Noisy Active Management

by

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Abstract

Lower skill of the active management industry can imply greater fee revenue, value added, and investor performance. Such outcomes arise in a competitive equilibrium in which portfolio choices of active managers partially echo those of noise traders and also contain manager-specific noise. Both sources of noise reduce managers' skill to identify mispriced securities and thereby produce alpha. However, lower skill also means a given amount of active management corrects prices less and thus competes away less alpha. The latter effect can outweigh managers' poorer portfolio choices, so that investors rationally allocate more to active management when its skill is lower.

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1. Introduction

Active investment managers seek positive alpha. That is, they seek to outperform a passive index by identifying and trading securities that are mispriced relative to correct fundamental values. For active management in aggregate to outperform, other market participants must underperform (e.g., Sharpe (1991)). The potentially underperforming segment is typically assumed to comprise "noise" traders whose asset demands deviate exogenously from those consistent with rational assessments of fundamental values.

What if noise also afflicts active managers' assessments of fundamental value? The noise might echo what is present in the demands of the noise traders, if that noise reflects pervasive sentiment to which professional managers might be susceptible. Such a scenario seems motivated, for example, by the empirical evidence of Dasgupta, Prat, and Verardo, (2011), Akbas, Armstrong, Sorescu, and Subrahmanyam (2015), Edelen, Ince, and Kadlec (2016), and DeVault, Sias, and Starks (2016). Or the noise might be idiosyncratic across managers and simply reflect manager-specific limitations and mistakes. Less noise in managers' beliefs means greater skill in correctly identifying mispriced assets. Pástor, Stambaugh, and Taylor (2015) estimate a proxy for such skill and find that its distribution across managers trends upward over the past three decades. In other words, their evidence indicates the active management industry has become more skilled.

Should the industry earn higher fee revenue if it is more skilled? Does it have higher value added if it is more skilled? (Value added, as defined by Berk and van Binsbergen (2015), equals alpha before fees, multiplied by assets under management.) If the industry is more skilled, are investors better or worse off in terms of the maximum Sharpe ratio they can achieve by combining active and passive management? As with many questions, the answer to these is "it depends." In this case it depends on factors including the magnitudes of noise trader demands and the noisiness of managers' beliefs. In the equilibrium developed and analyzed here, however, plausible scenarios have more skill accompanied by lower fee revenue, lower value added, and a lower maximum Sharpe ratio for investors. The explanation sketched below has essentially two ingredients: (i) skill has opposing effects on alpha and (ii) active management faces decreasing returns to scale.

First observe that skill exerts two opposing effects on alpha. On one hand, when active management has greater skill, it can better identify mispriced assets and thus better construct a portfolio yielding alpha. On the other hand, greater skill in identifying mispriced assets means that when active management in aggregate over-weights or under-weights an asset, the resulting price impact is more likely to be in the correct direction, toward the asset's fundamental value. Although price correction is presumably desirable from a societal perspective, enabling more efficient capital allocation, price correction has a downside for active management. Managers produce greater alpha if they can establish their active positions at prices that get corrected only later, after the positions are established. The more accurately that fundamentals are reflected in the equilibrium prices at which active managers establish positions, the lower is the resulting alpha.

The second ingredient of the explanation is decreasing returns to scale in active management. Investors who allocate their stock-market investment between a passive index fund and actively managed funds are attracted to the latter by positive alpha. Without positive alpha, investors receive no compensation for the non-benchmark volatility (tracking error) inherent to active management, so active management would receive no money from investors wanting to maximize their overall Sharpe ratio. As more money is allocated to active management, however, alpha declines, due to the increasingly greater trading costs funds incur with intermediaries when establishing larger active positions. Investors allocate enough to active management such that alpha declines to the level at which each investor's overall Sharpe ratio is maximized at that allocation. Alpha at that equilibrium level, although positive, is likely to be small, making the Berk and Green (2004) argument that alpha is driven to zero a reasonable approximation here in delivering the intuition for the role of decreasing returns to scale.[1]

Now consider how skill's two opposing effects on alpha combine with decreasing returns to scale. Suppose that there exists an allocation to active management at which lower skill of the industry helps alpha more, through less price correction, than alpha is hurt by managers' weaker ability to identify mispriced assets. That is, at that allocation, lower skill yields a higher alpha than does greater skill. Also suppose, at that allocation, the alphas associated with both skill levels are greater than zero (or, more precisely, greater than what investors require as compensation for tracking error). Then consider what happens to those alphas as the allocation to active management increases. With decreasing returns to scale, both alphas decline. The low-skill alpha starts its decline from a higher value, so by the time it reaches zero, the allocation to active management increases to a higher level than where the high-skill alpha reaches zero. In such a setting, which occurs under seemingly reasonable specifications of the model presented here, active management receives more money to manage if its skill is low than if it is high. For similar reasons involving the opposing effects of skill, lower skill

[1] See Pástor and Stambaugh (2012) and Stambaugh (2014) for models in which the equilibrium alpha with decreasing retuns to scale is positive but modest.

can imply higher value added and a greater maximum Sharpe ratio achievable by investors.

Active management corrects prices less when subject to either type of noise described earlier: a partial echoing of noise-trader demands as well as manager-specific noise. Not surprisingly, the more that managers' active positions echo noise-trader demands, the less that establishing those positions corrects the mispricing that noise traders originate. Manager-specific noise washes out across managers in that it does not affect the mispricing of one asset relative to another. At the same time, however, the overall aggressiveness of an individual manager's positions is limited by trading costs. The more such aggressiveness reflects manager-specific noise, which washes out of aggregate active positions, the less aggressive the aggregate positions become, and the less prices get corrected.

Including trading costs and the resulting decreasing returns to scale in a model of the active management industry seems desirable given empirical evidence of such costs' economic importance. For example, Edelen, Evans, and Kadlec (2007) conclude that trading costs present an important source of scale diseconomies for mutual funds, and Edelen, Evans, and Kadlec (2013) find that mutual funds' trading costs as a fraction of net asset value are comparable in magnitude to the funds' expense ratios. The latter result is consistent with the model presented here, which implies that funds' trading costs equal their fee revenues.

This study's interest centers on what happens in equilibrium when the active management industry as a whole possesses more skill versus less, so the model presented abstracts from differences in skill across managers.[2] The model essentially extends the setting in Stambaugh (2014) to one in which the beliefs of active managers are noisy in the ways described above. After presenting the model and its equilibrium implications in Section 2, the paper conducts a quantitative analysis of the model in Section 3, and then Section 4 concludes with final remarks.

2. Model

The model has four types of agents: active fund managers, investors, noise traders, and intermediaries. Active managers identify and exploit opportunities to trade many assets in order to outperform the market benchmark and, in particular, maximize their information ratios. Managers act competitively, conditioning on prices as well as their individual fund

[2]Equilibrium models with heterogeneously informed managers include Garcia and Vanden (2009) and Gârleanu and Pedersen (2017). Unlike here, managers in those models do not face liquidity costs and the resulting decreasing returns to scale.

sizes, and they face convex trading costs of deviating from benchmark asset weightings. There are many investors, all of whom allocate their stock-market wealth across the active funds and a market index fund so as to maximize the Sharpe ratio of the combination. Noise traders buy individual stocks directly and do not invest in either active funds or index funds. Intermediaries receive the trading costs incurred by active managers but otherwise play no role.

2.1. Stocks

The market contains N stocks, and the total supply of each stock equals one share. The model considers a single investment period. Stock i has share price p_i at the beginning of the investment period and value x_i at the end of the period (including dividends). A share in the market portfolio has end-of-period value $x_m = (1/N)\sum_{i=1}^N x_i$ and price $p_m = (1/N)\sum_{i=1}^N p_i$. For $i = 1, \ldots, N$,

$$x_i = \bar{x}_i + \eta_i, \tag{1}$$

where the η_i's have zero means and a risk structure given by

$$\eta_i = p_i z + p_m \epsilon_i, \quad i = 1, \ldots, N, \tag{2}$$

where $\mathrm{E}(z) = \mathrm{E}(\epsilon_i) = 0$, $\mathrm{Cov}(\epsilon_i, \epsilon_j) = 0$ for all $i \neq j$, and

$$\mathrm{Var}(\epsilon_i) = \left(\frac{p_i}{p_m}\right)\sigma^2, \quad i = 1, \ldots, N. \tag{3}$$

Also, $(1/N)\sum_{i=1}^N \epsilon_i \approx 0$, so that the rate of return on the market portfolio is well approximated as

$$r_m = \mu_m + z - 1, \tag{4}$$

where $\mu_m = \bar{x}_m / p_m$, with $\bar{x}_m = (1/N)\sum_{i=1}^N \bar{x}_i$. The rate of return on stock i is given by

$$r_i = \frac{x_i}{p_i} - 1 = \left(\frac{\bar{x}_i}{p_i}\right) + z + \left(\frac{p_m}{p_i}\right)\epsilon_i - 1, \tag{5}$$

from which we see that $\beta_i = \mathrm{Cov}(r_i, r_m)/\mathrm{Var}(r_m) = 1$. The market-adjusted return on stock i is

$$R_i = r_i - \beta_i r_m = \left(\frac{\bar{x}_i}{p_i}\right) - \mu_m + \left(\frac{p_m}{p_i}\right)\epsilon_i. \tag{6}$$

Conditional on its price p_i, stock i's expected market-adjusted return is equal to

$$\begin{aligned} \alpha_i &= \left(\frac{\bar{x}_i}{p_i}\right) - \mu_m \\ &= \mu_m\left(\frac{\bar{p}_i - p_i}{p_i}\right), \end{aligned} \tag{7}$$

where

$$\bar{p}_i = \bar{x}_i / \mu_m \tag{8}$$

is the expected end-of-period value discounted by the expected market return—the stock's CAPM fair value, given that $\beta_i = 1$. The conditional variance of stock i's market-adjusted return is

$$\sigma_i^2 = \left(\frac{p_m}{p_i} \right) \sigma^2. \tag{9}$$

2.2. Fund managers

There are M active fund managers. Each manager j expects end-of-period value for stock i to be $\tilde{x}_i^{(j)}$ instead of $\bar{x}_i$, $i = 1, \ldots, N$. Define

$$\tilde{p}_i^{(j)} = \frac{\tilde{x}_i^{(j)}}{\mu_m}. \tag{10}$$

Managers' expectations are of the form

$$\tilde{p}_i^{(j)} = (1 - \nu_1)\bar{p}_i + \nu_1 \hat{p}_i + \nu_2 \zeta_i^{(j)} \bar{p}_i, \tag{11}$$

with $0 \leq \nu_1 < 1$ and $\nu_2 > 0$, where $\hat{p}_i$ is the price that would arise in an equilibrium with no active management, so that prices would be determined solely by the asset demands of noise traders. The $\zeta_i^{(j)}$'s are purely idiosyncratic across both stocks and managers and cross-sectionally independent of both the $\bar{p}_i$'s and $\hat{p}_i$'s. For a given manager j, the $\zeta_i^{(j)}$'s have zero mean and unit variance across stocks. Noise is assumed not to affect expected return on the market portfolio, so

$$N p_m = \sum_{i=1}^{N} p_i = \sum_{i=1}^{N} \hat{p}_i = \sum_{i=1}^{N} \tilde{p}_i. \tag{12}$$

Although managers beliefs differ from each other to at least an infinitesimal degree (because $\nu_2 > 0$), managers are equally skilled, in that ν_1 and ν_2 are the same across managers. Given these noisy beliefs, manager j's assessed alpha for stock i is therefore

$$\tilde{\alpha}_i^{(j)} = \mu_m \left(\frac{\tilde{p}_i^{(j)} - p_i}{p_i} \right), \tag{13}$$

instead of the true alpha in equation (7).

Each manager can replicate the market index at zero cost but pays trading costs in order to deviate from those benchmark allocations. These costs represent compensation to liquidity-providing intermediaries for taking short-lived positions to facilitate the ultimate market clearing in a stock between managers and noise traders.[3] Specifically, define the active weight

$$\phi_i^{(j)} = \phi_{A,i}^{(j)} - \phi_{m,i}, \tag{14}$$

where $\phi_{A,i}^{(j)}$ is manager j's weight in stock i, and $\phi_{m,i}$ is stock i's weight in the market portfolio. (Note $\sum_{i=1}^N \phi_{A,i}^{(j)} = \sum_{i=1}^N \phi_{m,i} = 1$.) The value of stock i represented by the manager's benchmark deviation is $D_i^{(j)} = |\phi_i^{(j)}|W^{(j)}$, and the associated trading cost is denoted as $C_i^{(j)}$. The proportional trading cost is given by

$$\frac{C_i^{(j)}}{D_i^{(j)}} = c\delta_i^{(j)}, \tag{15}$$

where $\delta_i^{(j)}$ is the fraction of stock i's total market value represented by $D_i^{(j)}$, and c is a constant. In other words, the proportional trading cost is linear in the amount traded.

At the beginning of the period, manager j sets a proportional fee rate equal to $f^{(j)}$. Investors then decide to invest in aggregate the dollar amount $W^{(j)}$ with that manager, whose fee revenue is thus $f^{(j)}W^{(j)}$. Given $W^{(j)}$ and market prices, manager j chooses allocations across individual stocks to maximize his own perceived information ratio, defined as net alpha divided by the standard deviation of market-adjusted return.

2.3. Noise traders and investors

Aggregate stock-market wealth is owned by noise traders and investors. (The fund managers do not have any stock-market wealth.) The noise traders own fraction h of the total value of the stock market; the investors have aggregate stock-market wealth equal to the fraction $1 - h$ of total stock market value. The aggregate portfolio of the noise-traders has weight $\phi_{H,i}$ in stock i, with $\phi_{H,i}$ assumed to be exogenous and non-negative (no short selling by the noise traders).

Investors do not invest directly in individual stocks. Instead they allocate their stock-market investments across the active managers and a market index fund so as to maximize the Sharpe ratio of the resulting combination. Maximizing that Sharpe ratio implies that each investor chooses an allocation across active managers that produces the maximum information ratio of the resulting active portfolio. Each investor then allocates the fraction y

[3] In this interpretation, the trading cost is not viewed as a manager-specific price impact, such that if many other managers independently produce similar price impacts, the sum of such impacts aggregates to an implausibly large total price effect. Instead, one might imagine many intermediaries accessing different sources of liquidity or acting at slightly different times.

to that portfolio and the fraction $(1-y)$ to the index fund. Investors are assumed to correctly assess active managers' overall portfolio alphas and volatilities as well as the correlations among the M active managers' portfolios, and they take these parameters as given when making their individual allocation decisions. That is, the number of investors is assumed to be large relative to the number of funds (i.e., thousands of funds, millions of investors), and wealth is assumed to be sufficiently disperse across investors such that they treat their own individual decisions as having no effect on any fund's size ($W^{(j)}$ in equations (21) through (23)).

2.4. Equilibrium

An equilibrium occurs when, simultaneously,

1. managers' choices of active weights, $\left\{\phi_1^{(j)}, \phi_2^{(j)}, \ldots, \phi_N^{(j)}\right\}$, $j = 1, \ldots, M$, maximize each of their individually perceived information ratios,

2. investors' allocations across the M active funds maximize the correctly perceived information ratio of the resulting aggregate active portfolio,

3. investors' overall allocation, y, to that aggregate active portfolio maximizes their (correctly perceived) overall Sharpe ratio, and

4. stock prices, $\{p_1, p_2, \ldots, p_N\}$ satisfy the market-clearing condition

$$h\phi_{H,i} + (1-h)\phi_{S,i} = \phi_{m,i}, \quad i = 1, \ldots, N, \tag{16}$$

   where $\phi_{m,i} = p_i / \sum_{j=1}^N p_j$ is stock i's market weight, $\phi_{H,i}$ is the stock's weight in the aggregate stock portfolio of the noise traders, and $\phi_{S,i}$ is the stock's weight in investors' aggregate stock portfolio (which combines the index fund with the aggregate active portfolio).

Solving for an equilibrium first requires the solution to each manager's portfolio problem. The manager's portfolio choice, as well as the true properties of the resulting portfolio, are given by the following proposition.

