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A Theory of Takeover Bidding

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A Theory of Takeover Bidding[1]

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Abstract

In this paper, we present a model of takeover bidding. Grossman and Hart (1980a) argue that small shareholders in a diffusely held firm would hold on to their shares rather than tendering and earn the higher post takeover return. This "free rider" problem precludes profitable takeovers from ever occurring. However, this problem does not exist in our model. In equilibrium every type of the raider successfully hides her private information in the offer stage, thus earning positive profits. We also show that the higher the fraction of the shares the incumbent initially holds, the higher the equilibrium tender offer price. Contrary to Grossman and Hart (1980b), voluntary full disclosure of private information may not be the outcome even when misrepresenting is not feasible. Moreover, enforcement of disclosure laws may lead to a sub-optimal outcome. Finally, we discuss the design of the optimal corporate charter and show that the optimal choice of the supermajority rule depends on the incumbent's initial holdings.

JEL Classification: G34, C70, D70, D44

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1 Introduction

In this paper, we provide a new theoretical approach to corporate takeovers in a model of incomplete information. One of the questions involved in corporate acquisitions is whether a better management without any private benefits and initial holdings can take over a widely held firm. If so, what is the critical premium that needs to be paid to existing shareholders for a takeover to be successful? This paper provides a model to address both of these issues simultaneously. Moreover, we investigate the factors that play a role in an incumbent's resistance to a hostile takeover. This enables us to calculate the optimal charter which results in the "right" amount of resistance. We also analyze the welfare effects and the beneficiaries of disclosure laws. Finally, we discuss several issues concerning takeovers such as supermajority amendments and voluntary disclosure of information in the absence of positive disclosure laws.

Grossman and Hart's (1980a) work has led to a large literature on the "free rider" problem in takeovers. They argue that a raider without any initial holding and any private benefits of control cannot take over a diffusely held firm even if the value of the firm would be higher under the new management.[1] Whenever an outsider attempts a value increasing takeover, each infinitesimal shareholder demands the post takeover value for his share, knowing that his refusal to tender will not affect the outcome of the takeover attempt. Therefore, there does not exist an equilibrium in which a majority of the shares is tendered at a price lower than the post-takeover value. Thus, the outsider can never gain from a takeover and will not make any offer unless doing so is costless. Grossman and Hart (1981) extend this result and show that free rider problem persists even if the small shareholders are uncertain about the type of the raider.[2]

Since 1980 a large body of work has analyzed the robustness of the free rider problem.[3] Bagnoli and Lipman (1988) show that there does not exist any equilibrium in the Grossman and Hart model unless we relax the infinitesimal shareholder assumption. Consequently, they argue that not having an equilibrium in which the free

[1] In practice, it is not unusual to have outsider raiders (i.e., raiders without initial holdings). In Jennings and Mazzeo's (1993) sample of 647 observations of acquisition attempts only 15 percent of bidders own an initial stake.

[2] In their model, type stands for the manager's ability to increase the shareholder value.

[3] We do not intend to provide a full survey of this literature. We discuss only the papers we think are most related to this paper. See Hirshleifer (1995) for a more general discussion of this literature.

rider problem does not preclude profitable takeovers, fails to be significant since there is no equilibrium at all in the model. They conclude that without the infinitesimal shareholder assumption, the free rider problem does not preclude takeovers occurring with some positive probability. The key to their result is that each shareholder assigns positive probability to the event that he will be pivotal and, in equilibrium, randomizes in deciding whether to tender shares.[4] However, they also show that in this symmetric equilibrium, a raider's profit goes to zero as the number of shares gets large while the overall value of the firm remains constant, re-affirming the free rider problem.[5]

In order to analyze takeovers with small (infinitesimal) shareholders one has to avoid equilibrium non-existence problem in Grossman and Hart (1980a) resulted by unconditional offers. The approach that is mainly accepted is to restrict attention to conditional offers. [See, among others, Shleifer and Vishny (1986), Stulz (1988) Spatt (1989), and Hirshleifer and Titman (1990).]

Grossman and Hart (1980a) suggest dilution of post-takeover value of each non-tendered share, a form of private benefits of control for the raider, to overcome the free rider problem. They argue that in the absence of a large shareholder a value increasing dissident must have a significant level of private benefits of control.[6] Shleifer and Vishny (1986) take the alternative approach and argue that large minority shareholders may provide a solution to the free rider problem. Similar to the Grossman and Hart's dilution, having a large stake in the company is necessary in order to overcome the free rider problem according to their analysis. Therefore, there seems to be a consensus that a successful raider must have either private benefits of control or an initial holding to takeover a widely held firm within this framework.

Our first set of results has two contributions to this literature. First, we show

[4]See also Cornelli and Li (1998) for formation of non-infinitesimal shareholders through risk arbitrage.

[5]Similarly Holmstrom and Nalebuff (1992) show that the robustness of Grossman and Hart's (1980a) result depends on how one takes the limit. They show that divisibility of each share and different size of holdings by different shareholders increases the chance of a takeover. See also Noe (1995) for an alternative approach to this problem. In another related study, Harrington and Prokop (1993) show that the free rider problem persists as the discount factor approaches one in a setting in which there are sequential offers made by the raider.

[6]Minority freeze-out and two tier bids with a lower back-end price are essentially equivalent to dilution of post-takeover value of each non-tendered share. See Bradley, Desai and Kim (1988) for a discussion of two tier offers. In our model, it may not be optimal for the raider to use two tier bids.

that a value increasing raider without any private benefits of control and any initial holding can takeover a widely held firm. Second, our analysis does not suffer from equilibrium non-existence problem even with unconditional offers although we assume infinitesimal shareholders. Therefore, we are able to analyze the strategic behavior of small shareholders (i.e., the free rider problem) with unconditional offers. In our model, management can have two classes of benefits: private benefits of control and security benefits (i.e., benefits of being a shareholder).[7] There is a potential outsider (i.e., no initial holding) raider whose type (private and security benefits) is unknown to the shareholders. The raider makes her tender offer which can be countered by the incumbent's offer. The shareholders simultaneously make their decisions to tender to either the raider or the incumbent or to nobody. If the raider is unsuccessful, she can elect to engage in a proxy fight (or simply ask for an election). We show that in equilibrium a raider without any private benefits of control can hide her type (and thus post-takeover value of a share) resulting in a successful takeover, overcoming the free rider problem. The reason why small shareholders cannot demand the post-takeover value of the firm is that they are uncertain as to what the raider's motives are. The raider might be either a good (value increasing) manager or a bad one who is seeking private benefits of control.

Grossman and Hart (1980b) argue that if lying is illegal[8], the raider will reveal all of her private information voluntarily at the time of the offers.[9] Considering the fact that our results crucially depend on not revealing private information, it is appropriate to analyze the effects of a such law within our framework. We show that voluntary disclosure of information does not occur even if it is costless to do so. Therefore, as a secondary result, we establish that the argument, made by Grossman and Hart (1980b) on voluntary disclosure of information, is not robust to variations of private information.

There is ample evidence that shareholders receive a high premium in a successful takeover and the bidding for corporate acquisitions frequently moves in large jumps.[10]

[7] Either of these benefits can be zero.

[8] For the sake of the argument we will assume that this law can be enforced.

[9] See Okuno-Fujiwara et al. (1990) for a general treatment of strategic information revelation.

