Innovation, Competition, and Investment Timing*

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Abstract

In our model multiple innovators compete against each other by submitting investment proposals to an investor. The investor chooses the least expensive proposal and the timing of the investment. Innovators have to provide costly effort and they learn privately the cost of investing. Innovators' multiple effort costs have to be compensated for, but competition helps to erode innovators' informational rents. Consequently, competition leads to faster innovation, because the investor has less need to delay expensive investments. With endogenous number of innovators investment timing becomes first-best. When we introduce private benefits for winning the competition, innovators have an incentive to downplay investment costs. The investor may have to counter this by accelerating the timing of inexpensive investments. Increasing competition makes this problem even worse.

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1 Introduction

Innovations are essential for many firms, especially in electronics, information and communications technology (ICT), and healthcare sectors. According to Strategy&, a consulting firm part of PricewaterhouseCoopers, R&D investment by the 1,000 biggest spending public companies reached a record $647 billion in 2014.[1] Because of the significance of innovative activities, it is important to understand how to solve agency problems between innovators and firms. Innovators exert considerable amount of effort to come up with new solutions or products and have to be compensated for this. Innovators also need to share their knowledge with the firms that employ them and have to have proper incentives to do so truthfully. In this paper we study how competition between innovators help to solve these moral hazard and adverse selection issues that may hinder innovative activities. The scenario we have in mind is where innovators have to exert costly effort in order to come up with an investment proposal. While working on the project, innovators also learn privately how expensive it is to invest. The investor offers contracts that solve these agency problems. Our main results show that due to competition, innovators' compensation becomes less sensitive to the profits and that expensive investments take place earlier. With endogenous number of competitors, there is no need to delay investments and investment policy becomes first-best.

We employ a real options framework where investments are irreversible and the investor has to decide when to invest. An innovator has an incentive to inflate the costs if he thinks he will be awarded the contract: by declaring a high cost for the project the innovator can capture a difference between the declared and the true cost for himself. The investor can use two tools to solve this problem: provide higher compensation if the declared investment cost is low and delay the investment if the declared cost is high. By delaying the expensive investment the investor lowers the present value of the innovator's compensation, thus

[1]Strategy&: The Global Innovation 1000, available at http://www.strategyand.pwc.com/media/file/The-2014-Global-Innovation-1000_media-report.pdf

reducing the innovator's incentive to lie about the true cost of investing.

We formulate a principal-multiple agents model, in which the principal – the investor – can choose the number of agents – innovators – she can offer the contract to. The single agent case has been previously analyzed by Grenadier and Wang (2005), Mæland (2002), and Laffont and Martimort (2002, pp. 294-298). We add a feature from auction theory where agents compete in a "winner-take-all" fashion to obtain an incentive contract, as described in Klemperer (1999) and Laffont and Tirole (1987). A key insight in our model is that when innovators have to compete for a contract, their incentive to inflate costs is diminished. Each innovator would like to declare that the investment is expensive even if it is not the case and thus capture extra compensation for himself. This incentive to inflate the cost of investment is reduced by the presence of many innovators competing for the contract: by falsely declaring a high cost, an innovator would end up losing the contract to another agent that truthfully revealed his low cost.

Competition for contracts and the resulting erosion of informational rents has two implications: Firstly, the investment would not be delayed as much as it would have to be in the one agent case. As informational rents are decreasing in the number of innovators participating in the competition, it follows that investment delays are also decreasing in the number of innovators. Secondly, the winning innovator's compensation becomes less sensitive to the net cash flows from the investment. To induce the innovator not to inflate the cost the investor has to promise compensation that is increasing in the net cash flows. The need for this is thus reduced with competition.

We then proceed to show that when the investor can choose the number of innovators she will contract with, there are no investment delays, i.e., first-best investment policy is always achieved. Thus in equilibrium the informational rents have been completely eroded by competition. The agency problems are reduced to a pure moral hazard problem where the investor only needs to worry about providing incentives for the innovators to exert high effort. As a consequence, the winning innovator's compensation becomes completely insensitive to the cash flows. The reason is that in a competition the winner receives the effort costs of all the participating innovators. Thus winning the competition becomes so valuable that the expected value of inflating the investment cost is not enough to compensate for potentially losing the competition because of it.

We show that the optimal number of innovators is increasing in the value of the investment and decreasing in effort costs. With very high effort costs, a single innovator is enough to achieve first-best investment policy. Also, the more difficult the task, the more innovators should be invited to participate in the competition. The reason is that with an easy task (high *ex-ante* probability of a having a low cost investment), it is more likely that several innovators come up with low cost projects. Thus it is not worth the risk for any innovator to inflate the costs and likely lose the contract because of that. As a result, the investor can save some money by inviting fewer participants.

Interestingly, we also show that the optimal number of innovators is decreasing in the volatility of the project. When volatility increases, the value of the investment option increases for both the low cost and high cost projects. However, the difference in value between these projects decreases. As a result the innovators would have less of an incentive to inflate the costs, leading to shorter investment delays. To accommodate this, the investor has an incentive to decrease the number of agents.

In an extension of our model we introduce non-pecuniary private benefits the winning innovator receives if he manages to come up with a low cost investment proposal. In this situation the innovator who has a high cost proposal may have an incentive to lie and report a low cost investment in order to capture the private benefits, in contrast to our main model. Consequently, the investor may have to relinquish the first-best investment policy for the low cost proposal and invest early. Interestingly, competition makes the adverse selection problem worse and accelerates the timing of the low cost project. Thus our result that competition reduces the adverse selection problem is only applicable in the absence of private benefits.

We organize the rest of the paper as follows. Section 2 gives a literature review, whereas Section 3 outlines the model. Section 4 derives the investment triggers and expected compensations for innovators. Section 5 optimizes the investor's value with respect to the number of innovators invited to participate. Section 6 extends the model by including private benefits for the innovators. Section 7 concludes.

2 Related Literature

How to foster and provide incentives for innovations in the presence of agency conflicts and the role of competition among innovators has been an important topic for a long time in finance and economics literature. In their seminal paper Rotemberg and Saloner (1994) argue that narrow business strategies, which can be thought of as limiting competition between innovators, are good for providing *ex-ante* effort incentives, because narrow strategies serve as a commitment mechanism to adopt innovators' projects. In a similar vein, Fulghieri and Sevilir (2009a) provide a model where limiting competition (small VC portfolios) enhances incentives of entrepreneurs to exert effort *ex-ante*, but more competition (large and focused portfolios) improve the *ex-post* resource allocation. More competition is optimal when innovations are risky and their technologies are related, but less competition dominate when innovations have high expected returns.[2] Inderst, Mueller, and Münnich (2007) explore the incentive benefits of more competition in the form of constrained financing. The creation of shallow pockets forces innovative firms to work hard and compete against each other in order to receive scarce financing. Our paper provides a complimentary rationale for competition among innovators that doesn't rely on *ex-ante* effort incentives or *ex-post* resource allocation: competition for scarce resources eliminates informational rents innovators would

[2] In addition, Fulghieri and Sevilir (2009b) show that granting property rights for entrepreneurs provides *ex-ante* incentives and thus speeds up the process of innovation.

otherwise enjoy.

Our paper is also directly related to the literature on innovation or R&D contests. In an innovation contest the firm has an R&D problem and organizes a contest for outside agents to solve the issue. Following the literature on labor markets (see, e.g., the seminal contributions of Lazear and Rosen, 1981, and Nalebuff and Stiglitz, 1983), innovation contests are modelled as tournaments, where relative performance matters and the prize is fixed. The trade-off is between multiplication of costly efforts and improved likelihood of attaining good solutions. Innovation contests with free entry lead to underprovision of costly effort, as showed by Taylor (1995). Fullerton and McAfee (1999) argue that in order to mitigate the underprovision of effort, the optimal number of contestants should be two. The problem of a lower equilibrium effort can be mitigated by switching from fixed prize to performance contingent prize, as pointed out by Terwiesh and Xu (2008). Che and Gale (2003) further show that auctions, where agents bid for R&D contracts, outperform fixed-prize tournaments. We contribute to the literature of innovation contests by showing that the key benefit of contests is the reduction of informational rents that the agents are able to obtain.[3]

Our paper is also related to the large literature on investments as real options. Several papers have studied investment timing in a signalling framework, in contrast to our screening model. In Bouvard (2014) entrepreneurs may delay their investments in order to signal high quality. In Morellec and Schürhoff (2011) and Bustamante (2012) firms have an incentive to speed up investments in order to convey positive information and thus gain access to financing with more lucrative terms. Grenadier and Malenko (2011) provide a more general real options model with signaling where firms have either an incentive to speed up

[3] The auction model of Laffont and Tirole (1987) is set-up in a procurement context, where the government is the principal and private firms are the agents. Laffont and Tirole assume that each firm has private information about its future cost at the contracting stage and afterwards have to provide costly effort, whereas in our model private information is revealed after the effort is exerted. In addition McAfee and McMillan (1986) study auctions in a procurement context, but only with moral hazard.

investments or delay them. Firms will speed up investments if they benefit from highly valued projects, whereas they delay investments if they benefit from low valuations. We contribute to the real options literature by showing that competition helps to solve agency problems and consequently reduce investment distortions.

Product market competition has also an effect on investment timing. Grenadier (2002) shows that competition erodes the value of waiting to invest and firms invest at close to zero net present value threshold. However, Novy-Marx (2007) demonstrates that when firms' production technologies differ firms have an incentive to delay investments, even in the case that competition has eroded all the oligopoly profits. Lambrecht and Perraud (2003) discuss the trade-off between postponing the investment to maximize the option value and invest early to preempt competitors' from investing first and show that competition reduces monopoly rents of an investment option. Unlike in our paper, there are no agency problems in product market and investment timing literature.

3 Introducing the model

An investor (a principal) seeks to invest in an innovative project and invites n innovators (agents) to come up with project proposals. Initially we analyze the situation in which n is fixed, but in Section 5 we discuss how many innovators it is optimal to invite to the competition. At the time of the invitation, the investor announces that she will invest in one of the mutually exclusive proposed projects, and offers a pre-specified contract to all the innovators, stating that the innovator that reports the best project will actually be awarded the contract.

