The Rodney L. White Center for Financial Research

The Positive Role of Overconfidence and Optimism in Investment Policy

Simon Gervais
J. B. Heaton
Terrance Odean

15-02

The Wharton School
University of Pennsylvania

The Positive Role of Overconfidence and Optimism in Investment Policy*

by

Simon Gervais†
J. B. Heaton‡
Terrance Odean§

4 September 2002

*This paper is an updated version of a previous working paper, "Capital Budgeting in the Presence of Managerial Overconfidence and Optimism," by the same authors. Financial support by the Rodney L. White Center for Financial Research is gratefully acknowledged. The authors would like to thank Andrew Abel, Jonathan Berk, Domenico Cuoco, David Denis, Janice Eberly, Robert Goldstein, Peter Swan, and seminar participants at the 2000 meetings of the European Finance Association, the 2001 meetings of the American Finance Association, and the Wharton School for their comments and suggestions. Heaton acknowledges that the opinions expressed here are his own, and do not reflect the position of Bartlit Beck Herman Palenchar & Scott or its attorneys. All remaining errors are of course the authors' responsibility.

†Finance Department, Wharton School, University of Pennsylvania, Steinberg Hall - Dietrich Hall, Suite 2300, Philadelphia, PA 19104-6367, gervais@wharton.upenn.edu, (215) 898-2370.

‡Adjunct Associate Professor of Finance, Duke University Fuqua School of Business and Duke Global Capital Markets Center, and Associate, Bartlit Beck Herman Palenchar & Scott, 54 West Hubbard, Suite 300, Chicago, IL 60610, jb.heaton@bartlit-beck.com, (312) 494-4425.

§Haas School of Business, 545 Student Services #1900, University of California at Berkeley, Berkeley, CA 94720-1900, odean@haas.berkeley.edu, (510) 642-6767.

The Positive Role of Overconfidence and Optimism in Investment Policy

Abstract

We use a simple capital budgeting problem to contrast the decisions of overconfident, optimistic managers with those of rational managers. We reach the surprising conclusion that managerial overconfidence and optimism can increase the value of the firm. Risk-averse rational managers will postpone the decision to exercise real options longer than is in the best interest of shareholders. Overconfident managers underestimate the risk of potential projects and are therefore less likely to postpone the decision to undertake. Optimistic managers, too, undertake projects quickly. Thus moderately overconfident or optimistic managers make decisions that are in the better interest of shareholders than do rational managers. Overly overconfident or optimistic managers may be too eager to undertake projects. This tendency can sometimes be controlled by increasing hurdle rates for risky projects. While compensation contracts that increase the convexity of manager payoffs can be used to realign the decisions of a rational manager with those of shareholders, it is less expensive to simply hire a moderately overconfident manager. The gains from overconfidence and optimism will at times be sufficient that shareholders actually prefer an overconfident, optimistic manager with less ability to a rational manager with greater ability.

JEL classification codes: G31, L21.

1 Introduction

A vast experimental literature finds that individuals are usually optimistic (i.e., they believe outcomes favorable to themselves to be more likely than they actually are) and overconfident (i.e., they believe their knowledge is more precise than it actually is). Since optimism and overconfidence directly influence decision making, it is natural to ask how optimistic and overconfident managers will affect the value of the firm. Are managerial optimism and overconfidence sufficiently detrimental to firm value that shareholders should actively avoid hiring optimistic and overconfident managers? What possible benefits might optimistic and overconfident managers bring to the firm?

We use a simple model of capital budgeting to contrast the decisions of overconfident, optimistic managers with those of rational managers. We reach the surprising conclusion that managerial overconfidence and optimism can increase the value of the firm. Moderate overconfidence can align managers' preferences for risky projects more closely with those of shareholders. However, extreme managerial overconfidence and optimism are detrimental to the firm. Our analysis starts with the observation that many capital budgeting decisions can be viewed as decisions whether or not to exercise real options (Dixit and Pindyck, 1994). Because of their greater risk aversion, rational managers will postpone the decision to exercise real options longer than is in the best interest of shareholders. As Treynor and Black (1976) write:

> "If the corporation undertakes a risky new venture, the stockholders may not be very concerned, because they can balance this new risk against other risks that they hold in their portfolios. The managers, however, do not have a portfolio of employers. If the corporation does badly because the new venture fails, they do not have any risks except the others taken by the same corporation to balance against it. They are hurt by a failure more than the stockholders, who also hold stock in other corporations, are hurt."

Since overconfident managers believe that the uncertainty about potential project is less than it actually is, they are less likely to postpone the decision to undertake the project. Thus moderately overconfident managers make decisions that are in the better interest of shareholders than do rational managers. Overconfident managers also benefit the firm by expending more effort than rational managers, as they overestimate the value of that effort. Optimistic managers believe that the expected net present value of potential projects is greater than it actually is. Like overconfident managers, optimistic managers undertake projects more quickly than do rational managers. However, unlike overconfident managers, optimistic managers will sometimes undertake projects that actually have negative expected net present values. This error may be mitigated by raising hurdle rates.

While compensation contracts that increase the convexity of manager payoffs can be used to realign the decisions of a rational manager with those of shareholders, it may be less expensive to simply hire an overconfident, optimistic manager. The gains from overconfidence and optimism will at times be sufficient that shareholders actually prefer an overconfident, optimistic manager with less ability to a rational manager with greater ability. Extreme overconfidence or optimism is, however, always detrimental to the firm. Extremely overconfident or optimistic managers will perceive too little risk or too little chance of failure. They will greatly underestimate the option value of delaying a project or greatly overestimate the likelihood of success. When such individuals are put in charge of a firm's capital budgeting decisions, they will destroy that firm's value in the long run.

Our research helps to explain a puzzle in corporate finance. If rational individuals make better decisions than those influenced by behavioral biases, such as overconfidence, why are many CEOs overconfident (Audia, Locke and Smith, 2000; Malmendier and Tate, 2001)? This puzzle can be viewed from the perspective of the individual manager and that of the firm. Gervais and Odean (2001) demonstrate, in the context of investors, that the human tendency to take too much credit for success and attribute too little credit to chance can cause successful people to become overconfident. Thus, in a corporate setting, managers who successfully climb the corporate ladder to become CEOs are likely to also become overconfident. In the current paper, we address the puzzle of CEO overconfidence from the perspective of the firm. We show that it can be in the best interest of shareholders to hire managers (e.g., CEOs) who are overconfident.

Our paper proceeds as follows. Section 2 reviews some of the literature on optimism and overconfidence. Section 3 introduces a simple capital budgeting problem that is used throughout the paper to analyze the effects of behavioral biases on the value of the firm. The same section presents the first-best solution, which serves as a benchmark for later sections. Section 4 formally introduces the concepts of overconfidence and optimism, and shows how these individual traits can affect the value of the firm when a manager's sole intention is to maximize firm value. The principal-agent nature of the relationship between firm owners and managers is analyzed in section 5. This section shows how contracting interacts with manager biases to solve the firm's agency problems. In section 6, we show how our basic model can be extended to accommodate other forces that are likely to play a role in capital budgeting problems. Finally, section 7 discusses our findings and concludes. All the proofs are contained in the appendix.

2 Related Work

2.1 Experimental Studies

For this paper, we define optimism to be the belief that favorable future events are more likely than they actually are. Researchers find that, generally, individuals are unrealistically optimistic about future events. They expect good things to happen to themselves more often than to their peers (Weinstein, 1980; Kunda, 1987). For example, Ito (1990) reports that foreign exchange companies are more optimistic about how exchange rate moves will affect their firm than how they will affect others.[1] People overestimate their ability to do well on tasks and these overestimates increase when the task is perceived to be controllable (Weinstein, 1980) and when it is of personal importance (Frank, 1935). March and Shapira (1987) find that managers tend to believe that outcomes are largely controllable and that projects under their supervision are less risky than is actually the case. Finally, optimism is most severe among more intelligent individuals (Klaczynski and Fauth, 1996) and, we expect, most top managers are intelligent.

For this paper, we define overconfidence to be the belief that the precision of one's information is greater than it actually is, that is, one puts more weight on one's information than is warranted. Studies of the calibration of subjective probabilities find that individuals do tend to overestimate the precision of their information (Alpert and Raiffa, 1982; Fischhoff, Slovic and Lichtenstein, 1977).[2] Such overconfidence has been observed in many professional fields. Clinical psychologists (Oskamp, 1965), physicians and nurses (Christensen-Szalanski and Bushyhead, 1981; Baumann, Deber and Thompson, 1991), investment bankers (Staël von Holstein, 1972), engineers (Kidd, 1970), entrepreneurs (Cooper, Woo and Dunkelberg, 1988), lawyers (Wagenaar and Keren, 1986), negotiators (Neale and Bazerman, 1990), and managers (Russo and Schoemaker, 1992) have all been observed to exhibit overconfidence in their judgments.

The best established finding in the calibration literature is that people tend to be overconfident in answering questions of moderate to extreme difficulty (Fischhoff, Slovic and Lichtenstein, 1977;

[1]Over two years, the Japan Center for International Finance conducted a bi-monthly survey of foreign exchange experts in 44 companies. Each was asked for point estimates of future yen/dollar exchange rates. The experts in import-oriented companies expected the yen to appreciate (which would favor their company), while those in export-oriented companies expected the yen to fall (which would favor their company). People are even unrealistically optimistic about pure chance events (Marks, 1951; Irwin, 1953; Langer and Roth, 1975).

[2]See Lichtenstein, Fischhoff, and Phillips (1982) for a review of the calibration literature.

Lichtenstein, Fischhoff and Phillips, 1982; Yates, 1990; Griffin and Tversky, 1992). Exceptions to overconfidence in calibration are that people tend to be underconfident when answering easy questions, and they learn to be well-calibrated when predictability is high and when performing repetitive tasks with fast, clear feedback. For example, expert bridge players (Keren, 1987), race-track bettors (Dowie, 1976; Hausch, Ziemba and Rubinstein, 1981) and meteorologists (Murphy and Winkler, 1984) tend to be well-calibrated.

There are a number of reasons why we might expect the overconfidence of managers to exceed that of the general population. 1) Capital budgeting decisions can be quite complex. They often require projecting cash flows for a wide range of uncertain outcomes. Typically people are most overconfident about such difficult problems. 2) Capital budgeting decisions are not well suited for learning. Learning occurs "when closely similar problems are frequently encountered, especially if the outcomes of decisions are quickly known and provide unequivocal feedback" (Kahneman and Lovallo, 1993). But the major investment policy decisions we study here are not frequently encountered, outcomes are often delayed for long periods of time, and feedback is typically very noisy. Furthermore, it is often difficult for a manager to reject the hypothesis that every situation is new in important ways, allowing him to ignore feedback from past decisions altogether. Learning from experience is highly unlikely under these circumstances (Brehmer, 1980; Einhorn and Hogarth, 1978). 3) Unsuccessful managers are less likely to retain their jobs and be promoted. Those who do succeed are likely to become overconfident because of self-attribution bias. Most people overestimate the degree to which they are responsible for their own success (Miller and Ross, 1975; Langer and Roth, 1975; Nisbett and Ross, 1980). This self-attribution bias causes the successful to become overconfident (Daniel, Hirshleifer and Subrahmanyam, 1998; Gervais and Odean, 2001). 4) Finally, managers may be more overconfident than the general population because of selection bias. Those who are overconfident and optimistic about their prospects as managers are more likely to apply for these jobs. Firms, too, may select on the basis of apparent confidence and optimism, either because the applicant's overconfidence and optimism are perceived to be signs of greater ability or because, as in our model, shareholders recognize that it is less expensive to hire overconfident, optimistic managers who suit their needs than it is to hire rational managers who do so.

2.2 Overconfidence and Optimism in Finance

Recent studies explore the implications of overconfidence for financial markets. In Benos (1998), traders are overconfident about the precision of their own signals and their knowledge of the signals of others. De Long, Shleifer, Summers, and Waldmann (1991) demonstrate that overconfident traders can survive in markets. Hirshleifer, Subrahmanyam and Titman (1994) argue that overconfidence can promote herding in securities markets. Odean (1998) examines how the overconfidence of different market participants affects markets differently. Daniel, Hirshleifer, and Subrahmanyam (1998), and Gervais and Odean (2001) develop models in which, due to a self-attribution bias, overconfidence increases with success. Kyle and Wang (1997) and Wang (1997) argue that mutual funds may prefer to hire overconfident money managers, because overconfidence enables money managers to "pre-commit" to taking more than their share of duopoly profits. While we conclude that there are advantages to hiring overconfident managers in a corporate setting, our reasoning is quite different from that of Kyle and Wang (1997) and Wang (1997). These authors rely on assumptions about the timing of trading and information signals, and they require that competing money managers have knowledge of the information and overconfidence of each other. Our basic findings are based simply on the assumptions that shareholders are less risk-averse than are managers regarding the fate of the firm, and that overconfidence causes managers to perceive less risk than is there.

Fewer studies have looked at overconfidence in corporate settings. Roll (1986) suggests that overconfidence (hubris) may motivate many corporate takeovers. Kahneman and Lovallo (1993) argue that managerial overconfidence and optimism stem from managers taking an "inside view" of prospective projects. The inside view focuses on project specifics and readily anticipated scenarios while ignoring relevant statistical information such as "how often do projects like this usually succeed?" Heaton (2002) examines the implications of managerial optimism for the benefits and costs of free cash flow. He points out that in the corporate environment, irrational managers are not likely to be arbitraged away. Transactions costs for the most obvious "arbitrage" of managerial irrationality—the corporate takeover—are extremely large, due primarily to high legal and regulatory hurdles. The specialized investors who do pursue takeovers must bear very large idiosyncratic risks. These factors severely limit the power of arbitrage (Pontiff, 1996; Shleifer and Vishny, 1997). Consequently, there is no reason to believe that corporate financial decisions cannot manifest managerial irrationality within the large arbitrage bounds these limits create. Malmendier and Tate (2001) provide empirical evidence that optimistic managers invest more aggressively.

3 The Model

3.1 The Firm

An all-equity firm initially consists of half a dollar in cash, and is considering the possibility to invest that money in a risky project. At the beginning of the period, one such project becomes available. All risky projects return one or zero dollar with equal probabilities one period from now; we denote this end-of-period cash flow by $\tilde{v}$. For simplicity, we assume that the risk of these projects is completely idiosyncratic, and that the correct discount rate, the riskfree rate, is zero. Given this, the net present value of any risky project is exactly zero, and so the value of the firm is one half.

The potential value from a risky project comes from the possibility of acquiring information about it. This can be done in two stages: the firm can gather an imperfect signal about the project's payoff in the first stage, and a perfect signal in the second stage. Before each stage, the firm learns the probability that the project will still exist at the end of that stage. The cost of gathering information in this real options framework is therefore the potential loss of a project that is likely to be good. The qualitative implications of our model extend to real options settings in which the drawback to delaying exercise is foregone revenue from the project or an explicit cost to gathering additional information. Introducing these additional cash flows into the model, however, greatly complicates the formal analysis, without contributing intuition. We denote the probability that the project will still exist at the end of the first (second) stage by $\tilde{p}$ ($\tilde{q}$). We assume that $\tilde{p}$ and $\tilde{q}$ are uniformly distributed on $[0, 1]$ and are independent. These two variables can be thought of as describing the ease with which the firm can learn about the project's profitability. Alternatively, they capture the amount of competition that the firm faces when deciding whether to invest in a project immediately or to delay the decision. In that sense, a larger (smaller) probability that the project still exists represents a situation in which few (many) other firms are likely to undertake the project before more information can be gathered.