[10] Jarell et al. (1988) present evidence in this direction. See also Bradley (1980). Fishman (1988) argues that this is due to preemptive bidding caused by costly information acquisition. Daniel and Hirshleifer (1995), on the other hand, offers an explanation via a theory of costly sequential bidding.

Consistent with these stylized facts, in our model a raider pays a premium sufficient to persuade the shareholders to tender, although a good manager does not have to offer the full value in equilibrium. We also show that this premium is an increasing function of the fraction of shares held by the incumbent.

Regulators usually fear that consumers and small investors are taken advantage of due to their lack of information. Disclosure laws aim to remedy this problem. We investigate if these laws are designed correctly and whether they have any possible perverse effects not only for the overall economy but also for the very group it intends to protect. We analyze efficiency effects of the Disclosure Acts and conclude that these laws (if they could be enforced) may lead to sub-optimal outcomes.[11]

Finally, we isolate some other factors which play a role in takeovers: the fraction of the incumbent's shares and the parameters of the charter, so that one can design the optimal charter given an environment. These factors determine the intensity of the incumbent's resistance. Obviously, the incumbent's resistance to a hostile takeover has an important effect on the outcome. We show that although resistance generally helps the shareholders, it may lead to an undesirable outcome for the shareholders, as well as socially, once it is above a critical level. This leads us to the design of optimal corporate charters in a given environment.

Grossman and Hart (1988) and Harris and Raviv (1988) analyze the optimality of the one share-one vote and the simple majority rule. We take the one share-one vote rule as given and show that ex-post socially inefficient outcomes may arise under a supermajority rule. We also show that under the incomplete information assumption, the simple majority rule may no longer be socially optimal. Once we show that too much resistance may lead to inefficient outcomes and formalize the relationship between the intensity of resistance and the supermajority rule, we analyze the dependence of the optimal combination of the supermajority rule and incumbent's initial holdings on the parameters of the environment, both from the social and from the shareholders' point of view.[12]

Both Fishman and Daniel and Hirshleifer study auctions with a single selling shareholder as opposed to developing an explicit model of diffuse shareholders' behavior. Moreover, these models need at least two potential bidders in order to explain high premiums.

[11] See also Fishman and Hagerty (1995) who show that mandatory disclosure laws regarding insider trading may result in market manipulation.

[12] Our results may be viewed as complementary to that of Stulz (1988) who argue that an increase

The next section presents the core model. In Section 3, we characterize the equilibria and discuss the main features of the model. Section 4 analyzes the intensity of resistance. We discuss the possibility of voluntary disclosure of information in Section 5. Section 6 discusses the efficiency effects of the charter and outlines the optimal choice of supermajority rule in conjunction with incumbent's initial holdings. Section 7 includes our concluding remarks.

2 The Model

According to Grossman and Hart (1980), Bagnoli and Lipman (1988) and Holmstrom and Nalebuff (1992), the existence of a free rider problem and its preclusion of takeovers go hand in hand with the infinitesimal shareholder assumption. As mentioned above, Bagnoli and Lipman (1988) show that the free rider problem does not reduce the probability of a takeover to 0 if there are finitely many shareholders (i.e., a profitable takeover occurs with a positive probability). In this paper we show that, the free rider problem does not preclude takeovers at all even under the assumption of infinitesimal shareholders. For this reason, we assume a unit (Lebesgue) measure of non-atomic outsider shareholders indexed by $x \in [0, 1]$. Hence, initial holdings of outsider shareholders are defined by function $f : [0, 1] \to \mathcal{R}_+$.

Apart from the infinitesimal outsider shareholders, there is an incumbent who may hold a significant amount of the shares. Therefore, $\int_0^1 f(x)dx + f_0 = 1$, where f_0 is the fraction of the incumbent's initial holdings. However, for the time being, we assume f_0 to be zero in order to focus on the free rider problem. In Section 4, we relax this assumption and analyze the effects of a larger f_0. The charter requires the support of fraction $\gamma \geq \frac{1}{2}$ of the votes to capture the management, where each share has one vote. Therefore, an incumbent needs the support of only $1 - \gamma$ of the shares to remain in office.[13] We take γ to be $\frac{1}{2}$ in this section for simplicity and we will relax this

in the fraction of voting rights controlled by management increases the amount of premium target shareholders receive. He fixes the simple majority rule and hence does not attempt to find the optimal supermajority rule. He also assumes that expected fraction of the shares tendered is an increasing function of the premium. Furthermore, he requires tendering decisions not to depend on initial holding. Finally, he restricts attention to conditional offers.

[13] We will not specify any tie breaking rule, since in equilibrium,there will be no ties and introducing any such rules does not play a role as far as the results are concerned.

assumption later when we calculate the optimal choice of γ in Section 6. Under the incumbent, the present value of the dividends is p_0 and known to all the parties.[14] We also assume that the incumbent's private benefits of control, q_0, is 0. We will relax this assumption in Section 4 when we discuss the resistance in more detail.

Raiders differ in their ability to produce cash flows and their private benefits of control. Therefore, each raider may have a different reason for her attempt to takeover. Some would like to acquire a target just because they can generate higher cash flows whereas others are after high private benefits of control. When a possibility of a tender offer arises, before any action is taken, the shareholders do not know which kind of a raider they are dealing with. There is a bidder who can be either "good" (g) type or "bad" (b) type with probabilities α and $1-\alpha$, respectively. The present value of dividends is p_g under g and p_b under b. The bad type, b, also has a private benefit of control, q_b.[15] We assume $p_b + 2q_b > p_g > p_0 > p_b$ to make things interesting and simple.[16] For further use let us define $\bar{p} = \alpha p_g + (1-\alpha)p_b$.

For simplicity, we first assume that there is no cost of making a tender offer. We then show that each type of raider makes positive profits in equilibrium. In equilibrium there would be successful takeovers with positive profits even if there were a cost of making a tender offer that is lower than the expected profits of each type. As mentioned in the introduction, it is such a cost that precludes takeovers in other models in which raiders make zero profits because of the free rider problem.

As we have stated in the previous section we will focus on unconditional offers. Unconditional offer means that the bidder will buy even if the fraction of the tendered shares do not sum up to a prespecified level, θ_r. Restricted offer means that if the fraction of the tendered shares exceed θ_r, prorating takes place. Restricted offers are typically unconditional offers.[17] Any and all bid may be viewed as a special case of

[14] Obviously, p_0 may not be the same as the pre-takeover price of a share if a takeover is viewed as possible.

[15] The incumbent and g may also have private benefits but we assume away this possibility at this point. In Section 4, the possibility of private benefits of the incumbent will be discussed.

[16] The first inequality can be reversed without changing the main results. Such a restriction enables us to classify the set of equilibria in Section 3. Reversing this inequality (but still keeping $p_b + 2q_b \geq \alpha p_g + (1-\alpha)p_b$) results in additional pooling equilibria in which neither type makes a successful offer to earn strictly positive profit. Nevertheless, these equilibria fail our refinement requirement as we discuss in the appendix.

[17] See Comment and Jarrell (1987).

unconditional restricted offer with $\theta_r = 1$. Another common type of bid is a conditional tender offer. The offer to buy shares is conditional on the shareholders tendering at least some minimum amount. This amount is usually γ. In this model, if we allow the bidders to use both unconditional and conditional offers we get the same set of equilibria outcomes which we have when we allow only unconditional tender offers. Thus, it seems appropriate to focus on unconditional offers within this model.

The time structure is as follows:

$t = 0$ Nature chooses the type of the raider.