The quality of each project depends on the investment cost of the project. We assume that the investment cost for innovator i's project, K_i, can take one of two values, K^G or K^B, with $K^B - K^G > 0$. We interpret K^G as draw of a high quality (or a "good") project, i.e. a project with a low investment cost. Analogously, K^B refers to a high investment cost, which means that it is a low quality (or a "bad") project.

We assume that the investor is able to commit to the terms of the contract offered. All parties are risk neutral and have the same discount rate $r > 0$. Innovators do not have any initial wealth and that they have limited liability, implying that innovators' compensation has to be non-negative.

The innovators' projects are developed through two phases. In the first phase each innovator has to provide effort to come up with a proposal. By exerting effort, innovator i can influence the probability of the investment cost's level, K_i. Innovators can choose between two effort levels, high and low. Let q represent the probability of K^G when an innovator decides to exert high effort. If the innovator chooses to exert low effort, the innovator's probability of a good project is assumed to be zero. An innovator's cost of high effort is ξ, whereas the cost of low effort is zero. We assume that effort costs are non-pecuniary. Effort cannot be observed by the investor, and is therefore not contractible.

In the second phase the winner of the contract is selected based on the submitted business proposals. If the investor chooses to invest in project i at time t, the payoff from the project is equal to $X_t - K_i$, where X_t is a stochastic variable representing the project's cash flows that is observable to all parties, and K_i is innovator i's privately observed investment cost. More precisely, X_t is the time t value of future uncertain gross profits from a monopoly. It is driven by the process,

$$dX_t = \mu X_t dt + \sigma X_t dW_t, \tag{1}$$

where μ is the expected change in X_t per period, σ is the volatility, or standard deviation, per unit of time, and dW_t is the increment of a standard Wiener process. We assume that there is no traded asset that perfectly correlates with the cash flows from the project.

As X_t changes stochastically over time, we maximize the project value by finding the optimal time to invest in the project. This means that we allow for the possibility that

Figure 1: An overview of the stages of the model.

it may be optimal to postpone or accelerate the investment. The investment options are assumed to be perpetual. We assume $r > \mu$ to ensure that it will be optimal to invest at some future time.[4]

A summary of the timing stages of the model is given in Figure 1. The contract offered consists of a compensation scheme and an investment timing strategy.

Before we move on to discuss the optimal contract, we present the first-best case without agency problems. The first-best investment timing case will serve as a benchmark for our mixed hidden effort and private information problem.

3.1 First-best investment decisions: no hidden effort or private information (the benchmark case)

Let X be the current value of X_t. The function $V(X, K_i)$ denotes the value of a project with innovator i's investment cost when there is no unobservable action and no asymmetric information. The investment project is formulated as a real option: the project value is maximized by finding the optimal time to invest. Let the function $X^*(K_i)$ represent the value of future cash flows that triggers investment. This means that it is optimal to invest immediately when $X \geq X^*(K_i)$. If $X < X^*(K_i)$, the project value is maximized by postponing the investment until X_t hits the trigger $X^*(K_i)$. It is well known (shown in Brennan and Schwartz (1985), McDonald and Siegel (1986), and Dixit and Pindyck (1994), among others) that the project value then is given by

$$V(X, K_i) = \begin{cases} \left(\frac{X}{X^*(K_i)}\right)^{\beta} \left(X^*\left(K_i\right) - K_i\right) & \text{for } X < X^*\left(K_i\right) \\ X - K_i & \text{for } X \geq X^*\left(K_i\right) \end{cases} \tag{2}$$

[4] If the growth rate μ were larger than the discount rate r, it would always be optimal to postpone the investment.

where

$$X^*(K_i) = \frac{\beta}{\beta - 1} K_i, \tag{3}$$

and

$$\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + 2\frac{r}{\sigma^2}} > 1. \tag{4}$$

A proof of Eqs. (2)-(4) is given in Appendix A. The value function in Eq. (2) is the solution of the differential equation and the boundaries in Eqs. (A.1)-(A.4). The value of the constant β depends on the parameters r, μ, and σ. A higher volatility σ implies a lower β and higher value of the fraction $\frac{\beta}{\beta-1}$. Hence, more uncertainty means that the wedge between the investment cost K_i and trigger value for investment $X^*(K_i)$ gets larger. The intuition is that higher volatility makes it worthwhile to wait for a higher value of the stochastic value X_t before the investment is made. The term $(X/X^*(K_i))^{\beta}$ in Eq. (2) is a *discount factor* as it gives the present value of receiving one unit of money at the future time when the value of the stochastic variable reaches $X^*(K_i)$. For the rest of the presentation we assume that the current value of the underlying asset, X, is below the investment trigger for all values of the investment cost, K_i. This simplifies the presentation of the model without loss of generalization.

The value of the project in Eq. (2) is derived under the assumptions that the parties observe the investment cost of project i, and that innovator i is the winner of the contract. Initially, the investor and the innovators do not know whether their projects are of high or low quality. To increase the probability that at least one of the innovators' projects is of high quality, the investor can invite multiple innovators to submit business proposals. However, this comes at a cost as the investor has to compensate the innovators for their effort costs of preparing proposals. In the first-best case, the investor's optimization problem with respect to how many innovators, n, to invite to the contest is a trade-off between these two considerations. We will assume that it is optimal for the innovators to exert high effort.[5]

[5] If low effort were the optimal choice there would be no effort costs and hence no trade-off. Then it would

The model assumes that the investor has the bargaining power. Hence, in the first-best case the investor obtains the entire value of the project the investor selects, and the innovators are compensated only for their effort costs. The investor optimally prefers high quality projects instead of low quality. Let p_n represent the probability that there is at least one innovator with a K^G-type project, i.e., $p_n = 1 - (1-q)^n$. The investor's ex ante value of the investment opportunity is, for a given n, formulated by the value of a high quality investment opportunity weighted by the probability that there is at least one innovator with a high quality project, plus the value of a low quality project weighted by the probability that the investment opportunity is of low quality, minus the effort costs of all the innovators,

$$V_{FB}^{P}(X,n) = p_n \left(\frac{X}{X^*\left(K^G\right)}\right)^{\beta} \left(X^*\left(K^G\right) - K^G\right) + (1-p_n) \left(\frac{X}{X^*\left(K^B\right)}\right)^{\beta} \left(X^*\left(K^B\right) - K^B\right) - n\xi. \tag{5}$$

3.2 Setting with hidden effort and private information

Having defined the first-best benchmark in Section 3.1 we now go back to the setting in which the n innovators exert hidden effort and obtain private information about their respective investment costs. Similarly to Laffont and Tirole (1987) we organize the competition as a "winner-take-all" contract: The innovator awarded the contract shares the value of the project with the investor, whereas the other competitors receive nothing.

The contract between the winner and the investor specifies the investment timing strategy and how the value at the time of investment is shared between the winner and the investor. Let s^G be the winner's compensation at the time of investment if the winner reports the investment cost K^G, and s^B if the winner reports K^B.

The winning project is selected randomly from the pool of projects with the highest net value. Thus, at the stage when each innovator's private information is revealed, each

be optimal for the investor to invite an infinite number of innovators to the contest, as this would ensure that at least one innovator has a good project.

innovator's probability of being awarded the contract is represented by Y_n^G if the innovator announces a good project, or Y_n^B if the innovator reports a bad project. The subscript n indicates that there is n innovators competing for the project. The probability Y_n^B is equal to the probability that none of the other $n-1$ innovators reports K^G,

$$Y_n^B = \frac{1}{n}(1-q)^{n-1}. \tag{6}$$

For innovators of K^G-type, the probability of winning is equal to

$$Y_n^G = \sum_{i=0}^{n-1} \frac{1}{i+1} \binom{n-1}{i} q^i (1-q)^{n-1-i}. \tag{7}$$

To simplify notation we define each innovator's expected compensation, $S^G = Y_n^G s^G$ and $S^B = Y_n^B s^B$.

As each innovator ex ante has identical projects, the investor's portfolio of projects equals the investor's expected values from each project multiplied by the number of innovators competing for the contract, n,

$$V^P(X,n) = n\left[q\left(\frac{X}{X^G}\right)^\beta \left(Y_n^G(X^G-K^G)-S^G\right) + (1-q)\left(\frac{X}{X^B}\right)^\beta \left(Y_n^B(X^B-K^B)-S^B\right)\right]. \tag{8}$$

For a fixed n the investor maximizes her value $V^P(X,n)$ by finding optimal investment strategies, X^G and X^B, and compensation functions, S^G and S^B. This optimization problem is solved in Section 4. In Section 5 we solve the investor's problem of finding the optimal number of innovators to invite to submitting a project proposal.

4 Optimal investment trigger and expected compensation of each innovator

In Eqs. (9)-(15) we present the investor's optimization problem with respect to each innovator's project. The investor's value function to be optimized is formulated as

$$\max_{X^G,X^B,S^G,S^B} q\left(\frac{X}{X^G}\right)^\beta \left(Y_n^G(X^G-K^G)-S^G\right) + (1-q)\left(\frac{X}{X^B}\right)^\beta \left(Y_n^B(X^B-K^B)-S^B\right), \tag{9}$$

subject to the *ex ante* incentive compatibility and participation constraints (11)-(12), and the *ex post* incentive compatibility and participation constraints (13)-(15):

(i) The ex ante moral hazard constraint: The innovators first need to exert effort to get to know their respective investment costs. Each innovator's probability of developing a good project depends on his level of effort. The investor cannot observe innovators' effort choices. To give each innovator incentives for high effort, the investor needs to compensate the innovators such that the value of exerting high effort is higher than the value of exerting low effort,

$$q\left(\frac{X}{X^G}\right)^{\beta} S^G + (1-q)\left(\frac{X}{X^B}\right)^{\beta} S^B - \xi \geq \left(\frac{X}{X^B}\right)^{\beta} S^B. \tag{10}$$

The left-hand side of the equation represents an innovator's value of the project if he exerts high effort, whereas the right-hand side states the value of low effort. We rearrange and simplify the inequality as follows,

$$\left(\frac{X}{X^G}\right)^{\beta} S^G - \left(\frac{X}{X^B}\right)^{\beta} S^B - \frac{\xi}{q} \geq 0. \tag{11}$$

Eq. (11) will be denoted the *moral hazard* constraint of the investor's optimization problem.