Upon learning $\tilde{p}$, the imperfect signal that the firm can gather in the first stage is given by

$$\tilde{s} = \tilde{\varepsilon}\tilde{v} + (1 - \tilde{\varepsilon})\tilde{\eta},$$

where $\tilde{\eta}$ has the same distribution as $\tilde{v}$, but is independent from it, and $\tilde{\varepsilon}$ takes a value of one with probability $a \in (0, 1)$, and zero otherwise. This signal $\tilde{s}$ is more informative for larger values of a, as the true value of the project is then observed more often. The parameter a can in fact be interpreted as the ability of the individual making the capital budgeting decision, whom we refer to as the *manager* of the firm. At the same time that the manager observes $\tilde{s}$, he learns $\tilde{q}$, the probability that the project will still exist should he decide to keep gathering information (i.e., delay the decision to undertake the project for one more period). If he chooses to gather more information, the manager learns $\tilde{v}$ perfectly. In the event that the project disappears at any stage (probability $1-\tilde{p}$ in the first stage, and $1-\tilde{q}$ in the second stage), the firm's cash is simply invested at the riskfree rate until the end of the period; no other risky projects are available.

The sequence of events is illustrated in Figure 1. The manager of the firm makes up to three decisions (which are represented by open circles in the figure) during the period. At the outset, the first stage, he must choose whether to undertake the risky project, drop it, or gather some information about it. If information is gathered and if the project does not disappear while it is gathered, the manager makes his second-stage decision: the project can again be undertaken, dropped, or the manager can choose to gather more (i.e., in this case, perfect) information about it. If more information is gathered and the project remains available, the manager then chooses in the third and final stage—the last decision node—whether or not to undertake the project.

In this and the next sections, we assume that the manager's utility is a function of the firm's value exclusively. This is equivalent to assuming that the manager is compensated with firm's stock. In section 5, we take a closer look at the manager's incentives and analyze how more general compensation contracts can be used to align the manager's decisions with the interests of shareholders. This two-step approach allows us to disentangle the effects of risk aversion, behavioral biases, and compensation on decision-making.

Clearly, even if the project can be dropped in favor of a safe investment in the first two stages, this will never be considered by the manager: the worst possible outcome from gathering more information is that the risky project disappears; the safe investment can then be made anyway.[3] It is also clear that the decision to undertake or drop the risky project in the third stage is trivial: at that point, the risky project's payoff is known with certainty, and so the project will be undertaken if and only if $\tilde{v} = 1$. Effectively therefore, the manager makes active decisions in each of the first two stages only, and the decision involves a comparison between undertaking the project at that stage or acquiring more information about it. This simple two-period framework thus captures the idea that a firm may choose to wait to invest in a risky project (McDonald and Siegel, 1986), but waiting may be costly (Grenadier, 2002), since a good project may be lost to competition.

Suppose that, when making his second-stage decision, the manager knows that $\tilde{q}$ is close to

[3]This results from the fact that information can be gathered without the firm or manager incurring any direct monetary or effort costs. Such costs are considered in section 6.2.

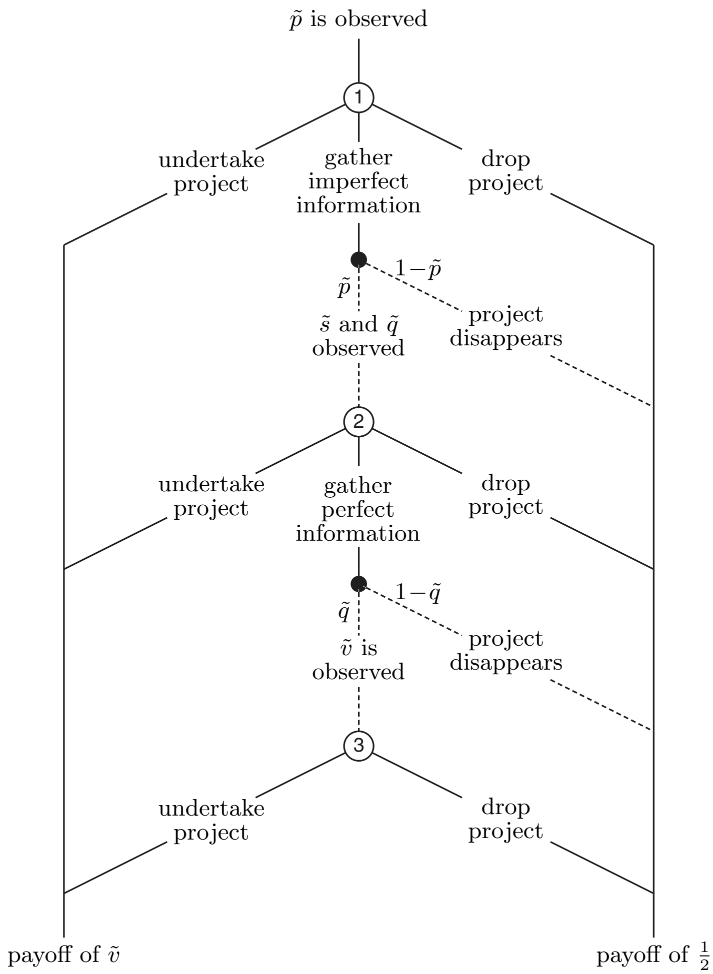


Figure 1: Sequence of events. The open circles represent stages at which the manager of the firm must make a decision; at each of these stages, the manager must decide whether to undertake the project, drop the project, or gather more information (only in the first two stages). The closed circles represent nodes at which random events occur: the project disappears with probability $\tilde{p}$ ($\tilde{q}$) in the first (second) of these nodes. At the end of the period, the firm will get its payoff either from the risky project ($\tilde{v}$) if the manager chose to undertake it at any point, or from the safe investment ($\frac{1}{2}$).

one—that is, the probability that the project will still be available next period is high. This means that risk in acquiring more information is low: the project will most likely still exist after $\tilde{v}$ is learned, and the decision whether to undertake it can then be made with perfect accuracy. At the other extreme, when $\tilde{q}$ is close to zero, the project is likely to disappear while $\tilde{v}$ is gathered. Unless the project is not worth undertaking using the information already known about it, there is no point in gathering more information about a project that will probably cease to exist. In the second stage therefore, the decision to gather more information will always be associated with larger values of $\tilde{q}$. In what follows, we use $\bar{Q}_s$, $s = 0, 1$, to generically denote the $\tilde{q}$-threshold above which more information will be gathered in the second stage after the decision-maker learns $\tilde{s} = s \in \{0, 1\}$ in the first stage. Similar reasoning leads to the conclusion that more information will be gathered only when $\tilde{p}$ is large enough in the first stage. We define the $\tilde{p}$-threshold of the first stage, $\bar{P}$, analogously. Thus the strategy of any decision-maker can be summarized by three information-gathering thresholds: $\bar{P}$, $\bar{Q}_1$, and $\bar{Q}_0$.

3.2 Updating, Firm Value, and First-Best

Given that the risk of the project available to the firm is purely idiosyncratic and that the riskfree rate is zero, we know that the value of the firm to its well-diversified, risk-neutral shareholders, is simply the expected value of its end-of-period cash flows. The expected value of the risky project evolves as more information is gathered about it. Thus, depending on when the project is undertaken, it will have a different value to the firm. Trivially, when the information about the risky project is perfect, the firm is worth one if $\tilde{v} = 1$ (the project is undertaken) and $\frac{1}{2}$ if $\tilde{v} = 0$ (the project is dropped). After $\tilde{s}$ is received however, the firm's value depends on the posterior probability of the risky project's success, and on what the manager does with the information. It is easy to verify that

$$\begin{aligned} \Pr\{\tilde{v} = 1 \mid \tilde{s} = 1\} &= \frac{1+a}{2} \equiv \pi_1, \quad \text{and} \\ \Pr\{\tilde{v} = 1 \mid \tilde{s} = 0\} &= \frac{1-a}{2} \equiv \pi_0, \end{aligned}$$

so that

$$\mathrm{E}[\tilde{v} \mid \tilde{s} = 1] = \frac{1+a}{2} > \frac{1}{2}, \quad \text{and} \tag{1}$$

$$\mathrm{E}[\tilde{v} \mid \tilde{s} = 0] = \frac{1-a}{2} < \frac{1}{2}. \tag{2}$$

Notice that π_1 (π_0) gets closer to one (zero) as a increases: more weight is put on the information when its precision is large. This also translates into more extreme assessments of the risky project's value, as shown in (1) and (2). The following lemma shows how the firm's value evolves with the information gathering process of the firm.

Lemma 3.1 *Suppose that the manager adopts $\tilde{q}$-thresholds of $\bar{Q}_s$, $s = 0, 1$, for the second stage. On average, after the manager learns that $\tilde{s} = s$ in the first stage, the firm is worth*

$$\bar{F}_s(\bar{Q}_s) \equiv \frac{1}{2}\left(1 + \frac{1}{2}\pi_s\right) + \left(\pi_s - \frac{1}{2}\right)\bar{Q}_s - \frac{1}{4}\pi_s \bar{Q}_s^2. \tag{3}$$

Note that $\bar{F}_s(\bar{Q}_s)$ does not represent the optimal or maximum value of the firm after the first stage. Instead it represents the value that is implied by a particular information gathering strategy chosen by the firm's manager. Of course, the firm's shareholders are ultimately interested in maximizing the initial value of the firm, which we calculate next. In the first stage, the manager either undertakes the project (when $\tilde{p} < \bar{P}$) or acquires $\tilde{s}$ (when $\tilde{p} \geq \bar{P}$), which will be one or zero with equal probabilities. We can use the expected future values of the firm derived in Lemma 3.1 to calculate the value of the firm at the outset.

Lemma 3.2 *Suppose that the manager adopts a $\tilde{p}$-threshold of $\bar{P}$ and $\tilde{q}$-thresholds of $\bar{Q}_s$, $s = 0, 1$. The initial value of the firm is then given by*

$$\bar{F}(\bar{P}, \bar{Q}_1, \bar{Q}_0) \equiv \frac{1}{4}\Big[\bar{F}_1(\bar{Q}_1) + \bar{F}_0(\bar{Q}_0) + 1\Big] - \frac{1}{4}\bar{P}^2, \tag{4}$$

where $\bar{F}_1(\cdot)$ and $\bar{F}_0(\cdot)$ are as calculated in (3).

This result shows how any capital budgeting strategy, which can be summarized with three thresholds $\{\bar{P}, \bar{Q}_1, \bar{Q}_0\}$, maps into a value for the firm. We start by characterizing the manager's set of decisions that will maximize this value. We refer to the value-maximizing strategy as the *first-best* strategy, and use a superscript "FB" to denote it. This strategy can be reached by assuming that the manager is a risk-neutral owner of the firm, as the personal objective of this owner is then precisely to maximize the firm's value. Alternatively, it can be viewed as the solution to the following maximization problem:

$$\left\{\bar{P}^{\text{FB}}, \bar{Q}_1^{\text{FB}}, \bar{Q}_0^{\text{FB}}\right\} = \underset{\{\bar{P}, \bar{Q}_1, \bar{Q}_0\} \in [0,1]^3}{\text{argmax}} \bar{F}(\bar{P}, \bar{Q}_1, \bar{Q}_0).$$

The following proposition solves for this first-best strategy and associated firm value.

Proposition 3.1 (First-Best) *The value of the firm is maximized with $\bar{P}^{\text{FB}} = 0$, $\bar{Q}_0^{\text{FB}} = 0$, and*

$$\bar{Q}_1^{\text{FB}} = \frac{\left(\pi_1 - \frac{1}{2}\right)}{\frac{1}{2}\pi_1} = \frac{2a}{1+a}. \tag{5}$$

With this strategy, the initial value of the firm is

$$\bar{F}^{\mathrm{FB}} = \frac{9}{16} + \frac{a^2}{8(1+a)}. \tag{6}$$

In the value-maximizing strategy, the firm always gathers information at the outset. This is intuitive: the ex ante values of both the risky project and the safe investment are exactly $\frac{1}{2}$; since the firm can always get this value of $\frac{1}{2}$ later by dropping the risky project in favor of the safe investment, acquiring information is always optimal in the first stage. A similar argument applies when the outcome of the first stage of information gathering is $\tilde{s} = 0$. In that case, the risky project is worth $\pi_0 = \frac{1-a}{2} < \frac{1}{2}$. The firm benefits from acquiring more information, since the worst possible scenario after doing so is again $\frac{1}{2}$.

After $\tilde{s} = 1$ is observed however, perfect information is gathered only with some probability, depending on the outcome of $\tilde{q}$. In particular, the risky project is then worth $\pi_1 = \frac{1+a}{2} > \frac{1}{2}$. When the risky project is almost sure to disappear, i.e., $\tilde{q}$ is close to zero, acquiring more information is foolish: the perfect information will be useless if the project cannot be undertaken after that information is learned. If on the other hand the project will most likely continue to exist, i.e., $\tilde{q}$ is close to one, acquiring $\tilde{v}$ makes sense, as a better informed decision can be made without much risk of losing the project. This is the tradeoff that leads the firm to undertake the project only when $\tilde{q} \geq \bar{Q}_1^{\mathrm{FB}}$. It is easy to show that $\bar{Q}_1^{\mathrm{FB}}$ is increasing in π_1 and a. Thus a firm with a high-ability manager is less willing to gather perfect information: the manager's imperfect information is already very informative, and so there is no point in further risking losing the project to competition. Thus a high-ability manager can make accurate decisions quickly.

There are a few things to notice about the first-best value of the firm, as calculated in (6). First, this value is also increasing in a: the firm directly benefits from the ability of its manager. Second, $\bar{F}^{\mathrm{FB}}$ is not only greater than $\frac{1}{2}$ but bounded away from it: the initial value of the firm, regardless of the ability of its manager, exceeds the present value of its initial opportunities, namely the risky project and the safe investment. This is because, even when $a = 0$, the firm can always choose to gather information for two stages, and make its decision about the risky project afterwards. On average, after the second stage, the project will still exist with probability $\mathrm{E}[\tilde{p}] \cdot \mathrm{E}[\tilde{q}] = \frac{1}{4}$. Since the project has a net value of $\frac{1}{2}$ half the time (payoff of $\tilde{v} = 1$ with an initial investment of $\frac{1}{2}$), the value created from information gathering without skill is $\frac{1}{4} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$. This is why the initial value of the firm is $\frac{1}{2} + \frac{1}{16} = \frac{9}{16}$ when the manager is unskilled.

3.3 The Effect of Risk Aversion

The first-best outcome maximizes the current value of the firm to its risk-neutral shareholders. To attain it, the firm's manager must not care about risk when making his capital budgeting decisions. However, capital budgeting decisions will be made by agents whose human capital is tied to the firm, e.g., the CEO of the firm. As pointed out by Jensen and Meckling (1976), this agent's risk aversion is likely to affect his decisions. Compensation contracts can be used to reduce these agency costs by realigning the objective of the firm's manager with those of the shareholders. We discuss these in section 5. For now, we concentrate on the problem of a risk-averse manager whose utility depends only upon the value of the firm. We show how the risk aversion of this decision-maker will affect his capital budgeting decisions and in turn the value of his firm.