Proposition 1. *Manager j's active weight in stock i is*

$$\phi_i^{(j)} = \tilde{a}^{(j)} \left(\frac{\tilde{p}_i^{(j)} - p_i}{p_m} \right), \tag{17}$$

where

$$\tilde{a}^{(j)} = \mu_m \left(\frac{f^{(j)} p_m}{cW^{(j)} \tilde{\psi}^{(j)}} \right)^{1/2}, \tag{18}$$

and

$$\tilde{\psi}^{(j)} = (1/p_m) \sum_{i=1}^{N} \left(\tilde{\alpha}_i^{(j)} \right)^2 p_i. \tag{19}$$

Trading costs incurred equal fee revenue:

$$\sum_{i=1}^{N} C_i^{(j)} = f^{(j)} W^{(j)}. \tag{20}$$

The resulting portfolio has alpha equal to

$$\alpha_A^{(j)} = \left(\frac{f^{(j)} \psi p_m}{cW^{(j)}} \right)^{1/2} \rho^{(j)} - 2f^{(j)}, \tag{21}$$

market-adjusted volatility equal to

$$\sigma_A^{(j)} = \sigma \left(\frac{f^{(j)} p_m}{cW^{(j)}} \right)^{1/2}, \tag{22}$$

and information ratio

$$I_A^{(j)} = \frac{\alpha_A^{(j)}}{\sigma_A^{(j)}} = \frac{1}{\sigma} \left[(\psi)^{1/2} \rho^{(j)} - 2 \left(cf^{(j)} W^{(j)} / p_m \right)^{1/2} \right], \tag{23}$$

where

$$\psi = (1/p_m) \sum_{i=1}^{N} \alpha_i^2 p_i, \tag{24}$$

and

$$\rho^{(j)} = \sum_{i=1}^{N} \tilde{\alpha}_i^{(j)} \alpha_i p_i \bigg/ \left[\left(\sum_{i=1}^{N} \left(\tilde{\alpha}_i^{(j)} \right)^2 p_i \right) \cdot \left(\sum_{i=1}^{N} \alpha_i^2 p_i \right) \right]^{1/2}. \tag{25}$$

Given the portfolios constructed by each of the M managers, the next proposition determines how investors allocate across those M portfolios, and it also gives the form of equilibrium pricing that this allocation implies.

Proposition 2. *In equilibrium, all managers receive equal fee revenue,*

$$f^{(j)} W^{(j)} = f^{(k)} W^{(k)}, \quad \textit{for all } j \textit{ and } k, \tag{26}$$

the price of stock i is of the form

$$p_i = \bar{p}_i \ + \ \theta(\hat{p}_i - \bar{p}_i), \tag{27}$$

with

$$\nu_1 < \theta < 1 \tag{28}$$

and θ constant for all i, and with

$$\hat{p}_i = N p_m \phi_{H,i} \tag{29}$$

being the price for stock i that would prevail in the absence of active management (i.e., when $y = 0$). The quantity ψ defined in equation (24) is given by

$$\psi = \theta^2 \mu_m^2 \sum_{i=1}^{N} \frac{[(\hat{p}_i - \bar{p}_i)/p_m]^2}{\bar{p}_i/p_m + \theta[(\hat{p}_i - \bar{p}_i)/p_m]}. \tag{30}$$

The quantities $\tilde{\psi}^{(j)}$ and $\rho^{(j)}$ defined in equations (19) and (25) are identical across j and given by

$$\tilde{\psi}^{(j)} = \tilde{\psi} = \left(\frac{\theta - \nu_1}{\theta}\right)^2 \psi \ + \ \nu_2^2 \mu_m^2 \left[\sum_{i=1}^{N} \frac{(\bar{p}_i/p_m)^2}{\bar{p}_i/p_m + \theta(\hat{p}_i - \bar{p}_i)/p_m}\right] \tag{31}$$

and

$$\rho^{(j)} = \rho = \left(\frac{\theta - \nu_1}{\theta}\right)\left(\frac{\psi}{\tilde{\psi}}\right)^{1/2}. \tag{32}$$

The correlation between the market-adjusted returns of any two managers is equal to ρ^2.

Because investors share the same objective, maximize the Sharpe ratio, each investor achieves that maximum by allocating the same fraction y to the aggregate portfolio of active management, which by the previous proposition is the portfolio giving each manager the same fee revenue. In solving for y in the competitive equilibrium, each investor takes as given the net alpha and non-market volatility of that active portfolio, denoted as α_A and σ_A. (The absence of a superscript "(j)" denotes an aggregate or common quantity.) Of course, in equilibrium, those quantities depend on investors' choice of y. As shown by Treynor and Black (1973), the optimal y from each individual investor's perspective is given by

$$y = \gamma \frac{\alpha_A}{\sigma_A^2}, \tag{33}$$

where

$$\gamma = \left(\frac{\mathrm{E}(r_m) - r_f}{\mathrm{Var}(r_m)}\right)^{-1}. \tag{34}$$

The values of α_A, σ_A, and y that deliver equation (33) in equilibrium are characterized by the following proposition.

Proposition 3. *In equilibrium, the aggregate portfolio of the active managers has net alpha equal to*

$$\alpha_A = \frac{f\bar{\sigma}^2}{\gamma N(1-h)(c/M)}, \tag{35}$$

and market-adjusted volatility equal to

$$\sigma_A = \frac{f\bar{\sigma}}{(\psi)^{1/2}\rho}\left(2 + \frac{\bar{\sigma}^2}{\gamma N(1-h)(c/M)}\right), \tag{36}$$

where f is the harmonic mean of fee rates,

$$f = \left(\frac{1}{M}\sum_{j=1}^{M}\frac{1}{f^{(j)}}\right)^{-1}, \tag{37}$$

and

$$\bar{\sigma}^2 = \sigma^2\left[\frac{(M-1)\rho^2+1}{M}\right]. \tag{38}$$

The fraction of investors' aggregate stock-market wealth allocated to active management is given by

$$y = \gamma\frac{\alpha_A}{\sigma_A^2} = \frac{1}{f}\,\rho^2\psi N(1-h)(c/M)\left(\frac{\gamma}{\bar{\sigma}^2 + 2\gamma N(1-h)(c/M)}\right)^2. \tag{39}$$

The aggregate active portfolio has information ratio equal to

$$I_A = \frac{\alpha_A}{\sigma_A} = \frac{\bar{\sigma}(\rho^2\psi)^{1/2}}{\bar{\sigma}^2 + 2\gamma N(1-h)(c/M)}. \tag{40}$$

The portfolio's active weight in stock i is given by

$$\phi_i = \left(\frac{\mu_m\sigma_A}{\bar{\sigma}\tilde{\psi}^{1/2}}\right)(\nu_1-\theta)\left(\frac{\hat{p}_i - \bar{p}_i}{p_m}\right). \tag{41}$$

Note from the second equality in (39) that because $1/f$ multiplies the right-hand-side expression, the quantity uniquely determined in equilibrium is yf, aggregate fee revenue per dollar of investor's stock-market wealth. This result is the aggregate analog of the statement in Proposition 2 that all managers receive equal fee revenue. In other words, the fee rates managers set do not affect either individual-fund or aggregate fee revenue. Observe also that both α_A and σ_A are proportional to the average fee, f, which drops out of the information ratio, I_A.

Imposing market clearing along with the above allocation to active management fully determines equilibrium prices, conditional on noise-trader demands:

Proposition 4. *In equilibrium, θ in the pricing equation (27) is the solution to*

$$\theta = \frac{1+\nu_1 q(\theta)}{1+q(\theta)}, \tag{42}$$

with

$$q(\theta) = \left(\frac{\theta - \nu_1}{\theta}\right)\left(\frac{\psi}{\tilde{\psi}}\right)\left(\frac{1-h}{h}\right)\left(\frac{\gamma\mu_m}{\bar{\sigma}^2/N + 2\gamma(1-h)(c/M)}\right), \tag{43}$$

and with θ entering the quantities ψ, $\tilde{\psi}$, and $\bar{\sigma}^2$ as given by equations (30), (31), and (38).

Given a specification of the noise-trader demands (as in the examples to be given below), the value of the scalar θ can be obtained by solving equation (42) numerically. That value of θ gives prices in an equilibrium in which the allocation to active management, y, is optimal for investors, in that it satisfies equation (33) with α_A and σ_A correctly assessed in that equilibrium. To understand why lower skill can result in a larger y in that equilibrium, it will also be instructive to consider what happens under an alternative equilibrium that imposes a given value of y instead of the value of y that is optimal for investors. The following proposition characterizes the prices and portfolio characteristics of active management that arise in such a setting.

Proposition 5. *For a given arbitrary positive allocation y to the aggregate portfolio of active management, equilibrium prices are given by equation (27) for θ as the solution to*

$$\theta = \frac{1 + \nu_1 q(\theta)}{1 + q(\theta)}, \tag{44}$$

with

$$q(\theta) = \frac{\mu_m}{h}\left(\frac{fyN(1-h)}{(c/M)\tilde{\psi}}\right)^{1/2} \tag{45}$$

and θ entering the quantity $\tilde{\psi}$ given by equation (31). The aggregate active portfolio has net alpha equal to

$$\alpha_A = \left(\frac{f\rho^2\psi}{(c/M)y(1-h)N}\right)^{1/2} - 2f, \tag{46}$$

with θ entering ψ as given by equation (30), market-adjusted volatility equal to

$$\sigma_A = \left(\frac{\bar{\sigma}^2 f}{Ny(1-h)(c/M)}\right)^{1/2}, \tag{47}$$

and active weight in stock i given by

$$\phi_i = \left(\frac{\mu_m\sigma_A}{\bar{\sigma}\tilde{\psi}^{1/2}}\right)(\nu_1 - \theta)\left(\frac{\hat{p}_i - \bar{p}_i}{p_m}\right). \tag{48}$$

Equation (45) can also be solved for θ numerically, again given a specification of noise-trader demands. Note that the expression for ϕ_i on the right-hand side of equation (48) is the same as that in equation (41), but the values of σ_A, appearing in both expressions, are given by different equations, i.e., (36) versus (47).

2.5. Active share and active position

Cremers and Petajisto (2009) propose *active share* as a measure summarizing the degree to which a portfolio's weights deviate from those of a benchmark portfolio. Applying their definition, active share of the aggregate active portfolio, with respect to the market benchmark, is computed $AS = (1/2)\sum_{i=1}^{N} |\phi_i|$. Substituting the expression for ϕ_i in equation (41) gives the model's implied active share,

$$AS = \frac{1}{2}\left(\frac{\sigma_A \mu_m}{\bar{\sigma}\tilde{\psi}^{1/2}}\right)|\nu_1 - \theta| \sum_{i=1}^{N} \left|\frac{\hat{p}_i - \bar{p}_i}{p_m}\right|. \tag{49}$$

Multiplying active share by the amount of money allocated to active management gives a measure one might term *active position*, which characterizes the economic magnitude of active share in dollar terms. Because the amount of money allocated to active management is equal to y times the dollar value of investors' aggregate stock-market wealth, the active position per dollar of that latter wealth is given by

$$AP = yAS. \tag{50}$$

For a given set of active portfolio weights, active management's impact on equilibrium prices is greater the larger the amount of money being deployed at those weights. As illustrated below, AP is useful in understanding active management's role in price correction.

2.6. Value added

Berk and van Binsbergen (2015) define the *value added* for an active fund as the value of its assets under management (AUM) multiplied by the fund's "gross" alpha, which is the net alpha that the fund's investors earn plus the fee rate they are charged. In the model here,

$$V_A = (\alpha_A + f)\,y \tag{51}$$

gives the aggregate value added across all funds, per dollar of investors' aggregate stock-market wealth.

3. Quantitative analysis

3.1. Parameter specifications

Table 1 lists the values for the model parameters used in the numerical examples below. The values for the first seven parameters—μ_m, γ, σ, N, c, M, and h—correspond to those used by Stambaugh (2014): The value of $\mu_m = 1.065$ is one plus the average return from 1980 through 2012 on the value-weighted NYSE/AMEX/NASDAQ portfolio, while the value of $\gamma = 0.7238$ is the variance of that portfolio over the same period (0.171^2) divided by the portfolio's average return in excess of the one-month Treasury Bill rate (0.0404). The value of $N = 6893$ is the average number of stocks on NYSE/AMEX/NASDAQ over the 1980–2012 period. The value of $\sigma = 0.188$ is the average annual cross-sectional mean idiosyncratic volatility for all CRSP stocks from 1981 through 2008, based on results reported by Brandt, Brav, Graham, and Kumar (2010). The calibration in Stambaugh (2014) gives a value for the ratio c/M, but this value is then translated into mid-sample (1996) separate values for c and M. The latter is first obtained by adding the number of active mutual funds in the dataset constructed by Pástor, Stambaugh, and Taylor (2015) to the number of institutions other than mutual funds filing Form 13F with the SEC, as provided by Thomson Reuters. The 1996 value for this estimate of M equals 2212, which when multiplied by the calibrated value for c/M implies a value of $c = 0.967$. With $c \approx 1$, the proportional cost of taking an active position in a stock is approximately equal to the position's fraction of the stock's total market capitalization. The value of $h = 0.20$ is roughly the current fraction of U.S. equity owned directly by individuals, as constructed by Stambaugh (2014), incorporating estimates reported by French (2008). As in Stambaugh (2014), direct holdings by individuals are entertained as representing the model's noise traders.

Active management's (harmonic) mean fee rate, f, is set at 1%, which is approximately the mid-sample trend value reported by Stambaugh (2014). Recall however that the quantity determined uniquely in equilibrium is yf, aggregate fee revenue per dollar of investors' stock-market wealth. The fee rate f is held constant throughout the analysis, so that y, the allocation to active management, is the quantity that varies across different equilibrium settings. This expositional focus on y is consistent with the active management industry's size being more commonly discussed in terms of AUM rather than fee revenue. The latter is really what implications about y should be interpreted to convey insofar as implications of skill for industry "size."

The amount of noise in managers' expectations is given by the parameters ν_1 and ν_2 in equation (11). Recall that ν_1 is the fraction of noise in the demands of noise traders that also pervades managers' expectations. Values of ν_1 entertained below cover this parameter's entire permissible range, $0 \leq \nu_1 < 1$. The value of ν_2 governs the magnitude of managers' idiosyncratic expectation errors, and recall $\nu_2 > 0$. Because the $\zeta_i^{(j)}$'s have unit variance across stocks, ν_2 represents a manager's typical valuation error relative to correct fundamental value. For much of the analysis below, the value of ν_2 ranges from 0.01 to 0.50, corresponding to typical valuation errors between 1% and 50% of fundamental value. In one case, however, much larger values of ν_2 are entertained, in order to explore more fully the effects of this component of managers' skill.

The remaining step in parameterizing the model is the specification of noise-trader demands, which are taken as exogenous. The approach taken here follows that in Stambaugh (2014). First, the price and payoff of each stock i are normalized by expected end-of-period value, so that $\bar{x}_i = \bar{x}_m$, and thus $\bar{p}_i = p_m$. The relative pricing error $(\hat{p}_i - \bar{p}_i)/p_m$ appearing in the mispricing measure ψ in equation (30) is then given by

$$\begin{aligned} (\hat{p}_i - \bar{p}_i)/p_m &= N\phi_{H,i} - \bar{x}_i/\bar{x}_m \\ &= v_i - 1 \end{aligned} \tag{52}$$

with

$$v_i = N\phi_{H,i}, \tag{53}$$

using equations (8) and (29) along with the relation $p_m = \bar{x}_m/\mu_m$. The mispricing measure ψ in equation (30) can then be written as

$$\psi = \theta^2 \mu_m^2 \sum_{i=1}^{N} \frac{(v_i - 1)^2}{1 + \theta(v_i - 1)}. \tag{54}$$

Similarly, equation (31) can then be written as

$$\tilde{\psi} = \left(\frac{\theta - \nu_1}{\theta}\right)^2 \psi \ + \ \nu_2^2 \mu_m^2 \sum_{i=1}^{N} \frac{1}{1 + \theta(v_i - 1)}. \tag{55}$$

Next, the cross-sectional distribution of the v_i's is approximated by a continuous Weibull density for v. The density is defined for $v \geq 0$, consistent with the assumption that noise-traders do not short. The Weibull distribution has two parameters, determining the distribution's scale and shape.[4] Because $\sum_{i=1}^{N} \phi_{H,i} = 1$, the scale is determined by $\mathrm{E}(v) = 1$, so there is one free parameter k that determines the distribution's shape. As k becomes large, the

[4] For a discussion of the Weibull distribution, see for example Johnson and Kotz (1970, chapter 20).

density concentrates around $v = 1$, yielding the completely diversified portfolio that puts equal weights on all stocks. As k becomes small, the mass concentrates toward zero and skewness increases, yielding an undiversified portfolio that puts low weights on most stocks and large weights on a relative few. Figure 1 displays densities corresponding to several values of k, including $k = 0.3$ and $k = 2$, which are the lowest and highest values entertained in the numerical investigation below. As Stambaugh (2014) discusses, numerous studies report evidence indicating that direct holdings of individuals are quite undiversified and exhibit significant commonality across individuals.[5] Commonality in holdings limits the extent to which the low diversification by individuals washes out when their holdings are aggregated, making the relatively low values of k plausible. In contrast, low commonality would likely result in a distribution of aggregate weights similar to the density displayed in Figure 1 for $k = 20$, corresponding to a relatively well diversified portfolio.