$t = 1$ The raider (r) makes an unconditional restricted tender offer consisting of a price p and a fraction $\theta_r \geq \frac{1}{2}$ of shares she is willing to buy.[18]

$t = 2$ The incumbent (i) either makes an unconditional restricted tender offer (p_i, θ_i) or does not act.

$t = 3$ The shareholders simultaneously either accept one and reject the other or reject both of the tender offers.

$t = 4$ Voting takes place between the incumbent and the raider, where each share has one vote. The raider takes over if she gets at least γ of the votes. Otherwise, the incumbent stays in power.

If a majority of the shares are tendered to a bidder (i.e., raider or incumbent) at $t = 3$, the outcome of the voting stage at $t = 4$ will be trivial in the sense that in equilibrium this bidder will choose to vote herself/himself into management. However, if no such majority is established, the small shareholders may vote for either the incumbent or the raider. In fact, each small shareholder is infinitesimal and has no effect on the outcome. Therefore, there are multiple equilibria of the voting subgame for each belief. Namely, small shareholders can all vote for the incumbent although they all believe that the raider is better or vice versa. Our results are robust to these variations and we will not put any restrictions on the set of equilibria of this subgame.[19] However, in equilibrium

[18] In Yılmaz (1997), we show that the raider never makes an offer with $\theta_r < \frac{1}{2}$ in an equilibrium of the dynamic version of this model, since she will be vulnerable to future control contests.

[19] See Lemma 1 for the characterization of equilibria of the voting subgame.

we never have a raider who is unsuccessful at the tendering stage winning a proxy contest. In other words, such subgames are off the equilibrium path.

We should also note that the sequencing of bids gives the incumbent a "second mover advantage". Prior to the incumbent's potential offer, a raider has to commit to a tender offer.[20] Therefore, it is possible for a raider to lose money in this process if she does not consider the possibility of a counter offer and fails. In fact, in equilibrium she never makes an offer which will be unsuccessful.

Let $g_k(.)$ denote the amount of shares a shareholder tenders to bidder k where $k \in \{i, r\}$. Obviously, we have $g_r(x) + g_i(x) \leq f(x)$ for all $x \in [0, 1]$ and $g_r(i) \leq f_0$. The shares bought from each shareholder (after prorating) is denoted by $\hat{g}_k(.) = \lambda_k g_k(.)$, where $\lambda_k = min\{\frac{\theta_k}{\int_0^1 g_k(x)dx + g_k(i)}, 1\}$ is the prorating coefficient. The amount of shares kept by shareholders are $\hat{f}(x) = f(x) - \hat{g}_i(x) - \hat{g}_r(x)$ and $\hat{f}(i) = f_0 - \hat{g}_r(i)$. The fraction of shares bought by the bidders are $\hat{\theta}_r = \int_0^1 \hat{g}_r(x)dx + \hat{g}_r(i)$ and $\hat{\theta}_i = \int_0^1 \hat{g}_i(x)dx$.

Let $v_k(.)$ denote the amount of votes used by a shareholder to support bidder k, where $v_r(x) + v_i(x) \leq \hat{f}(x)$ and $v_r(k) + v_i(k) \leq \hat{\theta}_k$. Let $\phi_k = \int_0^1 v_k(x)dx + v_k(i) + v_k(r)$ stand for the total fraction of votes received by the bidder k.

3 Equilibria

By an equilibrium of this game, we mean a perfect Bayesian equilibria. Common to games of incomplete information with observable actions, this model has both pooling and separating equilibria. The reason is that the perfect Bayesian equilibrium concept does not impose any restrictions on beliefs off the equilibrium path. Consequently, there is a large literature on refinement concepts in signaling games. Here, we will use a refinement concept (in spirit) identical to perfect sequential equilibrium (see Grossman and Perry (1986)). If the raider deviates from the equilibrium tender offer, the shareholders should form a posterior belief (calculated using the priors) such that the equilibrium outcome (given the posterior belief) in the subgame following the deviation should (weakly) benefit each type in the support of the formed belief. We define this refinement concept formally in the appendix and call an equilibrium that satisfies the refinement a refined equilibrium. In every separating equilibrium, the type

[20] We assume that these offers are registered and have to be honored.

b raider makes a successful tender offer with a price p_g, resulting in a positive profit for the raider. We should also note that there does not exist a separating equilibrium in which type g makes a profitable offer. If we reverse the assumption $p_b + 2q_b > p_g$ neither type can make positive profits in a separating equilibrium. Under either assumption no separating equilibrium satisfies the refinement concept.

All of the pooling equilibria have the same qualitative features. Both types of raider make a successful tender offer in which the price is between $\bar{p}$ and p_g and the fraction is at least $\frac{1}{2}$.[21] In the appendix, we provide a characterization of the complete set of equilibria and show that there is a unique refined equilibrium payoff. We show in the appendix that this refined equilibrium outcome is the least favorable for the shareholders out of the entire set of pooling perfect Bayesian Equilibrium outcomes.

In the remainder of this section, we will analyze the unique refined equilibrium outcome.

Proposition 1 *There exists an equilibrium in which both types of raider make an offer of* $(\bar{p}, \frac{1}{2})$ *and every shareholder tenders his share(s). Hence takeover takes place with probability* 1*. This is the least favorable equilibrium outcome for the shareholders among the set of pooling equilibria.*

Proof: See appendix. $\diamond$

The equilibrium payoffs characterized in Proposition 1 is the unique payoff satisfying the refined equilibrium concept. However, we will leave the discussion about uniqueness to the appendix and first talk more about the equilibrium behavior. One important feature of this result is that the transaction takes place at a price significantly lower than the post-takeover value once the raider, with higher cash flows, takes over. In other words, the small shareholders do not (or cannot) demand the entire surplus as they do in Grossman and Hart's model. The reason is the uncertainty about the post-takeover value of the firm which remains unraveled even after the tender offer. The shareholders do not know the post takeover value of their shares and are willing to tender at a price (weakly) greater than their expected value, given that a majority of the shares are bought by the raider.

[21] See figures 1 and 2 and propositions 8 and 9 for the set of pooling perfect Bayesian equilibrium outcomes under different assumptions.

Now let us see why each type of raider makes the offer $(\bar{p}, \frac{1}{2})$. The choice of $\bar{p}$ is clear, since this is the lowest price which can be successful. Type b has also no incentive to deviate from $\theta_r = \frac{1}{2}$, since she prefers to buy the minimum amount given that $p_b < \bar{p}$. We should note that the raider of type g benefits from obtaining a higher fraction θ_r in the offer (given shares are tendered), since each additional share costs less than post takeover value p_g. Therefore, type g may prefer an offer $(\bar{p}, \theta_r)$ where θ_r is greater than $\frac{1}{2}$. However, such an offer can signal that the raider is of type g. If this is the belief held by the shareholders, such an offer will be unsuccessful due to an attempt to "free ride". Hence type g does not find it profitable to deviate from the offer $(\bar{p}, \frac{1}{2})$.[22]