To keep the analysis of this and later sections tractable, we model risk aversion as a utility cost $r \geq 0$ that is incurred by the firm's manager when his firm is worth nothing, that is when the firm undertakes the risky project and this projects turns out to be bad. This cost effectively makes the firm's manager risk-averse: the three potential outcomes of the capital budgeting decisions, $\{0, \frac{1}{2}, 1\}$, will respectively yield $\{-r, \frac{1}{2}, 1\}$ in utility to the manager, making his utility function a convex function of the firm's end-of-period value. Note that, with this three-outcome specification, assuming more traditional utility functions will not change any of our results. Our specification, however, has the advantage that it allows us to solve for most results in the paper analytically. Note also that the cost r can alternatively be interpreted as the negative reputation that a manager acquires after running a firm down to the ground—the cost of getting fired, for example.

The risk-averse manager has the same information technology as the risk-neutral manager of section 3.2. His decisions only departs from first-best due to the fact that he is more reluctant to undertake the risky project with less than perfect information. Indeed, the risk-averse manager suffers more than the risk-neutral manager when the firm loses all of its value. Looking at Proposition 3.1, we see that it is only optimal for the risky project to be undertaken without perfect information when it is known that $\tilde{s} = 1$; this happen when $\tilde{q} < \bar{Q}_1^{\text{FB}}$. This is where the risk-averse manager's decisions will fail to maximize firm value, as the following result shows.

Proposition 3.2 *Suppose that the firm is managed by a single risk-averse individual with risk aversion $r \geq 0$. The information acquisition strategy of this manager is given by a $\tilde{p}$-threshold of $\bar{P}(r) \equiv 0$, and $\tilde{q}$-thresholds of $\bar{Q}_0(r) \equiv 0$ and*

$$\bar{Q}_1(r) \equiv \frac{2\big[a - (1-a)r\big]}{1+a} < \bar{Q}_1^{\text{FB}}. \tag{7}$$

With this strategy, the initial value of the firm is

$$\bar{F}(r) = \bar{F}^{\text{FB}} - \frac{(1-a)^2}{8(1+a)}\, r^2. \tag{8}$$

Since $\bar{Q}_1(r)$ decreases with r, we see that risk aversion makes the manager acquire more information, as $\tilde{q}$ will exceed the threshold more often. In fact, any value of r larger than $\frac{a}{1-a}$ makes $\bar{Q}_1(r)$ smaller than zero, and so would result in the manager always acquiring more information in the second stage. To avoid such an extreme effect of risk aversion and the corner solution that it introduces, we restrict our attention to values of r smaller than $\frac{a}{1-a}$ for the rest of the paper.[4]

As before, the firm's value $\bar{F}(r)$ is increasing in its manager's ability a. However, the firm's value is strictly decreasing in r. The loss of firm value results from the fact that acquiring perfect information in the second stage comes with a probability that the project will be lost to competition. In other words, for the risk-averse manager, the tradeoff between perfect and imperfect information is tilted towards the larger risk reduction that perfect information offers. The manager's utility gain from reducing risk does not transfer to the firm's shareholders.

4 The Role of Overconfidence and Optimism

Firm value is negatively affected by risk aversion. In this section, we show how managerial overconfidence and optimism may help restore firm value. In some cases, as we show, the first-best outcome may even be restored.

4.1 Definitions and Updating

Following the work of Daniel, Hirshleifer and Subrahmanyam (1998), Odean (1998), and Gervais and Odean (2001), we refer to overconfidence as the perception that private information is more precise and more reliable than it really is. In particular, we assume that the overconfident manager thinks that a is equal to $A \in [a, 1]$, the difference $A - a \in [0, 1-a]$ measuring the degree of overconfidence. Optimism, on the other hand, refers to the manager's ex ante view of the project. An optimistic manager thinks that the project is better than it really is. As in Malmendier and Tate (2001) and Heaton (2002), we assume that the manager thinks that the probability of a good outcome for the risky project ($\tilde{v} = 1$) is not $\frac{1}{2}$, but $B \in \left[\frac{1}{2}, 1\right]$, where $B - \frac{1}{2} \in \left[0, \frac{1}{2}\right]$ measures the

[4] Note that this has no impact on any of our results, which only get stronger as r increases. This assumption only allows us to analyze the effects of risk aversion without worrying about the fact that solutions have a different analytical form for different ranges of r.

degree of optimism. The following lemma shows how these biases will affect the way that imperfect information is interpreted by the manager.

Lemma 4.1 *An overconfident and optimistic manager thinks that*

$$\Pr_{\mathrm{b}}\Big\{\tilde{v}=1 \mid \tilde{s}=1\Big\} \;=\; A+(1-A)B \;\equiv\; \pi_1(A,B), \quad \textit{and} \tag{9}$$

$$\Pr_{\mathrm{b}}\Big\{\tilde{v}=1 \mid \tilde{s}=0\Big\} \;=\; (1-A)B \;\equiv\; \pi_0(A,B), \tag{10}$$

*where the subscript "*b*" refers to the fact that the manager is biased.*

Clearly, $\pi_1(A,B)$ is increasing in both A and B, whereas $\pi_0(A,B)$ is decreasing in A and increasing in B. This is intuitive. The overconfident manager puts too much weight on his information and, as a result, over-adjusts his beliefs towards his information. He therefore thinks that the project is better (worse) than it really is after observing $\tilde{s}=1$ ($\tilde{s}=0$). The optimistic manager, on the other hand, always thinks that the project is better than it really is. He does revise his beliefs upwards (downwards) after a positive (negative) signal but, the resulting posterior is still higher than it should be.

As before, the manager never considers dropping the risky project in favor of the safe investment in the first or second stages: the worst-case scenario from acquiring more information is that the risky project disappears, and the safe investment can then be made anyway. Thus the decision of the manager in each stage is to decide whether the project should be undertaken early, or more information should be gathered before he makes his mind up about the risky project. In the first stage, more information will be gathered if $\tilde{p}$ is large enough; in the second stage, more information will be gathered if $\tilde{q}$ is large enough, with the knowledge of the imperfect signal $\tilde{s}\in\{0,1\}$. So the manager's strategy can be fully described by the same three thresholds as before.

4.2 Overconfidence

Let us first concentrate on overconfidence, that is let us assume for now that the manager does not exhibit any optimism (i.e., $B=\frac{1}{2}$), but potentially some overconfidence (i.e., $A\geq a$). At the outset, this manager correctly assesses the odds of the risky project being successful; his overconfidence does not affect his priors, but only the way he processes information. This means that the overconfident manager, like the rational manager, always chooses to gather some information in the first stage: undertaking the project at the outset is worth $\frac{1}{2}(1)-\frac{1}{2}r$ to him, which is less than the $\frac{1}{2}$ that can be generated at any stage by making the safe investment. As discussed above, the manager's overconfidence makes him reach biased beliefs upon learning $\tilde{s}$. In particular, the manager thinks that the project is worse (better) than it really is after he observes $\tilde{s} = 0$ ($\tilde{s} = 1$). When $\tilde{s} = 0$ therefore, the overconfident manager views the project as less likely to be successful than an otherwise rational manager, and so he has nothing to lose by acquiring more information. On the other hand, when $\tilde{s} = 1$, the overconfident manager values the risky project more than an otherwise rational manager. As the following result shows, this affects his choice of a threshold for $\tilde{q}$.

Lemma 4.2 *Suppose that the firm is managed by an overconfident individual with risk aversion $r \geq 0$. The information acquisition strategy of this manager is given by thresholds of $\bar{P}^{\mathrm{OV}}(r, A) = 0$, $\bar{Q}_0^{\mathrm{OV}}(r, A) = 0$, and*

$$\bar{Q}_1^{\mathrm{OV}}(r, A) = \frac{2\big[A - (1 - A)r\big]}{1 + A}. \tag{11}$$

Notice that $\bar{Q}_1^{\mathrm{OV}}(r, A)$ is increasing in A. This is because an overconfident manager believes that his imperfect information is better than it actually is, and so tends to rely on imperfect information more that an otherwise rational manager. More precisely, for $\tilde{q} \in \big(\bar{Q}_1(r), \bar{Q}_1^{\mathrm{OV}}(r, A)\big]$, the overconfident manager chooses to rely on imperfect information, whereas an otherwise identical but rational manager would choose to gather perfect information before making a decision. Recall from section 3.3 that risk aversion has the opposite effect: when $\tilde{s} = 1$, the manager relies on perfect information more than is optimal for firm value. Therefore, it is possible for overconfidence to have a positive effect on firm value. This will be the case for example when $\bar{Q}_1^{\mathrm{OV}}(r, A) \in \big(\bar{Q}_1(r), \bar{Q}_1^{\mathrm{FB}}\big]$, that is when the manager's overconfidence offsets his risk aversion in such a way that his decisions become similar to that of a rational profit-maximizing manager or owner. This positive role of overconfidence is stated more precisely in the following proposition.

Proposition 4.1 *For any risk-averse manager, there is a level of overconfidence,*

$$A^* \equiv \frac{a + (1 + a)r}{1 + (1 + a)r} \in [a, 1], \tag{12}$$

such that the value of the firm is equal to the first-best value $\bar{F}^{\mathrm{FB}}$. The value of the firm is strictly increasing (decreasing) in A for $A < A^*$ ($A > A^*$).

The last part of this proposition has important implications. In particular, for a given level of risk aversion, it is always the case that some overconfidence helps restore some of the value that is lost to decisions made to reduce risk. This process is not monotonic. Too much overconfidence distorts the decision-making process in that the manager may over-rely on his imperfect information.

Still, it is possible for the decisions of a risk-averse manager to be close to profit-maximizing, even when he does not suspect it. This will be the case when the manager's overconfidence correctly counterbalances his risk aversion. This is the reason why we analyze compensation contracts separately: in some cases, changing the compensation of the decision-maker is not needed to restore the value lost to his risk aversion. As we shall see in section 5, this will have important implications for executive compensation.

Another implication of Proposition 4.1 is the fact that the ability to identify successful projects is not the only factor contributing to firm value. As the risk aversion of the decision-maker affects firm value, the behavioral traits of this decision-maker also possibly affect firm value. More than that, as we show next, a firm managed by an overconfident individual can be worth more than that managed by a rational individual with equal or even higher ability.

Proposition 4.2 *Suppose that the value of a firm managed by a risk-averse rational individual with ability a is $F < \bar{F}^{\mathrm{FB}}$. Then there exists a value $\bar{a} < a$ such that the firm with a manager of any ability $a' \in (\bar{a}, a)$ can be worth more than F.*

Note that this proposition does not say that the firm is *always* worth more with a lower-ability manager; the firm will be worth more only if the lower-ability manager has the right overconfidence. Again, this is because moderate levels of overconfidence effectively make the manager act as a profit maximizer.

4.3 Optimism

The possibility that the manager is optimistic (and no longer overconfident) potentially creates a problem.[5] The optimistic manager operates under the assumption that the project is intrinsically good, i.e., his beliefs are positively biased at the outset. As a result, in an effort to avoid losing this perceived project value to competition, the manager is tempted to undertake the risky project without gathering any information about it. This destroys firm value since value can only be created through information gathering.

Lemma 4.3 *Suppose that the firm is managed by an optimistic individual with risk aversion $r \geq 0$. The information acquisition strategy of this manager is such that $\bar{P}^{\mathrm{OP}}(r, B) = 0$ if and only if*

$$B \leq \frac{\frac{1}{2} + r}{1 + r}, \tag{13}$$

[5] In this section, we treat overconfidence and optimism separately in order to isolate the forces of each bias. The two biases are combined later on.

and $\bar{Q}_0^{\mathrm{OP}}(r,B) = 0$ *if and only if*

$$B \leq \frac{1}{1-a}\left(\frac{\frac{1}{2}+r}{1+r}\right). \tag{14}$$

Also, after this manager observes $\tilde{s} = 1$*, he uses a* $\tilde{q}$*-threshold of*

$$\bar{Q}_1^{\mathrm{OP}}(r,B) = \frac{1-2(1-a)(1-B)(1+r)}{a+(1-a)B}. \tag{15}$$

The decisions of the optimistic manager clearly fail to generate the first-best outcome when $\bar{P}^{\mathrm{OP}}(r,B)$ and $\bar{Q}_0^{\mathrm{OP}}(r,B)$ are strictly positive. This happens when B is large, that is when the manager exhibits extreme optimism. It is also easy to verify that $\bar{P}^{\mathrm{OP}}(r,B)$ and $\bar{Q}_0^{\mathrm{OP}}(r,B)$, when they exceed zero, are decreasing in a: the manager is particularly subject to his optimism when he cannot rely much on his ability to learn about projects; that is, from his perspective, there is little reason to acquire information about the risky project. As with overconfidence, we are interested in whether optimism can be used to realign the incentives of a risk-averse manager towards a profit-maximizing strategy. More precisely, for a given risk aversion $r > 0$, we would like to know whether some level B of optimism results in $\bar{P}^{\mathrm{OP}}(r,B) = 0$, $\bar{Q}_0^{\mathrm{OP}}(r,B) = 0$, and $\bar{Q}_1^{\mathrm{OP}}(r,B) = \bar{Q}_1^{\mathrm{FB}}$, as required by the first-best outcome. The following proposition shows that this is not possible.

Proposition 4.3 *Suppose that the firm is managed by an individual with risk aversion* $r \geq 0$*. Then the firm's value is strictly increasing in the manager's level of optimism* B *for*

$$B \leq \frac{\frac{1}{2}+r}{1+r}. \tag{16}$$

No level of optimism B *in* $\left[\frac{1}{2},1\right]$ *can generate the first-best outcome.*

The first part of this result says that some manager optimism, like manager overconfidence, makes the firm more valuable. However, unlike overconfidence, optimism cannot be used to generate the first-best outcome: the level B of optimism needed to make $\bar{Q}_1^{\mathrm{OP}}(r,B) = \bar{Q}_1^{\mathrm{FB}}$ is large enough to distort the decisions made by the manager at the outset, that is $\bar{P}^{\mathrm{OP}}(r,B)$ is then positive. Thus, for behavioral biases to be useful in realigning the decisions of risk-averse decision-makers, it is important that their effects target exclusively those of risk aversion. In this model, risk aversion creates an over-investment in information acquisition in the second stage; this is precisely the stage at which the effects of overconfidence are felt. Optimism on the other hand also affects the initial stage, and so distorts the overall decision-making process, even though it may be helpful in the second stage. As we show in section 6.2, this distortion is even worse when the decision-maker's effort is costly.

5 Compensation Issues

5.1 Managers and Compensation

So far, we have assumed that the capital budgeting decisions are made by a manager whose utility depends only on the value of the firm. This is equivalent to a manager whose sole form of compensation is company stock. While compensating managers with stock may motivate them to work harder, doing so can also, as we have seen, cause risk-averse managers to behave more cautiously than is in the best interests of shareholders. To better align the decisions of managers with the interests of shareholders, firms often offer managers compensation packages that include stock options. Such packages effectively "convexify" the managers' preferences, and align their incentives with those of the shareholders.