(ii) The ex ante participation constraint: To make sure that the innovators participate in the competition, each innovator's value of participating must be positive,

$$q\left(\frac{X}{X^G}\right)^{\beta} S^G + (1-q)\left(\frac{X}{X^B}\right)^{\beta} S^B - \xi \geq 0 \tag{12}$$

(iii) The ex post incentive compatibility constraints: When an innovator has exerted effort, he privately observes his investment cost level. To ensure that each innovator reports his true investment costs, the incentive compatibility constraints need to be satisfied.

If an innovator has a good project, and truthfully reports the low investment cost, his value is given by the present value of the expected compensation S^G. If an innovator with a good project is lying to keep the difference between the reported high investment cost, K^B, and the true low investment cost, K^G, his value will be given by $\left(\frac{X}{X^B}\right)^\beta \left(S^B + Y_n^G \left(K^B - K^G\right)\right)$. Thus, to ensure that the contract does not give incentives to lie, an innovator with a good project must have higher value from truthfully reporting K^G, than from lying,

$$\left(\frac{X}{X^G}\right)^\beta S^G \geq \left(\frac{X}{X^B}\right)^\beta \left(S^B + Y_n^B \Delta K\right), \tag{13}$$

where $\Delta K \equiv K^B - K^G > 0$. We denote the right-hand side value as the value of private information, or informational rents. Note that the investor has three tools at her disposal to reduce an innovator's value of private information. She can increase the investment trigger X^B at which an investment is made when an innovator has reported a bad project, thereby delaying the investment in the bad project. This reduces the innovator's value of reporting a high investment cost, as he would have to wait longer to realize his gain from a bad project. Secondly, the investor can increase the number of competitors. This reduces the K^G-type innovator's informational rents because of a reduced probability of being awarded the contract. Thus, the more competitors, the lower the value of exploiting private information. Thirdly, the investor can also reduce the compensation of an innovator reporting a bad project, S^B.

Analogously, the private information constraint of an innovator with a bad project is given by

$$\left(\frac{X}{X^B}\right)^\beta S^B \geq \left(\frac{X}{X^G}\right)^\beta \left(S^G - Y_n^G \Delta K\right), \tag{14}$$

where the left-hand side of the inequality represents the value of truthtelling, and the right-side is the value of lying. As the probability of winning is higher when reporting a good project than a bad project, a bad innovator may mimic a good innovator if the cost of lying is smaller than the increase in expected compensation. Lying is costly as the innovator is only remunerated for the low investment cost, K^G, but pays the true investment cost, K^B, when he makes the investment.

(iv) The ex post participation constraint: We introduce limited liability constraints of compensation, which means that expected compensation for both types of innovators must be positive,

$$S^G \geq 0, S^B \geq 0 \tag{15}$$

The optimization problem in Eqs. (9)-(15) can be simplified, as shown in Appendix B. Lemma 1 presents the results that leads to a simplified optimization problem.

Lemma 1 (i) *The expected compensation of an innovator with investment cost* K^G, S^G, *is strictly larger than zero.*

(ii) *The expected compensation of the* K^B*-type innovator,* S^B, *is equal to zero.*

(iii) *The ex ante participation constraint in Eq. (12) never binds as it is dominated by the hidden effort constraint in Eq. (11).*

(iv) *The private information constraints of innovators bad projects will never bind.*

(v) *At least one of the constraints in Eqs. (11) and (13) always binds.*

The intuition of (i) is that to ensure truthtelling the compensation of the good type, S^G, must be strictly larger than the compensation of the bad type, S^B. In (ii) the expected compensation of an innovator with investment cost K^B equals zero since there is no reason to give an innovator with the highest investment cost any informational rents. The explanation for (iv) is that it is too expensive for the investor to give incentives for high effort when the compensation is high enough to making the private information constraint of a bad project bind. The result in (v) says that compensation of a good innovator will either be determined by the moral hazard constraint, Eq. (11), or the private information constraint, Eq. (13).

Lemma 1 leaves us with the following simplified optimization problem for the investor,

$$\max_{X^G, X^B, S^G} q \left(\frac{X}{X^G}\right)^\beta \left(Y_n^G(X^G - K^G) - S^G\right) + (1-q) \left(\frac{X}{X^B}\right)^\beta Y_n^B(X^B - K^B), \tag{16}$$

subject to the private information constraint of the good innovator and the moral hazard constraint,

$$\left(\frac{X}{X^G}\right)^\beta S^G \geq \max\left\{\left(\frac{X}{X^B}\right)^\beta Y_n^B \Delta K, \frac{\xi}{q}\right\}. \tag{17}$$

The constraint in Eq. (17) replaces the constraints in Eqs. (11) and (13). If the first term in the max-operator in Eq. (17) has the higher value of the two terms in the operator, the value of a good innovator's compensation must be at least as large as his informational rents. If the second term is the larger one, the binding constraint is the cost of providing the innovators incentives to exert high effort. Note that if effort costs, ξ, were zero, there would be no moral hazard problem, and the optimization problem would be reduced to a pure private information problem. In this case the compensation would not be higher than each innovator's value of private information, equal to the first term in the max operator in Eq. (17).

Optimization of Eq. (16) subject to Eq. (17) results in the optimal investment triggers and corresponding compensation function in Proposition 1. We identify three scenarios with possible combinations of constraints: (*i*) only the private information constraint of a good innovator binds, (*ii*) the moral hazard constraint and the private information constraint of the good innovator bind simultaneously, and (*iii*) only the moral hazard constraint binds.

Proposition 1 *The optimal investment trigger of a good project is equal to the first-best trigger,* $X^{G*} = \frac{\beta}{\beta-1}K^G$.

Let PIG denote private information of a good innovator and MH moral hazard. The *optimal investment trigger of a low-quality project is given by*

$$X^{B^*} = \begin{cases} \frac{\beta}{\beta-1}\left(K^B + \frac{q}{1-q}\Delta K\right) & \text{if the PIG constraint binds} \\ \frac{\beta}{\beta-1}\left(K^B + \lambda_1 \frac{q}{1-q}\Delta K\right) & \text{if the PIG and MH constraints bind jointly} \\ \frac{\beta}{\beta-1}K^B & \text{if the MH constraint binds} \end{cases} \tag{18}$$

where λ_1 is a Lagrange multiplier that takes values between 0 and 1.

The value of the expected compensation of a good project equals

$$\left(\frac{X}{X^{G^*}}\right)^\beta S^{G^*} = \begin{cases} \left(\frac{X}{X^{B^*}}\right)^\beta Y_n^B \Delta K & \text{if the PIG constraint binds} \\ \left(\frac{X}{X^{B^*}}\right)^\beta Y_n^B \Delta K = \frac{\xi}{q} & \text{if the PIG and MH constraints bind jointly} \\ \frac{\xi}{q} & \text{if the MH constraint binds} \end{cases} \tag{19}$$

The regions are defined as follows:

- *The PIG constraint binds:* $\left(\frac{X}{\frac{\beta}{\beta-1}\left(K^B+\frac{q}{1-q}\Delta K\right)}\right)^\beta Y_n^B \Delta K > \frac{\xi}{q}$
- *The PIG and MH constraints bind:* $\left(\frac{X}{\frac{\beta}{\beta-1}\left(K^B+\frac{q}{1-q}\Delta K\right)}\right)^\beta Y_n^B \Delta K \leq \frac{\xi}{q} < \left(\frac{X}{\frac{\beta}{\beta-1}K^B}\right)^\beta Y_n^B \Delta K$
- *The MH constraint binds:* $\left(\frac{X}{\frac{\beta}{\beta-1}K^B}\right)^\beta Y_n^B \Delta K \leq \frac{\xi}{q} < \left(\frac{X}{\frac{\beta}{\beta-1}K^G}\right)^\beta Y_n^G \Delta K$

It is not profitable for the investor to offer the contract if $\frac{\xi}{q} \geq \left(\frac{X}{\frac{\beta}{\beta-1}K^G}\right)^\beta Y_n^G \Delta K$.

A proof of the results in Proposition 1 is given in Appendix B.

The optimal investment trigger of a good project will be first-best as the private information constraint of a bad project never binds. When the PIG constraint binds, the optimal investment trigger X^{B^*} in Proposition 1 implies a delay in the investment, as the investment trigger is higher than the first-best trigger $X^*\left(K^B\right) = \frac{\beta}{\beta-1}K^B$. There are no investment delays when the MH constraint is the only binding constraint.

Private information may induce innovators to report higher investment costs than the true ones. Hence, the investor has to leave them with rents to extract information about the true costs, as shown in Eq. (19) when only the PIG constraint binds. The investor reduces the present value of innovators' compensation by delaying the investment. The result is that the optimal investment trigger is higher than first-best, see Eq. (18) when only the PIG binds.

The trade-off between lower rents and an inefficient investment strategy is due to private information only. Adding moral hazard to the model mitigates the investment distortion. Since effort is costly, the investor has to leave rents to innovators to induce effort ex-ante. As long as the effort costs are lower than the innovators' value of private information, the same compensation is used both to ensure truth-telling ex-post and to induce effort ex-ante, but only the PIG constraint binds. If innovators' effort costs get sufficiently large, the rent-efficiency trade-off due to private information will be altered such that investment distortion is reduced. This is shown by the lower investment trigger in Eq. (18) when the MH and PIG constraints bind simultaneously. When effort costs are higher than the value of private information, the investor cannot push the innovators' rents down by delaying the investment. Hence, the investment timing becomes first-best, corresponding to the investment trigger in Eq. (18) when only the MH constraint is binding.

