want to issue such a security to buy back the existing claims, or will not be able to do so.

Date 1, $m = m_2$

In this case if the market conjectures that the bond will be converted, then the share price will be given by the right–hand side of (19). From (17) and (18), the manager will call to force conversion regardless of his private information and investors will convert when the security is called. If instead the manager tries to buy back the security and issue other claims S then, as above, the market attaches beliefs putting probability 1 on the type θ_i for whom $\alpha V_i - S_i$ is the maximum. No type of the manager will find such a deviation profitable.

Date 0

Given the call and conversion decisions of date 1 above, from (15) and (16) it follows that the market value of the security at date 0 will equal 1 dollar, the required outlay for the project. As a result, the manager, regardless of his private information, will be able to raise the required funds. The date 0 expected payoff of the existing shareholders will thus be equal to $V_i - 1 > A_i$ for each θ_i. Consequently, the manager will find it profitable to invest. Finally, we suppose that at date 0, if any type of the manager deviates by issuing some other security then the market puts probability 1 on type $\theta = \theta_1$. As a result, no type of the manager will find such a deviation profitable.■

Proof of Proposition 3

We begin our construction of the pooling equilibrium by considering the strategy of the uninformed analyst at date 1, on the equilibrium path. Since all types of the manager pool by issuing the same convertible S^*, neither the analyst nor the market learns nothing about θ from the date 0 financing decision. As a result, $\mu_i^0(S^*) = \lambda_i$ for all i. Since the informed analyst discloses the truth, this implies that the posterior probability that the market attaches to the analyst being informed after a message m_i is

$$\nu^1(m_i, S^*) = \frac{\lambda_i\gamma}{\lambda_i\gamma + \sigma_i(S^*)(1-\gamma)} \tag{34}$$

Since the uninformed analyst wants to maximize the posterior probability that he is informed, it follows that

$$\sigma_i(S^*) = \lambda_i \text{ and } \nu^1(m_i, S^*) = \gamma \text{ for all } i = 1, ..., N, \tag{35}$$

in equilibrium. To see this, note first that $\nu^1(m_i, S^*)$ cannot vary across messages m_i— if there exist messages m_i and m_j such that $\nu^1(m_i, S^*) > \nu^1(m_j, S^*)$, the uninformed analyst will strictly prefer to send message m_i (i.e., $\sigma_i(S^*) = 1$) implying that $\nu^1(m_i, S^*) = \frac{\lambda_i\gamma}{\lambda_i\gamma+(1-\gamma)} < 1 = \nu^1(m_j, S^*)$, a contradiction. So, we must have $\nu^1(m_i, S^*) = \kappa$ for some constant $\kappa \in [0, 1]$ for all $i = 1, ..., N$. From (34) we then obtain

$$\sigma_i(S^*)(1-\gamma)\kappa = \lambda_i\gamma(1-\kappa)$$

for all i. Since $\sum_i \sigma_i(S^*) = 1$, it follows that $\kappa = \gamma$ and $\sigma_i(S^*) = \lambda_i$ for all i.

Having established the equilibrium behavior of the uninformed analyst, we now turn to the date 1 market value of the firm $\overline{V}(m_i, S^*)$ after a message m_i. Note that

$$\mu_i^1(m, S^*) = \begin{cases} \gamma + (1-\gamma)\lambda_i & \text{if } m = m_i \\ (1-\gamma)\lambda_i & \text{otherwise} \end{cases} \tag{36}$$

Thus,

$$\overline{V}(m_i, S^*) = \gamma V_i + (1-\gamma)\overline{V} \tag{37}$$

Since $\overline{V}(m_i, S^*) = V_i^*$ for all i, the security S^* entitles the new shareholders to convert to α_i^* shares when the market value of the security is $\overline{V}(m_i, S^*)$.

Next, we turn to the choice of the equity shares α^*. Since $\sigma_i(S^*) = \lambda_i$ for all i, type θ_i of the manager knows that the analyst's message will be m_i with probability $\gamma + (1-\gamma)\lambda_i$ and will be equal to m_j with probability $(1-\gamma)\lambda_j$ for $j \neq i$. We want to choose α^* such that the expected value of the claims sold in equilibrium is equal to the outlay of 1, for each type of the manager. That is, α_i^* must solve:

$$[\gamma + (1-\gamma)\lambda_i]\alpha_i^* V_i + (1-\gamma)\sum_{j\neq i}\lambda_j\alpha_j^* V_i = 1,$$

for all $i = 1, ..., N$. Re–arranging we obtain,

$$\gamma\alpha_i^* + (1-\gamma)\sum_{j=1}^{N}\lambda_j\alpha_j^* = \frac{1}{V_i}, \tag{38}$$

for all $i = 1, ..., N$. Multiplying by λ_i and summing over i we obtain

$$\sum_{j=1}^{N}\lambda_j\alpha_j^* = \frac{1}{\widehat{V}} \tag{39}$$

Using (39) in (38) we obtain (24). It is easy to check that if $\gamma > \max[1 - \frac{\widehat{V}}{V_N}, \frac{\widehat{V}-V_1}{V_1(\widehat{V}-1)}]$ then $\alpha_i^* \in (0,1)$ for all i.

Given the equilibrium behavior derived above, the date 0 expected value of the claims sold by type θ_i of the manager is seen to be equal to 1, by construction. Thus, the date 0 market value of the security will also equal 1 and the expected payoff to the old claimholders will equal $V_i - 1 > A_i$ for all $i = 1, ..., N$. This implies that no type of the manager will prefer to under–invest.

Note that the manager is allowed to buy back the security S^* in the market by issuing some other security after a message m_i. For any security S that is issued to buy back the convertible, we assume that the market puts probability 1 on the type θ_j for whom $\alpha_i^* V_j - S_j$ is the maximum. Given such beliefs, it is straightforward to check that all types of the manager will either not want to issue such a security to buy back the existing claims, or will not be able to do so.

It remains to check that no type of the manager will want to deviate at date 0 by issuing a different security S'. We suppose that if any such security S' is issued by any type of the manager then the market attaches probability 1 to type θ_1, i.e., $\mu_1^0(S') = 1$. It follows that $\mu_1^1(m, S') = 1$ for all m so that $\overline{V}(m, S') = V_1$ for all m. Given such beliefs, it is straightforward to verify that no type of the manager will find such a deviation profitable, and we omit the details. ■

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