The analogs of equations (54) and (55) in terms of the continuous v are

$$\psi = \theta^2 \mu_m^2 N \ \mathrm{E}\left\{\frac{(v-1)^2}{1+\theta(v-1)}\right\} \tag{56}$$

and

$$\tilde{\psi} = \left(\frac{\theta - \nu_1}{\theta}\right)^2 \psi \ + \ \nu_2^2 \mu_m^2 N \ \mathrm{E}\left\{\frac{1}{1+\theta(v-1)}\right\}. \tag{57}$$

Applying equation (29) and the same normalization as above in which $\bar{p}_i = p_m$ allows active share in equation (49) to be written as

$$\begin{aligned} AS &= \frac{1}{2}\left(\frac{\sigma_A \mu_m}{\bar{\sigma}\tilde{\psi}^{1/2}}\right)(\theta - \nu_1)\sum_{i=1}^{N}|N\phi_{H,i} - 1| \\ &= \frac{1}{2}\left(\frac{\sigma_A \mu_m}{\bar{\sigma}\tilde{\psi}^{1/2}}\right)(\theta - \nu_1)\sum_{i=1}^{N}|v_i - 1|, \end{aligned} \tag{58}$$

and the analog in terms of the continuous v becomes

$$AS = \frac{N}{2}\left(\frac{\sigma_A \mu_m}{\bar{\sigma}\tilde{\psi}^{1/2}}\right)(\theta - \nu_1)\mathrm{E}\{|v-1|\}. \tag{59}$$

3.2. Equilibrium quantities

Figures 2 plots various equilibrium quantities as a function of ν_1, which is the fraction of aggregate noise-trader noise present in managers' expectations. In other words, managers'

[5] Studies presenting evidence of poor diversification include Blume, Crockett, and Friend (1974), Lease, Lewellen, and Schlarbaum (1974), Blume and Friend (1975), Kelly (1995), Polkovnichenko (2005), and Goetzmann and Kumar (2008). Evidence of significant commonality in individuals' stock holdings is reported by Feng and Seasholes (2004), Dorn, Huberman, and Sengmueller (2008), Barber and Odean (2008), and Barber, Odean, and Zhu (2009).

skill in correctly identifying mispricing is greater when ν_1 is lower. The range of ν_1 in Figure 2 is from 0.0 to 0.3. Each panel displays plots for two values of the manager-specific noise parameter ν_2, 0.01 and 0.50, with lower skill corresponding to the larger value. First note from Panel A that as ν_1 increases $1-\theta$ declines. Also, for a given ν_1, a higher value of ν_2 produces a lower value of $1-\theta$. In other words, lower skill of active management, whether due to a greater ν_1 or a greater ν_2, means less correction of the mispricing that would otherwise exist if there were no active management. These results are unsurprising. More interesting, in Panel B, is that as ν_1 increases so does y, so that investors allocate more to active management when its skill is lower. Note that they also allocate more when lower skill reflects a higher ν_2. As will be discussed a bit later, allocating more to active management when its skill is lower owes chiefly to there being less price correction in that case.

Consistent with less price correction is that active management's aggregate portfolio departs less in equilibrium from the benchmark. This result occurs not only when the departure is characterized in terms of portfolio weights, as summarized by active share displayed in Panel C, but it also occurs in Panel D for the aggregate active position. Recall from equation (50) that active position is active share multiplied by the amount invested in active management, with the latter amount stated per dollar of investors' stock market wealth. When skill declines, even though the amount invested in active management increases, active share declines sharply enough to make active position decline. With active management taking less aggressive dollar positions, it is not surprising that there is less correction of mispricing.

The less aggressive aggregate active positions taken when skill is lower reflect roles for both ν_1 and ν_2. When ν_1 is higher, meaning managers' beliefs echo more of the aggregate noise-trader demand, the resulting equilibrium prices appear more correct to managers, and thus managers take less aggressive active positions. When ν_2 is higher, meaning there is more manager-specific noise, more of the aggressiveness in each manager's positions reflects that noise, which diversifies away across managers, so the aggregate active portfolio becomes less aggressive.

Figure 3 plots various performance-related measures versus ν_1, over the same range of that parameter as in Figure 2. We see in Panel A that the equilibrium net alpha declines as skill decreases, but so does the tracking error in Panel B. The ratio of alpha to the tracking error, which is the information ratio, I_A, plotted in Panel C, increases as ν_1 goes from 0 to about 0.2, at which point I_A then begins to decrease as ν_1 increases further. The effect of ν_2 on I_A also flips at about that point, with the higher ν_2 producing a higher I_A at the lower ν_1 values and a lower I_A at the higher ν_1 values. In other words, depending on how much noise-trader noise pervades managers' beliefs, lower skill can either improve or hurt active management's information ratio in equilibrium. As is well known (e.g. Treynor and Black (1973)), a higher value of that information ratio implies a higher Sharpe ratio for investors' optimal combination of active management and passive indexing.

Panel D of Figure 3 plots active management's value added in equation (51). Observe that value added is higher at the lower levels of skill, i.e., at the higher values of ν_1 and ν_2. Even though α_A is lower at lower skill levels, the active allocation y, plotted in Panel B of Figure 2, increases sharply enough as skill declines that the product $(\alpha + f)y$ increases as skill declines. In other words, greater value added for the active management industry need not correspond to a greater level of skill within the industry.

Figure 4 repeats selected plots from Figures 2 and 3 but with the range for ν_1 extended to 1.0 (actually just below, as $\nu_1 < 1$). The degree of mispricing correction (Panel A) falls monotonically as skill declines. Also, reassuringly, as ν_1 gets sufficiently large, and thus as managers' skill gets sufficiently low, active management receives a lower allocation (Panel B), has a lower information ratio (Panel C), and produces lower value added (Panel D). In other words, active management's size, information, and value added are hump shaped with respect to skill. The hump in the information ratio is much less pronounced, however. The hump displayed over the shorter range for ν_1 in Figure 3 is easily missed in Figure 4. In other words, over a significant skill range, a lower level of skill makes investors worse off while it increases the active management industry's size and value added.

3.3. Opposing skill effects

As discussed earlier, there are essentially two opposing effects of skill. On one hand, greater skill means active management can more accurately identify mispricing and thereby better construct an alpha-producing portfolio. On the other hand, when active management's positions reflect greater skill, more price correction results from managers establishing those positions in aggregate. More correction in the prices at which active positions are established leaves less correction to occur later and produce alpha. Either of these two effects of skill can outweigh the other.

The opposition of the two effects becomes apparent when observing that ρ and ψ appear as the product $\rho^2\psi$ in a number of equilibrium quantities, including the active allocation, y, in equation (39) and the information ratio, I_A, in equation (40). The quantity $\rho^{(j)}$ for manager j is defined in equation (25), and recall from Proposition 2 that $\rho^{(j)}$ equals the same value, ρ, for all managers. From equation (25) we see that ρ summarizes the extent to which managers' assessed alphas correspond to true alphas: the closer the correspondence, the closer ρ is to 1. That is, ρ depends on managers' ability to correctly identify mispriced assets, which is the first of the two skill effects noted above. The value of ψ, defined in equation (24), is a mispricing measure, in that it aggregates the magnitudes of true alphas present in equilibrium. For a given set of noise-trader demands, greater price correction by active management implies a lower value of ψ. In this sense, ψ captures the second effect of skill noted above. A lower level of manager skill, i.e., a greater ν_1 or ν_2, produces a higher ρ and a lower ψ, but the product $\rho^2\psi$ can be either higher or lower.

The opposing effects of skill are especially evident if the size of the active management industry is held fixed. The product $\rho^2\psi$ also appears on the right-hand of equation (46), which gives the net alpha that results in an equilibrium in which the allocation to active management, y, is fixed at an arbitrary value. Note from equation (46) that when y is held constant, the effect of skill on α_A enters solely through $\rho^2\psi$; the higher that product, the higher is α_A. For y fixed arbitrarily at 0.7 (roughly active management's current market share), Figure 5 plots $1-\theta$, ψ, ρ, and α_A as functions of ν_1. As in the previous figures, plots are displayed for two values of ν_2: 0.01 and 0.50.[6] We see that as ν_1 increases (and thus skill declines), there is less price correction, as reflected by both $1-\theta$ in Panel A and the mispricing measure ψ in Panel B. We also see in Panel C that the correlation measure ρ decreases as ν_1 increases. Finally we see in Panel D that α_A, and therefore $\rho^2\psi$, first increases and then declines as ν_1 increases from 0 to 1. At the higher values of ν_1 (i.e., lower levels of skill), α_A is hurt less by the additional mispricing correction (i.e., lower ψ) than α_A is helped by managers' better portfolio choices (i.e., higher ρ). The opposite is true at the lower values of ν_1. In other words, for those higher levels of skill, lowering skill raises α_A at a given allocation to active management, y.

3.4. Decreasing returns to scale

Once we see that lower skill can give a higher α_A at an arbitrary y, as above, the next step is to understand why lower skill then also gives a larger equilibrium y, as in Panel B of Figure 2. Key here is the concept of decreasing returns to scale in active management, introduced by Berk and Green (2004) at the fund level and by Pástor and Stambaugh (2012) at the

[6] As in Figure 4, the role of ν_2 is more difficult to discern from the plots, given that the curves for the two values of that parameter plot very closely to each other. The effects of increasing ν_2 are examined in more detail in a later subsection.

industry level: The more money there is under management, the lower is the alpha. In the model here, decreasing returns are evident at both the fund and industry level. At the fund level, decreasing returns follow from $W_{(j)}$ appearing in the denominator of $\alpha_A^{(j)}$ in equation (21). At the industry level, decreasing returns follow from y appearing in the denominator of α_A in equation (46).

Suppose that at an arbitrary $y = y_0$, the α_A's associated with two skill levels are both higher than investors require, in that y_0 is less than the resulting right-hand side of equation (33) under either skill level. Also assume that, as the previous subsection shows is possible, the α_A at the lower skill level exceeds that at the higher skill level. Now consider what happens as y increases up to the point that equation (33) is satisfied. Because the low-skill α_A starts at a higher value than the high-skill α_A, the low-skill α_A essentially has farther to fall before it drops to the point at which equation (33) is satisfied. As a result, the equilibrium y is greater under low skill than under high skill.

The above intuition, for simplicity, abstracts from the fact that the right-hand side of equation (33) is proportional to α_A/σ_A^2 rather than simply α_A. Even though y also appears in the denominator of σ_A in equation (47), the value of σ_A remains positive as y increases, whereas α_A, and thus α_A/σ_A^2, eventually become negative as y increases. Thus, the intuition for the effect of y on either quantity is essentially the same. It is worth noting that in general, though, investors allocate enough to active management to bring α_A to a rather modest value. In that sense, as observed previously by Pástor and Stambaugh (2012), the zero-alpha condition imposed in Berk and Green (2004), while technically inconsistent with a positive allocation to active management in a general equilibrium, can nevertheless be a reasonable approximation for practical purposes.

Insights into the source and effects of decreasing returns to scale are provided by Figure 6, which displays plots of equilibrium quantities under fixed values of y ranging from 0 to 2. Each panel contains plots under four different pairs of values of ν_1 and ν_2, representing different skill levels. First note, in Panel A, that $1 - \theta$ depends on the skill level but is relatively insensitive to y under the model parameterizations considered here. That is, the degree of price correction does not depend greatly on the size of the active management industry. Consistent with this result is the fact that the aggregate active position in Panel B is similarly insensitive to y. When managers are given more money to manage, they become less aggressive in terms of their active portfolio weights, as characterized by active share, so the product of active share and y, equal to active position in Panel B (cf. equation 50), does not increase greatly as y increases. To understand why the active weights become less aggressive, recall from Proposition 1 (equation 20) that an outcome of managers' optimization is that fee revenue equals trading costs. With convex trading costs, as in equation (15), increasingly smaller increments in active dollar positions are required to keep trading costs equal to fee revenue, which increases in proportion to y. The modest increases in active dollar positions then translate to declines in the active weights.[7]

If price correction and active dollar positions do not change much as the allocation to active management, y, increases, then neither do gross active dollar profits, before trading costs and fees. Because trading costs and fees both increase in proportion to y, however, dollar profits net of costs and fees then decline as y increases. Value added, given in equation (51), adds back the fees but not the trading costs. As a result, value added declines as y increases, as illustrated in Panel C of Figure 6. Of course the information ratio, I_A, whose numerator, α_A, deducts both fees and cost while also dividing dollar profits by assets under management, must also decline as those assets increase. That result is illustrated in Panel D, which shows I_A declining as y increases.

3.5. Manager-specific noise

In some of the previous results, such as in Figures 4 and 5, the effect of manager-specific noise is somewhat difficult to discern, because the plots using different value of ν_2 are often close together. Figure 7 better isolates that parameter's role by plotting various equilibrium quantities versus ν_2. For this analysis, the noise-trader parameter is set to $k = 0.3$, so that noise-trader demands are noisier than in the analysis above, conducted with $k = 2$. (Recall that the distributions of noise-trader weights under both parameters are displayed in Figure 1.) As ν_2 increases, we see in Panel A that $1 - \theta$ declines, meaning there is less price correction when skill is lower, consistent with all of the previous results. In Panel B, active management receives a greater allocation as ν_2 increases, illustrating, as before, that lower skill can mean a larger active management industry. The information ratio, I_A, plotted in Panel C, declines monotonically as ν_2 increases, so here we do not see a repeat of the earlier cases in which I_A increases as skill declines. Moreover, Panel C plots I_A versus ν_2 using two different values of ν_1, and we see that I_A is higher under the lower value of ν_1, again in contrast to the previous cases going the other way under the higher value of k. Consistent with earlier results, however, we see that value added, plotted in Panel D, declines as ν_2

[7] A related result is noted by Stambaugh (2014), who observes that the presence of the square root of $W^{(j)}$ in the denominator of $\tilde{a}^{(j)}$ in equation (18) implies that each additional dollar received by a manager is deployed less actively than the previous one.

increases over the range plotted, up to $\nu_2 = 0.50$.

As shown earlier in Figure 4, the active management industry's size and value added decline as skill declines, once the noise parameter ν_1 reaches sufficiently high values, i.e., as that dimension of skill gets low enough. Is the same true for ν_2, as the amount of manager-specific noise gets sufficiently high? Figure 8 plots the same quantities as in Figure 7, but with the range of ν_1 greatly extended. Panels B and D reveal that, indeed, as manager-specific noise gets sufficiently high, lower levels of that skill dimension also produce smaller industry size and value added. Notice, however, that the values of ν_2 at which those quantities begin to decline with respect to ν_2 are quite high, roughly in the range of 30 to 70 (i.e., 3000 to 7000 percent). The fact that the plots eventually decline is somewhat reassuring from a theoretical perspective, in that active management should ultimately shrink as skill gets sufficiently low. At the same time, though, such high amounts of manager-specific noise seem rather unlikely, so it would seem that from a practical perspective, at least in this parameterization of the model, greater manager-specific noise implies a larger active management industry in equilibrium.

Recall from Table 1 that the model's parameterization here includes specifying $M = 2212$ as the number of managers (funds). Therefore, with that many managers, even a large amount of manager-specific noise essentially diversifies away across managers, thereby having little impact on active management's ability in aggregate to identify mispriced securities. Greater manager-specific noise does, however, lower the aggressiveness of each manager's positions, thereby lowering the aggressiveness of active management's overall positions and reducing equilibrium price correction.