At this point one should note the effects of the voting stage. If one thinks of a takeover as a bargaining problem, Grossman and Hart (1980a) model is simply about a large agent who makes a "take it or leave it" to infinitely many small agents. If a majority of small agents does not accept, the surplus is wasted although the accepting minority receives what they are offered. If a majority accepts, then any member of the minority who does not accept the offer, receive the entire surplus for his share. If small shareholders believe that majority is accepting the offer, then it is optimal not to accept. Similarly, if small shareholders believe that a majority rejects the offer, it is optimal to accept. Therefore, this "take it or leave it" creates the equilibrium non-existence problem. The driving force in finite models of Bagnoli and Lipman (1988) and Holmstrom and Nalebuff (1992) is the ability of the raider making such a take it or leave it offer and therefore forcing the shareholders to tender at any price over the current price since rejecting the offer leaves the shareholders with the possibility of reducing the post-takeover price for each share. However, such a tool in this model whenever there is complete information would be "too" strong. If the shareholders know the exact value of each share under the raider's management, they would not tender at any price below this value; in the case of such a low offer, they would simply vote for the raider and offer her reservation wage for compensation. Therefore, the free rider problem would be even stronger in the sense that it precludes takeovers occurring even in finite shareholder cases. In our model, the shareholders cannot demand the entire

[22] We provide the rigorous analysis of the proof of this proposition and supporting four lemmas in the appendix.

surplus since they do not have complete information about the type of the raider.[23] Therefore, the voting stage in our model introduces a balance in the bargaining process, thus solving the equilibrium non-existence problem and ending the dichotomy between the infinitesimal and finite shareholder cases.[24]

There are several implications of the above proposition. First of all, in contrast to Grossman and Hart (1980), the raider acquires a sufficient number of shares to govern at a price at which she can make positive profits even after the cost of taking over. More importantly, this result does not depend on the measure of each share, i.e., we do not need finitely many shareholder. Third, the tender price includes a premium over the pre-takeover value even if there is no additional cost or second potential raider. Fourth, the asymmetric information prevents the strategic behavior of shareholders from having negative impact on shareholder value. We already know that in the absence of asymmetric information the free rider problem precludes takeovers, thus reducing shareholder value. If one interprets the Disclosure Acts as a requirement for the raider to reveal any private information (relevant to the post-takeover value of the firm), implementation of the Disclosure Acts or any other revelation mechanism of this kind may have perverse effects.[25] Namely, it reduces the shareholder value.

4 More Resistance by the Incumbent

In the previous section, we saw that the threat of a counter offer pushes the price to $\bar{p}$. Therefore, the shareholders benefit from the possibility of resistance.[26] Nevertheless, it is clear that too much resistance may have undesired effects for non-management shareholders, since it may reduce the possibility of a takeover. In this section, we will relax the assumptions $f_0 = q_0 = 0$. This enables us to work towards characterization of

[23] But what they can do is to demand the expected value of the firm under the new management.

[24] Although the voting stage is crucial in this model, one can eliminate it without altering the results in similar settings. See Yılmaz (1997).

[25] We should also note that such laws will have an implementability problem in non-trivial settings. Although we do not intend to fully address the mechanism design problem and analyze which level of revelation could be achieved by the SEC, we will shortly discuss the case where lying is illegal (under the assumption that this could be somehow enforced) in Section 5.

[26] In an empirical study Jennings and Mazzeo (1993) show that resistance by the incumbent and premium received by the shareholders are positively correlated.

the optimal corporate charter, once we relax $\gamma = \frac{1}{2}$ in Section 6. We will first analyze the effects of various factors on resistance, including the fraction of the incumbent's initial shares (or equivalently the stock options) and the incumbent's private benefit of control. Then in Section 6 relaxing $\gamma = \frac{1}{2}$ will enable us to find the optimal combination of supermajority rule and incumbent's initial holdings resulting in the "right" amount of resistance.

Generally, there are several tools which can be used by management against a hostile takeover attempt. However, we focus on counter offers and supermajority amendments. Obviously, one reason for resistance is incumbent's private benefits of control. It is natural to see more resistance by an incumbent who has higher private benefits. Nevertheless, any shareholder without any private benefit may also be tempted to resist as we have seen in the previous section in line with the free rider problem.

Consequently, the incumbent may have two different reasons for accumulating shares: To take advantage of potential high cash flows of the raider or to defeat a takeover attempt to protect his private benefits of control. The former is the motive we try to capture in the previous section, since the private benefits of the incumbent is assumed to be negligible until Section 4. Clearly, there exists a high enough level of private benefits of control for the incumbent so that his motive becomes protecting his private benefits by defeating a hostile takeover attempt.[27]

As we have seen in the previous section, in order to have a successful takeover, the raider has to bid high enough to prevent the incumbent to make an offer to buy additional shares. Therefore, if a raider wants to have a successful offer, then the tender offer (p, θ_r) has to be lucrative enough so that the incumbent does not find it in his interest to resist for either of the reasons we discussed above. Thus, $\bar{p}$ is the minimum price the raider has to pay in order to succeed against an incumbent with low level of private benefit control. Similarly, $\hat{p}$ stands for the minimum successful price against an incumbent with high level of private benefit of control. Let $\bar{q}_0$ denote the critical level of incumbent's private benefit of control, $\hat{p}$ is greater than $\bar{p}$ whenever q_0 is greater than $\bar{q}_0$.

Once the fraction of shares that are tendered exceed γ, prorating takes place. There-

[27] If this is the case, then the incumbent needs to have only $1 - \gamma$ of the shares. Therefore, when we relax $\gamma = \frac{1}{2}$ in Section 6 and allow supermajority rules, we will see that higher γ strengthens this kind of resistance.

fore, some shares are returned to the shareholders. The ex-ante expected value of a share returned to a shareholder following a successful takeover attempt is $\bar{p}$. Therefore, this does not affect the behavior of the incumbent with low private benefits of control. However, if the incumbent has high private benefits of control, prorating results in a loss for the incumbent. This loss is higher when f_0, the fraction of the incumbent's initial holdings, is higher. Therefore, the larger f_0 is, the greater incentive incumbent has to resist the takeover.[28] Moreover, the number of shares which he needs to buy is small once the ex-ante ownership is large for the incumbent. This second effect also helps the incumbent to pay a higher premium for the remaining $\frac{1}{2} - f_0$ shares. Therefore, whenever f_0 is higher the raider has to pay a higher price in order to prevent a counter offer. This intuition is formulized in Proposition 4.

Similarly, θ_r, the fraction of the shares the raider offers to buy, also affects the price, since different fractions lead to different ratios in prorating. Clearly, higher θ_r means a smaller loss due to prorating. Therefore, the raider can reduce the tender offer price as long as he increases the fraction of shares he is willing to buy. Consequently, equilibrium tender offer price is a function of both θ_r and f_0 as characterized by Proposition 2 below.

Proposition 2 *Let* $p^* = max\{\bar{p}, \hat{p}\}$ *where* $\hat{p}(\theta_r) = \frac{(1-\gamma)p_0 - f_0(1-\theta_r)\bar{p} + q_0}{1-\gamma-f_0(1-\theta_r)}$. *For any* θ_r, p^* *is the lowest possible price such that the tender offer* (p^*, θ_r) *leads to a successful takeover attempt in a pooling equilibrium.*

Proof: See appendix. ◇

It is important to note that a higher equilibrium price is not necessarily better for the shareholders. The lowest successful price p^* is a decreasing function of θ_r, since the incumbent finds it less attractive to resist once his loss due to prorating gets smaller. Therefore, a low θ_r results in a higher lower bound for equilibrium prices. However, this higher price comes at the cost of only small fraction of each share being honored due to prorating. The remaining fraction $1 - \theta_r$ of each share will have an expected value of $\bar{p}$ which is much smaller than the equilibrium tender price. The following proposition formalizes this trade-off.

[28]We assume that $f_0 < \frac{1}{2}$.