The results are illustrated in Figure 2, where investment triggers are graphed as functions of the effort cost ξ. The lower horizontal line illustrates the first-best investment trigger, and the upper horizontal parts of the other trigger curves are equal to the second-best investment trigger when only the private information constraint of a good innovator binds.[6] In the interim region where both the private information constraint of a good innovator and the moral hazard constraint bind simultaneously, the second-best investment trigger is pushed towards the first-best trigger as the effort cost ξ increases and the Lagrange multiplier λ_1 goes from 1 to 0. The value of λ_1 is determined by the condition that the expected compensation must satisfy the equality $\frac{\xi}{q} = \left(\frac{X}{X^{B*}}\right)^{\beta} Y_n^B \Delta K$. Figure 2 illustrates that the intervals of effort costs where the investment policy is inefficient is smaller the more

[6] The parameter values of the numerical illustrations are given in Appendix C.

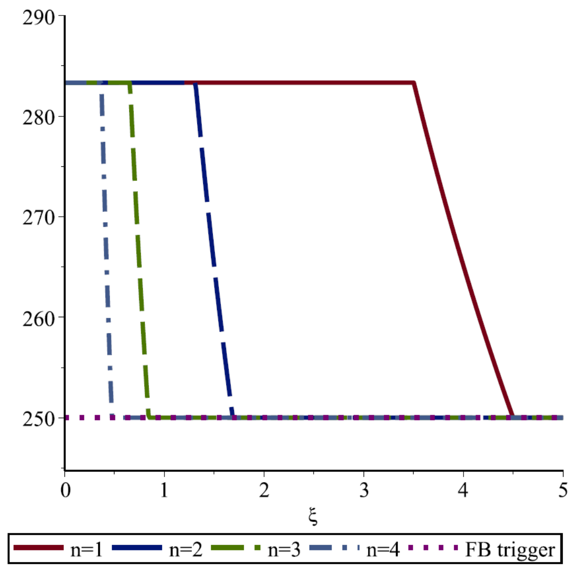

Figure 2: The investment trigger $X^{\acute{B}^*}$ as a function of effort cost ξ. The dotted, lowest horizontal line represents the first-best investment trigger. The four other curves represent optimal triggers given agency problems for various n. The investment distortion is largest when private information is the only binding constraint, represented by the upper horizontal parts of the curves. In the interval where both the private information and the moral hazard constraints bind, the investment triggers are decreasing. When only the moral hazard constraint binds, the triggers are equal to first-best. The figure illustrates that as the number of competitors increases, the optimal investment policy is pushed towards the first-best.

competitors there are. Thus, competition speeds up innovation and leads to less investment distortions.

Recall that S^{G^*}represents the expected compensation of each innovator, adjusted for the probability of winning the contract, Y_n^G. The expected compensation of the winner is thus given by $s^{G^*} = S^{G^*}/Y_n^G$. The winner's present value of compensation, derived from Eq. (19), is then equal to

$$\left(\frac{X}{X^{G^*}}\right)^{\beta} s^{G^*} = \begin{cases} \left(\frac{X}{X^{B^*}}\right)^{\beta} \frac{Y_n^B}{Y_n^G} \Delta K & \text{if the PIG constraint binds} \\ \left(\frac{X}{X^{B^*}}\right)^{\beta} \frac{Y_n^B}{Y_n^G} \Delta K = \frac{\xi}{qY_n^G} & \text{if the PIG and MH constraints bind jointly} \\ \frac{\xi}{qY_n^G} & \text{if the MH constraint binds} \end{cases} \tag{20}$$

The winner's compensation as a function of the effort cost ξ is given in Figure 3 for various n. We see a reduction in the winner's compensation for higher n in the intervals where the private information constraint binds. The figure also shows that the winner's compensation is increasing in n in the interval where the effort cost is a binding constraint. The figure illustrates the trade-off the investor faces, between reducing value of private information by inviting more innovators to compete, and the extra cost added by having to compensate for those innovators for their costs.

The compensation functions in Eqs. (19) and (20) are increasing and convex in the underlying project value, X, in the region of parameter values where private information is a binding constraint, but are independent of X when the moral hazard constraint is binding. Thus, when the moral hazard constraint is binding, no extra compensation is needed even for very valuable investment projects.

5 Optimal number of innovators

So far we have kept n fixed and optimized the investor's value in Eq. (8) only with respect to investment triggers and compensation of the innovators. However, an investor will also

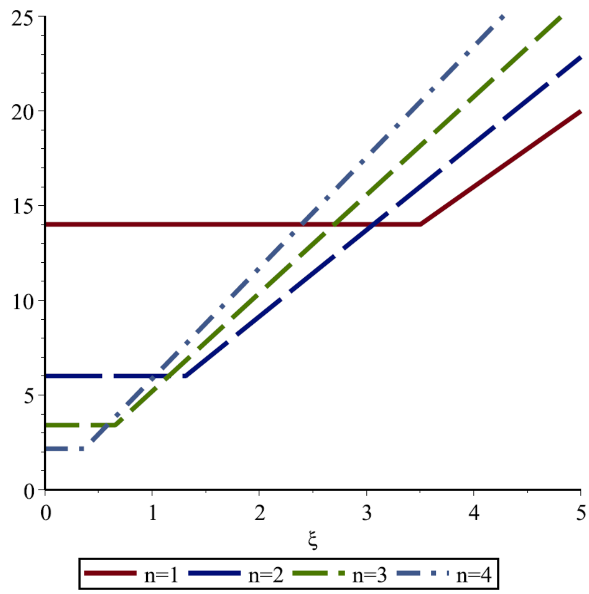

Figure 3: The winner's compensation s^{G^*} for various n as a function of effort cost ξ. Each curve is independent of ξ when private information is the only binding constraint, corresponding to the horizontal parts of the curves. Each curve is increasing in ξ in the intervals where moral hazard binds. The figure illustrates that informational rents are decreasing in n. In the intervals where moral hazard is a binding constraint, the winner's compensation increases in n.

consider how many innovators to invite to come up with costly project proposals. An interesting question is then if the optimal choice of n will imply that the private information constraint is binding, resulting in second-best investment policy, or if it results in first-best investment policy. The optimal n will be a trade-off of the following factors: The more competitors the investor invites to participate, the higher is the probability that there is at least one competitor with a good project. In addition, the more competitors there are, the lower are the informational rents, as these are competed away. On the other hand, the more competitors there are, the higher the cost of compensating all the innovators for their efforts.

To find the optimal n, we maximize the investor's value in Eq. (8), given the optimal contract in Proposition 1. To focus on the dependence on n we let $X^{B^*} = X^{B^*}(n)$ and $V^P = V^P(n)$, such that the investor's value is formulated as

$$V^P(n) = n\left[q\left(\frac{X}{X^{G^*}}\right)^{\beta}\left(Y_n^G\left(X^{G^*} - K^G\right) - S^{G^*}\right) + (1-q)\left(\frac{X}{X^{B^*}(n)}\right)^{\beta} Y_n^B\left(X^{B^*}(n) - K^B\right)\right]. \tag{21}$$

Recall that $p_n = 1-(1-q)^n$ is the probability that the winner of the contract is of K^G-type. The probability can also be expressed as $p_n = nqY_n^G$. Moreover, we have the relationship $1 - p_n = n(1-q)Y_n^B$. Thus, we can rewrite Eq. (21) as

$$V^P(n) = p_n\left(\frac{X}{X^{G^*}}\right)^{\beta}\left(\left(X^{G^*} - K^G\right) - s^{G^*}\right) + (1-p_n)\left(\frac{X}{X^{B^*}(n)}\right)^{\beta}\left(X^{B^*}(n) - K^B\right). \tag{22}$$

We analyze the problem of optimizing Eq. (22) by evaluating the optimization problem for the different regions defined in Proposition 1. For each region we analyze whether the investor's value is increasing or decreasing in n, by differentiating Eq. (22) with respect to n. In Appendix D we show that as long as the private information constraint of a good innovator is binding, the value for the investor is increasing in n. In the region where only the moral hazard constraint is binding, we find two opposing effects: a higher number of innovators increases the probability of at least one high-quality project in the set of the submitted proposals, but also increases the total effort costs. Our results are summed up in Proposition 2.

Proposition 2 *It is optimal for the investor to choose n so that first-best investment triggers are reached. The optimal number of innovators is equal to*

$$n^* = \frac{\ln\left(\frac{\xi}{-\ln(1-q)\left[\left(\frac{X}{X^*(K^G)}\right)^{\beta}(X^*(K^G)-K^G)-\left(\frac{X}{X^*(K^B)}\right)^{\beta}(X^*(K^B)-K^B)\right]}\right)}{\ln(1-q)}. \tag{23}$$

Eq. (23) is well defined for combinations of parameter values such that the fraction in the logarithmic expression in the nominator gives values between 0 and 1. If ξ goes to zero, the optimal number of innovators goes to infinity, i.e. in a pure adverse selection model it is always advantageous to invite more innovators. The results are illustrated in Figures 4, where the solid curve represents the value for the investor under agency problems, and the dotted curve is the first-best benchmark. Note that only in the interval of n values where private information is a binding constraint, the solid curve is below the dotted one, as only in this interval we have a second-best investment strategy. Note also that when the investor is allowed to choose the number of innovators, there is no need to make the assumption that the investor has to make a commitment to a second-best investment strategy. Thus optimal number of innovators ensures that contracts are renegotiation-proof.

In Appendix E we discuss the optimality conditions when we restrict n to be discrete and show that Proposition 2 is still valid when we ensure that there is at least one value of n in the moral hazard region.