3.6. Noise-trader noise

Figure 9 plots various equilibrium quantities versus the noise-trader parameter, k, over the range from 0.3 or 2. Those endpoints are the two values of k used in the above analyses. The quantities plotted are the same as in Figures 7 and 8, and each panel contains plots for four different pairs of values for ν_1 and ν_2 (the same four pairs used in Figure 6). Panel A reveals that as noise trader demands get less noisy (k gets larger), active management corrects a smaller fraction of the mispricing that would exist without active management (i.e., $1 - \theta$ declines). Lower values of either ν_1 or ν_2, corresponding to greater skill, result in greater price correction, consistent with the earlier analyses.

Less noise-trader noise results in an active management industry that is smaller (Panel

B), has less value added (Panel C), and has a lower information ratio (Panel D). Throughout the range of k considered, both the industry's size and its value added are greater when skill is lower, in that the higher of the two values of both ν_1 and ν_2 produce greater values of those quantities. At the values of k closer to 0.3, lower skill produces a lower information ratio, I_A. At the values of k closer to 2, the effect of skill on I_A is not discernible in Panel D, as the four curves plot essentially on top of each other. As revealed in the earlier analysis in Figure 3, which is constructed using $k = 2$ and a more magnified scale for I_A, the effects of both ν_1 and ν_2 on I_A then go in the other direction, with lower skill producing a higher I_A.

4. Conclusions

Suppose that the active management industry has indeed become more skilled over time, consistent with the evidence of Pástor, Stambaugh, and Taylor (2015). Those authors suggest education and technology, for example, could be part of that story. The recent trend toward quantitatively managed "smart-beta" products might even be construed as a self-proclaimed increase in the industry's skill (or at least its "smartness"). The results here show that an increase in skill can imply a smaller equilibrium amount of active management.

In that sense, an upward trend in skill represents a potential challenge for the industry in addition to the one identified by Stambaugh (2014), which is the downward trend in direct equity ownership by individuals, a potential source of noise trading. That is, if not only the presence of noise traders declines, but the mispricing they induce is more skillfully identified, then active management can face a doubly strong headwind in maintaining its presence in the money management industry. Of course an industry composed of many competing managers cannot decide to calm that headwind by becoming less skilled; applying more skill is surely in each manager's individual interest. Glode, Green, and Lowery (2012) model another setting in which increases in the skills of competing financial firms do not benefit the industry in equilibrium.

From the model's perspective, the amount of active management can decline through a loss of AUM, a drop in the fee rate, or both. The product of those quantities, fee revenue, is what the model's equilibrium determines uniquely. In fact both market share and fee rates have declined over recent decades, as noted by Stambaugh (2014). In the case of equity mutual funds, for example, over the period from 2001 to 2016, active management lost 15% in its share of total AUM while reducing its fee (expense ratio) by 30 basis points.[8]

[8] The fraction of total equity mutual fund assets under active management went from 90% to 75%,

For settings in which there is a negative relation between skill and industry size, there is at least an imperfect analogy to the situation faced by any industry that gets more efficient at producing a good or service for which the capacity to consume is relatively constrained. The more efficient the industry becomes in exploiting its productive resources, the less of those resources it needs to employ. A notable example comes from agriculture, where efficiency gains play a big role in that sector's accounting for a much smaller share of the U.S. economy than it once did. The capacity for consuming the active management industry's output is constrained in the sense that the industry can do no more than drive its equilibrium net alpha to (nearly) zero. Being more skilled in identifying mispriced assets can enable the industry to accomplish that job with less resources.

As noted at the outset, given the focus here on implications of the overall skill level of the industry, differences in skill across managers are suppressed for simplicity. Berk and van Binsbergen (2015) propose value added as a skill measure in essentially a cross-sectional context, used to distinguish skill across managers. The results of this study, showing that value added and skill can be inversely related at the industry level, do not speak to the measure's cross-sectional application. Of the two opposing effects of skill on alpha—accuracy in portfolio choices versus the degree of equilibrium price correction—the latter effect is shared by all managers whereas the former differs across managers with different skill levels. Thus, in a more complicated version of the model with skill heterogeneity, it seems conceivable that value added could relate positively to skill in the cross section while having a negative relation to the industry's overall skill.

while the asset-weighted average expense ratio of active equity funds went from 1.1% to 0.8%. (Investment Company Institute, 2017).

Appendix

Proof of Proposition 1: First note that before fees and costs, the manager's market-adjusted rate of return is $\sum_{i=1}^{N} \phi_{A,i}^{(j)} R_i = \sum_{i=1}^{N} \phi_i^{(j)} R_i$, since $\sum_{i=1}^{N} \phi_{m,i} R_i = 0$. The fee reduces this rate of return by $f^{(j)}$. The manager's trading cost for asset i is

$$C_i^{(j)} = c\delta_i^{(j)} |\phi_i^{(j)}| W^{(j)} = c\left(|\phi_i^{(j)}| W^{(j)}/p_i\right) |\phi_i^{(j)}| W^{(j)}, \tag{A1}$$

so trading costs reduce the manager's rate of return by $\left(\sum_{i=1}^{N} C_i^{(j)}\right)/W^{(j)} = cW^{(j)} \sum_{i=1}^{N} (\phi_i^{(j)})^2/p_i$. The manager's net market-adjusted return is therefore

$$R_A^{(j)} = \sum_{i=1}^{N} \phi_i^{(j)} R_i - f^{(j)} - cW^{(j)} \sum_{i=1}^{N} (\phi_i^{(j)})^2/p_i, \tag{A2}$$

so the manager perceives his portfolio as having net alpha equal to

$$\tilde{\alpha}_A^{(j)} = \sum_{i=1}^{N} \phi_i^{(j)} \tilde{\alpha}_i^{(j)} - f^{(j)} - cW^{(j)} \sum_{i=1}^{N} (\phi_i^{(j)})^2/p_i. \tag{A3}$$

The manager perceives (correctly) the volatility of the portfolio's non-market return as

$$\sigma_A^{(j)} = \sigma \left[\sum_{i=1}^{N} \left(\phi_i^{(j)}\right)^2 \left(\frac{p_m}{p_i}\right)\right]^{1/2}, \tag{A4}$$

where the last equality uses equation (9) and the property that the R_i's are mutually uncorrelated. Define the N-element vectors α, $\tilde{\alpha}^{(j)}$ and $\phi^{(j)}$, whose i-th elements equal α_i, $\tilde{\alpha}_i^{(j)}$ and $\phi_i^{(j)}$ respectively. Also define the $N \times N$ matrix P with i-th diagonal element equal to p_i and all non-diagonal elements equal to zero. The portfolio parameters in equations (A3) and (A4) can then be rewritten as

$$\tilde{\alpha}_A^{(j)} = \phi^{(j)\prime} \tilde{\alpha}^{(j)} - f^{(j)} - cW^{(j)} \phi^{(j)\prime} P^{-1} \phi^{(j)} \tag{A5}$$

and

$$\sigma_A^{(j)} = p_m^{1/2} \sigma \left(\phi^{(j)\prime} P^{-1} \phi^{(j)}\right)^{1/2}. \tag{A6}$$

The manager chooses the vector of active weights $\phi^{(j)}$ to maximize $\tilde{I}_A^{(j)} = \tilde{\alpha}_A^{(j)}/\sigma_A^{(j)}$ subject to $\iota' \phi^{(j)} = 0$, where $\iota' = (1\ 1\ \cdots 1)$. (Recall that the competitive manager takes $\tilde{\alpha}^{(j)}$, P, and $W^{(j)}$ as given, i.e., unaffected by his portfolio choice.) The corresponding Lagrangian is

$$\mathcal{L} = \frac{\phi^{(j)\prime} \tilde{\alpha}^{(j)} - f^{(j)} - cW^{(j)} \phi^{(j)\prime} P^{-1} \phi^{(j)}}{p_m^{1/2} \sigma \left(\phi^{(j)\prime} P^{-1} \phi^{(j)}\right)^{1/2}} - \tilde{\xi}(\iota' \phi^{(j)}), \tag{A7}$$

Differentiating with respect to $\phi^{(j)}$ and multiplying through by $p_m^{1/2} \sigma \left(\phi^{(j)\prime} P^{-1} \phi^{(j)}\right)^{1/2}$ gives

$$\tilde{\alpha}^{(j)} - 2cW^{(j)} P^{-1} \phi^{(j)} - \frac{\phi^{(j)\prime} \tilde{\alpha}^{(j)} - f^{(j)} - cW^{(j)} \phi^{(j)\prime} P^{-1} \phi^{(j)}}{\phi^{(j)\prime} P^{-1} \phi^{(j)}} P^{-1} \phi^{(j)} - \xi \iota = 0, \tag{A8}$$

where ξ is the rescaled Lagrange multiplier. Multiplying through by P and rearranging gives

$$\phi^{(j)} = \left(cW^{(j)} + \frac{\phi^{(j)\prime}\tilde{\alpha}^{(j)} - f^{(j)}}{\phi^{(j)\prime} P^{-1} \phi^{(j)}} \right)^{-1} P(\tilde{\alpha}^{(j)} - \xi\iota). \tag{A9}$$

It follows readily from (12) that the market-weighted combination of the $\tilde{\alpha}_i^{(j)}$'s is zero: $\iota' P \tilde{\alpha}^{(j)} = 0$. Therefore, since $\iota'\phi^{(j)} = 0$, multiplying both sides of equation (A9) by ι' implies $\xi = 0$, and thus

$$\phi^{(j)} = \left(cW^{(j)} + \frac{\phi^{(j)\prime}\tilde{\alpha}^{(j)} - f^{(j)}}{\phi^{(j)\prime} P^{-1} \phi^{(j)}} \right)^{-1} P\tilde{\alpha}^{(j)}. \tag{A10}$$

Multiplying both sides of equation (A10) by $\phi^{(j)\prime} P^{-1}$ and rearranging gives

$$\phi^{(j)\prime} P^{-1} \phi^{(j)} = \frac{f^{(j)}}{cW^{(j)}}, \tag{A11}$$

which implies equation (20), since from equation (A1) total costs equal $c\left(W^{(j)}\right)^2 \phi^{(j)\prime} P^{-1} \phi^{(j)}$. Substituting equation (A11) into equation (A10) gives

$$\phi^{(j)} = \frac{f^{(j)}}{cW^{(j)} \phi^{(j)\prime} \tilde{\alpha}^{(j)}} P\tilde{\alpha}^{(j)}. \tag{A12}$$

Multiplying both sides of equation (A12) by $\tilde{\alpha}^{(j)\prime}$ and then solving for $\phi^{(j)\prime}\tilde{\alpha}^{(j)}$ gives

$$\phi^{(j)\prime}\tilde{\alpha}^{(j)} = \left(\frac{f^{(j)}}{cW^{(j)}}\right)^{1/2} \left(\tilde{\alpha}^{(j)\prime} P \tilde{\alpha}^{(j)}\right)^{1/2} = \left(\frac{f^{(j)} p_m}{cW^{(j)}}\right)^{1/2} \left(\tilde{\psi}^{(j)}\right)^{1/2}, \tag{A13}$$

which when substituted into equation (A12) gives

$$\phi^{(j)} = \left(\frac{f^{(j)} p_m}{cW^{(j)}}\right)^{1/2} \left(\tilde{\psi}^{(j)}\right)^{-1/2} \frac{1}{p_m} P\tilde{\alpha}^{(j)}, \tag{A14}$$

equivalent to equation (17). From equations (A2), (A11), and (A14), the true net alpha of the manager's portfolio is given by

$$\begin{aligned}
\alpha_A^{(j)} &= \phi^{(j)\prime}\alpha - 2f^{(j)} \\
&= \left(\frac{f^{(j)} p_m}{cW^{(j)}}\right)^{1/2} \left(\tilde{\psi}^{(j)}\right)^{-1/2} \frac{1}{p_m} \alpha' P \tilde{\alpha}^{(j)} - 2f^{(j)} \\
&= \left(\frac{f^{(j)} \psi p_m}{cW^{(j)}}\right)^{1/2} \underbrace{\psi^{-1/2} \left(\tilde{\psi}^{(j)}\right)^{-1/2} \frac{1}{p_m} \alpha' P \tilde{\alpha}^{(j)}}_{\rho^{(j)}} - 2f^{(j)},
\end{aligned} \tag{A15}$$

which is equivalent to equation (21), with $\tilde{\psi}^{(j)}$, ψ, and $\rho^{(j)}$ defined as in equations (19), (24). and (25). Substituting from equation (A11) into equation (A6) gives equation (22). Equation (23) follows directly when dividing equation (21) by equation (22).

Proof of Proposition 2: Each investors wishes to combine the M active funds to produce the highest information ratio. Because managers' active weights contain manager-specific noise, benchmark-adjusted returns are imperfectly correlated across funds, and the covariance matrix of the M fund returns is nonsingular. The weights on funds that the maximize the information ratio obey the relation

$$\omega = (\iota'\Sigma_A^{-1}\alpha)^{-1}\Sigma_A^{-1}\alpha, \tag{A16}$$

where Σ_A is the $M \times M$ variance-covariance matrix of the returns on the M active funds, α is the $M \times 1$ vector whose j-th element is $\alpha_A^{(j)}$, and ι is the $M \times 1$ vector $[1\ 1\ \cdots\ 1]'$, and ω denotes the $M \times 1$ vector whose j-th element, $\omega^{(j)}$, is the weight of fund j in the aggregate portfolio of active management. To solve for an equilibrium in this case, the proof conjectures and then verifies a solution in which each manager receives the same fee revenue in equilibrium and $\tilde{\psi}^{(j)}$ is the same across managers. That is, if $f^{(j)}W^{(j)} = g$ denotes the common level of fee revenue received by each manager j, then the weight of fund j in the aggregate portfolio of active managers is

$$\omega^{(j)} = \frac{W^{(j)}}{\sum_{j=1}^{M} W^{(j)}} = \frac{g/f^{(j)}}{\sum_{j=1}^{M} g/f^{(j)}} = \frac{f}{Mf^{(j)}}, \tag{A17}$$

where f is the harmonic mean of fee rates,

$$f = \left(\frac{1}{M}\sum_{j=1}^{M}\frac{1}{f^{(j)}}\right)^{-1}, \tag{A18}$$

and

$$\tilde{\psi}^{(j)} = \tilde{\psi}, \qquad j = 1, \ldots, M. \tag{A19}$$

This choice for ω will be shown to satisfy (A16) under the values of Σ_A and α that result from the equilibrium prices in such an allocation. Those equilibrium prices also imply equal $\tilde{\psi}^{(j)}$'s across funds.

From equations (17), (18), (A17), and (A19), the aggregate active portfolio's weight in stock i is

$$\begin{aligned}
\phi_i = \sum_{j=1}^{M}\omega_i^{(j)}\phi^{(j)} &= \sum_{j=1}^{M}\frac{f}{Mf^{(j)}}\mu_m\left(\frac{f^{(j)}p_m}{cW^{(j)}\tilde{\psi}}\right)^{1/2}\left(\frac{\tilde{p}_i^{(j)} - p_i}{p_m}\right) \\
&= \frac{f\mu_m}{M}\left(\frac{p_m}{c\tilde{\psi}g}\right)^{1/2}\sum_{j=1}^{M}\left(\frac{\tilde{p}_i^{(j)} - p_i}{p_m}\right),
\end{aligned} \tag{A20}$$

and then substituting from equation (10) gives

$$\begin{aligned}
\phi_i &= \frac{f\mu_m}{M}\left(\frac{p_m}{c\tilde{\psi}g}\right)^{1/2}\sum_{j=1}^{M}\left(\frac{\bar{p}_i - p_i}{p_m} + \nu_1\frac{\hat{p}_i - \bar{p}_i}{p_m} + \nu_2\frac{\zeta_i^{(j)}\bar{p}_i}{p_m}\right) \\
&= f\mu_m\left(\frac{p_m}{c\tilde{\psi}g}\right)^{1/2}\left(\frac{\bar{p}_i - p_i}{p_m} + \nu_1\frac{\hat{p}_i - \bar{p}_i}{p_m} + \frac{\nu_2\bar{p}_i M}{p_m}\left[\frac{1}{M}\sum_{j=1}^{M}\zeta_i^{(j)}\right]\right) \\
&= \mu_m\left(\frac{f}{(c/M)\tilde{\psi}y(1-h)N}\right)^{1/2}\left(\frac{\bar{p}_i - p_i}{p_m} + \nu_1\frac{\hat{p}_i - \bar{p}_i}{p_m}\right),
\end{aligned} \tag{A21}$$

recalling that the $\zeta_i^{(j)}$'s average to zero across managers and substituting

$$g = fy(1-h)p_m N/M, \tag{A22}$$

which follows from the fact that since the aggregate value of the stock market is Np_m, the aggregate value of the amount invested in active management is equal to $y(1-h)(Np_m) = \sum_{j=1}^{M} W^{(j)} = \sum_{j=1}^{M} g/f^{(j)} = Mg/f$, using equation (A18).