Proposition 3 *p^* is a weakly decreasing function of θ_r.*

Proof: See appendix. $\diamond$

Now we turn to the discussion how the incumbent's initial holdings and private benefits of control changes the equilibrium premium and hence the welfare of shareholders. As we have explained above higher initial holdings by the incumbent results in higher prices since the resistance is greater. The private benefits of control has a similar effect. Higher private benefits of control increases the incentive of the incumbent to resist, resulting in higher equilibrium prices. Therefore, for any θ_r, if we look at the least favorable equilibrium outcome for the shareholders, then the premium they receive in that equilibrium increases with both f_0 and q_0. The following proposition characterizes the effects of f_0 and q_0 on the equilibrium outcome.

Proposition 4 *Both p^* and the expected shareholder value in lowest price equilibrium, $p^*\theta_r + \bar{p}(1-\theta_r)$, are (weakly) increasing functions of both the incumbent's initial holdings, f_0 and private benefit of control, q_0.*

Proof: See appendix. $\diamond$

This result will be useful in Section 6 once we start discussing how to maximize the shareholder value.

5 Voluntary Disclosure of Private Information

We have stated earlier that if disclosure of all private information is enforced, then we will have the free rider problem back. Whenever the shareholders know that the raider is of type g, they will not tender their shares at a price lower than p_g. This would result in no takeovers and hence a welfare loss. As a result, the enforcement of disclosure laws can harm shareholders in certain situations.

However in this section, we take a different approach to disclosure laws following Grossman and Hart (1980b). They basically ask what would happen if there were no disclosure laws but only a law against lying. They conclude that even in the absence of any positive disclosure laws (a law which requires disclosure of relevant private information), the bidder will fully disclose all the information at the time of the raid as long as lying is illegal.[29] To see why this could be true, let us consider two types of raiders (both without any private benefits of control), one of which creates higher cash flows than the other one. Obviously the high-value type would benefit from being seen as the low-value type since the shareholders will then agree to tender at the low type's value. However, it is common knowledge that such a misrepresentation is not possible since lying is illegal. Therefore, the only possible pooling would be making no announcements for both types. However, this kind of pooling cannot be an equilibrium either, since not telling her type is never a best response for the low-value type unless the beliefs are such that the raider is low-value type in the absence of announcements. However, such a belief is not possible on the equilibrium path since the high-value type will not make any announcements once the beliefs are set this way. Therefore, low-value type will make a voluntary announcement.

Unfortunately, as we will see in the next proposition, this result does not extend to multi-dimensional type spaces, even ones as simple as the one in our model. To analyze this, we add a stage to our game, in which the raider has a chance to announce her type before the tendering period. Moreover we will assume that lying is illegal and this law is enforceable.

One may think that in our model the same result should hold since at first sight the low-value type (i.e. type b) may seem like she would benefit from revealing her type. After all, once the shareholders know that she is of type b, there would be no reason why they should not tender for any price over p_0. However, a possible resistance by the incumbent may reverse this outcome. In our model, the reason why the bad type may not reveal her type is that she wants to take advantage of restricted offers. If she can hide her type, the returned shares after prorating have a higher expected value. Therefore, pooling across types results in a less intense resistance. Hence, it may be optimal for type b not to disclose the private information.

For the low q_0 case, there are two forces affecting the intensity of the resistance, which are working against each other. Resistance due to the belief that the firm's expected value is $\bar{p}$ no longer exists once it is known that the raider is of type b. This value is now p_b, which is strictly less than $\bar{p}$. Therefore, the resistance will be

[29] We have some reservations about the enforceability of such a law against lying. Nevertheless, for the sake of the argument let us assume that there is no implementability problem.

weakened in this way if the raider is known to be of type b. This is in fact the only force in Grossman and Hart (1980b). However, there is a second factor in our model; the shares that are returned to the shareholders due to prorating will have a value of p_b compared to $\bar{p}$ of the pooling equilibrium. Therefore, the resistance to prorating will be higher once type b reveals her identity.

For the high q_0 case, the only force is the one that increases resistance is due to prorating, since the other force does not exist by the definition of incumbent with high private benefits. (He would like to stay in control independent of the raider's type. Thus, the expected post takeover value is irrelevant.) Therefore, for any set of other parameters of the model, the incumbent with high private benefits will resist more once he realizes that the raider is of type b.

In the next proposition we formalize the arguments above and show that even if lying is illegal, there may be no voluntary disclosure of private information.

Proposition 5 *If the incumbent has high private benefits of control, $p^* = \hat{p}$, then there is no voluntary disclosure of information. Otherwise, there is no voluntary disclosure of information only if $\frac{(1-\gamma)p_0 - f_0(1-\gamma)p_b}{(1-\gamma)(1-f_0)} > \bar{p}$.*

Proof: See appendix. ◇

6 Optimal Supermajority Rule

In order to work towards an optimal supermajority rule, we have to analyze how the equilibrium behavior is affected by the supermajority rule, γ. For this purpose, we will relax the assumption of $\gamma = \frac{1}{2}$ and allow γ to take any value between $\frac{1}{2}$ and 1.

If the price is higher than $\bar{p}$, then the incumbent buys any share only to save his private benefits of control. Therefore, he will buy only up to $1-\gamma$ fraction of the shares since each additional share costs more than its post takeover value, p_0. Consequently, the premium that can be afforded in order to save private benefits is higher, resulting in a higher equilibrium price for a higher supermajority rule. The following proposition states the effect of a higher supermajority rule on the equilibrium prices:

Proposition 6 *For any θ_r, p^* is a (weakly) increasing function of the supermajority rule, γ.*

Proof: See appendix. $\diamond$

Now we can proceed with our analysis of optimal supermajority rule. We present two examples to show that optimality of a supermajority rule depends on several factors.

Example 1: An incumbent with low private benefits Let the supermajority rule γ be equal to $\frac{1}{2}$. [This is the socially optimal choice according to Grossman and Hart and Harris and Raviv under mild conditions.] Now let $p_b + 2q_b > p_g > p_0 > p_b + q_b > p_b$, $\alpha p_g + (1-\alpha)(p_b+q_b) > p_0+q_0$ and f_0 and $q_0 = 0$ are small. In other words, both types can afford to pay $\bar{p}$ and the incumbent has low private benefits of control. Finally, the expected social welfare is higher if the takeover attempt is successful.

In this example, the set of equilibrium outcomes is very similar to the one in Figure 1. Hence, according to Proposition 1, the shareholders tender their shares to the offer $(\bar{p}, \frac{1}{2})$ in the unique refined equilibrium outcome. This is a socially optimal outcome, since a raider generates an expected value larger than the incumbent does. However, this is not the best outcome for the shareholders. If we increase the supermajority rule the price may exceed $\bar{p}$. Therefore, we must choose $\gamma > \frac{1}{2}$ if we are concerned with shareholder value maximization.[30]

If we modify our example to allow $\alpha p_g + (1-\alpha)(p_b+q_b) < p_0+q_0$ (i.e., takeovers are expected to destroy value), simple majority rule will not be even socially optimal. If this is the case, the optimal charter should allow any anti-takeover measure to avoid a takeover. However, we will make the following assumption so that we restrict attention to cases in which takeovers (weakly) increase social welfare.

Assumption 1 *The expected post takeover social welfare, $\alpha p_g + (1-\alpha)(p_b + q_b)$, is (weakly) greater than $p_0 + q_0$.*

Given the previous example one might think that a high supermajority rule may maximize both shareholder value and social welfare as long as Assumption 1 is satisfied. The following example shows that such a corporate charter may have perverse effects.