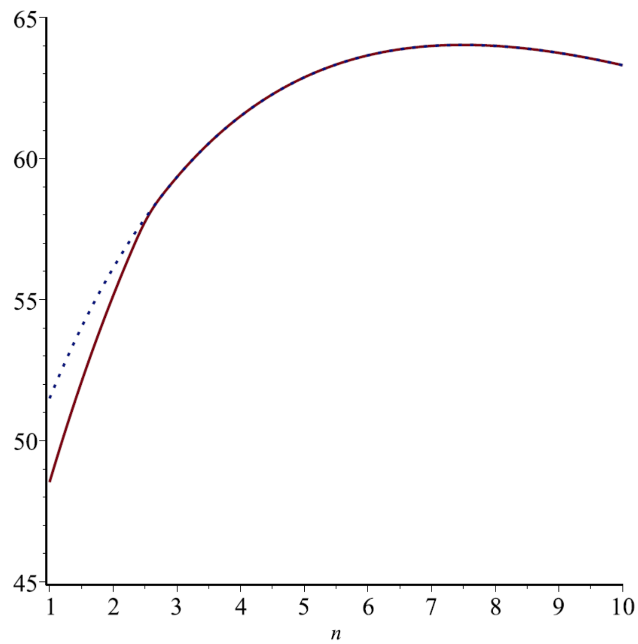

Figure 4: The project value for the investor as a function of n. The dotted curve represents the investor's project value as a function of the number of innovators, n, in the first-best case. The solid curve is the investor's value when she faces agency problems. In the interval of n where private information is a binding constraint, the investor's value is lower than the first-best value.

Next we present comparative statics results for the optimal number of innovators.

Proposition 3 *Comparative statics results for the optimal number of innovators:*

(*i*) n^* *increases in* X*.*

(*ii*) n^* *decreases in* σ*.*

(*iii*) n^* *decreases in* q*.*

(*iv*) n^* *decreases in* ξ*.*

The comparative statics results are proved in Appendix F. Proposition 3 *(i)* says that n^* increases the more "in-the-money" the investment option is: the more valuable a good project is compared to the bad project, the higher is the value of increasing the probability that the winner of the contract has a good project. The model has two uncertainty parameters, σ and q. Volatility σ measures the riskiness of the project cash flows and probability q measures how difficult the task is (low q implies a difficult task). Note that the two uncertainty measures have opposite effect on n^*. The optimal n is decreasing in σ, as stated in Proposition 3 *(ii)*. The explanation is that the difference in the value between good and bad projects decreases in σ. This makes it less important to invite many innovators, as it is less important to ensure that there is at least one good project among the investment proposals. Proposition 3 *(iii)* implies that n^* is higher the more uncertain it is that at least one of the innovators has a good project. Finally, Proposition 3 *(iv)* means that the larger the effort cost for each innovator, the more expensive it is for the investor to attract innovators to the competition.

Boudreau et al. (2011) study 9,661 competitions from Top-Coder, a contest platform where elite software developers participate in competitions to create novel algorithms. Boudreau et al. find that more competitors lead to better contest performance only when problems are highly uncertain. This empirical result is consistent with our comparative statics result that optimal number of innovators is decreasing in q, i.e. increasing in the difficulty of the task. As far as we know, our other comparative statics results have not been empirically examined.

6 Model with private benefits

So far we have assumed that the winning innovator receives just monetary compensation. In this section we relax this assumption and allow the innovator to receive non-pecuniary private benefits in addition of receiving monetary compensation. We assume that the winner receives private benefits at the time the investment of a good project is made. Being able to come up with a good project enhances the innovator's reputation that would be valuable in the future when the innovator negotiates his next contract.[7]

As the innovators' private benefits are non-pecuniary, the investor's maximization problem is identical to Eq. (9). Let b denote non-pecuniary private benefits realized at the investment time. The private information and moral hazard constraints in Eqs. (11)-(14) are changed in the following way:

(i) The private benefits of a good project gives an innovator an additional incentive to exert high effort, and results in the moral hazard constraint,

$$q\left(\frac{X}{X^G}\right)^{\beta}(S^G + Y_n^G b) + (1-q)\left(\frac{X}{X^B}\right)^{\beta} S^B - \xi \geq \left(\frac{X}{X^B}\right)^{\beta} S^B. \quad (24)$$

The left-hand side of the equation represents an innovator's value of the project if he exerts high effort, whereas the right-hand side states the value of low effort. We rearrange the inequality as follows,

$$\left(\frac{X}{X^G}\right)^{\beta}(S^G + Y_n^G b) - \left(\frac{X}{X^B}\right)^{\beta} S^B - \frac{\xi}{q} \geq 0. \quad (25)$$

[7] Alternatively, we could assume that the winner received private benefits both after good and bad projects. We would obtain similar results in this case, if we also assumed that the time value of private benefits declined faster than the time value of monetary compensation.

(ii) Correspondingly, the ex ante participation constraint is given by,

$$q\left(\frac{X}{X^G}\right)^{\beta}(S^G+Y_n^G b)+(1-q)\left(\frac{X}{X^B}\right)^{\beta}S^B-\xi\geq 0. \tag{26}$$

We immediately see that the participation constraint in Eq. (26) is dominated by the moral hazard constraint in Eq. (24), as $\left(\frac{X}{X^B}\right)^{\beta}S^B\geq 0$.

(iii) The private information constraint of a good innovator that receives private benefits from investment is given by,

$$\left(\frac{X}{X^G}\right)^{\beta}(S^G+Y_n^G b)\geq\left(\frac{X}{X^B}\right)^{\beta}\left(S^B+Y_n^B\Delta K\right). \tag{27}$$

We assume that an innovator reaps reputational benefits if outsiders believe that he is of good type, even if the innovator falsely reports the quality of his project. By reporting a good project the innovator signals that he is a talented innovator and thus in the future it is worthwhile to hire him for other projects. Thus, the private information constraint of an innovator with a bad project equals

$$\left(\frac{X}{X^B}\right)^{\beta}S^B\geq\left(\frac{X}{X^G}\right)^{\beta}(S^G+Y_n^G(b-\Delta K)). \tag{28}$$

The limited liability constraints, $S^G\geq 0$ and $S^B\geq 0$, are identical to Eq. (15). Note that for sufficiently high private benefits, i.e., when

$$\left(\frac{X}{K^B\frac{\beta}{\beta-1}}\right)^{\beta}Y_n^G b-\left(\frac{X}{K^B\frac{\beta}{\beta-1}}\right)^{\beta}Y_n^B\Delta K>0, \tag{29}$$

none of the agency constraints in Eqs. (25)-(28) or (15) bind. Hence, in this situation non-pecuniary private benefits removes the agency problem, which means that the investment triggers X^G and X^B are first-best, and expected compensations S^G and S^B are zero.

Below we study the situation when the private benefits are small enough to ensure $S^G>0$. This is reasonable to assume, since there are very few projects where innovators would be willing to work for free. Similarly to the situation of no private benefits in Section 4, the expected compensation for the bad project, S^B, is equal to zero. The results in Proposition 4 are proved in Appendix G.

Proposition 4 *For $S^G > 0$, we have the following results: The optimal investment trigger of a high-quality project is formulated as*

$$X^{G^*} = \begin{cases} \frac{\beta}{\beta-1}\left(K^G - b\right) & \begin{cases} \text{if only the PIG constraint binds,} \\ \text{if the PIG and MH constraints bind jointly,} \\ \text{if only the MH constraints bind,} \end{cases} \\ \frac{\beta}{\beta-1}\left(K^G - \lambda_2 \Delta K - b\right) & \text{when the MH and the PIB constraints bind,} \end{cases} \tag{30}$$

where λ_2 is a Lagrange multiplier, determined by requiring that Eqs. (25) and (28) hold with equality. The optimal investment trigger of a low-quality project is given by

$$X^{B^*} = \begin{cases} \frac{\beta}{\beta-1}\left(K^B + \frac{q}{1-q}\Delta K\right) & \text{if only the PIG constraint binds,} \\ \frac{\beta}{\beta-1}\left(K^B + \frac{q}{1-q}\lambda_1 \Delta K\right) & \text{if the PIG and the MH constraints bind,} \\ \frac{\beta}{\beta-1} K^B & \begin{cases} \text{if only the MH constraint binds, or} \\ \text{if the MH and PIB constraints bind jointly,} \end{cases} \end{cases} \tag{31}$$

where λ_1 is a Lagrange multiplier, determined by requiring that Eqs. (25) and (27) hold with equality. The value of the expected compensation of a good project equals

$$\left(\frac{X}{X^{G^*}}\right)^{\beta} S^{G^*} = \begin{cases} \left(\frac{X}{X^{B^*}}\right)^{\beta} Y_n^B \Delta K - \left(\frac{X}{X^{G^*}}\right)^{\beta} Y_n^G b & \text{if the PIG constraint binds} \\ \frac{\xi}{q} - \left(\frac{X}{X^{G^*}}\right)^{\beta} Y_n^G b & \begin{cases} \text{if the PIG and MH constraints bind jointly,} \\ \text{if only the MH constraint binds,} \\ \text{if the MH and PIB constraints bind jointly.} \end{cases} \end{cases} \tag{32}$$

The region where the MH and PIB constraints bind simultaneously is given by $\frac{\xi}{q} \geq \left(\frac{X}{\frac{\beta}{\beta-1}(K^G - b)}\right)^{\beta} Y_n^G \Delta K$. The other regions are as defined in Proposition 1.

Proposition 4 shows that private benefits have two main effects. As the innovator receives private benefits from the investment the innovator's compensation, in Eq. (32), is lower. In addition, as innovators with good projects are willing to invest for less compensation, it is optimal for the investor to let them invest earlier than first-best, as Eq. (30) shows. This means that private benefits introduces a second-best investment trigger for a good project as well.

Note that with private benefits the private information constraint of a bad project may bind together with the moral hazard constraint. The reason is that when the effort cost is sufficiently high, the compensation of an innovator that reports a good project increases, which may make it worthwhile for an innovator with a bad project to report a good one. To avoid untruthful reporting, it is optimal for the investor to choose the investment of a good project to take place even earlier than when the PIG or MH constraints bind, as Eq. (30) shows. Moreover, when the PIB constraint binds, increasing the number of competitors distorts the investment strategy further, as shown in Appendix H. In other words, here competition leads to higher investment distortions.