The investors' aggregate weight in stock i is

$$\phi_{S,i} = (1-y)\phi_{m,i} + y\phi_{A,i} = (1-y)\phi_{m,i} + y(\phi_{m,i} + \phi_i) = \phi_{m,i} + y\phi_i. \tag{A23}$$

where, $\phi_{A,i}$, the total weight of stock i in the active-management portfolio, equals the market weight, $\phi_{m,i}$, plus the aggregate active weight, ϕ_i. Substituting the last expression above for $\phi_{S,i}$ into the market-clearing condition in equation (16) gives

$$h\phi_{H,i} + (1-h)(\phi_{m,i} + y\phi_i) = \phi_{m,i}, \tag{A24}$$

or

$$h(\phi_{H,i} - \phi_{m,i}) = -y(1-h)\phi_i. \tag{A25}$$

Substituting the identities $\phi_{H,i} = \hat{p}_i/(Np_m)$ and $\phi_{m,i} = p_i/(Np_m)$ as well as the expression for ϕ_i in equation (A21) then gives

$$\begin{aligned} h\left(\frac{\hat{p}_i}{Np_m} - \frac{p_i}{Np_m}\right) &= -y(1-h)\mu_m \left(\frac{f}{(c/M)\tilde{\psi}y(1-h)N}\right)^{1/2} \left(\frac{\bar{p}_i - p_i}{p_m} + \nu_1 \frac{\hat{p}_i - \bar{p}_i}{p_m}\right) \\ &= -\mu_m \left(\frac{fy(1-h)}{(c/M)N\tilde{\psi}}\right)^{1/2} \left(\frac{\bar{p}_i - p_i}{p_m} + \nu_1 \frac{\hat{p}_i - \bar{p}_i}{p_m}\right). \end{aligned} \tag{A26}$$

Dividing through by h and multiplying by Np_m gives

$$\hat{p}_i - p_i = -\frac{\mu_m}{h}\left(\frac{fyN(1-h)}{(c/M)\tilde{\psi}}\right)^{1/2} \left[\bar{p}_i - p_i + \nu_1\left(\hat{p}_i - \bar{p}_i\right)\right], \tag{A27}$$

which can be rewritten as

$$p_i = \bar{p}_i + \theta\left(\hat{p}_i - \bar{p}_i\right), \tag{A28}$$

where

$$\theta = \left[1 + \nu_1 \frac{\mu_m}{h}\left(\frac{fyN(1-h)}{(c/M)\tilde{\psi}}\right)^{1/2}\right] \left[1 + \frac{\mu_m}{h}\left(\frac{fyN(1-h)}{(c/M)\tilde{\psi}}\right)^{1/2}\right]^{-1}. \tag{A29}$$

Equation (A29) is of the form

$$\begin{aligned} \theta &= \frac{1 + \nu_1 q(\theta)}{1 + q(\theta)} \\ &= \left(\frac{1}{1+q(\theta)}\right) 1 \ + \ \left(\frac{q(\theta)}{1+q(\theta)}\right)\nu_1 \end{aligned} \tag{A30}$$

with

$$q(\theta) = \frac{\mu_m}{h}\left(\frac{fyN(1-h)}{(c/M)\tilde{\psi}}\right)^{1/2}, \tag{A31}$$

and $q(\theta) > 0$ for $y > 0$, implying a solution for θ must obey the inequalities in (28).

Substituting from equations (7) and (27) into equation (24) gives equation (30). Substituting from equations (11), (13), and (27) into equation (19) gives

$$\begin{aligned}
\tilde{\psi}^{(j)} &= \frac{\mu_m^2}{p_m}\sum_{i=1}^{N}\frac{(\tilde{p}_i^{(j)}-p_i)^2}{p_i} \\
&= \frac{\mu_m^2}{p_m}\sum_{i=1}^{N}\frac{\left[(\nu_1-\theta)(\hat{p}_i-\bar{p}_i)+\nu_2\zeta_i^{(j)}\bar{p}_i\right]^2}{\bar{p}_i+\theta(\hat{p}_i-\bar{p}_i)} \\
&= (\nu_1-\theta)^2\mu_m^2\sum_{i=1}^{N}\frac{[(\hat{p}_i-\bar{p}_i)/p_m]^2}{\bar{p}_i/p_m+\theta[(\hat{p}_i-\bar{p}_i)/p_m]}+\nu_2^2\frac{\mu_m^2}{p_m}\sum_{i=1}^{N}\frac{\bar{p}_i^2}{\bar{p}_i+\theta(\hat{p}_i-\bar{p}_i)}\left(\zeta_i^{(j)}\right)^2 \\
&\quad +2(\nu_1-\theta)\nu_2\frac{\mu_m^2}{p_m}\sum_{i=1}^{N}\frac{(\hat{p}_i-\bar{p}_i)\bar{p}_i}{\bar{p}_i+\theta(\hat{p}_i-\bar{p}_i)}\zeta_i^{(j)} \\
&= \frac{(\nu_1-\theta)^2}{\theta^2}\psi+\nu_2^2\frac{\mu_m^2}{p_m}N\left[\frac{1}{N}\sum_{i=1}^{N}\frac{\bar{p}_i^2}{\bar{p}_i+\theta(\hat{p}_i-\bar{p}_i)}\right]\left[\frac{1}{N}\sum_{i=1}^{N}\left(\zeta_i^{(j)}\right)^2\right] \\
&\quad +2(\nu_1-\theta)\nu_2\frac{\mu_m^2}{p_m}N\left[\frac{1}{N}\sum_{i=1}^{N}\frac{(\hat{p}_i-\bar{p}_i)\bar{p}_i}{\bar{p}_i+\theta(\hat{p}_i-\bar{p}_i)}\right]\left[\frac{1}{N}\sum_{i=1}^{N}\zeta_i^{(j)}\right] \qquad \text{(A32)} \\
&= \frac{(\nu_1-\theta)^2}{\theta^2}\psi+\nu_2^2\frac{\mu_m^2}{p_m}N\left[\frac{1}{N}\sum_{i=1}^{N}\frac{\bar{p}_i^2}{\bar{p}_i+\theta(\hat{p}_i-\bar{p}_i)}\right][1] \\
&\quad +2(\nu_1-\theta)\nu_2\frac{\mu_m^2}{p_m}N\left[\frac{1}{N}\sum_{i=1}^{N}\frac{(\hat{p}_i-\bar{p}_i)\bar{p}_i}{\bar{p}_i+\theta(\hat{p}_i-\bar{p}_i)}\right][0] \\
&= \left(\frac{\theta-\nu_1}{\theta}\right)^2\psi \;+\; \nu_2^2\mu_m^2\left[\sum_{i=1}^{N}\frac{(\bar{p}_i/p_m)^2}{\bar{p}_i/p_m+\theta(\hat{p}_i-\bar{p}_i)/p_m}\right], \qquad \text{(A33)}
\end{aligned}$$

which is identical across j, thus giving equation (31). The equality in (A32) follows from the assumption that the $\zeta_i^{(j)}$'s are independent of the $\bar{p}_i$'s and $\hat{p}_i$'s across i, so that the mean of the product is the product of the means. Applying again the assumed properties of the $\zeta_i^{(j)}$'s along with equations (7), (11), (13), (25), (19), (24), and (A28), gives

$$\begin{aligned}
\left(\tilde{\psi}\psi\right)^{1/2}\rho^{(j)} &= \frac{\mu_m^2}{p_m}\sum_{i=1}^{N}\frac{\left(\tilde{p}_i^{(j)}-p_i\right)(\bar{p}_i-p_i)}{p_i} \\
&= \frac{\mu_m^2}{p_m}\sum_{i=1}^{N}\frac{\left[(1-\nu_1)\bar{p}_i+\nu_1\hat{p}_i+\nu_2\zeta_i^{(j)}\bar{p}_i-\bar{p}_i-\theta(\hat{p}_i-\bar{p}_i)\right](\bar{p}_i-p_i)}{p_i} \\
&= \frac{\mu_m^2}{p_m}\sum_{i=1}^{N}\frac{\left[(\theta-\nu_1)(\bar{p}_i-\hat{p}_i)+\nu_2\zeta_i^{(j)}\bar{p}_i\right](\bar{p}_i-p_i)}{p_i} \\
&= \frac{\mu_m^2}{p_m}\sum_{i=1}^{N}\frac{\theta-\nu_1}{\theta}\frac{(\bar{p}_i-p_i)^2}{p_i}+\nu_2\frac{\mu_m^2}{p_m}\sum_{i=1}^{N}\zeta_i^{(j)}\frac{\theta(\bar{p}_i-\hat{p}_i)\bar{p}_i}{\bar{p}_i+\theta\left(\hat{p}_i-\bar{p}_i\right)} \\
&= \frac{\theta-\nu_1}{\theta}\frac{\mu_m^2}{p_m}\sum_{i=1}^{N}\frac{(\bar{p}_i-p_i)^2}{p_i}+0 \\
&= \frac{\theta-\nu_1}{\theta}\psi, \qquad \text{(A34)}
\end{aligned}$$

yielding equation (32).

Next is to derive the Σ_A that results under the above pricing. Combining equations (11), (17), and (A28) gives manager j's active weight in stock i as

$$\begin{aligned}
\phi_i^{(j)} &= \tilde{a}^{(j)}\left(\frac{\tilde{p}_i^{(j)}-p_i}{p_m}\right)\\
&= \frac{\tilde{a}^{(j)}}{p_m}\left[\bar{p}_i+\nu_1(\hat{p}_i-\bar{p}_i)+\nu_2\zeta_i^{(j)}\bar{p}_i-\bar{p}_i-\theta\,(\hat{p}_i-\bar{p}_i)\right]\\
&= \frac{\tilde{a}^{(j)}}{p_m}\left[(\theta-\nu_1)(\bar{p}_i-\hat{p}_i)+\nu_2\zeta_i^{(j)}\bar{p}_i\right]. \qquad \text{(A35)}
\end{aligned}$$

Because market-adjusted returns are uncorrelated across stocks, the covariance between the market-adjusted returns of managers j and k is

$$\begin{aligned}
\sigma_A^{(j,k)} &= \sum_{i=1}^{N}\phi_i^{(j)}\phi_i^{(k)}\mathrm{Var}(R_i)\\
&= \sum_{i=1}^{N}\frac{\tilde{a}^{(j)}}{p_m}\left[(\theta-\nu_1)(\bar{p}_i-\hat{p}_i)+\nu_2\zeta_i^{(j)}\bar{p}_i\right]\frac{\tilde{a}^{(k)}}{p_m}\left[(\theta-\nu_1)(\bar{p}_i-\hat{p}_i)+\nu_2\zeta_i^{(k)}\bar{p}_i\right]\left(\frac{p_m}{p_i}\right)\sigma^2\\
&= \sigma^2\frac{\tilde{a}^{(j)}\tilde{a}^{(k)}}{p_m}\left[(\theta-\nu_1)^2\sum_{i=1}^{N}\frac{(\bar{p}_i-\hat{p}_i)^2}{p_i}+(\theta-\nu_1)\nu_2\sum_{i=1}^{N}\frac{(\bar{p}_i-\hat{p}_i)\bar{p}_i}{p_i}\left(\zeta_i^{(j)}+\zeta_i^{(k)}\right)\right.\\
&\qquad\left.+\,\nu_2^2\bar{p}_i^2\sum_{i=1}^{N}\frac{\zeta_i^{(j)}\zeta_i^{(k)}}{p_i}\right]\\
&= \sigma^2\frac{\tilde{a}^{(j)}\tilde{a}^{(k)}}{p_m}\left[(\theta-\nu_1)^2\sum_{i=1}^{N}\frac{(\bar{p}_i-\hat{p}_i)^2}{p_i}+(\theta-\nu_1)\nu_2\sum_{i=1}^{N}\frac{(\bar{p}_i-\hat{p}_i)\bar{p}_i}{\bar{p}_i+\theta\,(\hat{p}_i-\bar{p}_i)}\left(\zeta_i^{(j)}+\zeta_i^{(k)}\right)\right.\\
&\qquad\left.+\,\nu_2^2\bar{p}_i^2\sum_{i=1}^{N}\frac{\zeta_i^{(j)}\zeta_i^{(k)}}{\bar{p}_i+\theta\,(\hat{p}_i-\bar{p}_i)}\right]\\
&= \sigma^2\frac{\tilde{a}^{(j)}\tilde{a}^{(k)}}{p_m}\left[(\theta-\nu_1)^2\sum_{i=1}^{N}\frac{(\bar{p}_i-\hat{p}_i)^2}{p_i}+0+0\right], \qquad \text{(A36)}
\end{aligned}$$

where the zeros reflect the assumption that, conditional on the $\bar{p}_i$'s and $\hat{p}_i$'s, the $\zeta_i^{(j)}$'s and $\zeta_i^{(k)}$'s are mean zero and mutually uncorrelated across managers. From equations (7), (24), and (A28),

$$\sum_{i=1}^{N}\frac{(\bar{p}_i-\hat{p}_i)^2}{p_i}=\frac{1}{\theta^2}\sum_{i=1}^{N}\frac{(\bar{p}_i-p_i)^2}{p_i}=\frac{p_m\psi}{\mu_m^2\theta^2}, \qquad \text{(A37)}$$

which, along with equation (18), allows $\sigma_A^{(j,k)}$ in (A36) to be written as

$$\begin{aligned}
\sigma_A^{(j,k)} &= \sigma^2\frac{1}{p_m}\mu_m\left(\frac{f^{(j)}p_m}{cW^{(j)}\tilde{\psi}}\right)^{1/2}\mu_m\left(\frac{f^{(k)}p_m}{cW^{(k)}\tilde{\psi}}\right)^{1/2}(\theta-\nu_1)^2\frac{p_m\psi}{\mu_m^2\theta^2}\\
&= \sigma^2\left(\frac{f^{(j)}}{W^{(j)}}\right)^{1/2}\left(\frac{f^{(k)}}{W^{(k)}}\right)^{1/2}\left(\frac{p_m\psi}{c\tilde{\psi}}\right)\left(\frac{\theta-\nu_1}{\theta}\right)^2. \qquad \text{(A38)}
\end{aligned}$$

Dividing $\sigma_A^{(j,k)}$ by $\sigma_A^{(j)}\sigma_A^{(k)}$, using equation (22), gives the correlation between the returns on funds j and k as

$$\begin{aligned}
\rho^{(j,k)} &= \sigma^2 \left(\frac{f^{(j)}}{W^{(j)}}\right)^{1/2} \left(\frac{f^{(k)}}{W^{(k)}}\right)^{1/2} \left(\frac{p_m \psi}{c\tilde{\psi}}\right) \left(\frac{\theta - \nu_1}{\theta}\right)^2 \left[\sigma \left(\frac{f^{(j)} p_m}{cW^{(j)}}\right)^{1/2}\right]^{-1} \left[\sigma \left(\frac{f^{(k)} p_m}{cW^{(k)}}\right)^{1/2}\right]^{-1} \\
&= \left(\frac{\psi}{\tilde{\psi}}\right) \left(\frac{\theta - \nu_1}{\theta}\right)^2 \\
&= \rho^2. \qquad \text{(A39)}
\end{aligned}$$

The resulting variance-covariance matrix of fund returns is therefore

$$\Sigma_A = \Lambda \left[(1-\rho^2) I + \rho^2 \iota\iota'\right] \Lambda, \qquad \text{(A40)}$$

where Λ denotes the diagonal matrix whose j-th diagonal element is $\sigma_A^{(j)}$.