[30] A low level of resistance does not create a socially inefficient outcome as long as a pooling equilibrium with a successful takeover survives.

Example 2: An incumbent with high private benefit Let us assume that $p_g > p_b + 2q_b > p_0 > p_b$. The incumbent's private benefits and initial holdings could be so large that if we use a high supermajority rule, type b is driven out of the market. Therefore, the raider of type g faces the free rider problem and does not make any tender offer in equilibrium. This outcome is not only socially inefficient but also undesirable for the shareholders. Therefore, this example shows that optimal choice of γ depends on the incumbent's initial holdings and private benefits of control.

We should also note that socially inefficient outcomes do not necessarily imply a sub-optimal result for the shareholders. To see this, it suffices to consider a raider of type b with very high private benefits so that she can separate herself from the other type regardless of the choice of the supermajority rule. If f_0 and q_0 are high enough, only type b can make a successful takeover. Obviously, this is a socially sub-optimal outcome. However, shareholders may prefer a very high price with probability $1 - \alpha$) to a lower price with probability 1. In this case a socially inefficient level of resistance (and a charter which allows it) is optimal for the shareholders. This is clearly a very special and concocted case. In fact, we will make the following assumption and restrict attention to cases in which possibility of takeovers by type g always increases shareholder wealth.

Assumption 2 *The highest shareholder wealth with type g driven out of market,* $(1 - \alpha)(p_b + q_b) + \alpha p_0$, *is less than* p_g, *the highest shareholder wealth that can be achieved by pooling.*

We also would like to assume away the trivial case in which the incumbent has very high benefits and initial holdings such that he can stay in control independent of the supermajority rule.

Assumption 3 *There exist a supermajority rule,* γ, *and an offer* $p^*(\theta_r, \gamma)$ *such that the incumbent cannot drive either type of the raider out of the market:* $Min\{p_g, p_b + \frac{q_b}{\theta_r}\} > p^*(\theta_r, \gamma)$.

This leaves us to state that there is a the critical combination of γ and f_0 so that the charter results in the right amount of resistance.

Proposition 7 *For a given f_0, there exists an interval of supermajority rules $(\frac{1}{2}, \bar{\gamma}(f_0))$ such that for all $\gamma \in (\frac{1}{2}, \bar{\gamma}(f_0))$, there exists a socially optimal equilibrium outcome. Furthermore, for any $\gamma > \bar{\gamma}(f_0)$, there is no socially optimal equilibrium. Similarly, for a given γ there exists a $\bar{f}_0(\gamma)$ such that all $f_0 < \bar{f}_0(\gamma)$ are socially optimal and all $f_0 > f_0\bar{(}\gamma)$ are socially sub-optimal.*

Proof: See appendix. $\diamond$

As f_0 gets larger the intensity of resistance continuously increases, so that after a critical level takeover becomes non-profitable for at least one type of the raider. Hence, the possibility of a pooling equilibrium is zero for a higher f_0. Therefore the raider with high cash flows cannot takeover, resulting in a socially inefficient outcome. From Proposition 3, we also know that shareholder value increases as f_0 rises. Therefore, the highest f_0 that does not cause a socially inefficient outcome is optimal for shareholders.

7 Conclusion

We noted that the literature on takeovers mostly focuses on widely held firms following Grossman and Hart (1980a). Unfortunately, earlier papers exclude unconditional bids from any (equilibrium) analysis of strategic behavior of small (i.e. infinitesimal) shareholders due to non-existence of equilibrium in the presence of such offers. The other feature of this literature is the consensus that a value increasing raider without any initial holdings (toehold) or a private benefit of control cannot take over a widely held company and make positive profits.

We use a framework with incomplete information in which small shareholders are uncertain as to what the raider's motives are. One possible type of a raider would like to acquire a target just because she can generate higher cash flows whereas other type is after high private benefits of control. Consequently, small shareholders cannot demand the post takeover value with higher cash flows since they are not certain that such cash flows can be attained under the new management. Therefore, in contrast to the free rider problem, we show that a raider lacking both a toehold and a private benefit of control, can be successful in a hostile takeover. Moreover, the analysis allow conditional bids as well as unconditional ones.

In addition, we revisit the result by Grossman and Hart (1980b) that the bidders always disclose their private information voluntarily as long as misrepresentation is illegal. We show that this argument does not hold if the raider can have various motives for not revealing her identity.

We also formulate the intensity of incumbent's resistance in terms of the parameters of the corporate charter and his initial holdings. We show that higher initial holdings as well as a higher private benefit of control by the incumbent result in a higher premium for the shareholders. We also show that a higher supermajority rule used by the charter does not necessarily translate into higher equilibrium prices.

After characterizing the intensity of resistance by the incumbent, we finally discuss corporate charter design. We show that the optimal supermajority rule depends on the initial holdings as well as private benefit of control of the incumbent.

Appendix

We begin with a characterization of all equilibria of the model in Section 2, i.e, $f_0 = q_0 = 0$ and $\gamma = \frac{1}{2}$.

Let μ_1 to be the beliefs formed by the shareholders after the tender offer in stage one about the probability of raider being type g. Lemmas 1 to 3 characterize the subgame starting with the tendering stage. Let π stand for the expected return of a share given an equilibrium behavior in the subgame starting with the tendering stage.

Lemma 1 *Table 1 summarizes the equilibrium outcomes of the voting subgame.*

Share Distribution	Stage 4	Beliefs
$\hat{\theta}_r \geq \gamma$	RW	Any belief
$\hat{\theta}_i \geq \gamma$	(i) RL (ii) RW	(i) $\mu_1 p_g + (1-\mu_1)p_b < p_0$ (ii) $\mu_1 p_g + (1-\mu_1)p_b > p_0$
$\hat{\theta}_r < \gamma$ & $\hat{\theta}_i < \gamma$ & $\hat{\theta}_r + \hat{\theta}_i \geq \gamma$	(i) RL (ii) RW	$\mu_1 p_g + (1-\mu_1)p_b < p_0$ Any belief
$\hat{\theta}_r < \gamma$ & $\hat{\theta}_i < \gamma$ & $\hat{\theta}_r + \hat{\theta}_i < \gamma$	(i) RL (ii) RW	Any belief Any belief

Table 1: The set of equilibria of the voting subgame.[31]

<u>*Proof:*</u> If $\hat{\theta}_r \geq \gamma$, then $\phi_r \geq \gamma$, since both types of raider vote for herself given that her vote is sufficient in voting stages. The other shareholders' vote cannot alter the outcome; therefore, any action is a (weak) best response for them.

If $\hat{\theta}_i \geq \gamma$, then the incumbent's votes are sufficient to control the outcome. Therefore, if $\mu_1 p_g + (1-\mu_1)p_b > p_0$ then the incumbent votes for the raider at $t = 4$.

If $\hat{\theta}_r < \gamma$ and $\hat{\theta}_i < \gamma$ but $\hat{\theta}_r + \hat{\theta}_i \geq \gamma$, then the votes of incumbent and the raider guarantees a takeover as long as the incumbent has high beliefs, i.e., $\mu_1 p_g + (1-\mu_1)p_b > p_0$. Therefore, the incumbent must have low beliefs in order to raider to lose. However, the raider can win for any belief since the sum of small shareholders and the raider is

[31] RW and RL stand for "raider wins" and "raider loses", respectively.

sufficient. Each small shareholder has no effect, so any action is a best response for him.