In this section, we concentrate on the convex part of these compensation packages, and argue that such convexity is less needed when the manager is known to be overconfident or optimistic. To make our point, we use the same framework as in previous sections, but assume that the firm chooses the compensation package from which the manager derives his utility. As before, we assume that the manager's risk aversion is captured by a certainty equivalent cost of $r > 0$ that reduces his compensation when the risky project fails. The end-of-period value of the firm is 0, $\frac{1}{2}$ or 1 in the low, medium and high states. Let us denote the manager's compensation in each of these states by $\{0, \Delta_{\text{M}}, \Delta_{\text{M}} + \Delta_{\text{H}}\}$, in which we have used the fact that compensation in the valueless state cannot be positive. For example, with this notation, a compensation package consisting of s stocks and c at-the-money call options is denoted by $\Delta_{\text{M}} = \frac{1}{2}s$ and $\Delta_{\text{H}} = \frac{1}{2}(s+c)$. More generally, Δ_{H} measures the convexity of the compensation package, as any convex (concave) compensation is characterized by $\Delta_{\text{H}} > \Delta_{\text{M}}$ ($\Delta_{\text{H}} < \Delta_{\text{M}}$).

5.2 Perfect Realignment

We are initially interested in characterizing the compensation package that will make the manager act like a maximizer of firm profits. For now, we ignore the fact that the manager's compensation effectively reduces the firm's value; we tackle this problem in section 5.3.

Proposition 5.1 *Suppose that the manager hired by the firm is characterized by a risk aversion of r, ability a, overconfidence $A \geq a$, and optimism $B \geq \frac{1}{2}$. Then the compensation package $(\Delta_{\text{M}}, \Delta_{\text{H}})$ that realigns his incentives with those of the risk-neutral shareholders must satisfy*

$$\Delta_{\text{H}} = \frac{1 - \pi_1(A, B)}{\pi_1(A, B)} \frac{\pi_1}{1 - \pi_1} (\Delta_{\text{M}} + r) \equiv \Delta_{\text{H}}^{\text{FB}}(\Delta_{\text{M}}). \tag{17}$$

Notice that the high-state compensation Δ_{H} required to make the manager act in the best interest of the shareholders is increasing in r. In fact, when $A = a$ (no overconfidence) and $B = \frac{1}{2}$ (no optimism), equation (17) reduces to $\Delta_{\text{H}} = \Delta_{\text{M}} + r$, which implies that the compensation package that perfectly realigns the incentives of the manager is convex (since Δ_{H} is then larger than Δ_{M}). This makes sense: convexity of the compensation contract is required to make the manager less subject to the conservatism brought about by his risk aversion.

The interesting aspect of (17) is the fact that the right-hand side of the equation is decreasing in both A and B. This means that less convexity is required to realign the incentives of an overconfident or optimistic risk-averse manager than an otherwise identical rational manager. The intuition is simple: biased managers have a natural tendency to overcome the effects of their risk aversion, and so outside incentives are not needed quite as much. This observation may have important implications if including stock options in the compensation of a firm's top managers is expensive. In particular, it may be worthwhile for a firm to actively seek out or promote individuals who are likely to display some overconfidence or optimism, as this will reduce the need for the firm to complement their compensation with expensive stock options. We conjecture that this will be especially the case for firms involved in volatile industries since, for these firms, stock options are expensive and manager risk aversion is particularly hurtful. Alternatively, if firms base the option compensation they offer managers on the assumption that the managers are rational, the firms will end up paying overconfident, optimistic managers more than is in the best interests of the firm.

The compensation calculated in Proposition 5.1 realigns the incentives of the manager in the second stage after he observes $\tilde{s} = 1$, that is it makes him choose a threshold of $\bar{Q}_1^{\text{FB}}$ for $\tilde{q}$ in that state. It remains to be shown whether such a compensation schedule also realigns the manager's incentives in the other states, namely in the first stage and in the second stage after $\tilde{s} = 0$ is observed. The following result shows that, although managerial overconfidence can always be realigned through compensation, managerial optimism can pose problems.

Proposition 5.2 *Suppose that the manager hired by the firm is characterized by a risk aversion of r and ability a. It is then possible to perfectly realign his incentives with those of the risk-neutral shareholders if*

$$A \geq \frac{2a}{1+a}, \tag{18}$$

or

$$B \leq \frac{A(1-a)}{2a(1-A)}. \tag{19}$$

The right-hand side of (19) monotonically increases from $\frac{1}{2}$ to infinity as A increases from a to one. This means that perfect realignment is always possible when the manager is not optimistic ($B = \frac{1}{2}$). Neither (18) or (19) is satisfied when $A = a$ and $B > \frac{1}{2}$: optimism alone cannot be realigned. Indeed, as shown in the proof to Proposition 5.2, an optimistic manager offered a contract satisfying (17) will choose to undertake the risky project in the first stage when $\tilde{p}$ is small enough. Interestingly, a manager who is both optimistic *and* overconfident can be realigned if his optimism is small enough or his optimism is large enough. This is because the temptation to undertake a project in the first stage (caused by optimism) is reduced by the perceived prospect of obtaining precise information for the second stage's decision (as a result of overconfidence).

5.3 Value Maximization

The fact that the manager's incentives can be realigned perfectly with those of the shareholders is important, but not necessarily useful. After all, the shareholders seek to maximize firm value *after* taking into account the compensation that is paid to the firm's employees. In this principal-agent framework therefore, it is not enough to maximize the profits that the firm's projects generate; manager compensation, a cost to the firm, must also be taken into consideration. In particular, it may be too costly for the firm to compensate the manager enough to perfectly realign his incentives.

Suppose that the firm's manager is characterized by a risk aversion of r, ability a, overconfidence A, and optimism B. Let us denote the firm's expected profits, before it pays any compensation to the manager, by $\bar{\Pi}(\Delta_{\text{M}}, \Delta_{\text{H}})$, where it is understood that the compensation that will be paid to the manager is given by

$$\tilde{w}(\Delta_{\text{M}}, \Delta_{\text{H}}) = \begin{cases} \Delta_{\text{M}} + \Delta_{\text{H}}, & \text{if the risky project is successful} \\ \Delta_{\text{M}}, & \text{if the risky project is not undertaken} \\ 0, & \text{otherwise.} \end{cases}$$

The objective of the firm's owners is to maximize firm value, that is the owners look for

$$\{\Delta^*_{\text{M}}, \Delta^*_{\text{H}}\} = \underset{\{\Delta_{\text{M}}, \Delta_{\text{H}}\}}{\text{argmax}} \; \bar{\Pi}(\Delta_{\text{M}}, \Delta_{\text{H}}) - \text{E}\big[\tilde{w}(\Delta_{\text{M}}, \Delta_{\text{H}})\big]. \tag{20}$$

Notice that this maximization problem does not include a participation constraint on the part of the manager. We have implicitly assumed that the manager has a low enough reservation utility that his participation constraint is always satisfied. We do this for two reasons. First, the participation constraint, which serves as a reduced-form representation for the other opportunities available to the manager, only complicates the analysis without adding to the intuition that we try to convey in this paper. In fact, the participation constraint of a potentially biased individual may have perverse effects which are beyond the scope of this paper; for example, highly biased individuals will accept any contractual term because they always think that the most favorable outcomes are going to occur. Assuming away the manager's participation constraint simply allows us to avoid such effects. Second, since it is the size relationship between Δ_{M} and Δ_{H} that affects the manager's incentives in this problem, the omission of a participation constraint implies that Δ_{M} will be equal to zero in the maximization problem (20).[6] This greatly simplifies the analysis.

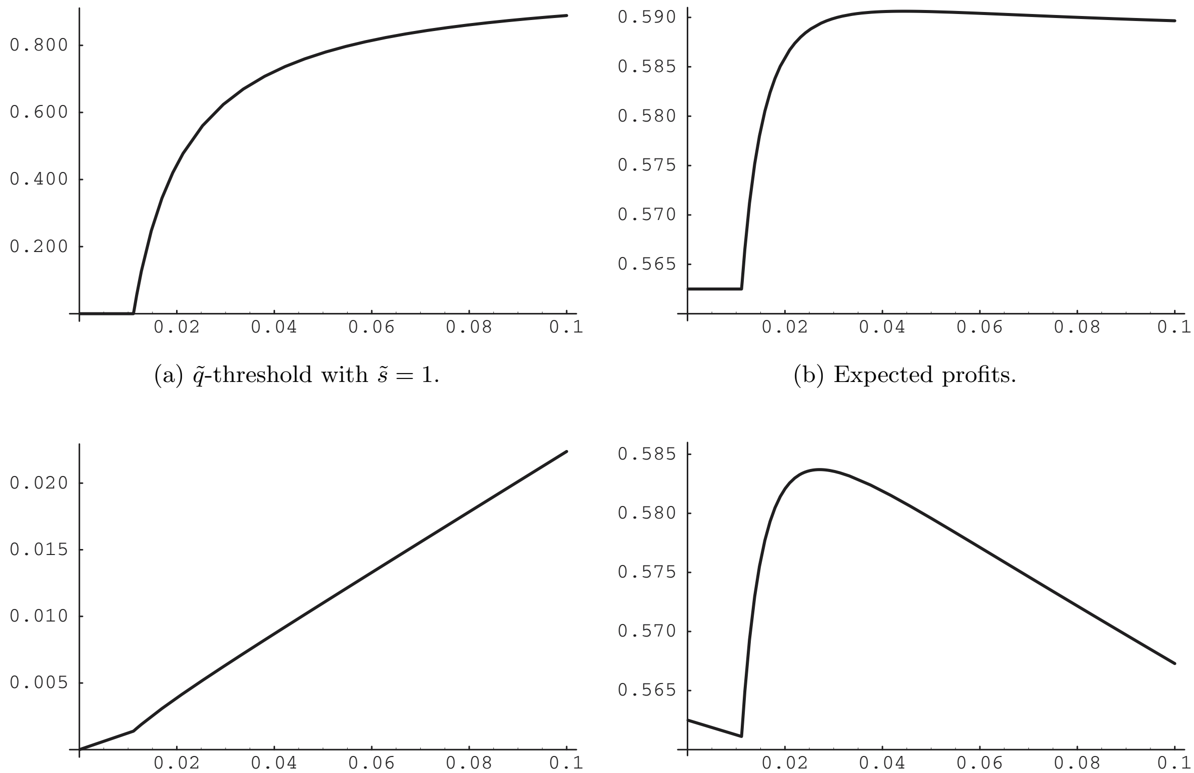
Proposition 5.3 *Suppose that the manager hired by the firm is characterized by a risk aversion of r, ability a, overconfidence $A \geq a$, and optimism $B \geq \frac{1}{2}$ such that (18) or (19) holds. If the firm's value can be improved by the manager, then the compensation package $(\Delta_{\mathrm{M}}^*, \Delta_{\mathrm{H}}^*)$ that maximizes this value is such that $\Delta_{\mathrm{M}}^* = 0$ and*

$$\frac{1-\pi_1(A,B)}{\pi_1(A,B)}\, r < \Delta_{\mathrm{H}}^* < \frac{1-\pi_1(A,B)}{\pi_1(A,B)}\, \frac{\pi_1}{1-\pi_1}\, r. \tag{21}$$

Unfortunately, it is not possible to write down a simple expression for Δ_{H}^*, as the first-order condition for the owners' maximization problem is equivalent to a cubic equation. Notice however that the right-hand side of (21) is equal to $\Delta_{\mathrm{H}}^{\mathrm{FB}}(0)$ derived in Proposition 5.1; it is therefore always optimal for the firm to compensate their manager less than what perfect realignment and first-best decision-making would require. Instead the firm's owners are willing to give up some profits, as long as they can expect to save on manager compensation. This can be seen through a numerical example, which is depicted in Figure 2. As the firm increases the manager's compensation in the high state (and so the convexity of the compensation), the manager tends to acquire less and less information in the second stage after observing $\tilde{s} = 1$. This can be seen in Figure 2(a), where we plot the decision threshold that the manager uses with different compensation contracts.[7] As depicted in Figure 2(b), it is clearly the case that the firm's expected profits increase with this better alignment. These expected profits reach a maximum at $\Delta_{\mathrm{H}} = \Delta_{\mathrm{H}}^{\mathrm{FB}} \approx 0.044$, and then decrease with Δ_{H}. Figure 2(c) shows how the expected compensation that is paid to the manager monotonically increases with Δ_{H}. The difference between these last two curves yields the firm's value, which is plotted in Figure 2(d). The maximum firm value is reached at $\Delta_{\mathrm{H}} = \Delta_{\mathrm{H}}^* \approx 0.027$, which is clearly below $\Delta_{\mathrm{H}}^{\mathrm{FB}}$.

[6] The manager's participation constraint can always be satisfied by increasing Δ_{M} appropriately, while adjusting Δ_{H} to keep the ratio of Δ_{H} to $\Delta_{\mathrm{M}} + r$ constant.

[7] The threshold is equal to zero for Δ_{H} close to zero: the high-state compensation is then not large enough to push the manager to undertake the risky project without perfect information. This also explains the flat part of Figure 2(b).



(a) $\tilde{q}$-threshold with $\tilde{s} = 1$.

(b) Expected profits.

(c) Expected compensation.

(d) Firm value.

Figure 2: These graphs show (a) the $\tilde{q}$-threshold with $\tilde{s} = 1$, (b) expected firm profits, (c) expected manager compensation, and (d) firm value as functions of the high-state compensation Δ_{H} of the manager ($\Delta_{\mathrm{M}} = 0$). All graphs were generated using the following parameter values: $a = 0.6$, $A = 0.8$, $B = 0.5$, $r = 0.1$.

Before finishing this section, let us ask the following question: what happens to firm value if the manager hired by the firm is biased (i.e., overconfident or optimistic) when the owners initially thought him rational? Since the contract offered to this manager is then clearly suboptimal, it is tempting to conclude that firm value will be reduced by this ignored bias. However, the following result shows that this may not be the case: it is possible that a manager hired with a contract written for a rational manager creates more firm value.

Proposition 5.4 *Suppose that all potential managers have risk aversion r and ability a. Suppose that the firm's owners hire their manager under the assumption that he is rational (i.e, $A = a$ and $B = \frac{1}{2}$), and offer him the compensation contract that will maximize firm value. The value of the firm is then increasing in A and B.*

In sum, when the shareholders make their decisions about the appointment of the firm's top managers, they cannot think only about hiring the most qualified person (i.e., the individual with the highest ability). Some thoughts must be given to the underlying incentives that this person requires, especially since his natural incentives may be different from those of the shareholders' objective to maximize value. Aspects of an individual's personality that will help realign his incentives with those of shareholders in a costless manner are sometimes welcome. In fact, in some cases, these personality traits will be as important as ability.

6 Other Considerations

The simple capital budgeting problem analyzed in this paper is meant to capture some basic forces that are likely to be present in reality. Undoubtedly, many other factors play a role in the capital budgeting process of any firm. In this section, we take a look at some of these factors, and discuss how they are likely to change or reinforce the results. To make our points as intuitive as possible, we again ignore compensation issues and revert back to the analysis of a risk-averse manager whose utility depends only upon the value of the firm. The additional forces that we identify in this framework should be equally relevant when compensation contracts are considered.