In the model without private benefits the optimal n is found in the region where only the moral hazard constraint is binding. Will this be the case when we have private benefits too? It turns out that the optimal number of innovators will be found either in the region where only moral hazard is a binding constraint, or when the private information constraint of a bad innovator binds together with moral hazard, see Appendix I. Hence, with private benefits a possible equilibrium is that the private information constraint of a bad innovator may be binding, resulting in too early investment.

7 Conclusion

We have presented an investment problem involving both moral hazard and private information in a real options framework where one of the control variables is when to invest. In our screening model the investor designs the contract so that the innovators have an incentive to truthfully reveal their information and provide high effort. In order to elicit information revelation high cost investments are delayed when the investor contracts only with one innovator. Competition among innovators alters this result dramatically: we show that when the investor can choose the number of innovators optimally, investment options are exercised so that first-best investment trigger is always achieved. Consequently, all the informational rents are competed away and the winner of the competition is only compensated for the effort costs. Thus the investor is able to capture the full upside potential of the investments. When we introduce non-pecuniary private benefits for the winning innovator who reports a low cost investment, in order to elicit truthful information revelation low cost investments may have to be exercised too early. In contrast to our main model, competition doesn't lead to first-best investment policy, it only makes informational rent extraction problem worse. Thus in high prestige contexts, competition may have a dark side.

Appendix

A Value of the investment project in the first-best case: Proof of Eqs. (2)-(4)

As shown by Dixit and Pindyck (1994), among many others, the ordinary differential equation of the project value $V(X, \cdot)$ can be formulated as

$$\frac{1}{2}\sigma^2 X^2 V_{XX} + \mu X V_X - rV = 0, \tag{A.1}$$

where V_X and V_{XX} are the first and second order derivatives of the value function $V(X, K_i)$ with respect to X. Boundary conditions of the value are

$$V\left(X^*\left(K_i\right), K_i\right) = X^*\left(K_i\right) - K_i, \tag{A.2}$$

$$V_X\left(X^*\left(K_i\right), K_i\right) = 1, \tag{A.3}$$

$$V\left(0, K_i\right) = 0. \tag{A.4}$$

The boundary condition in Eq. (A.2) tells us that at the investment trigger, $X^*(K_i)$, the project value must be equal to the payoff from the project when investment takes place. Eq. (A.3) is an optimality condition, ensuring that the investment trigger $X^*(K_i)$ is determined so as to maximize the value of the investment option. Eq. (A.4) is a boundary condition, saying that when the value of future cash flows, X, is zero, the value of the investment project is zero too. The solution of the differential equation in Eq. (A.1), subject to the boundary conditions in Eqs. (A.1)-(A.4), is given in Eqs. (2)-(4).

B The investor's optimization problem in Eqs. (9)-(15): Deriving the optimal contract for each innovator

Proof of Lemma 1 (i): The expected compensation of an innovator who reports a good project is always strictly positive, as we have the following relationships,

$$\left(\frac{X}{X^G}\right)^\beta S^G \geq \left(\frac{X}{X^B}\right)^\beta \left(S^B + Y_n^B \Delta K\right) \geq \left(\frac{X}{X^B}\right)^\beta Y_n^B \Delta K > 0, \tag{B.1}$$

for $X > 0$. The first inequality in Eq. (B.1) follows from the incentive constraint of an innovator with a good project in project, in Eq. (13). The second inequality is a result of the limited liability constraint in Eq. (15).

The ex ante participation constraint in Eq. (12) will never bind, as the right-hand side of Eq. (10), $\left(\frac{X}{X^B}\right)^\beta S^B$, is positive and therefore dominates the right-hand side of Eq. (11).

The optimization problem is solved using the method of Kuhn-Tucker. We formulate the Lagrangian function as follows:

$$\begin{aligned} L \;=\; & \left(\frac{X}{X^G}\right)^\beta \left[Y_n^G \left(X^G - K^G\right) - S^G\right] + \frac{(1-q)}{q} \left(\frac{X}{X^B}\right)^\beta \left[Y_n^B \left(X^B - K^B\right) - S^B\right] \\ & + \lambda_1 \left[\left(\frac{X}{X^G}\right)^\beta S^G - \left(\frac{X}{X^B}\right)^\beta \left(S^B + Y_n^B \Delta K\right)\right] \\ & + \lambda_2 \left[\left(\frac{X}{X^B}\right)^\beta S^B - \left(\frac{X}{X^G}\right)^\beta \left(S^G - Y_n^G \Delta K\right)\right] \\ & + \lambda_3 \left[\left(\frac{X}{X^G}\right)^\beta S^G - \left(\frac{X}{X^B}\right)^\beta S^B - \frac{\xi}{q}\right] \\ & + \lambda_4 S^B \end{aligned} \tag{B.2}$$

The first-order conditions of the decision variables S^G, S^B, X^G, and X^B are given by

$$\frac{\partial L}{\partial S^G} = 0 \Longrightarrow \lambda_1 - \lambda_2 + \lambda_3 = 1 \tag{B.3a}$$

$$\frac{\partial L}{\partial S^B} = 0 \Longrightarrow \left(\frac{X}{X^B}\right)^\beta \left[-\lambda_1 + \lambda_2 - \lambda_3 - \frac{1-q}{q}\right] + \lambda_4 = 0 \tag{B.3b}$$

$$\frac{\partial L}{\partial X^G} = 0 \Longrightarrow X^G = \frac{\beta\left[Y_n^G\left(K^G - \lambda_2 \Delta K\right) + \left(1 - \lambda_1 + \lambda_2 - \lambda_3\right) S^G\right]}{(\beta - 1)\, Y_n^G} \tag{B.3c}$$

$$\frac{\partial L}{\partial X^B} = 0 \Longrightarrow X^B = \frac{\beta\left[Y_n^B\left[K^B + \lambda_1 \frac{q}{1-q}\Delta K\right] + \left[\frac{q}{1-q}\left(\lambda_1 - \lambda_2 + \lambda_3\right) + 1\right] S^B\right]}{(\beta - 1)\, Y_n^G} \tag{B.3d}$$

The investment triggers given by (B.3c)-(B.3d) can be simplified. For the investment trigger in (B.3c) the second term in the nominator equals zero as the first-order condition implies $1 - \lambda_1 + \lambda_2 - \lambda_3 = 0$. The investment trigger of the innovator with the good project can thus be simplified to

$$X^G = \frac{\beta\left(K^G - \lambda_2 \Delta K\right)}{(\beta - 1)}. \tag{B.4}$$

We use the first-order condition in Eq. (B.3a) to evaluate and simplify the first-order condition in Eq. (B.3d). This leads to $\lambda_4 = \frac{1}{q}\left(\frac{X}{X^B}\right)^\beta > 0$. Thus, for the Kuhn-Tucker conditions to hold, we need $S^B = 0$, as is stated in Proposition 1 (*ii*). The condition for the investment trigger is therefore

$$X^B = \frac{\beta\left(K^B + \lambda_1 \frac{q}{1-q}\Delta K\right)}{(\beta - 1)}. \tag{B.5}$$

The ex post incentive compatibility constraints in Eqs. (13)-(14) cannot bind simultaneously: If Eq. (13) binds, the present value of the expected compensation is given by $\left(\frac{X}{X^G}\right)^\beta S^G = \left(\frac{X}{X^B}\right)^\beta Y_n^B \Delta K$. If Eq. (14) is binding, the expected compensation is equal to $\left(\frac{X}{X^G}\right)^\beta S^G = \left(\frac{X}{X^G}\right)^\beta Y_n^G \Delta K$. We have the relationships $X^B > X^G$, and therefore $\left(\frac{X}{X^G}\right)^\beta > \left(\frac{X}{X^B}\right)^\beta$, and $Y_n^G > Y_n^B$. This means that $\left(\frac{X}{X^G}\right)^\beta Y_n^G \Delta K > \left(\frac{X}{X^B}\right)^\beta Y_n^B \Delta K$, and thus the two constraints cannot bind at the same time. This gives Lemma 1 (*iv*).

We end up with the following possible combinations of binding and not binding constraints that need to be checked:

(a) Only PIG is a binding constraint: $\lambda_1 > 0,\ \lambda_2 = 0,\ \lambda_3 = 0.$ Evaluation of Eq. (B.3a) gives $\lambda_1 = 1$. The result is that

$$X^B = \frac{\beta}{\beta - 1}\left(K^B + \frac{q}{1-q}\Delta K\right) \tag{B.6}$$

$$X^G = \frac{\beta}{\beta - 1}K^G \tag{B.7}$$

$$S^G = \left(\frac{X^G}{X^B}\right)^{\beta} Y_n^B \Delta K \tag{B.8}$$

(b) The PIG and MH constraints bind simultaneously: $\lambda_1 > 0,\ \lambda_2 = 0,\ \lambda_3 > 0.$ This scenario gives the following values for the decision variables:

$$X^B = \frac{\beta}{\beta - 1}\left(K^B + \lambda_1\frac{q}{1-q}\Delta K\right) \tag{B.9}$$

$$X^G = \frac{\beta}{\beta - 1}K^G \tag{B.10}$$

$$S^G = \left(\frac{X^G}{X^B}\right)^{\beta} Y_n^B \Delta K = \left(\frac{X^G}{X}\right)^{\beta}\frac{\xi}{q} \tag{B.11}$$

(c) The MH constraint binds: $\lambda_1 = 0,\ \lambda_2 = 0,\ \lambda_3 > 0.$ The incentive compatibility constraint in Eq. (B.3a) implies $\lambda_3 = 1$. Both investment triggers will in this case be equal to first-best investment policy, whereas the expected compensation functions of innovators with good and bad projects, respectively, are given by the moral hazard constraint, $\left(\frac{X}{X^G}\right)^{\beta} S^G - \frac{\xi}{q} = 0$. The result is that

$$X^B = \frac{\beta}{\beta - 1}K^B \tag{B.12}$$

$$X^G = \frac{\beta}{\beta - 1}K^G \tag{B.13}$$

$$S^G = \left(\frac{X^G}{X}\right)^{\beta}\frac{\xi}{q} \tag{B.14}$$

(d) The PIB constraint binds: $\lambda_1 = 0,\ \lambda_2 > 0,\ \lambda_3 = 0$. This is an impossible scenario as the first-order condition of S^G would result in a negative value of the lambda multiplicator, $\lambda_2 = -1$. This contradicts the necessary conditions for the Kuhn-Tucker optimization problem.