The rest of the argument also from the fact that each small shareholders has no affect on the outcome. ◇

Lemma 2 *Let (p_1, θ_1) and (p_2, θ_2) be the two tender offers where $p_1 > p_2$. If (p_1, θ_1) is not successful at stage 3, then no shares are tendered to (p_2, θ_2).*

Proof: Suppose that Lemma 2 does not hold. Take any shareholder with $g_2(.) > 0$. His expected return from tendering (after prorating) $\lambda_2 p_2 + (1 - \lambda_2)\pi$ is at most p_2, since otherwise he would hold on to his share and do better. Such a shareholder would therefore prefer to tender at price p_1 and receive $p_1 > p_2$. ◇

Lemma 3 *Let (p_1, θ_1) and (p_2, θ_2) be the two tender offers. If $p_1 > max\{p_2, \pi\}$, then (p_1, θ_1) will be successful.*

Proof: Suppose $p_1 > max\{p_2, \pi\}$ but (p_1, θ_1) is not successful. By Lemma 2, no shares are tendered to (p_2, θ_2). Therefore, there exists a shareholder with $\hat{f}(.) > 0$, but could tender $\hat{f}(.) > 0$ without prorating (i.e., $\lambda_1 = 1$) at a price p_1. Keeping the share has an expected return of $\pi < p_1$. Therefore, he is better off by tendering to (p_1, θ_1). ◇

Let us consider the subgame following tender offer prices, both (weakly) less than $\mu_1 p_g + (1 - \mu_1)p_b$. Let $\mathcal{O}$ be an equilibrium outcome of this subgame such that

t=3 not tender to any offer,

t=4 RW.

Lemma 4 *Following an offer price $p_r < \mu_1 p_g + (1 - \mu_1)p_b$ the incumbent makes a counter offer $(\mu_1 p_g + (1 - \mu_1)p_b, \theta_i)$ which enables him to buy at least γ of the shares whenever $\mathcal{O}$ is not the equilibrium outcome in the following subgame. Otherwise, he is indifferent between making an offer with $p_i \leq \mu_1 p_g + (1 - \mu_1)p_b$ and not making any offer.*

Proof: First we show that not countering such a low offer cannot be an equilibrium behavior whenever $\mathcal{O}$ is not the equilibrium outcome in the following subgame. If $\mathcal{O}$ is not the equilibrium outcome in the following subgame, by Lemma 1 π is lower than $\mu_1 p_g + (1-\mu_1)p_b$ and so is p_r by assumption. By Lemma 3, we know that the shareholders would tender to any price greater than $\hat{\pi} = max\{\pi, p_r\}$. Therefore, the incumbent is inclined to make such a counter offer to make positive profits, since we know from Lemma 1 that the value of each share $\pi = \mu_1 p_g + (1-\mu_1)p_b$ if $\hat{\theta}_i \geq \gamma$. Second, we show that a counter offer price is $p_i = \mu_1 p_g + (1-\mu_1)p_b$ in equilibrium. We know that if $\hat{\theta}_i \geq \gamma$, then $\pi = \mu_1 p_g + (1-\mu_1)p_b$. Therefore, for a successful offer we must have $p_i \geq \mu_1 p_g + (1-\mu_1)p_b$ in equilibrium. The second part of the Lemma is self explanatory. ◇

Proposition 1 *There exists an equilibrium in which both types of raider make an offer of* $(\bar{p}, \frac{1}{2})$ *and every shareholder tenders his share(s). Hence takeover takes place with probability* 1. *This is the least favorable equilibrium outcome for the shareholders among the set of pooling equilibria.*

Proof: Let's fix the beliefs in the following way: the beliefs are equal to the prior beliefs for all offers of the type $(p, \frac{1}{2})$[32]. For all other offers (any offer which has fraction greater than $\frac{1}{2}$) let $\mu_1 = 1$. Now we are left to check whether there is any profitable deviation by any party. For any offer of the raider of the type (p, θ_r) where $p < \bar{p}$, the incumbent will not make a counter offer by Lemma 4. We also know that this would suffice to make the raider's attempt unsuccessful. Therefore, the raider offers $\bar{p}$. A similar argument shows that the raider will not offer any θ_r other than $\frac{1}{2}$. The incumbent will not find it profitable to make a counter offer with a price higher than $\bar{p}$, since any share that he holds will have an expected value of $\bar{p}$. Therefore he tenders all of his shares. The last argument applies to the other shareholders as well.[33] The final statement of the proposition is immediate. ◇

Proposition 8 *If* $f_0 = 0$, $\gamma = \frac{1}{2}$ *and* $p_b + 2q_b > p_g$, *then in every equilibrium in which both types of the raider make an offer of* (p, θ_r) *and sufficiently many shares are tendered, where* $p \in [\bar{p}, \frac{\frac{1}{2}p_g + (\theta_r - \frac{1}{2})p_b}{\theta_r}]$ *and* $\theta_r \in [\frac{1}{2}, 1]$. *Hence takeover takes place with probability one.*

[32]This seems plausible considering the fact that no tender offer price is a dominated action for either type. However, we should note that fixing the beliefs this way is not crucial. Any μ_1 greater than the prior beliefs, will support this equilibrium.

[33]Due to the indifference of shareholders, there are also asymmetric equilibria in which majority of shareholders tender and minority of shareholders keep their shares.

Proof: Take any (p, θ_r) where $p \in [\bar{p}, p_g]$ and $\theta_r \in [\frac{1}{2}, 1]$. Let $\mu_1 = 1$ for any belief following any offer other than the equilibrium offer (p, θ_r).[34] To prove that this is an equilibrium we can use an argument identical to that in proof of Proposition 1. Therefore, we proceed to show that no other pooling equilibrium exists. We know from Lemma 4 that any offer with a price lower than $\bar{p}$ cannot be successful. Thus we cannot have a price less than $\bar{p}$ in an equilibrium. For any $\theta_r \geq \frac{1}{2}$, the raider of type b is indifferent between pooling and separating at a tender offer price $\frac{\frac{1}{2}p_g + (\theta_r - \frac{1}{2})p_b}{\theta_r}$. Therefore, we cannot have pooling equilibria with a price greater than $\frac{\frac{1}{2}p_g + (\theta_r - \frac{1}{2})p_b}{\theta_r}$. $\diamond$

Proposition 9 *If $f_0 = 0$, $\gamma = \frac{1}{2}$ and $p_b + 2q_b < p_g$, then for every (p, θ_r) where $p \in [\bar{p}, p_b + \frac{q_b}{\theta_r}]$ and $\theta_r \in [\frac{1}{2}, 1]$, there exists an equilibrium in which both types of raider make an offer of (p, θ_r) and sufficiently many shares are tendered. Hence takeover takes place with probability one.*

Proof: Similar to the previous proof. $\diamond$

Proposition 10 *If $f_0 = 0$, $\gamma = \frac{1}{2}$ and $p_b + 2q_b > p_g$, there exists a separating equilibrium in which type b makes an offer of $(p_g, \frac{1}{2})$ whereas type g makes an offer with a price (weakly) less than p_g. Every shareholder tenders his share(s) to the offer (p_g, γ).*

Proof: Let $\mu_1 = 1$ for any belief following any offer other than the equilibrium offer $(p_g, \frac{1}{2})$. The rest of the argument is identical to the proof of Proposition 1. $\diamond$

Refined Equilibria

The refinement concept extends the definition of Perfect Sequential Equilibrium by Grossman and Perry (1986). Our model has multi periods in which several uninformed players take actions following a "message" by the informed player. Therefore, in contrast to Grossman and Perry (1986) the payoff of the informed player cannot be solely determined by a single uninformed player's reaction to the message. Consequently, we use an equilibrium of the subgame following a message rather than a single action of a single uninformed player. In order to define the refinement concept rigorously, we will

[34] We should note that one can support this equilibrium with any belief $\mu_1 \geq \frac{p' - p_0}{p_g - p_0}$ following an offer $(p', \theta'_r) \neq (p, \theta_r)$.

first introduce some notation. Let $U(i,(p,\theta_r),\sigma)$ be the utility function for the raider of type $i \in I$. Let σ stand for the equilibrium strategy profile in the subgame following a tender offer (p,θ_r). The prior beliefs about the type of raider is represented by a probability distribution $\pi(.)$. Fix a perfect Bayesian equilibrium outcome, and let $\bar{U}(i)$ be type i's expected payoff.