6.1 Discount Rates

Sections 4.2 and 4.3 assume that it is possible to pick the manager's overconfidence and optimism in order to remove the negative effects of risk aversion. Of course, like risk aversion, overconfidence and optimism are personality traits that are impossible to change for a given individual. For example, if the manager's overconfidence does not satisfy the condition for perfect realignment as described in Proposition 4.1, firm value will simply not be maximized.

Fortunately, one variable in the capital budgeting problem can be adjusted, namely the discount rate, or hurdle rate. Indeed, so far, we have assumed that the discount rate used by the firm in its capital budgeting process is that prescribed by capital markets. If the capital-budgeter is prescribed a different rate, his decisions will change with that rate. Let us denote by $\delta \in \left(\frac{1}{2}, 1\right]$ the discount factor that the manager is told to apply to the risky project's cash flows, where $\delta = 1$ corresponds to using the riskfree rate as the hurdle rate.[8] In present value terms therefore, risky projects offer end-of-period cash flows of zero or δ with equal probabilities.

[8] The discount factor is assumed to be larger than $\frac{1}{2}$ so that the present value of the risky project can, with enough weight on the outcome of $\tilde{v} = 1$, be larger than $\frac{1}{2}$, the current cash value of the firm.

The effect of this discount factor is to make the risky project less appealing. So, like risk aversion, the discount factor makes the manager reluctant to take on a project. For a rational manager, this is bad news, as the discount factor exacerbates the effects of his risk aversion. However, as the following proposition shows, the discount factor can be used to realign the incentives of a manager in some cases.

Proposition 6.1 *Suppose that the manager hired by the firm is characterized by a risk aversion of r, ability a, overconfidence $A \geq a$, and optimism $B \geq \frac{1}{2}$ such that (18) or (19) holds. Then the firm can generate the first-best outcome by setting the risky project's discount factor equal to*

$$\delta^* = \frac{1}{2} + \frac{\pi_1\big[1 - \pi_1(A, B)\big]}{2(1 - \pi_1)\pi_1(A, B)}(1 + 2r). \tag{22}$$

It is easy to verify that δ^* is decreasing in A and B: the larger the manager's bias, the larger the hurdle rate that needs to be imposed on him by the firm in order to realign his incentives.[9] This has important implications for the hiring and firing of decision-makers, especially in environments where it is difficult and costly to precisely decipher the personality traits of individuals (risk aversion, overconfidence, optimism) before they are hired to make decisions. In such situations, it may be more productive for the firm to learn the personality traits of individuals while they perform their duties and accordingly adjust the problems they are asked to solve (e.g., by adjusting the suggested hurdle rate), than firing and replacing them with a new set of individuals with unknown personality traits.

It is well-known that individuals who remain with one firm acquire firm-specific capital and thus become more valuable to that firm (see, e.g., Becker, 1962). The above result may point to another positive aspect of a long-term relationship between a firm and its employees: the relationship enables the firm to learn the personal characteristics of its decision-makers more precisely, and thus to better frame the problems they are asked to solve for more effective value maximization.

6.2 Costly Effort

So far, we have assumed that the benefits from gathering information get impounded into firm value without any cost. In particular, we have assumed that the manager's effort to create value for his firm is costless. This is unlikely to be the case in reality: when looking at his options, the manager will typically weigh in the effort that gathering more information entails. To illustrate

[9] In fact, when A and B are close to a and $\frac{1}{2}$ respectively, δ^* is larger than one, that is the project has to be made *more* appealing by the firm for the manager's incentives to be realigned. The prescription of a hurdle rate smaller than the riskfree rate is avoided if and only if $r \leq \frac{\pi_1(A,B) - \pi_1}{2\pi_1[1 - \pi_1(A,B)]}$.

the effect of effort costs, let us assume that the manager incurs a utility cost of $e \in \left(0, \frac{1}{4}\right)$ when he decides to acquire more information in the first stage.[10] To isolate the effects that costly effort has on the manager's decisions, we further assume that the manager is risk-neutral (i.e., $r = 0$). As before, we assume that the manager is potentially overconfident ($A \geq a$) or optimistic ($B \geq \frac{1}{2}$).

There is always a (strictly) positive probability that this manager will choose to undertake the risky project in the first stage of the capital budgeting process instead of gathering information. Indeed, when $\tilde{p}$ is close to zero, the payoff from gathering more information is close to $\frac{1}{2} - e < \frac{1}{2}$: the project is likely to disappear, despite the effort exerted by the manager. Undertaking the project on the other hand yields the manager a payoff of $B \geq \frac{1}{2}$. Thus the manager's incentives are never perfectly realigned with those of the shareholders. Of course, since firm value can only be created through information gathering about projects, a larger commitment to effort by the manager is beneficial to the shareholders. As the following proposition shows, overconfidence tends to be a natural force fostering that commitment, whereas optimism deters the manager from exerting an effort.

Proposition 6.2 *Suppose that the firm's manager is risk-neutral and must incur a utility cost of $e > 0$ to gather information in the first stage of the capital budgeting process. The information-gathering threshold of the manager in the first stage ($\bar{P}$) is decreasing in the manager's overconfidence level (A) and increasing in his optimism level (B).*

A lower information threshold translates into more frequent costly gathering of information by the manager in the first stage. In that sense, overconfidence commits the manager to a higher level of effort. This is due to the fact that the overconfident manager overestimates the value of his information, and so is less reluctant to "invest some utility" into gathering it: for him, the effort cost appears small relative to the expected gain from the information he gathers. This idea that some personal biases may help individuals self-motivate is also explored by Bénabou and Tirole (2002).

One should be careful about extrapolating their or our results to all types of personal bias however. As the last part of Proposition 6.2 illustrates, optimism does not have the same positive effects as overconfidence; instead, it reduces the level of effort exerted by the manager. Indeed, an optimistic manager fails to see the value of gathering information; for him, the project is good at the outset, and so there is no need to invest any effort into convincing himself about the project's

[10]For the purpose of this discussion, we assume that acquiring more information in the second stage is costless to the manager. Adding such a cost would only reinforce the point we are about to make. Also, we restrict the manager's cost of effort to be below $\frac{1}{4}$, as a larger cost always results in the manager not exerting any effort.

value.

7 Conclusion

Because managers cannot diversity their human capital, they tend to be more conservative than owners would like them to be when making capital budgeting decisions for the firm. Thus managers may wait too long to exercise real options when quickly undertaking such projects is optimal from the owners' perspective. Traditionally, owners use incentives packages, such as stock options, to align the decision of managers more closely with their own interests. However, overconfidence and optimism make the manager more willing to undertake risky projects, and so can align the decisions of managers with the interests of owners.

In addition to aligning the decisions of managers with the interests of shareholders, overconfidence has the advantage that it motivates managers to expend more effort. Optimistic managers, however, may so exaggerate the chances of success that they undertake negative present value projects. One way in which firms can counteract such excessive optimism, is to set the firm's internal discount rate (i.e., hurdle rate) artificially high. While we have treated overconfidence and optimism separately in our analysis, these two traits will often go hand in hand. In addition to influencing capital budgeting decisions and effort, these traits may affect manager's behavior in other ways that benefit the firm. Overconfident, optimistic people tend to be happier, more popular, more willing to help others, and more willing to persist in tasks (Taylor and Brown, 1988).

Our paper helps explain the puzzle of why, if rational decision-making dominates biased decision-making, CEOs are often biased towards overconfidence. Shareholders may simply be better off hiring moderately overconfident, optimistic managers (e.g., CEOs) than paying rational managers additional incentives to change their decision patterns. Shareholders may even prefer overconfident, optimistic managers to rational managers who have more ability.

Appendix A

Proof of Lemma 3.1

Suppose that the firm learns $\tilde{s} = s \in \{0,1\}$ and $\tilde{q} = q \in [0,1]$ in the first stage. If the risky project is undertaken in the second stage, it has a probability of π_s of being worth one and a probability of $1-\pi_s$ of being worthless, for an expected value of π_s. If instead more information is gathered in the second stage, the project will be undertaken only if it does not disappear and $\tilde{v} = 1$; the payoff is then one. Otherwise (i.e., if the project disappears or $\tilde{v} = 0$), the safe investment is made for a payoff of $\frac{1}{2}$. Thus the value of the firm when more information is gathered in the second stage is

$$\begin{aligned}
&\Pr\{\text{project exists}\}\left[\Pr\{\tilde{v}=1 \mid \tilde{s}=s\}(1) + \Pr\{\tilde{v}=0 \mid \tilde{s}=s\}\frac{1}{2}\right] + \Pr\{\text{project disappears}\}\frac{1}{2} \\
&\quad = q\left[\pi_s(1) + (1-\pi_s)\frac{1}{2}\right] + (1-q)\frac{1}{2} \\
&\quad = \frac{1}{2} + \frac{1}{2} q\,\pi_s.
\end{aligned}$$

As mentioned in section 3.1, any decision-maker will always elect to gather more information in the second stage when $\tilde{q}$ is larger than some value $\bar{Q}_s$. Thus the value of the firm for each possible value of the signal $\tilde{s}$ is given by

$$\begin{aligned}
\bar{F}_s(\bar{Q}_s) &\equiv \int_0^{\bar{Q}_s} \pi_s\,dq + \int_{\bar{Q}_s}^1 \left(\frac{1}{2} + \frac{1}{2} q\,\pi_s\right) dq \\
&= \pi_s \bar{Q}_s + \frac{1}{2}\big(1-\bar{Q}_s\big) + \frac{1}{4}\pi_s\big(1-\bar{Q}_s^2\big),
\end{aligned}$$

which is equal to (3). ∎

Proof of Lemma 3.2

The acquisition of information in the first stage will result in $\tilde{s} = 1$ or $\tilde{s} = 0$, which are equally likely. At the outset therefore, when it is known that $\tilde{p} = p$, the value of the firm conditional on the decision to acquire some information about the risky project is given by

$$\begin{aligned}
&\Pr\{\text{project exists}\}\Big[\Pr\{\tilde{s}=1\}\,\bar{F}_1(\bar{Q}_1) + \Pr\{\tilde{s}=0\}\,\bar{F}_0(\bar{Q}_0)\Big] + \Pr\{\text{project disappears}\}\frac{1}{2} \\
&\quad = p\left[\frac{1}{2}\bar{F}_1(\bar{Q}_1) + \frac{1}{2}\bar{F}_0(\bar{Q}_0)\right] + (1-p)\frac{1}{2} \\
&\quad = \frac{1}{2}p\Big[\bar{F}_1(\bar{Q}_1) + \bar{F}_0(\bar{Q}_0)\Big] + \frac{1}{2}(1-p) \\
&\quad = \frac{1}{2} + \frac{1}{2}p\Big[\bar{F}_1(\bar{Q}_1) + \bar{F}_0(\bar{Q}_0) - 1\Big].
\end{aligned}$$

If on the other hand the risky project is undertaken before any information about it is acquired, the firm is worth the ex ante value of the project, that is $\frac{1}{2}$. As argued in section 3.1, it is always the case that the manager ends up gathering information for outcomes of $\tilde{p}$ larger than some threshold $\bar{P}$. The initial value of the firm conditional on its information acquisition policy can then be calculated as

$$\begin{aligned}\bar{F}(\bar{P},\bar{Q}_1,\bar{Q}_0) &\equiv \int_0^{\bar{P}} \frac{1}{2}\,dp + \int_{\bar{P}}^{1} \left\{\frac{1}{2} + \frac{1}{2}p\Big[\bar{F}_1(\bar{Q}_1) + \bar{F}_0(\bar{Q}_0) - 1\Big]\right\} dp \\ &= \frac{1}{2}\bar{P} + \frac{1}{2}(1-\bar{P}) + \frac{1}{2}\Big[\bar{F}_1(\bar{Q}_1) + \bar{F}_0(\bar{Q}_0) - 1\Big]\frac{1-\bar{P}^2}{2},\end{aligned}$$

where $\bar{F}_1(\cdot)$ and $\bar{F}_0(\cdot)$ are as calculated in (3). This last expression reduces to (4). ∎

Proof of Proposition 3.1

The value of the firm derived in Lemma 3.2 is clearly decreasing in $\bar{P}$, so that $\bar{P}^{\text{FB}} = 0$. Since $\pi_0 = \frac{1-a}{2} < \frac{1}{2}$, it is straightforward to show that $\bar{F}_0(\bar{Q}_0)$ calculated in Lemma 3.1 is decreasing in $\bar{Q}_0 \in [0,1]$, so that $\bar{Q}_0^{\text{FB}} = 0$. Finally, $\bar{F}_1(\bar{Q}_1)$, also calculated in Lemma 3.1, is maximized at $\bar{Q} = \bar{Q}_1^{\text{FB}}$, as shown in (5). With this strategy, we can use Lemma 3.1 to show that

$$\begin{aligned}\bar{F}_0(0) &= \frac{1}{2}\left(1 + \frac{1}{2}\pi_0\right) = \frac{1}{8}(5-a), \quad \text{and} \\ \bar{F}_1(\bar{Q}_1^{\text{FB}}) &= \frac{1}{2}\left(1 + \frac{1}{2}\pi_1\right) + \left(\pi_1 - \frac{1}{2}\right)\bar{Q}_1^{\text{FB}} - \frac{1}{4}\pi_1(\bar{Q}_1^{\text{FB}})^2 \\ &= \frac{1}{2}\left(1 + \frac{1}{2}\pi_1\right) + \frac{\left(\pi_1 - \frac{1}{2}\right)^2}{\pi_1} = \frac{1}{8}(5+a) + \frac{a^2}{2(1+a)}.\end{aligned}$$

Lemma 3.2 can then be used with $\bar{P} = 0$, $\bar{Q}_0 = 0$ and $\bar{Q}_1 = \bar{Q}_1^{\text{FB}}$ to calculate (6). ∎

Proof of Proposition 3.2

Since the manager is reluctant to undertake the risky project early, he will choose $\bar{P} = 0$ and $\bar{Q}_0 = 0$ for the same reason that the risk-neutral manager chose these thresholds in section 3.2. Suppose now that the manager with risk aversion $r \geq 0$ observes $\tilde{s} = 1$ and $\tilde{q} = q \in [0,1]$ in the first stage. If the risky project is undertaken, it has a probability of π_1 of being worth one and a probability of $1-\pi_1$ of being worthless; the manager's expected utility from this option is $\pi_1 - (1-\pi_1)r$. Suppose instead that, knowing that the risky project will disappear with probability q, the manager chooses to acquire a perfect signal. Clearly, the risky project is undertaken if $\tilde{v} = 1$, and dropped in favor of the safe investment if $\tilde{v} = 0$; these will occur with probabilities π_1 and

$1-\pi_1$ respectively. The utility loss from a bad risky project is then completely avoided, and the manager's expected utility from this option is

$$\begin{aligned}
&\Pr\{\text{project exists}\}\left[\Pr\{\tilde{v}=1\mid\tilde{s}=1\}(1)+\Pr\{\tilde{v}=0\mid\tilde{s}=1\}\frac{1}{2}\right]+\Pr\{\text{project disappears}\}\frac{1}{2}\\
&\quad= q\left[\pi_1(1)+(1-\pi_1)\frac{1}{2}\right]+(1-q)\frac{1}{2}\\
&\quad=\frac{1}{2}+\frac{1}{2}q\,\pi_1.
\end{aligned}$$