(e) The MH and PIB constraints bind simultaneously: $\lambda_1 = 0,\ \lambda_2 > 0,\ \lambda_3 > 0$. Evaluation of the first-order condition in Eq. (B.3a) leads to $\lambda_2 = \lambda_3 - 1$, which means that we need $\lambda_3 > 1$ to satisfy the condition $\lambda_2 > 0$. Evaluation of the first-order condition w.r.t. S^B, Eq. (B.3b), using the condition $\lambda_2 = \lambda_3 - 1$ from Eq. (B.3a), gives $\lambda_4 = \frac{1}{q}\left(\frac{X}{X^G}\right)^{\beta}$. The decision variables are thus given by

$$X^B = \frac{\beta}{\beta-1} K^B \tag{B.15}$$

$$X^G = \frac{\beta}{\beta-1}\left(K^G - \lambda_2 \Delta K\right) \tag{B.16}$$

$$S^G = Y_n^G \Delta K = \left(\frac{X^G}{X}\right)^{\beta} \frac{\xi}{q} \tag{B.17}$$

We need to check if the investor will choose to give the innovators incentives for high effort in this scenario, i.e., if the investor's expected value of value of exerting high effort is larger than the value of exerting low effort,

$$q\left(\frac{X}{X^G}\right)^{\beta}\left[Y_n^G\left(X^G - K^G\right) - S^G\right] + (1-q)\left(\frac{X}{X^B}\right)^{\beta} Y_n^B\left(X^B - K^B\right) \geq \left(\frac{X}{X^B}\right)^{\beta}\left(X^B - K^B\right),$$

where the left-hand side of the inequality is the investor's value when he gives the innovators incentives to exert high effort, and the right-hand side is the value of giving incentives to exerting low effort. Evaluation of the inequality using Eq. (B.17), leads to the requirement

$$\left(\frac{X}{X^G}\right)^{\beta}\left(X^G - K^B\right) \geq \left(\frac{X}{X^B}\right)^{\beta}\left(X^B - K^B\right).$$

This inequality will never hold as the right-hand side will always be more valuable than the left-hand side. The reason is that the investment trigger in the option value of the right-hand side is the optimal trigger given the cost K^B. All other triggers (including X^G) will give lower values of the investment option. In other words, it is never optimal for the investor to give the innovators incentives to exert high effort if that results in early investment compared to the first-best investment.

C Parameter values in numerical illustrations

Value of observable asset	X	150
Risk-adjusted drift	μ	0
Volatility	σ	0.2
Risk-free interest rate	r	0.04
Investment cost of high quality project	K^G	75
Investment cost of low quality project	K^B	125
Probability of high quality if high effort	q	0.25
Cost of high effort	ξ	1
Resulting values:		
Beta multiple	β	2
First-best investment trigger of high quality project	$X^*\left(K^G\right)$	150
First-best investment trigger of low quality project	$X^*\left(K^B\right)$	250
Investment trigger of bad project when only private information binds	X_B^*	283.33

D Effect on the investor's value of the project from an increase in n

The probability that the investor will contract with a good innovator, $p_n = 1 - (1-q)^n$, increases in n, as $\frac{\partial p_n}{\partial n} = -(1-q)^n \ln(1-q) \geq 0$.

The derivative of the investor's value function in Eq. (22) with respect to n is evaluated for each of the regions, the *PIG region*, the *joint PIG and MH region*, and the *MH region*. For the PIG region the derivative of the value function in Eq. (22), evaluated for optimal value of s^{G^*}, is positive,

$$\frac{\partial V^P(n)}{\partial n} = \frac{\partial p_n}{\partial n}\left[\left(\frac{X}{X^*(K^G)}\right)^\beta (X^*(K^G) - K^G) - \left(\frac{X}{X^{B^*}}\right)^\beta (X^{B^*} - K^B - \frac{q}{1-q}\Delta K)\right] \geq 0. \tag{D.1}$$

In the *joint PIG and MH region* the optimal investment trigger X^{B^*} depends on n. However, X^{B^*} decreases monotonically in n between the upper boundary $\frac{\beta}{\beta-1}\left(K^B + \frac{q}{1-q}\Delta K\right)$ and the lower boundary $\frac{\beta}{\beta-1}K^B$. Both boundary values are independent of n, and the derivative of the investor's value will therefore be identical to the result in Eq. (D.1).

For the *MH region* differentiating Eq. (22) with respect to n results in the following

$$\frac{\partial V^P(n)}{\partial n} = \frac{\partial p_n}{\partial n}\left[\left(\frac{X}{X^*(K^G)}\right)^\beta (X^*(K^G) - K^G) - \left(\frac{X}{X^{B^*}}\right)^\beta (X^{B^*} - K^B)\right] - \xi, \tag{D.2}$$

which can both be positive or negative, depending on whether the first or the last term is the largest one.

E Discrete n

To ensure that the optimal number of innovators are found in the *moral hazard region* when n is discrete we require that there is at least one value of n in this region:

Condition 1 *We assume that parameter values are given such that there is at least one value of n in the* moral hazard region*.*

For discrete n we find the optimal n by increasing the number of innovators in the *moral hazard region* as long as the additional value of inviting one more innovator to participate in the contest is higher than the cost of inviting him, i.e., we increase n as long as

$$\Delta p_n \left\{\left(\frac{X}{X^*\left(K^G\right)}\right)^\beta \left(X^*\left(K^G\right) - K^G\right) - \left(\frac{X}{X^*\left(K^B\right)}\right)^\beta \left(X^*\left(K^B\right) - K^B\right)\right\} - \xi \geq 0, \tag{E.1}$$

where $\Delta p_n \equiv p_n - p_{n-1} > 0$. Since Δp_n is positive and decreasing in n, and the last term is a negative constant, we find an optimum in the *moral hazard region*.

F Comparative statics for n^*

- The derivative of n^* with respect to X is given by $\frac{dn^*}{dX} = -\frac{\beta}{\ln(1-q)X} > 0$.

- Here we show that n^* decreases in σ: Differentiation of n^* with respect to β leads to

$$\frac{\partial n^*}{\partial \beta} = A\left[\left(K^G\right)^{1-\beta} \cdot \ln\left(\frac{X}{\frac{\beta}{\beta-1}K^G}\right) - \left(K^B\right)^{1-\beta} \cdot \ln\left(\frac{X}{\frac{\beta}{\beta-1}K^{GB}}\right)\right], \tag{F.1}$$

where

$$A = \frac{\left(X\frac{\beta-1}{\beta}\right)^{\beta}}{-\ln\left(1-q\right)\left[\left(\frac{X}{X^*(K^G)}\right)^{\beta}K^G - \left(\frac{X}{X^*(K^B)}\right)^{\beta}K^B\right]} > 0.$$

The term in brackets in Eq. (F.1) is positive, as the exponential of the term is larger than 1,

$$\exp\left\{A\left[\left(K^G\right)^{1-\beta} \cdot \ln\left(\frac{X}{\frac{\beta}{\beta-1}K^G}\right) - \left(K^B\right)^{1-\beta} \cdot \ln\left(\frac{X}{\frac{\beta}{\beta-1}K^{GB}}\right)\right]\right\} = \frac{\left(\frac{X}{X^*(K^G)}\right)^{A\cdot\left(K^G\right)^{1-\beta}}}{\left(\frac{X}{X^*(K^B)}\right)^{A\cdot\left(K^B\right)^{1-\beta}}} > 1,$$

which means that the derivative in Eq. (F.1) is positive. Hence we find that

$$\frac{dn^*}{d\sigma} = \frac{\partial n^*}{\partial \beta}\frac{d\beta}{d\sigma} < 0, \tag{F.2}$$

since $\frac{\partial n^*}{\partial \beta} > 0$, and $\frac{d\beta}{d\sigma} < 0$.

- The derivative of n^* with respect to q can be written as

$$\frac{dn^*}{dq} = \frac{n^* + \frac{1-q}{q} + \ln(1-q)^{-1}}{\ln(1-q)\,(1-q)} < 0. \tag{F.3}$$

since $n^* + \frac{1-q}{q} + \ln(1-q)^{-1} > 0$ for $q \in (0,1)$ and $n^* \geq 1$.