To solve for the implied optimal allocation across funds, invert the right-hand side of (A40) and substitute for Σ^{-1} in equation (A16), giving

$$\omega = s_1 \Lambda^{-1} \left[\frac{1}{1-\rho^2} I - \frac{\rho^2 (1-\rho^2)}{1 + (M-1)\rho^2} \iota\iota'\right] \Lambda^{-1} \alpha, \qquad \text{(A41)}$$

where $s_1 = (\iota' \Sigma_A^{-1} \alpha)$. Observe, using equations (21) and (22), that

$$\begin{aligned}
\left(\sigma_A^{(j)}\right)^{(-1)} \alpha_A^{(j)} &= \frac{1}{\sigma} \left(\frac{f^{(j)} p_m}{cW^{(j)}}\right)^{-1/2} \left[\left(\frac{f^{(j)} \psi p_m}{cW^{(j)}}\right)^{1/2} \rho - 2 f^{(j)}\right] \\
&= \frac{1}{\sigma} \left[\psi^{1/2} - 2 \left(\frac{p_m}{c f^{(j)} W^{(j)}}\right)\right] \\
&= \frac{1}{\sigma} \left[\psi^{1/2} - 2 \left(\frac{p_m}{cg}\right)\right], \qquad \text{(A42)}
\end{aligned}$$

which does not depend on j, so

$$\Lambda^{-1} \alpha = s_2 \iota, \qquad \text{(A43)}$$

where s_2 is the (scalar) value on the right-hand side of (A42). Substituting for $\Lambda^{-1}\alpha$ in equation (A41) gives

$$\begin{aligned}
\omega &= s_3 \Lambda^{-1} \left[\frac{1}{1-\rho^2} I - \frac{\rho^2(1-\rho^2)}{1+(M-1)\rho^2} \iota\iota'\right] \iota \\
&= s_3 \left[\frac{1}{1-\rho^2} - \frac{\rho^2 (1-\rho^2) M}{1 + (M-1)\rho^2}\right] \Lambda^{-1} \iota \\
&= s_4 \Lambda^{-1} \iota, \qquad \text{(A44)}
\end{aligned}$$

where s_3 equals $s_1 s_2$ and s_4 equals s_3 times the bracketed (scalar) value in equation (A44). Therefore,

$$\begin{aligned}
\omega^{(j)} &= s_4 \left[\sigma_A^{(j)}\right]^{-1} = \frac{s_4}{\sigma} \left(\frac{f^{(j)} p_m}{cW^{(j)}}\right)^{-1/2} = \frac{s_4}{\sigma} \left(\frac{c f^{(j)} W^{(j)}}{p_m}\right)^{1/2} \frac{1}{f^{(j)}} = \frac{s_4}{\sigma} \left(\frac{cg}{p_m}\right)^{1/2} \frac{1}{f^{(j)}} \\
&\propto \frac{1}{f^{(j)}}, \qquad \text{(A45)}
\end{aligned}$$

thereby verifying the the conjectured allocation in equation (A17).

Proof of Proposition 3: Given the weights in (A17), the alpha of the aggregate active portfolio is given by

$$\begin{aligned}
\alpha_A &= \sum_{j=1}^{M} \omega^{(j)} \alpha_A^{(j)} \\
&= \sum_{j=1}^{M} \frac{f}{Mf^{(j)}} \left[\left(\frac{f^{(j)} \psi p_m}{cW^{(j)}} \right)^{1/2} \rho - 2f^{(j)} \right] \\
&= f \sum_{j=1}^{M} \frac{1}{M} \left(\frac{\psi p_m}{cf^{(j)} W^{(j)}} \right)^{1/2} \rho - 2f \\
&= f \left[\left(\frac{\psi p_m}{cg} \right)^{1/2} \rho - 2 \right] \\
&= f \left[\left(\frac{\psi}{(c/M) f y (1-h) N} \right)^{1/2} \rho - 2 \right].
\end{aligned} \tag{A46}$$

Note from equation (A44) and the identity $\iota'\omega = 1$ that

$$\omega = (\iota' \Lambda^{-1} \iota)^{-1} \Lambda^{-1} \iota, \tag{A47}$$

so pre- and post-multiplying Σ_A in equation (A40) by this expression for ω gives the variance of the aggregate active portfolio as

$$\begin{aligned}
\sigma_A^2 &= \omega' \Sigma_A \omega \\
&= (\iota' \Lambda^{-1} \iota)^{-2} \iota' \Lambda^{-1} \left(\Lambda \left[(1-\rho^2) I + \rho^2 \iota \iota' \right] \Lambda \right) \Lambda^{-1} \iota \\
&= (\iota' \Lambda^{-1} \iota)^{-2} M \left[(M-1)\rho^2 + 1 \right] \\
&= \left[\sum_{j=1}^{M} \left(\sigma_A^{(j)} \right)^{-1} \right]^{-2} M \left[(M-1)\rho^2 + 1 \right] \\
&= \sigma^2 \left[\sum_{j=1}^{M} \frac{1}{f^{(j)}} \left(\frac{p_m}{cf^{(j)} W^{(j)}} \right)^{-1/2} \right]^{-2} M \left[(M-1)\rho^2 + 1 \right] \\
&= \sigma^2 \left[\frac{\sum_{j=1}^{M} \frac{1}{f^{(j)}}}{M} \right]^{-2} \left(\frac{p_m}{cg} \right) \left[\frac{(M-1)\rho^2 + 1}{M} \right] \\
&= \sigma^2 f^2 \frac{1}{(c/M) f y (1-h) N} \left[\frac{(M-1)\rho^2 + 1}{M} \right] \\
&= \frac{\bar{\sigma}^2 f}{(c/M) y (1-h) N}.
\end{aligned} \tag{A48}$$

Applying the equilibrium condition $y = \gamma(\alpha_A / \sigma_A^2)$ in equation (33) requires that y solves, using equations (A46) and (A48),

$$y = \gamma f \left[\left(\frac{\psi}{(c/M) f y (1-h) N} \right)^{1/2} \rho - 2 \right] \left[\frac{\bar{\sigma}^2 f}{[(c/M) y (1-h) N]} \right]^{-1}. \tag{A49}$$

The solution for y, given by equation (39), can be readily verified after rewriting equation (A49) as

$$y = \frac{\gamma}{\bar{\sigma}^2}\left[y^{1/2}\left(\frac{(c/M)(1-h)N}{f}\right)^{1/2}\left(\rho^2\psi\right)^{1/2} - 2y(c/M)(1-h)N\right]. \tag{A50}$$

Substituting fy from equation (39) into the last expression in equation (A46) gives

$$\alpha_A = f\left[\psi^{1/2}\rho\left((c/M)\rho^2\psi N(1-h)(c/M)\left(\frac{\gamma}{\bar{\sigma}^2+2\gamma N(1-h)(c/M)}\right)^2(1-h)N\right)^{-1/2} - 2\right],$$

and simplifying gives equation (35). The first equality in equation (36) is given by the square root of the last expression in equation (A48). Substituting y from equation (39) into that expression gives

$$\sigma_A = \bar{\sigma}f\left(\rho^2\psi N(1-h)(c/M)\left(\frac{\gamma}{\bar{\sigma}^2+2\gamma N(1-h)(c/M)}\right)^2(1-h)N(c/M)\right)^{-1/2},$$

and simplifying gives equation (36). It is straightforward to verify that dividing the right-hand side of equation (35) by the rightmost expression in equation (36) gives equation (40). Equation (41) is obtained by substituting the pricing relation in (A28) into equation (A21), giving the aggregate active weight in stock i as

$$\begin{aligned}
\phi_i &= \mu_m\left(\frac{f}{(c/M)\tilde{\psi}y(1-h)N}\right)^{1/2}\left(\frac{-\theta(\hat{p}_i-p_i)}{p_m}+\nu_1\frac{\hat{p}_i-\bar{p}_i}{p_m}\right)\\
&= \mu_m\left(\frac{f}{(c/M)\tilde{\psi}y(1-h)N}\right)^{1/2}(\nu_1-\theta)\left(\frac{\hat{p}_i-\bar{p}_i}{p_m}\right)\\
&= \left(\frac{\mu_m\sigma_A}{\bar{\sigma}\tilde{\psi}^{1/2}}\right)(\nu_1-\theta)\left(\frac{\hat{p}_i-\bar{p}_i}{p_m}\right),
\end{aligned} \tag{A51}$$

where the last equality uses equation (36).

Proof of Proposition 4:

Substituting for fy from equation (39),

$$fy = \rho^2\psi N(1-h)(c/M)\left(\frac{\gamma}{\bar{\sigma}^2+2\gamma N(1-h)(c/M)}\right)^2,$$

into equation (A29) yields

$$\begin{aligned}
\theta &= \left[1+\nu_1\frac{\mu_m}{h}\left(\frac{\rho^2\psi N(1-h)(c/M)\left(\frac{\gamma}{\bar{\sigma}^2+2\gamma N(1-h)(c/M)}\right)^2 N(1-h)}{(c/M)\tilde{\psi}}\right)^{1/2}\right]\\
&\quad\times\left[1+\frac{\mu_m}{h}\left(\frac{\rho^2\psi N(1-h)(c/M)\left(\frac{\gamma}{\bar{\sigma}^2+2\gamma N(1-h)(c/M)}\right)^2 N(1-h)}{(c/M)\tilde{\psi}}\right)^{1/2}\right]^{-1}
\end{aligned}$$

$$
\begin{aligned}
&= \left[1+\nu_1\left(\frac{\rho^2\psi}{\tilde{\psi}}\right)^{1/2}\frac{(1-h)}{h}\left(\frac{\gamma\mu_m}{\bar{\sigma}^2/N+2\gamma(1-h)(c/M)}\right)\right] \\
&\quad\times\left[1+\left(\frac{\rho^2\psi}{\tilde{\psi}}\right)^{1/2}\frac{(1-h)}{h}\left(\frac{\gamma\mu_m}{\bar{\sigma}^2/N+2\gamma(1-h)(c/M)}\right)\right]^{-1} \\
&= \left[1+\nu_1\left(\frac{\theta-\nu_1}{\theta}\right)\left(\frac{\psi}{\tilde{\psi}}\right)\frac{(1-h)}{h}\left(\frac{\gamma\mu_m}{\bar{\sigma}^2/N+2\gamma(1-h)(c/M)}\right)\right] \\
&\quad\times\left[1+\left(\frac{\theta-\nu_1}{\theta}\right)\left(\frac{\psi}{\tilde{\psi}}\right)\frac{(1-h)}{h}\left(\frac{\gamma\mu_m}{\bar{\sigma}^2/N+2\gamma(1-h)(c/M)}\right)\right]^{-1},
\end{aligned}
\tag{A52}
$$

where the last equality applies equation (32).

Proof of Proposition 5: Equations (44) and (45) come directly from equation (A29). Equations (46) and (47) are the same as equations (A46) and (A48). Equation (48) is obtained by substituting from equation (27) into equation (A21) and then simplifying by substituting σ_A using equation (47).

Table 1

Parameter Values Used in the Quantitative Analysis

Description	Parameter	Values
Expected gross return on the stock market	μ_m	1.065
Market price of risk	γ	0.7238
Conditional idiosyncratic volatility of individual stocks	σ	0.188
Number of stocks in the market	N	6893
Proportional trading cost $\div$ fraction of market cap. traded	c	0.967
Number of active managers (funds)	M	2212
Fraction of stock market owned by noise traders	h	0.2
Active management's (harmonic) mean fee rate	f	0.01
Fraction of noise-trader noise in managers' beliefs	ν_1	[0, 1)
Standard deviation of manager-specific relative valuation errors	ν_2	(0, 300]
Noise-trader noise parameter (smaller value $\Leftrightarrow$ more noise)	k	[0.3, 2]

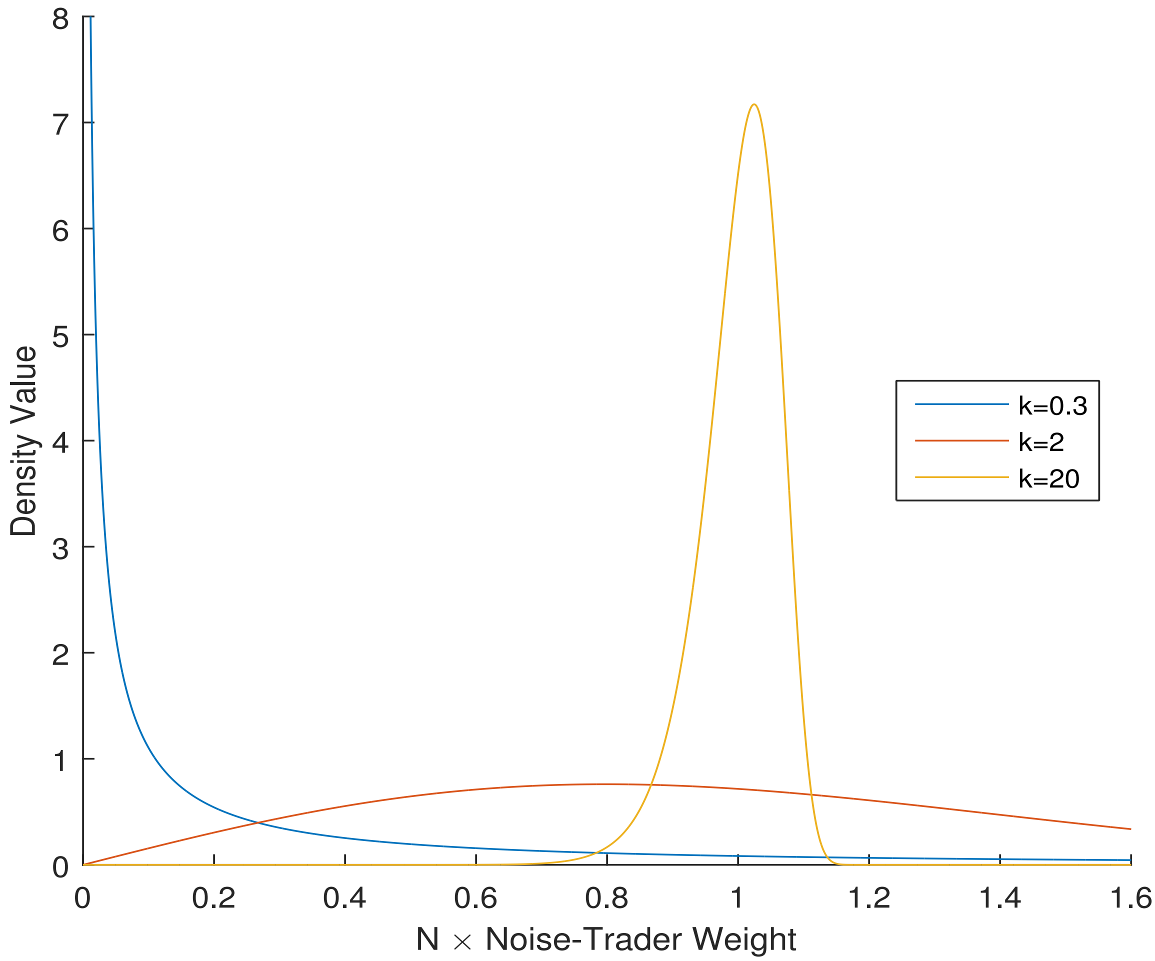

Figure 1. Noise Trading Densities. The figure plots alternative specifications of a Weibull density for approximating the cross-sectional distribution of $N\phi_{H,i}$, where N is the number of stocks in the market and $\phi_{H,i}$ is the aggregate weight that noise traders place in stock i. All densities have 1.0 as the mean and differ with respect to the shape parameter k.