Clearly, the manager will choose to acquire more information when $\frac{1}{2}+\frac{1}{2}q\,\pi_1\geq\pi_1-(1-\pi_1)r$, that is when

$$q\geq\frac{\frac{1}{2}-(1-\pi_1)(1+r)}{\frac{1}{2}\pi_1}=\frac{2\big[a-(1-a)r\big]}{1+a}\equiv\bar{Q}_1(r).$$

Lemma 3.2 can then be used with $\bar{P}=0$, $\bar{Q}_0=0$ and $\bar{Q}_1=\bar{Q}_1(r)$ to calculate (8). ∎

Proof of Lemma 4.1

Using Bayes' rule, we have

$$\begin{aligned}
\Pr_{\mathrm{b}}\big\{\tilde{v}=1\mid\tilde{s}=1\big\} &= \frac{\Pr_{\mathrm{b}}\big\{\tilde{s}=1\mid\tilde{v}=1\big\}\Pr_{\mathrm{b}}\big\{\tilde{v}=1\big\}}{\Pr_{\mathrm{b}}\big\{\tilde{s}=1\mid\tilde{v}=1\big\}\Pr_{\mathrm{b}}\big\{\tilde{v}=1\big\}+\Pr_{\mathrm{b}}\big\{\tilde{s}=1\mid\tilde{v}=0\big\}\Pr_{\mathrm{b}}\big\{\tilde{v}=0\big\}}\\
&= \frac{\big[A+(1-A)B\big]B}{\big[A+(1-A)B\big]B+(1-A)B(1-B)}\\
&= A+(1-A)B,
\end{aligned}$$

and

$$\begin{aligned}
\Pr_{\mathrm{b}}\big\{\tilde{v}=1\mid\tilde{s}=0\big\} &= \frac{\Pr_{\mathrm{b}}\big\{\tilde{s}=0\mid\tilde{v}=1\big\}\Pr_{\mathrm{b}}\big\{\tilde{v}=1\big\}}{\Pr_{\mathrm{b}}\big\{\tilde{s}=0\mid\tilde{v}=1\big\}\Pr_{\mathrm{b}}\big\{\tilde{v}=1\big\}+\Pr_{\mathrm{b}}\big\{\tilde{s}=0\mid\tilde{v}=0\big\}\Pr_{\mathrm{b}}\big\{\tilde{v}=0\big\}}\\
&= \frac{(1-A)(1-B)B}{(1-A)(1-B)B+\big[A+(1-A)(1-B)\big](1-B)}\\
&= (1-A)B.
\end{aligned}$$

This completes the proof. ∎

Proof of Lemma 4.2

The manager chooses $\bar{P}=0$ and $\bar{Q}_0=0$ for the reasons mentioned in the paragraph preceding the lemma. Suppose now that the manager with risk aversion $r\geq 0$ and overconfidence $A\geq a$ observes $\tilde{s}=1$ and $\tilde{q}=q\in[0,1]$ in the first stage. If the risky project is undertaken, this manager assesses that it has a probability of $\pi_1(A, \frac{1}{2})$ of being worth one and a probability of $1 - \pi_1(A, \frac{1}{2})$ of being worthless; the manager's expected utility from this option is $\pi_1(A, \frac{1}{2}) - \big[1 - \pi_1(A, \frac{1}{2})\big] r$. Suppose instead that, knowing that the risky project will disappear with probability $\tilde{q} = q$, the manager chooses to acquire a perfect signal. Clearly, the risky project is undertaken if $\tilde{v} = 1$, and dropped in favor of the safe investment if $\tilde{v} = 0$; according to this manager, these will occur with probabilities $\pi_1(A, \frac{1}{2})$ and $1 - \pi_1(A, \frac{1}{2})$ respectively. The utility loss from a bad risky project is then completely avoided, and the manager's expected utility from this option is

$$\begin{aligned}
&\Pr\{\text{project exists}\} \left[\Pr_{\mathrm{b}}\{\tilde{v} = 1 \mid \tilde{s} = 1\}(1) + \Pr_{\mathrm{b}}\{\tilde{v} = 0 \mid \tilde{s} = 1\} \frac{1}{2} \right] + \Pr\{\text{project disappears}\} \frac{1}{2} \\
&\quad = q \left\{ \pi_1(A, \tfrac{1}{2})(1) + \big[1 - \pi_1(A, \tfrac{1}{2})\big] \frac{1}{2} \right\} + (1 - q)\frac{1}{2} \\
&\quad = \frac{1}{2} + \frac{1}{2} q\, \pi_1(A, \tfrac{1}{2}).
\end{aligned}$$

Clearly, the manager will choose to acquire more information when

$$\frac{1}{2} + \frac{1}{2} q\, \pi_1(A, \tfrac{1}{2}) \geq \pi_1(A, \tfrac{1}{2}) - \big[1 - \pi_1(A, \tfrac{1}{2})\big] r,$$

that is when

$$q \geq \frac{\frac{1}{2} - \big[1 - \pi_1(A, \frac{1}{2})\big](1 + r)}{\frac{1}{2}\pi_1(A, \frac{1}{2})} = \frac{2\big[A - (1 - A)r\big]}{1 + A} \equiv \bar{Q}_1^{\mathrm{OV}}(r, A).$$

This completes the proof. ∎

Proof of Proposition 4.1

The value of the firm will be equal to the first-best value if the manager chooses a $\tilde{q}$-threshold of $\bar{Q}_1^{\mathrm{FB}}$, as derived in Proposition 3.1, after he observes $\tilde{s} = 1$. From Lemma 4.2, we see that this will occur when $\bar{Q}_1^{\mathrm{OV}}(r, A) = \bar{Q}_1^{\mathrm{FB}}$, that is when

$$\frac{2\big[A - (1 - A)r\big]}{1 + A} = \frac{2a}{1 + a}.$$

Solving for A in this last expression, we get (12).

To show how the firm's value changes with A, we use the expression for the firm's value calculated in Lemma 3.2 along with the overconfident manager's information thresholds calculated in Lemma 4.2. The resulting firm value is given by

$$\frac{1}{16} \left[9 + \frac{4a(A - r + Ar)}{1 + A} - \frac{2(1 + a)(A - r + Ar)^2}{(1 + A)^2} \right].$$

Tedious but straightforward calculations show that this expression is increasing (decreasing) in A for $A < A^*$ ($A > A^*$). ∎

Proof of Proposition 4.2

We know from Proposition 4.1 that a manager with ability a and overconfidence A^* restores the first-best outcome, that is a firm value of $\bar{F}^{\text{FB}}$, as derived in Proposition 3.1. So a firm value exceeding F can be obtained with a manager of ability a' as long as

$$\frac{9}{16} + \frac{(a')^2}{8(1+a')} > F,$$

or equivalently, as long as

$$a' > \frac{1}{4}\left(-9 + 16F + \sqrt{9 - 160F + 256F^2}\right) \equiv \bar{a}.$$

This completes the proof. ∎

Proof of Lemma 4.3

Suppose that the manager with risk aversion $r \geq 0$ and optimism $B \geq \frac{1}{2}$ observes $\tilde{s} = s \in \{0, 1\}$ and $\tilde{q} = q \in [0, 1]$ in the first stage. If the risky project is undertaken, this manager assesses that it has a probability of $\pi_s(a, B)$ of being worth one and a probability of $1 - \pi_s(a, B)$ of being worthless; the manager's expected utility from this option is $\pi_s(a, B) - \big[1 - \pi_s(a, B)\big]r$. Suppose instead that, knowing that the risky project will disappear with probability $\tilde{q} = q$, the manager chooses to acquire a perfect signal. Clearly, the risky project is undertaken if $\tilde{v} = 1$, and dropped in favor of the safe investment if $\tilde{v} = 0$; according to this manager, these will occur with probabilities $\pi_s(a, B)$ and $1 - \pi_s(a, B)$ respectively. The utility loss from a bad risky project is then completely avoided, and the manager's expected utility from this option is

$$\begin{aligned}
&\Pr\{\text{project exists}\}\left[\Pr\nolimits_{\text{b}}\{\tilde{v} = 1 \mid \tilde{s} = 1\}(1) + \Pr\nolimits_{\text{b}}\{\tilde{v} = 0 \mid \tilde{s} = 1\}\frac{1}{2}\right] + \Pr\{\text{project disappears}\}\frac{1}{2} \\
&= q\left\{\pi_s(a, B)(1) + \big[1 - \pi_s(a, B)\big]\frac{1}{2}\right\} + (1-q)\frac{1}{2} \\
&= \frac{1}{2} + \frac{1}{2}q\,\pi_s(a, B).
\end{aligned}$$

Clearly, the manager will choose to acquire more information when

$$\frac{1}{2} + \frac{1}{2}q\,\pi_s(a, B) \geq \pi_s(a, B) - \big[1 - \pi_s(a, B)\big]r,$$

that is when

$$q \geq \frac{\frac{1}{2} - \big[1 - \pi_s(a,B)\big](1+r)}{\frac{1}{2}\pi_s(a,B)}.$$

We can replace $\pi_s(a,B)$ by the values calculated in Lemma 4.1 for $s = 0, 1$; this yields

$$\bar{Q}_0^{\text{OP}}(r,B) = \frac{1 - 2\big[1 - (1-a)B\big](1+r)}{(1-a)B}, \tag{23}$$

and (15). Clearly $\bar{Q}_0^{\text{OP}}(r,B) \leq 0$ (which, since $\tilde{q}$ cannot be negative, is equivalent to $\bar{Q}_0^{\text{OP}}(r,B) = 0$) if and only if $1 - 2\big[1 - (1-a)B\big](1+r) > 0$, which is equivalent to (14).

For the manager to set his $\tilde{p}$-threshold equal to zero, it has to be the case that he prefers gathering information in the first stage however close $\tilde{p}$ is to zero. When $\tilde{p}$ is arbitrarily close to zero, the manager expects the project not to exist after gathering information in the first stage, and so the riskfree investment to be made; his expected utility is then $\frac{1}{2}$. If on the other hand, the manager chooses to undertake the project at the outset, he expects it to be successful with probability B; his expected utility is then $B - (1-B)r$. Information gathering is preferable if $\frac{1}{2} \geq B - (1-B)r$, that is if (13) is satisfied. ∎

Proof of Proposition 4.3

A necessary condition for the first-best outcome to be generated is that $\bar{Q}_1^{\text{OP}}(r,B)$, as derived in Lemma 4.3, is equal to $\bar{Q}_1^{\text{FB}}$, as derived in Proposition 3.1. Solving for B yields

$$B = \frac{\frac{1}{2} + r(1+a)}{1 + r(1+a)} \equiv B^*.$$

Since this B^* is greater than $\frac{\frac{1}{2}+r}{1+r}$, we know from Lemma 4.3 that $\bar{P}^{\text{OP}}(r,B^*) > 0$, and so first-best cannot be achieved.

For $B \leq \frac{\frac{1}{2}+r}{1+r} < \frac{1}{1-a}\left(\frac{\frac{1}{2}+r}{1+r}\right)$, Lemma 4.3 tells us that $\bar{P}^{\text{OP}}(r,B) = \bar{Q}_0^{\text{OP}}(r,B) = 0$. Using these values, along with (15), in (4), we find after some manipulations that the firm's value is equal to

$$\bar{F}^{\text{OP}}(r,B) \equiv \frac{1}{16}\left\{9 + \frac{2a\big[1 - 2(1+r)(1-a)(1-B)\big]}{a + B - aB} + \frac{(1+a)\big[1 - 2(1+r)(1-a)(1-B)\big]^2}{2(a+B-aB)^2}\right\}.$$

Differentiation with respect to B and some manipulations yield

$$\frac{\partial \bar{F}^{\text{OP}}(r,B)}{\partial B} = \frac{(1-a)^2(1+2r)\big[1 + 2r + 2ar - 2B(1+r+ar)\big]}{16(a+B-aB)^3}. \tag{24}$$

For $B \leq \frac{\frac{1}{2}+r}{1+r}$, we have

$$1 + 2r + 2ar - 2B(1+r+ar) \geq 1 + 2r + 2ar - \frac{1+2r}{1+r}(1+r+ar) = \frac{ar}{1+r} > 0,$$

and so (24) is greater than zero, that is the firm's value is increasing in B. ∎

Proof of Proposition 5.1

Suppose that the manager learns $\tilde{s} = 1$ and $\tilde{q} = q \in [0, 1]$ in the first stage. If the risky project is undertaken in the second stage, the manager assesses that it has a probability of $\pi_1(A, B)$ of being worth one and a probability of $1 - \pi_1(A, B)$ of being worthless. Given a compensation package that pays him $\Delta_{\text{M}} + \Delta_{\text{H}}$ for a successful project and zero for an unsuccessful project, his expected utility from undertaking the project is

$$\pi_1(A, B)(\Delta_{\text{M}} + \Delta_{\text{H}}) - \big[1 - \pi_1(A, B)\big] r. \tag{25}$$

If instead more information is gathered in the second stage, the project will be undertaken only if it does not disappear and $\tilde{v} = 1$; the manager's utility is then $\Delta_{\text{M}} + \Delta_{\text{H}}$. Otherwise (i.e., if the project disappears or $\tilde{v} = 0$), the safe investment is made, and the manager's utility is then Δ_{M}. Thus, if the manager chooses to gather more information, his expected utility is

$$\begin{aligned}
&\Pr\{\text{project exists}\}\Big[\Pr_{\text{b}}\{\tilde{v} = 1 \mid \tilde{s} = 1\}(\Delta_{\text{M}} + \Delta_{\text{H}}) \\
&\qquad\qquad + \Pr_{\text{b}}\{\tilde{v} = 0 \mid \tilde{s} = 1\}\Delta_{\text{M}}\Big] + \Pr\{\text{project disappears}\}\Delta_{\text{M}} \\
&= q\Big\{\pi_1(A, B)(\Delta_{\text{M}} + \Delta_{\text{H}}) + \big[1 - \pi_1(A, B)\big]\Delta_{\text{M}}\Big\} + (1 - q)\Delta_{\text{M}} \\
&= \Delta_{\text{M}} + q\,\pi_1(A, B)\Delta_{\text{H}}.
\end{aligned} \tag{26}$$

The manager will therefore choose to gather information if (26) exceeds (25), that is if

$$q \geq 1 - \frac{1 - \pi_1(A, B)}{\pi_1(A, B)} \cdot \frac{\Delta_{\text{M}} + r}{\Delta_{\text{H}}}. \tag{27}$$

For the first-best outcome to be generated, it is necessary that this threshold be equal to $\bar{Q}_1^{\text{FB}}$, as derived in Proposition 3.1. After rearrangement, this yields (17). ∎

Proof of Proposition 5.2

The $\tilde{q}$-threshold after the manager observes $\tilde{s} = 0$ is derived the same way that the $\tilde{q}$-threshold with $\tilde{s} = 1$ was derived in the proof of Proposition 5.1. In fact, we can simply replace the subscript of one by zero in (27): the manager will choose to gather more information if

$$q \geq 1 - \frac{1 - \pi_0(A, B)}{\pi_0(A, B)} \cdot \frac{\Delta_{\text{M}} + r}{\Delta_{\text{H}}}. \tag{28}$$

For first-best to obtain, it has to be the case that the manager always gathers more information in this state, that is first-best requires that

$$1 - \frac{1-\pi_0(A,B)}{\pi_0(A,B)} \cdot \frac{\Delta_{\mathrm{M}} + r}{\Delta_{\mathrm{H}}} \leq 0,$$

or, equivalently, that

$$\Delta_{\mathrm{H}} \leq \frac{1-\pi_0(A,B)}{\pi_0(A,B)} \, (\Delta_{\mathrm{M}} + r). \tag{29}$$

We know from Proposition 5.1 that Δ_{M} and Δ_{H} must satisfy (17), so that (29) reduces to

$$\frac{1-\pi_1(A,B)}{\pi_1(A,B)} \, \frac{\pi_1}{1-\pi_1} \leq \frac{1-\pi_0(A,B)}{\pi_0(A,B)}.$$

Using $\pi_0(A,B)$ and $\pi_1(A,B)$ from Lemma 4.1 and $\pi_1 = \frac{1+a}{2}$ in this expression yields

$$\frac{(1-A)(1-B)}{A+(1-A)B} \, \frac{1+a}{1-a} \leq \frac{A+(1-A)(1-B)}{(1-A)B}$$

which, after some reductions is equivalent to

$$2(1-A)(1-B)B \leq \frac{A}{a} \, \frac{1-a}{1-A}. \tag{30}$$

Since $A \geq a$, the right-hand side of this last expression is greater than one. Since $B \in [\frac{1}{2}, 1]$, we have $2(1-B)B \leq \frac{1}{2}$, implying that the left-hand side of (30) is smaller than $\frac{1}{2}$. Thus (30) is always true: when the manager's compensation satisfies (17), he always gathers more information after observing $\tilde{s} = 0$ in the first stage.