- The derivative of n^* with respect to ξ equals

$$\frac{dn^*}{d\xi} = \frac{1}{\ln(1-q)\xi} < 0. \tag{F.4}$$

G The optimal contract for each innovator when innovators have private benefits

The Lagrangian function when we extend the model in Appendix B to include private benefits:

$$
\begin{aligned}
L \; = \; & \left(\frac{X}{X^G}\right)^\beta \left[Y_n^G\left(X^G - K^G\right) - S^G\right] + \frac{(1-q)}{q}\left(\frac{X}{X^B}\right)^\beta \left[Y_n^B\left(X^B - K^B\right) - S^B\right] \qquad \text{(G.1)} \\
& +\lambda_1\left[\left(\frac{X}{X^G}\right)^\beta (S^G + Y_n^G b) - \left(\frac{X}{X^B}\right)^\beta \left(S^B + Y_n^B \Delta K\right)\right] \\
& +\lambda_2\left[\left(\frac{X}{X^B}\right)^\beta S^B - \left(\frac{X}{X^G}\right)^\beta \left(S^G + Y_n^G(b - \Delta K)\right)\right] \\
& +\lambda_3\left[\left(\frac{X}{X^G}\right)^\beta (S^G + Y_n^G b) - \left(\frac{X}{X^B}\right)^\beta S^B - \frac{\xi}{q}\right] \\
& +\lambda_4 S^B \qquad \text{(G.2)}
\end{aligned}
$$

The first-order condition of the Lagrange function with respect to S^G leads to

$$\frac{\partial L}{\partial S^G} = 0 \Longrightarrow \lambda_1 - \lambda_2 + \lambda_3 - 1 = 0. \tag{G.3}$$

Likewise, the first-order condition with respect to S^B is given by

$$\frac{\partial L}{\partial S^B} = 0 \Longrightarrow \left(\frac{X}{X^B}\right)^\beta \left[-\lambda_1 + \lambda_2 - \lambda_3 - \frac{1-q}{q}\right] + \lambda_4 = 0. \tag{G.4}$$

We evaluate Eq. (G.4) using the equality in Eq. (G.3) and find that $\lambda_4 = \left(\frac{X}{X^B}\right)^\beta \frac{1}{q} > 0$, resulting in $S_B = 0$. The first-order condition of Eq. (G.1) with respect to the investment triggers are then given by

$$\frac{\partial L}{\partial X^G} = 0 \Longrightarrow X^G = \frac{\beta}{\beta - 1}(K^G - \lambda_2 \Delta K - b) \tag{G.5}$$

and

$$\frac{\partial L}{\partial X^B} = 0 \Longrightarrow X^B = \frac{\beta}{\beta - 1}(K^B + \lambda_1 \frac{q}{1-q}\Delta K). \tag{G.6}$$

Before we start evaluating all the possible combinations of binding and unbinding constraints, we note that the following combination is not possible: By the same arguments as for the model without private benefits in Appendix B, the two private information constraints in Eqs. (27)-(28) cannot bind simultaneously.

In the following we investigate the possible combinations of binding and not binding constraints.

a) Only PIG is a binding constraint which means that $\lambda_1 > 0$, $\lambda_2 = 0$, $\lambda_3 = 0$: Evaluation of the first-order condition in Eq. (G.4) implies $\lambda_1 = 1$, which again leads to the following optimal investment triggers when we evaluate Eqs. (G.3)-(G.4): $X^{G^*} = \frac{\beta}{\beta-1}(K^G - b)$ and $X^{B^*} = \frac{\beta}{\beta-1}(K^B + \frac{q}{1-q}\Delta K)$. The expected compensation is given by $\left(\frac{X}{X^{G^*}}\right)^\beta S^{G^*} = \left(\frac{X}{X^{B^*}}\right)^\beta Y_n^B \Delta K - \left(\frac{X}{X^{G^*}}\right)^\beta Y_n^G b$.

b) The PIG and MH constraints bind simultaneously, i.e. $\lambda_1 > 0$, $\lambda_2 = 0$, $\lambda_3 > 0$: $X^{G^*} = \frac{\beta}{\beta-1}(K^G - b)$ and $X^{B^*} = \frac{\beta}{\beta-1}(K^B + \lambda_1\frac{q}{1-q}\Delta K)$. The multiplier λ_1 is determined together with X^{B^*} by setting Eqs. (25) and (28) equal to zero and solve the two equalities with respect to λ_1 and X^{B^*}. The expected compensation is given by $\left(\frac{X}{X^{G^*}}\right)^\beta S^{G^*} = \frac{\xi}{q} - \left(\frac{X}{X^{G^*}}\right)^\beta Y_n^G b = \left(\frac{X}{X^{B^*}}\right)^\beta Y_n^B \Delta K - \left(\frac{X}{X^{G^*}}\right)^\beta Y_n^G b$.

c) Only the MH constraint binds, i.e. $\lambda_1 = 0$, $\lambda_2 = 0$, $\lambda_3 > 0$: $X^{G^*} = \frac{\beta}{\beta-1}(K^G - b)$ and $X^{B^*} = \frac{\beta}{\beta-1}K^B$. The expected compensation is given by $\left(\frac{X}{X^{G^*}}\right)^\beta S^{G^*} = \frac{\xi}{q} - \left(\frac{X}{X^{G^*}}\right)^\beta Y_n^G b$.

d) The MH and the PIB constraints bind, i.e. $\lambda_1 = 0$, $\lambda_2 > 0$, $\lambda_3 > 0$: $X^{G^*} = \frac{\beta}{\beta-1}(K^G - \lambda_2\Delta K - b)$ and $X^{B^*} = \frac{\beta}{\beta-1}K^B$. The expected compensation is given by $\left(\frac{X}{X^{G^*}}\right)^\beta S^{G^*} = \frac{\xi}{q} - \left(\frac{X}{X^{G^*}}\right)^\beta Y_n^G b = (\frac{X}{X^G})^\beta Y_n^G(\Delta K - b)$. This scenario is a possible outcome as long as the investor's expected value of value of exerting high effort is larger than the value of exerting low effort,

$$q\left(\frac{X}{X^{G^*}}\right)^\beta \left[Y_n^G\left(X^{G^*} - K^G\right) - S^G\right] + (1-q)\left(\frac{X}{X^B}\right)^\beta Y_n^B\left(X^B - K^B\right) \geq \left(\frac{X}{X^B}\right)^\beta \left(X^B - K^B\right),$$

i.e., for parameter values such that

$$\left(\frac{X}{X^{G^*}}\right)^{\beta} Y_n^G \left(X^{G^*} - K^B + b\right) - \left(\frac{X}{X^B}\right)^{\beta} Y_n^B \left(X^B - K^B\right) \geq 0, \tag{G.7}$$

where $X^{G^*} = X \left(\frac{Y_n^G \Delta K q}{\xi}\right)^{\frac{1}{\beta}}$. We cannot find an analytical upper value for the joint MH and PIB region, the inequality in (G.7) has to be solved for ξ numerically.

H With private benefits: Effect on X^{B^*} from an increase in n

The optimal investment trigger for a good project when both the MH and PIB constraints bind is given by Eq. (30), $X^{G^*} = \frac{\beta}{\beta-1}\left(K^G - \lambda_2 \Delta K - b\right)$, where λ_2 is determined by requiring that Eqs. (25) and (28) hold with equality. Evaluation of equality for these two constraints implies

$$\left(\frac{X}{X^G}\right)^{\beta} Y_n^G \left(\Delta K - b\right) = \frac{\xi}{q}\left(\frac{X}{X^G}\right)^{\beta} Y_n^G b.$$

Reformulation means that the optimal investment trigger in Eq. (30) alternatively can be written as

$$X^{G^*} = \frac{X}{\left(\frac{\xi}{Y_n^G \Delta K q}\right)^{\left(\frac{1}{\beta}\right)}}. \tag{H.1}$$

The optimal investment trigger in Eq. (H.1) depends on n only through the probability Y_n^G. As we know that $\frac{dY_n^G}{dn} < 0$, we get the result that

$$\frac{dX^{G^*}}{dn} = \frac{X \frac{dY_n^G}{dn}}{\left(\frac{\xi}{Y_n^G \Delta K q}\right)^{\left(\frac{1}{\beta}\right)} \beta Y_n^G} < 0.$$

I Optimal n in model with private benefits

The proof that n^* is found in the region where only moral hazard is a binding constraint is similar to the corresponding proof for the model without private benefits, in Appendix D.

The derivative of the investor's value function in Eq. (22) with respect to n is evaluated for each of the regions, the *PIG region*, the *joint PIG and MH region*, the *MH region* and the *joint MH and PIB region*.

For the *PIG region* the derivative of the value function in Eq. (22), evaluated for optimal value of s^{G^*}, is positive,

$$\frac{\partial V^P(n)}{\partial n} = \frac{\partial p_n}{\partial n}\left[\left(\frac{X}{X^{G^*}}\right)^{\beta}(X^{G^*} - K^G + b) - \left(\frac{X}{X^{B^*}}\right)^{\beta}(X^{B^*} - K^B - \frac{q}{1-q}\Delta K)\right] \geq 0. \tag{I.1}$$

The analysis for *joint PIG and MH region* is similar to the analysis of this region in Appendix D: X^{B^*} decreases monotonically in n between the upper boundary $\frac{\beta}{\beta-1}\left(K^B + \frac{q}{1-q}\Delta K\right)$ and the lower boundary $\frac{\beta}{\beta-1}K^B$. Both boundary values are independent of n, and the derivative of the investor's value will therefore be identical to the result in Eq. (I.1).

In the *MH region* the investment triggers X^{G^*} and X^{B^*} do not depend on n. For this region differentiating Eq. (22) with respect to n results in

$$\frac{\partial V^P(n)}{\partial n} = \frac{\partial p_n}{\partial n}\left[\left(\frac{X}{X^{G^*}}\right)^{\beta}(X^{G^*} - K^G + b) - \left(\frac{X}{X^{B^*}}\right)^{\beta}(X^{B^*} - K^B)\right] - \xi, \tag{I.2}$$

which can both be positive or negative, depending on whether the first or the last term is the largest one.

The effect of n is ambiguous in the *joint MH and PIB region* too. Here the optimal investment trigger, X^{G^*}, depend on n, as $X^{G^*} = X\left(\frac{Y_n^G q\Delta K}{\xi}\right)^{\frac{1}{\beta}}$. Hence, we get

$$\begin{aligned}\frac{\partial V^P(n)}{\partial n} &= \frac{\partial p_n}{\partial n}\left[\left(\frac{X}{X^{G^*}}\right)^{\beta}(X^{G^*} - K^G + b) - \left(\frac{X}{X^{B^*}}\right)^{\beta}(X^{B^*} - K^B)\right] \\ &\quad + p_n\left[\frac{dX^{G^*}}{dn}\cdot\left(\frac{X}{X^{G^*}}\right)^{\beta}\left(1 - \frac{\beta\left(X^{G^*} - K^B + b\right)}{X^{G^*}}\right)\right] - \xi, \end{aligned} \tag{I.3}$$

which also can be positive or negative since $\frac{dX^*(K^G)}{dn} < 0$ in this region. Thus, if parameter values are such that the *joint MH and PIB region* is a possible outcome, it may happen that the optimal n is found in this region.

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