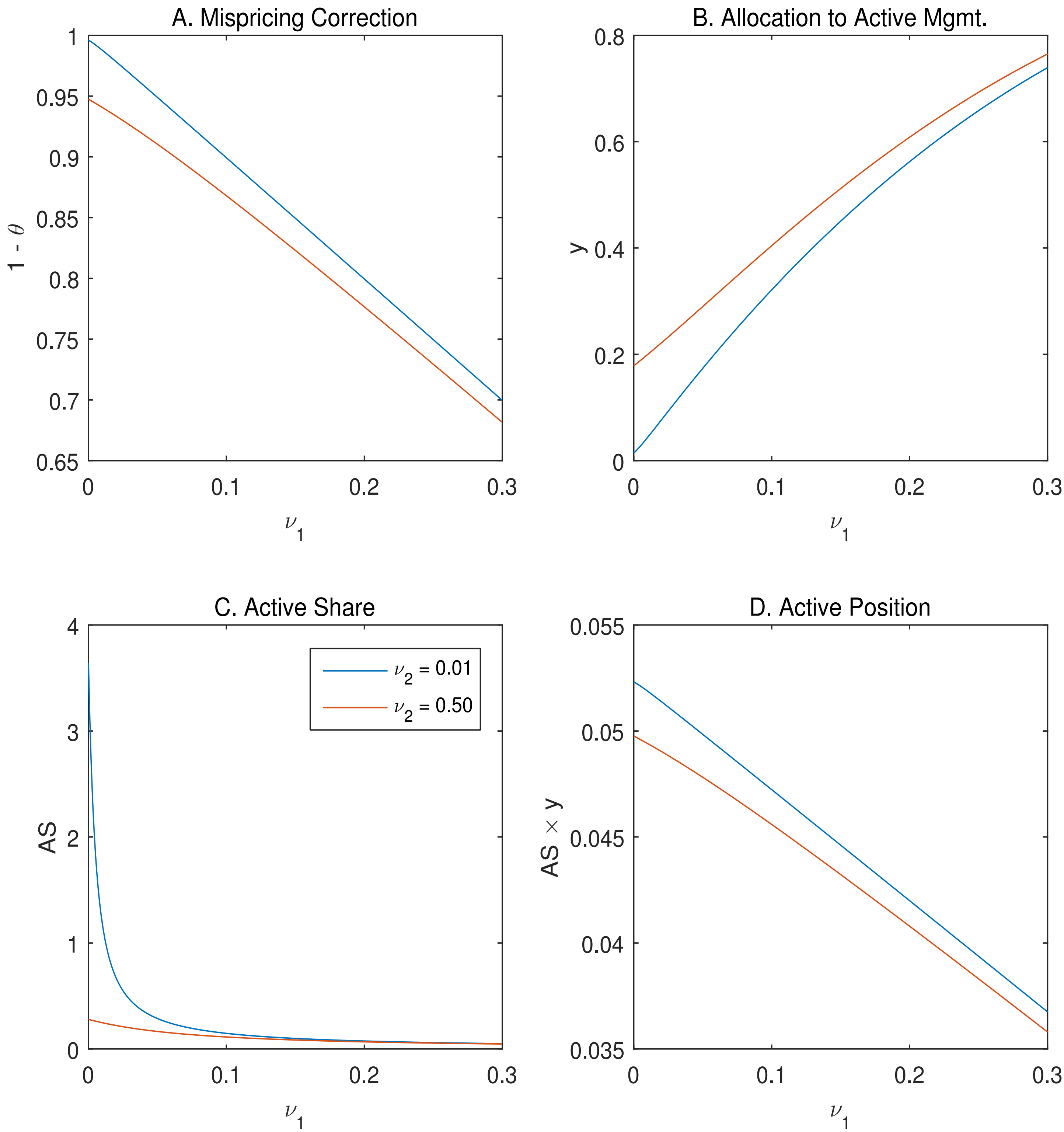

Figure 2. In equilibrium, the degree of mispricing correction, and active management's allocation, active share, and active position versus ν_1. The parameter ν_1 represents the fraction of noise-trader distortions present in active manager's beliefs, and ν_2 is the standard deviation of each manager's idiosyncratic deviations of beliefs relative to fundamental value. The noise-trader distribution has parameter $k = 2$.

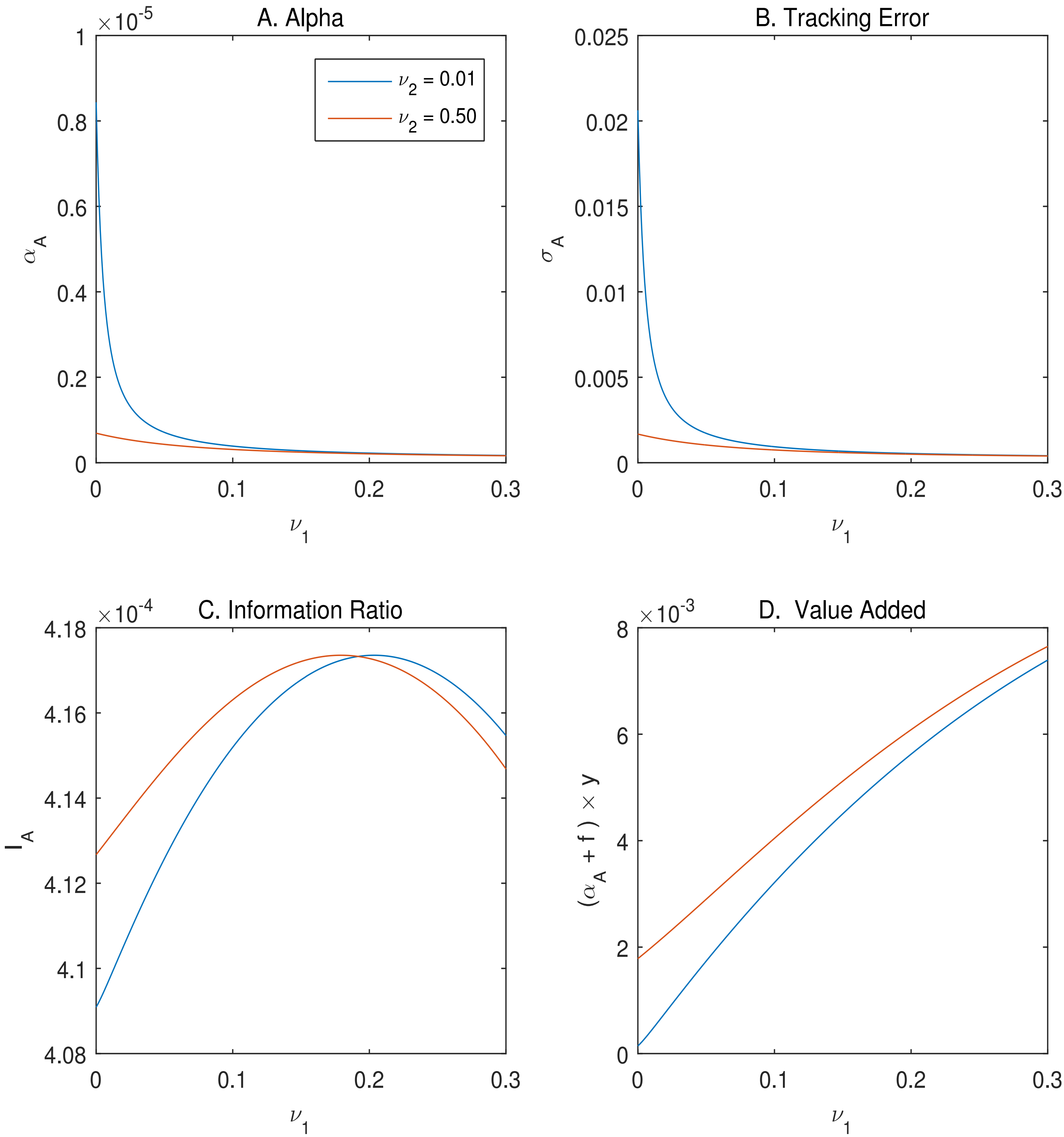

Figure 3. In equilibrium, active management's net alpha, standard deviation of benchmark-adjusted return (tracking error), information ratio, and value added versus $\boldsymbol{\nu_1}$. The parameter ν_1 represents the fraction of noise-trader distortions present in active manager's beliefs, and ν_2 is the standard deviation of each manager's idiosyncratic deviations of beliefs relative to fundamental value. The noise-trader distribution has parameter $k = 2$.

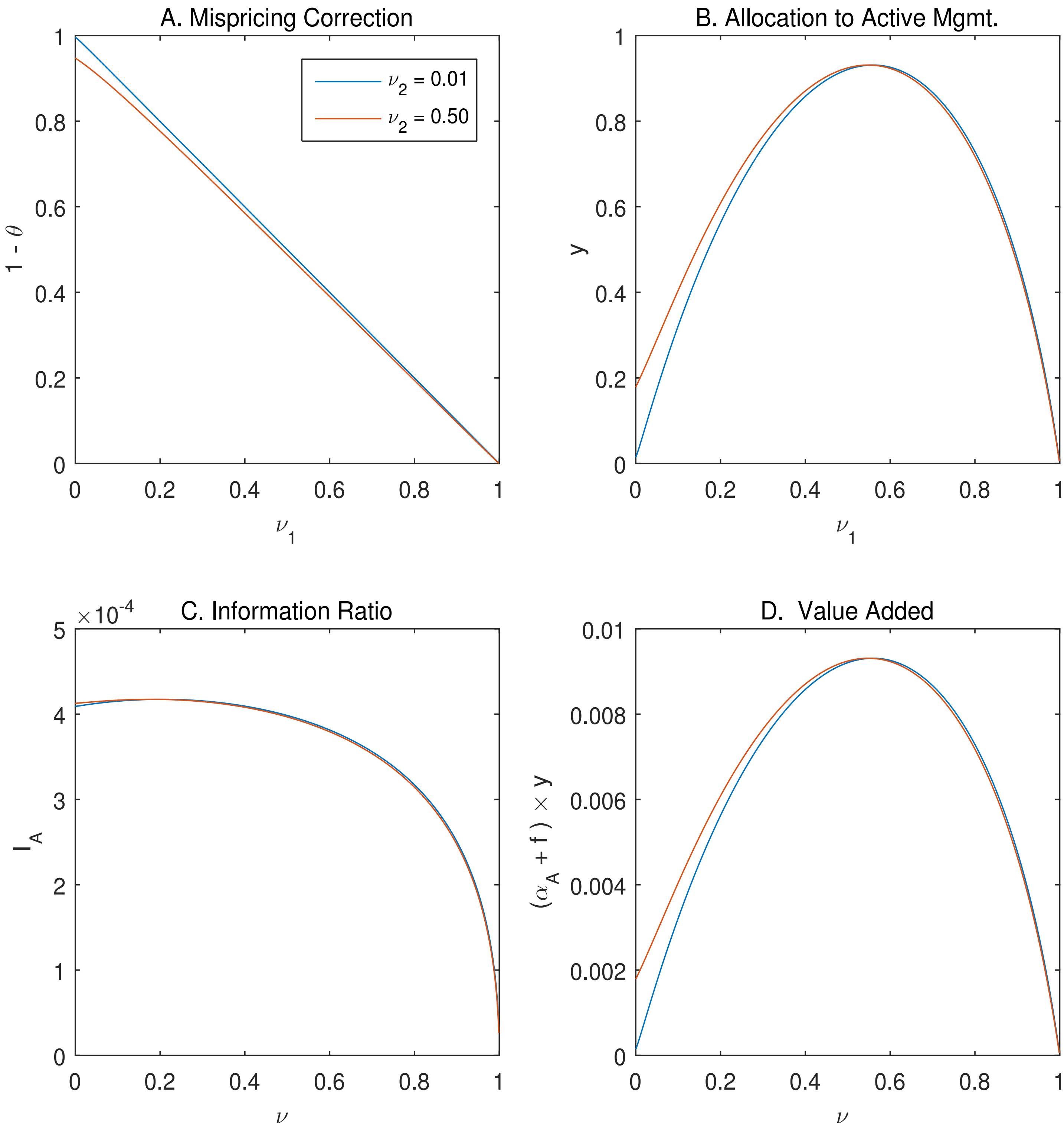

Figure 4. In equilibrium, the degree of mispricing correction, and active management's allocation, information ratio, and value added versus ν_1 over an expanded range. The parameter ν_1 represents the fraction of noise-trader distortions present in active manager's beliefs, and ν_2 is the standard deviation of each manager's idiosyncratic deviations of beliefs relative to fundamental value. The noise-trader distribution has parameter $k = 2$.

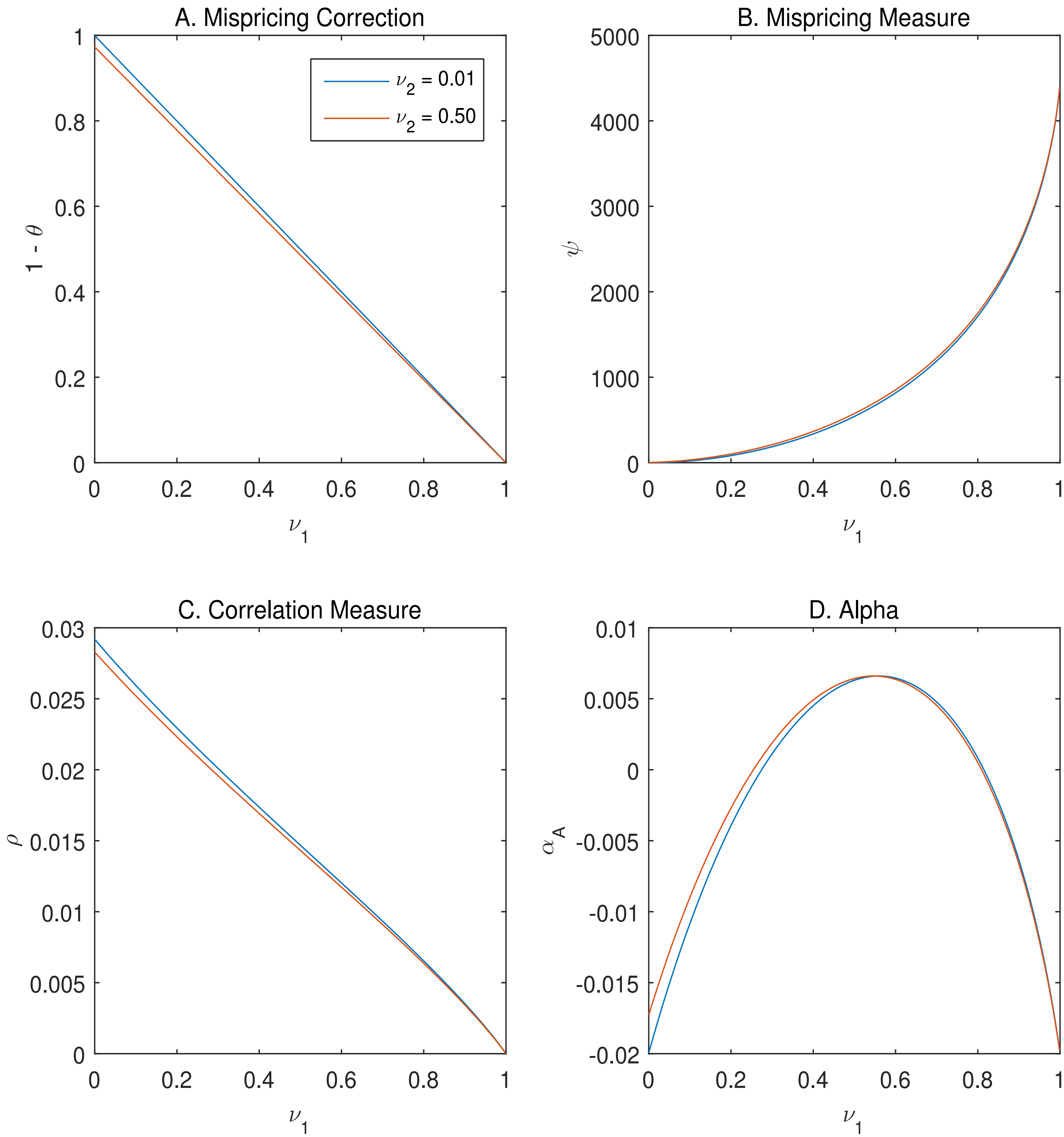

Figure 5. For equilibrium prices under a fixed (non-optimal) 70% allocation to active management, the degree of mispricing correction, the resulting mispricing measure (ψ), and active management's correlation measure (ρ) and net alpha versus ν_1. The parameter ν_1 represents the fraction of noise-trader distortions present in active manager's beliefs, and ν_2 is the standard deviation of each manager's idiosyncratic deviations of beliefs relative to fundamental value. The noise-trader distribution has parameter $k = 2$.