Let us now turn to the manager's problem in the first stage. Suppose that $\tilde{p} = p$. For simplicity, let us denote the $\tilde{q}$-threshold calculated in (27) by $\bar{Q}_1$. Given (25) and (26), the manager's expected utility conditional on observing $\tilde{s} = 1$ is given by

$$\begin{aligned} \bar{U}_1(\bar{Q}_1) \;&\equiv\; \int_0^{\bar{Q}_1} \Big\{ \pi_1(A,B)(\Delta_{\mathrm{M}} + \Delta_{\mathrm{H}}) - \big[1-\pi_1(A,B)\big] r \Big\} \, dq + \int_{\bar{Q}_1}^{1} \Big[\Delta_{\mathrm{M}} + q\, \pi_1(A,B) \Delta_{\mathrm{H}} \Big] dq \\ &=\; \Big\{ \pi_1(A,B)(\Delta_{\mathrm{M}} + \Delta_{\mathrm{H}}) - \big[1-\pi_1(A,B)\big] r \Big\} \bar{Q}_1 + \Delta_{\mathrm{M}}(1-\bar{Q}_1) + \pi_1(A,B)\Delta_{\mathrm{H}} \frac{1-\bar{Q}_1^2}{2} \end{aligned}$$

The manager always gathers more information after observing $\tilde{s} = 0$, so that his expected utility conditional on observing $\tilde{s} = 0$ is given by

$$\bar{U}_0 \equiv \int_0^1 \Big[\Delta_{\mathrm{M}} + q\, \pi_0(A,B) \Delta_{\mathrm{H}} \Big] dq = \Delta_{\mathrm{M}} + \frac{1}{2} \pi_0(A,B) \Delta_{\mathrm{H}}.$$

Thus, before the manager observes $\tilde{s}$, his expected utility from gathering information in the first stage is

$$\begin{aligned}
&\Pr\{\text{project exists}\}\Big[\Pr\{\tilde{s}=1\}\,\bar{U}_1(\bar{Q}_1)+\Pr\{\tilde{s}=0\}\,\bar{U}_0\Big]+\Pr\{\text{project disappears}\}\,\Delta_{\text{M}}\\
&\qquad = p\Big[B\bar{U}_1(\bar{Q}_1)+(1-B)\bar{U}_0\Big]+(1-p)\Delta_{\text{M}}. \qquad (31)
\end{aligned}$$

If instead the manager undertakes the project, his expected utility is given by

$$B(\Delta_{\text{M}}+\Delta_{\text{H}})-(1-B)r. \qquad (32)$$

The manager will therefore choose to gather more information if (31) exceeds (32), that is if

$$\begin{aligned}
p &\geq \frac{B\Delta_{\text{H}}-(1-B)(\Delta_{\text{M}}+r)}{B\bar{U}_1(\bar{Q}_1)+(1-B)\bar{U}_0-\Delta_{\text{M}}}\\
&= \frac{2\big[B\Delta_{\text{H}}-(1-B)(\Delta_{\text{M}}+r)\big]}{\Delta_{\text{H}}\big[B\pi_1(A,B)+(1-B)\pi_0(A,B)\big]+2B\Big\{\Delta_{\text{H}}\pi_1(A,B)-\big[1-\pi_1(A,b)\big](\Delta_{\text{M}}+r)\Big\}}\\
&= -\frac{2(1+a)\big[A(1-a)-2aB(1-A)\big]}{(1-A)B\Big\{1+2a+a^2\big[1+4B+4A(1-B)\big]\Big\}} \qquad (33)
\end{aligned}$$

where the last equality is obtained after replacing Δ_{H} by $\Delta_{\text{H}}^{\text{FB}}(\Delta_{\text{M}})$ derived in Proposition 5.1, $\pi_0(A,B)$ and $\pi_1(A,B)$ by their values in Lemma 4.1, and $\bar{Q}_1$ by its value in (27). The first-best outcome is obtained when (33) is negative or, equivalently, when the expression in brackets in the numerator is positive. It is straightforward to verify that this will be the case when (18) or (19) are satisfied. ∎

Proof of Proposition 5.3

The fact that $\Delta_{\text{M}}^{*}=0$ is clear from the discussion preceding the proposition. Let us determine the range for Δ_{H} in which the manager will always choose to gather more information in the first stage and in the second stage after observing $\tilde{s}=0$; this is the value of Δ_{H} that makes the manager indifferent between undertaking the project and gathering information when $\tilde{p}$ is arbitrarily close to zero.[11] When $\tilde{p}=0$, gathering more information always results in the project disappearing, and the manager's payoff is then $\Delta_{\text{M}}=0$. If instead the project is undertaken, the manager's payoff is $B(\Delta_{\text{M}}+\Delta_{\text{H}})-(1-B)r=B\Delta_{\text{H}}-(1-B)r$. Thus the manager will set $\bar{Q}_0=\bar{P}=0$ as long as $B\Delta_{\text{H}}-(1-B)r\leq 0$, that is as long as $\Delta_{\text{H}}\leq\frac{(1-B)}{B}r$. Increasing Δ_{H} past this range is foolish

[11] The manager is always more tempted to undertake the project in the first stage than in the second stage after he observes $\tilde{s}=0$.

for the firm's shareholders: we know that the firm's profits will be smaller as $\bar{P}$ and $\bar{Q}_0$ increase, and this will be done with a higher compensation for the manager. So, we need to calculate and maximize the firm's value for $\Delta_{\text{H}} \in \left[0, \frac{(1-B)}{B} r\right]$.

We know from the proof of Proposition 5.1 that the manager sets his $\tilde{q}$-threshold equal to the right-hand side of (27) when $\tilde{s} = 1$. Of course, when this quantity is negative, the manager effectively always gathers more information until he learns $\tilde{v}$. When that is the case, the manager's ability never affects the decision process and never gets impounded into firm value: the shareholders are better off not hiring the manager at all. For the manager to create any value therefore, it has to be the case that the right-hand side of (27) is strictly greater than zero; this happens when the first inequality in (21) is satisfied. Thus we know that $\Delta^*_{\text{H}} \in \left[\frac{1-\pi_1(A,B)}{\pi_1(A,B)} r, \frac{(1-B)}{B} r\right]$.

In this range for Δ_{H}, the firm's profits are given by $\bar{F}(0, \bar{Q}_1, 0)$, as derived in Lemma 3.2. Similarly, we can calculate the expected compensation that the firm will pay the manager. Since $\bar{Q}_0 = \bar{P} = 0$, and since the project has a probability of $\text{E}[\tilde{p}] = \frac{1}{2}$ of surviving the first stage, the compensation that the firm expects to pay the manager is equal to

$$
\begin{aligned}
\text{E}\big[\tilde{w}(0, \Delta_{\text{H}})\big] &= \frac{1}{2}\left[\Pr\{\tilde{s}=0\}\int_0^1 q\pi_0\, dq + \Pr\{\tilde{s}=1\}\left(\int_0^{\bar{Q}_1} \pi_1 dq + \int_{\bar{Q}_1}^1 q\pi_1 dq\right)\right]\Delta_{\text{H}} \\
&= \frac{1}{2}\left[\frac{1}{2}\left(\frac{1}{2}\pi_0\right) + \frac{1}{2}\left(\pi_1\bar{Q}_1 + \pi_1\frac{1-\bar{Q}_1^2}{2}\right)\right]\Delta_{\text{H}} \\
&= \frac{1}{8}\left[\pi_0 + \pi_1 + 2\pi_1\bar{Q}_1 - \pi_1\bar{Q}_1^2\right]\Delta_{\text{H}}.
\end{aligned}
$$

Thus the firm's value when the manager's compensation is Δ_{H} in the high state and zero otherwise is given by

$$
\begin{aligned}
\bar{V}(\Delta_{\text{H}}) &\equiv \bar{F}(0, \bar{Q}_1, 0) - \frac{1}{8}\left[\pi_0 + \pi_1 + 2\pi_1\bar{Q}_1 - \pi_1\bar{Q}_1^2\right]\Delta_{\text{H}} \\
&= \frac{1}{16}\Bigg\{6 + 2\pi_1 + (\pi_0 + 2\pi_1)(1 - 2\Delta_{\text{H}}) \\
&\qquad + \frac{2(1-\pi_1)\big[1-\pi_1(A,B)\big]r}{\Delta_{\text{H}}\,\pi_1(A,B)} - \frac{\pi_1\big[1-\pi_1(A,B)\big]^2 r^2(1-2\Delta_{\text{H}})}{\Delta_{\text{H}}^2\big[\pi_1(A,B)\big]^2}\Bigg\},
\end{aligned} \tag{34}
$$

where we have used $\bar{Q}_1 = 1 - \frac{1-\pi_1(A,B)}{\pi_1(A,B)} \cdot \frac{r}{\Delta_{\text{H}}}$ from (27) to write the last line (some tedious manipulations were skipped as well).

The first-order condition for firm value maximization is obtained by differentiating (34) with respect to $\Delta_{\rm H}$ and setting the resulting expression equal to zero:[12]

$$\begin{aligned} 0 &= r^2\pi_1\big[1-\pi_1(A,B)\big](1-\Delta_{\rm H}) \\ &\qquad -r(1-\pi_1)\pi_1(A,B)\big[1-\pi_1(A,B)\big]\Delta_{\rm H} - (\pi_0+2\pi_1)\big[\pi_1(A,B)\big]^2\Delta_{\rm H}^3. \end{aligned} \tag{35}$$

It is easy to see that the right-hand side of (35) is positive at $\Delta_{\rm H} = 0$, negative at $\Delta_{\rm H} = 1$, and decreases monotonically from one to the other. This means that there is a unique solution for $\Delta_{\rm H}$ in $(0,1)$. To verify that this solution is smaller than $\frac{1-\pi_1(A,B)}{\pi_1(A,B)}\,\frac{\pi_1}{1-\pi_1}\,r$, let us use this value for $\Delta_{\rm H}$ in the right-hand side of (35). After some manipulations, this yields

$$\frac{-r^3\big[1-\pi_1(A,B)\big]^3\pi_1^2\big[1-(2-\pi_0)\pi_1+3\pi_1^2\big]}{\pi_1(A,B)(1-\pi_1)^3}. \tag{36}$$

Clearly, this quantity has the same sign as its numerator which, using $\pi_0 = \frac{1-a}{2}$, $\pi_1 = \frac{1+a}{2}$ and $\pi_1(A,B) = A + (1-A)B$, is equal to

$$-\frac{1}{8}r^3(1-A)^3(1-B)^3(1+a)^2(2+a+a^2) < 0.$$

Thus (36) is negative which, from the discussion following (35), implies that the compensation $\Delta_{\rm H}^*$ that maximizes firm value is smaller than $\frac{1-\pi_1(A,B)}{\pi_1(A,B)}\,\frac{\pi_1}{1-\pi_1}\,r$. ∎

Proof of Proposition 5.4

The firm value derived in (34) can be differentiated with respect to A and B:

$$\frac{\partial\bar{V}(\Delta_{\rm H})}{\partial A} = \frac{(1-B)r\Big\{r\pi_1\big[1-\pi_1(A,B)\big](1-2\Delta_{\rm H}) - \pi_0\pi_1(A,B)\Delta_{\rm H}\Big\}}{8\big[\pi_1(A,B)\big]^3\Delta_{\rm H}^2}, \tag{37}$$

$$\frac{\partial\bar{V}(\Delta_{\rm H})}{\partial B} = \frac{(1-A)r\Big\{r\pi_1\big[1-\pi_1(A,B)\big](1-2\Delta_{\rm H}) - \pi_0\pi_1(A,B)\Delta_{\rm H}\Big\}}{8\big[\pi_1(A,B)\big]^3\Delta_{\rm H}^2}. \tag{38}$$

Both derivatives are positive for

$$\Delta_{\rm H} < \frac{r\pi_1\big[1-\pi_1(A,B)\big]}{\pi_0\pi_1(A,B)+2r\pi_1\big[1-\pi_1(A,B)\big]}.$$

For a rational manager ($A = a$ and $B = \frac{1}{2}$), this condition simplifies to $\Delta_{\rm H} < \frac{r}{1+2r}$. So the result will be proved if we can establish that the value-maximizing compensation of a rational manager

[12] The second-order condition can be easily verified.

is smaller than $\frac{r}{1+2r}$. With $A = a$ and $B = \frac{1}{2}$, the first-order condition (35) derived in the proof of Proposition 5.3 reduces to

$$0 = r^2(1-a)^2(1-\Delta_{\text{H}}) - r(1-a)^2\Delta_{\text{H}} - (3+4a+a^2)\Delta_{\text{H}}^3.$$

As before, the right-hand side of this condition is greater than zero for $\Delta_{\text{H}} = 0$, smaller than zero for $\Delta_{\text{H}} = 1$, and monotonically decreasing in between. Using $\Delta_{\text{H}} = \frac{r}{1+2r}$ in the right-hand side yields, after some manipulations,

$$\frac{2r^3\left[-1-3a+2r(1+r)(1-a)^2\right]}{(1+2r)^3} < -1-3a+2a = -(1+a) < 0,$$

where $r < \frac{a}{1-a}$ was used to get the first inequality. Thus the value-maximizing compensation for the rational manager is smaller than $\frac{r}{1+2r}$. This implies that the value of the firm is increasing in A and B. ∎

Proof of Proposition 6.1

Suppose that the manager observes $\tilde{s} = 1$ and $\tilde{q} = q$ in the first stage. The payoff he expects from undertaking the project in the second stage is

$$\Pr_{\text{b}}\big\{\tilde{v} = 1 \mid \tilde{s} = 1\big\}\delta - \Pr_{\text{b}}\big\{\tilde{v} = 0 \mid \tilde{s} = 1\big\}r = \pi_1(A,B)\delta - \big[1 - \pi_1(A,B)\big]r. \tag{39}$$

If instead, the manager chooses to gather more information, he will undertake the project if $\tilde{v} = 1$ (for a payoff of δ), and drop it otherwise (for a payoff of $\frac{1}{2}$); his expected payoff is then given by

$$\begin{aligned}
&\Pr\{\text{project exists}\}\left[\Pr_{\text{b}}\{\tilde{v}=1 \mid \tilde{s}=1\}\,\delta + \Pr_{\text{b}}\{\tilde{v}=0 \mid \tilde{s}=1\}\,\frac{1}{2}\right] + \Pr\{\text{project disappears}\}\,\frac{1}{2} \\
&\quad = q\left\{\pi_1(A,B)\delta + \big[1-\pi_1(A,B)\big]\frac{1}{2}\right\} + (1-q)\frac{1}{2} \\
&\quad = \frac{1}{2} + \left(\delta - \frac{1}{2}\right) q\,\pi_1(A,B).
\end{aligned}$$

The manager gathers more information if this last expression exceeds (39), that is if

$$q \geq \frac{(\delta - \frac{1}{2}) - \big[1-\pi_1(A,B)\big](\delta + r)}{(\delta - \frac{1}{2})\pi_1(A,B)}. \tag{40}$$

For first-best to obtain, this last quantity has to be equal to $\bar{Q}_1^{\text{FB}}$, as calculated in Proposition 3.1. This will be achieved by setting the discount factor equal to (22). Using techniques similar to the ones used in previous proofs, it is straightforward to verify that the manager will choose $\bar{P} = \bar{Q}_0 = 0$ as long as (18) or (19) holds. ∎

Proof of Proposition 6.2

The $\tilde{q}$-thresholds used by the manager in the second stage (in which no effort cost is incurred) can be derived the same way they were derived in section 4. In fact, using the same arguments as in the proof of Lemma 4.3, it can be shown that the manager chooses

$$\bar{Q}_1 = \frac{2\left[\pi_1(A,B) - \frac{1}{2}\right]}{\pi_1(A,B)} \tag{41}$$

and, as long as $B \le \frac{1}{2(1-a)}$, he chooses $\bar{Q}_0 = 0$. These values can be used to calculate the threshold used by the manager in the first stage.[13]

Suppose that $\tilde{p} = p$ in the first stage. Conditional on getting a positive signal ($\tilde{s} = 1$), the manager's expected utility is equal to[14]

$$\bar{U}_1(\bar{Q}_1) \equiv \pi_1(A,B)\bar{Q}_1 + \frac{1}{2}(1-\bar{Q}_1) + \frac{1}{2}\pi_1(A,B)\frac{1-\bar{Q}_1^2}{2}.$$

Conditional on getting a negative signal ($\tilde{s} = 0$), the manager's expected utility is equal to

$$\bar{U}_0 \equiv \frac{1}{2} + \frac{1}{4}\pi_0(A,B).$$

Thus, if the manager chooses to gather more information in the first stage, his expected utility is given by

$$\begin{aligned} &\Pr\{\text{project exists}\}\Big[\Pr\{\tilde{s}=1\}\,\bar{U}_1(\bar{Q}_1) + \Pr\{\tilde{s}=0\}\,\bar{U}_0\Big] + \Pr\{\text{project disappears}\}\,\frac{1}{2} - e \\ &= p\Big[B\bar{U}_1(\bar{Q}_1) + (1-B)\bar{U}_0\Big] + (1-p)\frac{1}{2} - e. \end{aligned} \tag{42}$$

If instead the manager undertakes the project right away, his expected utility is equal to

$$\Pr_{\mathrm{b}}\{\tilde{v}=1\}(1) = B.$$

The manager will therefore choose to gather more information if (42) is greater than B, that is if

$$\begin{aligned} p &\ge \frac{B - \frac{1}{2} + e}{B\bar{U}_1(\bar{Q}_1) + (1-B)\bar{U}_0 - \frac{1}{2}} \\ &= \frac{4\left(B - \frac{1}{2} + e\right)}{B\pi_1(A,B) + (1-B)\pi_0(A,B) + 4B\left[\pi_1(A,B) - \frac{1}{2}\right]\bar{Q}_1 - B\pi_1(A,B)\bar{Q}_1^2} \\ &= \frac{4\left(B - \frac{1}{2} + e\right)\pi_1(A,B)}{(1-B)\pi_0(A,B)\pi_1(A,B) + B\Big\{4\left[\pi_1(A,B) - \frac{1}{2}\right]^2 + \left[\pi_1(A,B)\right]^2\Big\}}, \end{aligned} \tag{43}$$

[13] The fact that $\bar{Q}_0$ is positive for B larger than $\frac{1}{2(1-a)}$ is unimportant, since we are only interested in departures from rationality (i.e., $B = \frac{1}{2}$) in this result.

[14] This is the same as $\bar{U}_1(\bar{Q}_1)$ in the proof of Proposition 5.2, except that Δ_{M} and Δ_{H} have both been replaced by $\frac{1}{2}$, and $r = 0$.

where we used (41) to obtain the last equality. To establish the result, we need to differentiate (43) first with respect to A, and then with respect to B. After setting $A = a$ and $B = \frac{1}{2}$ in the resulting expressions, we find

$$-\frac{16a(2+a)e}{(1+a+2a^2)^2} < 0,$$

and

$$\frac{1+2a+3a^2+2a^3-2e(1+6a+a^2)}{(1+a+2a^2)^2},$$

which is greater than zero for $e < \frac{1}{4}$. ∎

References

Alpert, M., and H. Raiffa, 1982, "A Progress Report on the Training of Probability Assessors," in *Judgment Under Uncertainty: Heuristics and Biases*, eds. D. Kahneman, P. Slovic, and A. Tversky, Cambridge and New York: Cambridge University Press, 294-305.

Audia, P. G., E. A. Locke, and K. G. Smith, 2000, "The Paradox of Success: An Archival and a Laboratory Study of Strategic Persistence Following Radical Environmental Change," *Academy of Management Journal*, 43, 837-853.

Baumann, A. O., R. B. Deber, and G. G. Thompson, 1991, "Overconfidence Among Physicians and Nurses: The 'Micro-Certainty, Macro-Uncertainty' Phenomenon," *Social Science and Medicine*, 32, 167-174.

Becker, G. S., 1962, "Investment in Human Capital: A Theoretical Analysis," *Journal of Political Economy*, 70, 9-49.

Bénabou, R., and J. Tirole, 2002, "Self-Confidence and Personal Motivation," *Quarterly Journal of Economics*, 117, 871-915.

Benos, A., 1998, "Aggressiveness and Survival of Overconfident Traders," *Journal of Financial Markets*, 1, 353-383.

Brehmer, B., 1980, "In One Word: Not from Experience," *Acta Psychologica*, 45, 223-241.

Christensen-Szalanski, J. J., and J. B. Bushyhead, 1981, "Physicians' Use of Probabilistic Information in a Real Clinical Setting," *Journal of Experimental Psychology: Human Perception and Performance*, 7, 928-935.

Cooper, A. C., C. Y. Woo, and W. C. Dunkelberg, 1988, "Entrepreneurs' Perceived Chances for Success," *Journal of Business Venturing*, 3, 97-108.

Daniel, K., D. Hirshleifer, and A. Subrahmanyam, 1998, "A Theory of Overconfidence, Self-Attribution, and Security Market Under- and Over-reactions," *Journal of Finance*, 53, 1839-1885.

De Long, J., A. Shleifer, L. Summers, and R. Waldmann, 1991, "The Survival of Noise Traders in Financial Markets," *Journal of Business*, 64, 1-19.

Dixit, A. K., and R. S. Pindyck, 1994, *Investment under Uncertainty*, Princeton, New Jersey: Princeton University Press.

Dowie, J., 1976, "On the Efficiency and Equity of Betting Markets," *Economica*, 43, 139-150.

Einhorn, H. J., and R. M. Hogarth, 1978, "Confidence in Judgment: Persistence of the Illusion of Validity," *Psychological Review*, 85, 395-416.

Fischhoff, B., P. Slovic, and S. Lichtenstein, 1977, "Knowing with Certainty: The Appropriateness of Extreme Confidence," *Journal of Experimental Psychology*, 3, 552-564.

Frank, J. D., 1935, "Some Psychological Determinants of the Level of Aspiration," *American Journal of Psychology*, 47, 285-293.

Gervais, S., and T. Odean, 2001, "Learning To Be Overconfident," *Review of Financial Studies*, 14, 1-27.

Grenadier, S., 2002, "Option Exercise Games: An Application to the Equilibrium Investment Strategies of Firms," *Review of Financial Studies*, 15, 691-721.

Griffin, D., and A. Tversky, 1992, "The Weighing of Evidence and the Determinants of Confidence," *Cognitive Psychology*, 24, 411-435.

Hausch, D. B., W. T. Ziemba, and M. Rubinstein, 1981, "Efficiency of the Market for Racetrack Betting," *Management Science*, 27, 1435-1452.

Heaton, J. B., 2002, "Managerial Optimism and Corporate Finance," *Financial Management*, 31, 33-45.

Hirshleifer, D., A. Subrahmanyam, and S. Titman, 1994, "Security Analysis and Trading Patterns When Some Investors Receive Information Before Others," *Journal of Finance*, 49, 1665-1698.

Irwin, F. W., 1953, "Stated Expectations as Functions of Probability and Desirability of Outcomes," *Journal of Personality*, 21, 329-335.

Ito, T., 1990, "Foreign Exchange Rate Expectations: Micro Survey Data," *American Economic Review*, 80, 434-449.

Jensen, M. C., and W. H. Meckling, 1976, "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure," *Journal of Financial Economics*, 3, 305-360.

Kahneman, D. and D. Lovallo, 1993, "Timid Choices and Bold Forecasts: A Cognitive Perspective on Risk Taking," *Management Science*, 39, 17-31.

Keren, G. B., 1987, "Facing Uncertainty in the Game of Bridge: A Calibration Study," *Organizational Behavior and Human Decision Processes*, 39, 98-114.

Kidd, J. B., 1970, "The Utilization of Subjective Probabilities in Production Planning," *Acta Psychologica*, 34, 338-347.

Klaczynski, P. A., and J. M. Fauth, 1996, "Intellectual Ability, Rationality, and Intuitiveness as Predictors of Warranted and Unwarranted Optimism for Future Life Events," *Journal of Youth and Adolescence*, 25, 755-773.

Kunda, Z., 1987, "Motivated Inference: Self-Serving Generation and Evaluation of Causal Theories," *Journal of Personality and Social Psychology*, 53, 636-647.

Kyle, A., and F. A. Wang, 1997, "Speculation Duopoly With Agreement to Disagree: Can Overconfidence Survive the Market Test?," *Journal of Finance*, 52, 2073-2090.

Langer, E., and J. Roth, 1975, "Heads I Win, Tails It's Chance: The Illusion of Control as a Function of the Sequence of Outcomes in a Purely Chance Task," *Journal of Personality and Social Psychology*, 32, 951-955.

Lichtenstein, S., B. Fischhoff, and L. Phillips, 1982, "Calibration of Probabilities: The State of the Art to 1980," in *Judgment Under Uncertainty: Heuristics and Biases*, eds. D. Kahneman, P. Slovic, and A. Tversky, Cambridge and New York: Cambridge University Press, 306-334.

Malmendier, U., and G. Tate, 2001, "CEO Overconfidence and Corporate Investment," Working Paper, Harvard University.

March, J. G., and Z. Shapira, 1987, "Managerial Perspectives on Risk and Risk Taking," *Management Science*, 33, 1404-1418.

Marks, R., 1951, "The Effect of Probability, Desirability, and 'Privilege' on the Stated Expectations of Children," *Journal of Personality*, 19, 332-351.

McDonald, R., and D. Siegel, 1986, "The Value of Waiting to Invest," *Quarterly Journal of Economics*, 101, 707-727.

Miller, D., and M. Ross, 1975, "Self-serving Biases in Attribution of Causality: Fact or Fiction?," *Psychological Bulletin*, 82, 213-225.

Murphy, A. H., and R. L. Winkler, 1984, "Probability Forecasting in Meteorology," *Journal of the American Statistical Association*, 79, 489-500.

Neale, M. A., and M. H. Bazerman, 1990, *Cognition and Rationality in Negotiation*, New York: The Free Press.

Nisbett, R. E., and L. Ross, 1980, *Human Inference: Strategies and Shortcomings of Social Judgment*, Englewood Cliffs, N.J.: Prentice-Hall.

Odean, T., 1998, "Volume, Volatility, Price, and Profit When All Traders Are Above Average," *Journal of Finance*, 53, 1887-1934.

Oskamp, S., 1965, "Overconfidence in Case-Study Judgments," *Journal of Consulting Psychology*, 29, 261-265.

Pontiff, J., 1996, "Costly Arbitrage: Evidence from Closed-End Funds," *Quarterly Journal of Economics*, 111, 1135-1151.

Roll, R., 1986, "The Hubris Hypothesis of Corporate Takeovers," *Journal of Business*, 59, 197-216.

Russo, J. E., and P. J. H. Schoemaker, 1992, "Managing Overconfidence," *Sloan Management Review*, 33, 7-17.

Shleifer, A., and R. W. Vishny, 1997, "The Limits of Arbitrage," *Journal of Finance*, 52, 35-55.

Staël von Holstein, C.-A. S., 1972, "Probabilistic Forecasting: An Experiment Related to the Stock Market," *Organizational Behavior and Human Performance*, 8, 139-158.

Treynor, J. L., and F. Black, 1976, "Corporate Investment Decisions," In *Modern Developments in Financial Management*, ed. Stewart C. Myers, New York: Praeger, 310-327.

Wagenaar, W., and G. B. Keren, 1986, "Does the Expert Know? The Reliability of Predictions and Confidence Ratings of Experts," eds. E. Hollnagel, G. Maneini, and D. Woods, *Intelligent Decision Support in Process Environments*, Berlin: Springer, 87-107.

Wang, A., 1997, "Overconfidence, Delegated Fund Management, and Survival," Working Paper, Columbia University.

Weinstein, N. D., 1980, "Unrealistic Optimism About Future Life Events," *Journal of Personality and Social Psychology*, 39, 806-820.

Yates, J. F., 1990, *Judgment and Decision Making*, Englewood Cliffs, New Jersey: Prentice Hall.

The Rodney L. White Center for Financial Research

The Wharton School
University of Pennsylvania
3254 Steinberg Hall-Dietrich Hall
3620 Locust Walk
Philadelphia, PA 19104-6367

(215) 898-7616
(215) 573-8084 Fax
http://finance.wharton.upenn.edu/~rlwctr

The Rodney L. White Center for Financial Research is one of the oldest financial research centers in the country. It was founded in 1969 through a grant from Oppenheimer & Company in honor of its late partner, Rodney L. White. The Center receives support from its endowment and from annual contributions from its Members.

The Center sponsors a wide range of financial research. It publishes a working paper series and a reprint series. It holds an annual seminar, which for the last several years has focused on household financial decision making.

The Members of the Center gain the opportunity to participate in innovative research to break new ground in the field of finance. Through their membership, they also gain access to the Wharton School's faculty and enjoy other special benefits.