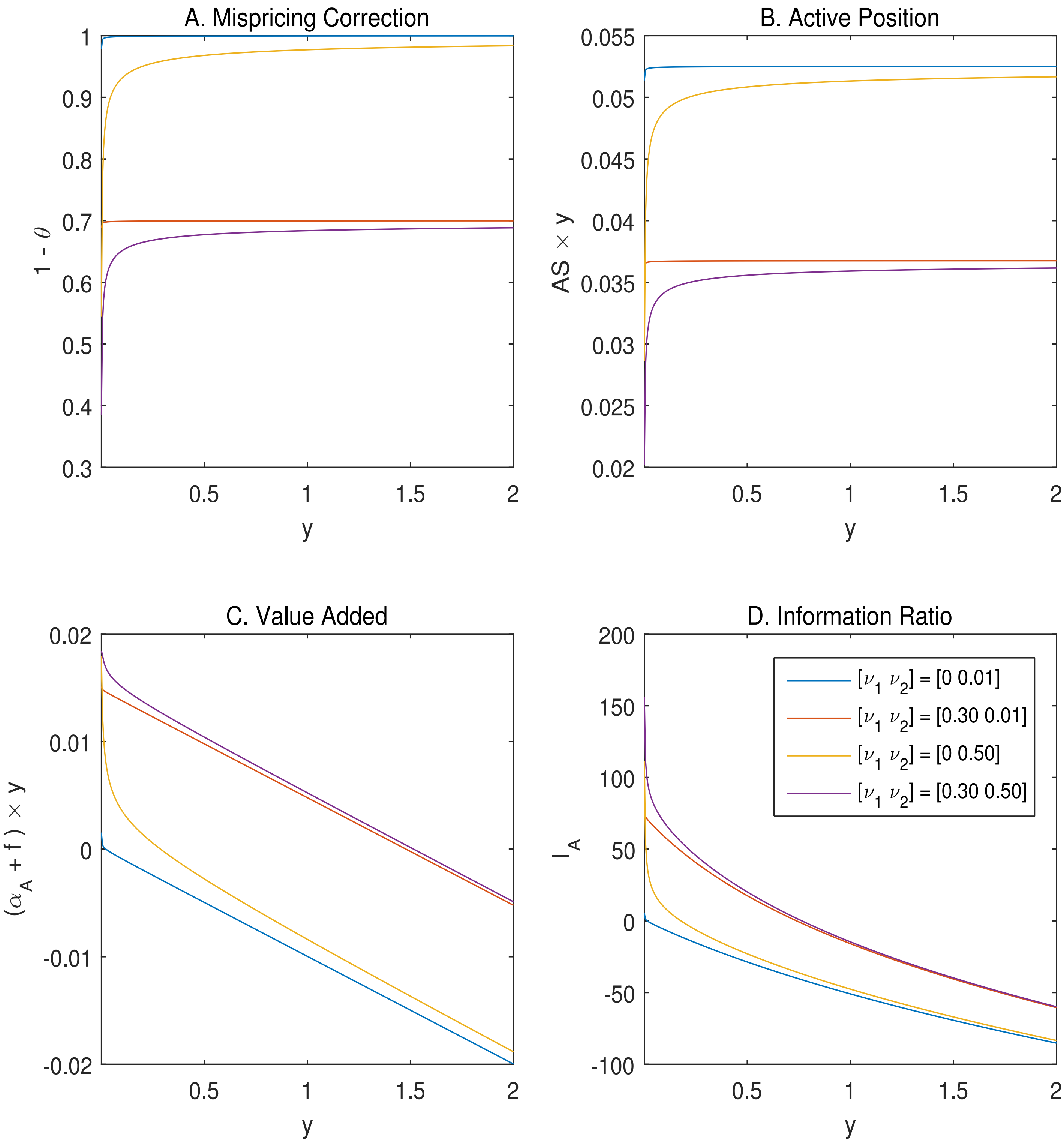

Figure 6. For equilibrium prices under a given (non-optimal) allocation, *y*, to active management, the degree of mispricing correction and active management's active position, value added, and information ratio . The parameter ν_1 represents the fraction of noise-trader distortions present in active manager's beliefs, and ν_2 is the standard deviation of each manager's idiosyncratic deviations of beliefs relative to fundamental value. The noise-trader distribution has parameter $k = 2$.

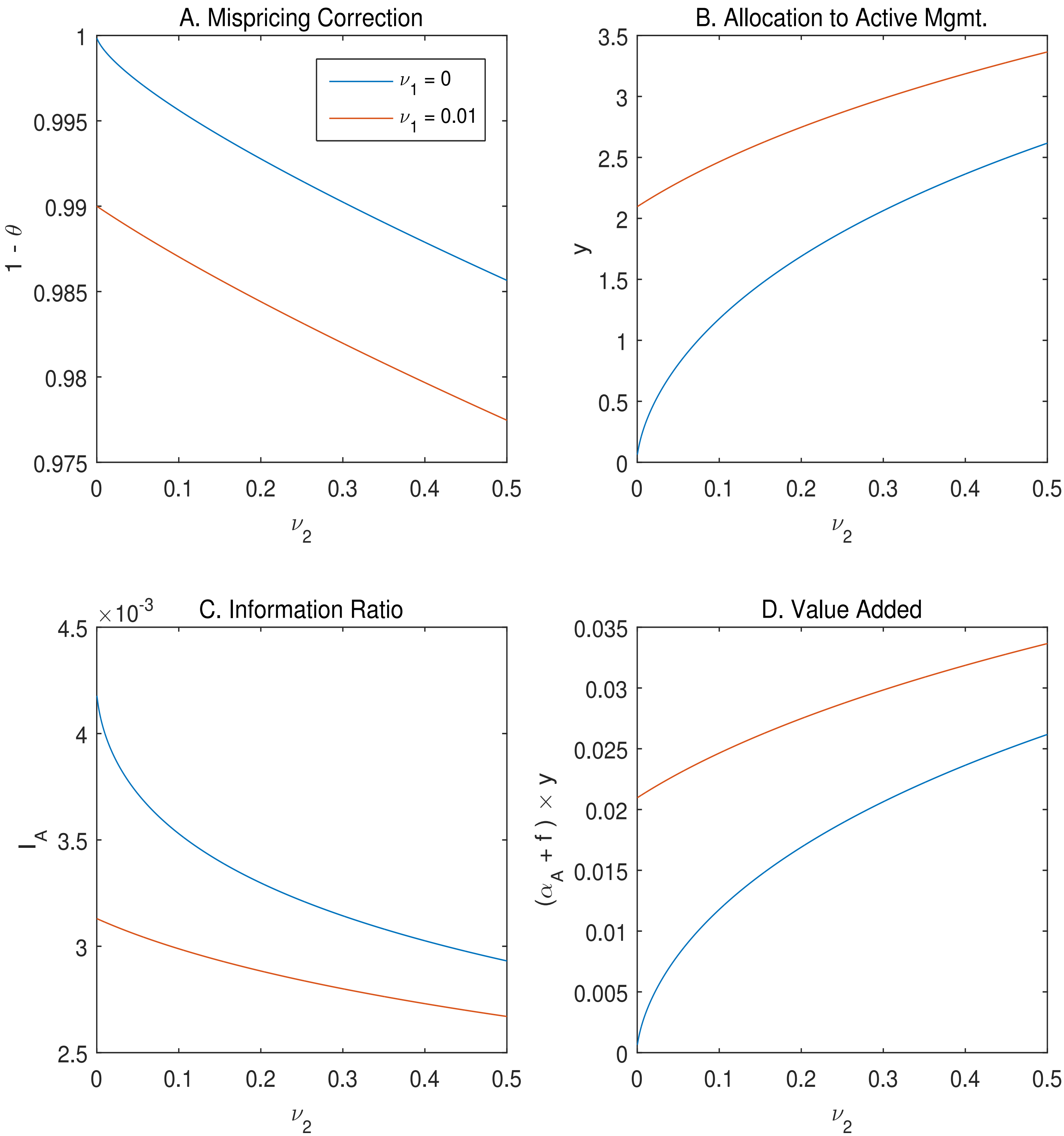

Figure 7. In equilibrium, the degree of mispricing correction, and active management's allocation, information ratio, and value added versus ν_2. The parameter ν_1 represents the fraction of noise-trader distortions present in active manager's beliefs, and ν_2 is the standard deviation of each manager's idiosyncratic deviations of beliefs relative to fundamental value. The noise-trader distribution has parameter $k = 0.3$.

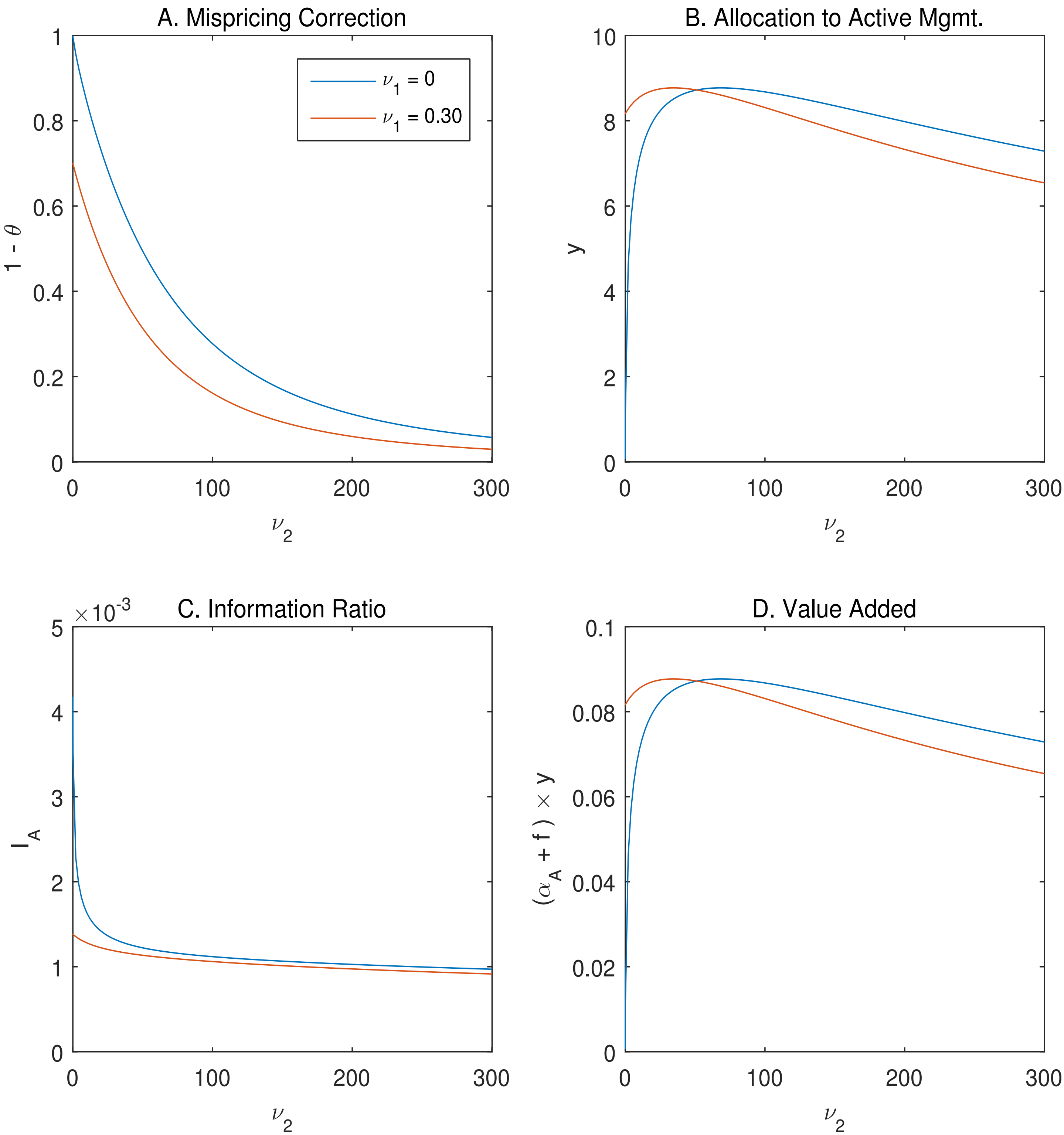

Figure 8. In equilibrium, the degree of mispricing correction, and active management's allocation, information ratio, and value added versus $\boldsymbol{\nu_2}$ over an expanded range. The parameter ν_1 represents the fraction of noise-trader distortions present in active manager's beliefs, and ν_2 is the standard deviation of each manager's idiosyncratic deviations of beliefs relative to fundamental value. The noise-trader distribution has parameter $k = 0.3$.

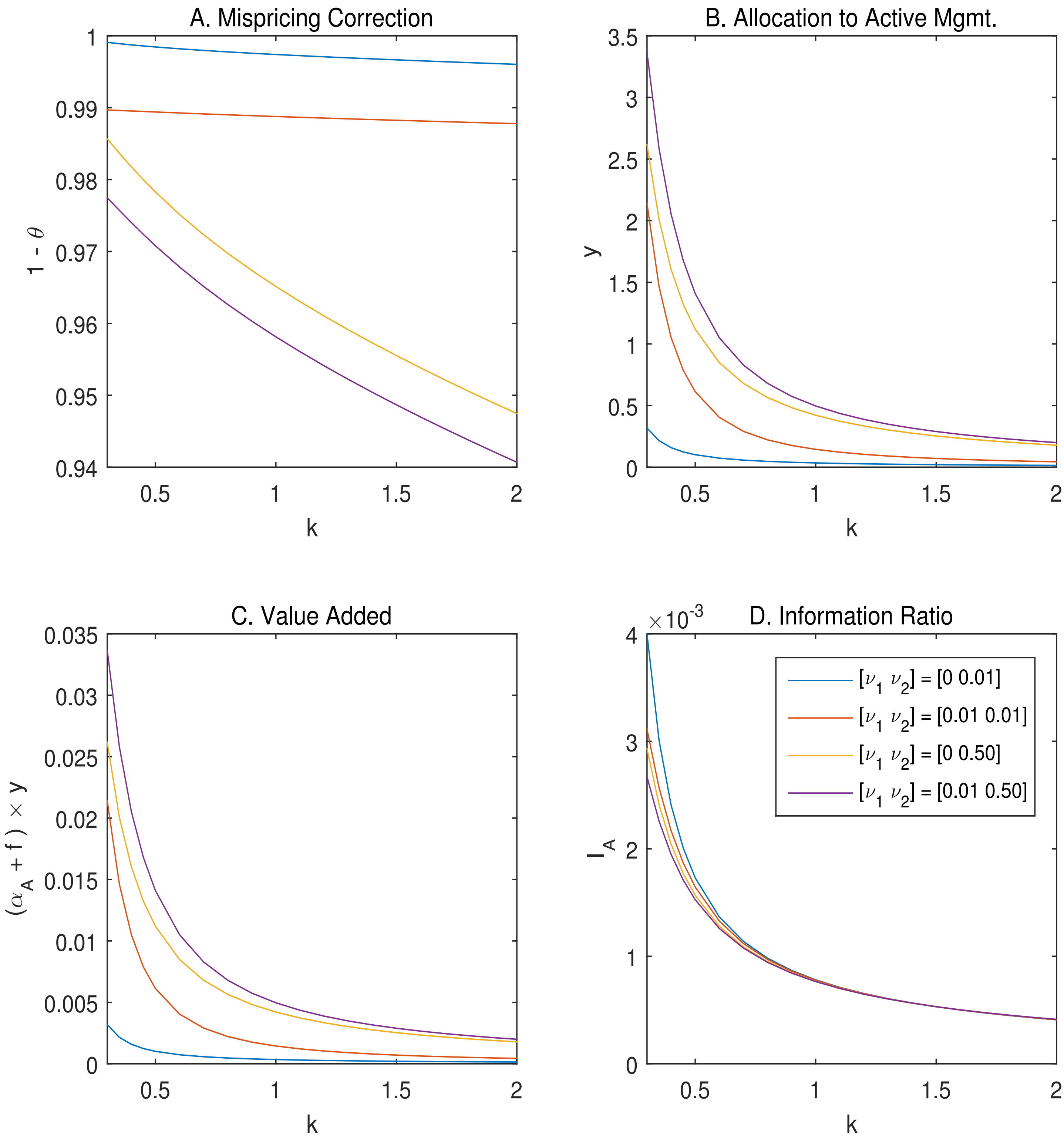

Figure 9. In equilibrium, the degree of mispricing correction and active management's allocation, active position, and information ratio versus *k*. The noise in noise-traders's demands is decreasing in the parameter k. The parameter ν_1 represents the fraction of noise-trader distortions present in active manager's beliefs, and ν_2 is the standard deviation of each manager's idiosyncratic deviations of beliefs relative to fundamental value.

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