Definition 1 *(Restriction on Beliefs) For any out off equilibrium path tender offer (p,θ_r) if there exist a non-empty set $J \subseteq I$ such that:*

$$U(i,(p,\theta_r),\sigma((p,\theta_r),\pi_J)) \geq \bar{U}(i) \text{ for all } i \in J,$$

$$U(i,(p,\theta_r),\sigma((p,\theta_r),\pi_J)) \leq \bar{U}(i) \text{ for all } i \in I \setminus J,$$

then $\mu = \pi_J$, where $\pi(J) > 0$, $i \in J$ and $\pi_J(i) = \frac{\pi(i)}{\sum_{j \in J} \pi(j)}$.

We say that the equilibrium outcome $\bar{U}$ fails the refined equilibrium concept if there does not exist beliefs μ which satisfy the criterion above and at the same time support $\bar{U}$.

Proposition 11 *For $f_0 = 0$ and $\gamma = \frac{1}{2}$, in every perfect Bayesian equilibria which satisfies the refined equilibrium concept, both type of raiders make an offer $(\bar{p}, \frac{1}{2})$ and every shareholder tenders his share(s).*

Proof: We partition our analysis into two:

Take any candidate equilibrium outcome $(p,\theta_r) > (\bar{p}, \frac{1}{2})$.[35] To support this equilibrium outcome we must have $\mu_1 > \alpha$ following any out of equilibrium tender offer (p_1, θ_1) such that $(\bar{p}, \frac{1}{2}) < (p_1, \theta_1) < (p, \theta_r)$. Otherwise, both types of the raider (for $p_1 > \bar{p}$) or type b (for $\theta_1 > \frac{1}{2}$) will deviate and make an offer involving either a lower price or a lower fraction, respectively. Therefore, we can find the non-empty subset $J \subseteq \{b, g\}$ for both cases such that the equilibrium fails the test above, where $J = \{b, g\}$ if $p_1 > \bar{p}$ and $J = \{b\}$ if $\theta_1 > \frac{1}{2}$. This concludes the proof of the first half.

Take any candidate equilibrium outcome $(p,\theta_r) \not> (\bar{p}, \frac{1}{2})$. We know from Lemma 4 that such an offer will be countered by the incumbent if $p < \bar{p}$ resulting in zero profit

[35] We say $(x_1, ..., x_n) > (y_1, ..., y_n)$ if and only if there exists $i \in \{1, ..., n\}$ such that $x_i > y_i$ and $x_j \geq y_j$ for all $j \in \{1, ..., n\} \setminus \{i\}$.

for the raider. Any offer with $\theta_r < \frac{1}{2}$ and $p \geq \bar{p}$ creates a loss for raider of type b. Thus any such offer is also non-profitable for type g in equilibrium. Therefore any offer $(p, \theta_r) \not> (\bar{p}, \frac{1}{2})$ means at most zero profits for both types of raider. For this to be an equilibrium outcome all the potentially profitable offers $(p, \theta_r) > (\bar{p}, \frac{1}{2})$ result in $\mu_1 = 1$ so that all such offers are rejected or countered. However, as we have shown above such off the equilibrium path beliefs fail our criterion. $\diamond$

Proposition 2 *Let* $p^* = max\{\bar{p}, \hat{p}\}$ *where* $\hat{p}(\theta_r) = \frac{(1-\gamma)p_0 - f_0(1-\theta_r)\bar{p} + q_0}{1-\gamma-f_0(1-\theta_r)}$. *For any* θ_r, p^* *is the lowest possible price such that the tender offer* (p^*, θ_r) *leads to a successful takeover attempt in a pooling equilibrium.*

Proof: We know from Lemma 2 that shareholders other than the incumbent will tender their shares if an offer price exceeds $\bar{p}$. In this proof we will use this fact as well as Lemma 4. The basic idea behind Lemma 4 is to find the raider's offer price which makes the incumbent indifferent between tendering and counter bidding. At this point we need to modify Lemma 4 and find this critical level of price, when we relax the assumption of $f_0 = q_0 = 0$. The lowest possible price the incumbent will accept in a pooling equilibrium whenever he has low private benefits of control is $\bar{p}$ as in Proposition 1. The incumbent is indifferent between making a counter offer and tendering when the equation below holds whenever he has high private benefits of control:

$$f_0[\theta_r\hat{p} + (1-\theta_r)\bar{p}] = -(1-\gamma-f_0)\hat{p} + (1-\gamma)p_0 + q_0$$

Solving for $\hat{p}$, we get $\hat{p} = \frac{(1-\gamma)p_0 - f_0(1-\theta_r)\bar{p} + q_0}{1-\gamma-f_0+f_0\theta_r}$. The incumbent will employ the strategy which gives him a higher level of expected value. Hence his resistance leads to either $\tilde{p}$ or $\hat{p}$ whichever is higher. Therefore, $p^* = max\{\bar{p}, \hat{p}\}$ is the lowest possible price which leads to a successful takeover in equilibrium.

Now we need to prove that there exists a $\bar{q}_0 > 0$ such that $q_0 > \bar{q}_0 \Leftrightarrow \hat{p} > \bar{p}$. Clearly, $\bar{p} > \hat{p}$ for any $\bar{p} > p_0$ whenever $q_0 = f_0 = 0$. Looking at the partial derivatives, we have:

$$\frac{\partial \bar{p}}{\partial q_0} = 0 < \frac{1}{1-\gamma-f_0(1-\theta_r)} = \frac{\partial \hat{p}}{\partial q_0}$$

Therefore, there exists a $\bar{q}_0 > 0$ such that for every $q_0 > \bar{q}_0$, $\hat{p} > \bar{p}$. $\diamond$

Proposition 3 p^* *is a weakly decreasing function of* θ_r.

Proof:

$$\frac{\partial \hat{p}}{\partial \theta_r} = \frac{f_0\bar{p}[1-\gamma-f_0(1-\theta_r)] - f_0[(1-\gamma)p_0 - f_0(1-\theta_r)\bar{p} + q_0]}{[1-\gamma-f_0(1-\theta_r)]^2}$$

Note that the second term in the numerator is $f_0\hat{p}[1-\gamma-f_0(1-\theta_r)]$. Therefore, first term is less than the second term since $\hat{p} > \bar{p}$ whenever $p^* = \hat{p}$. Therefore, $\frac{\partial \hat{p}}{\partial \theta_r} < 0$. We also have $\frac{\partial \bar{p}}{\partial \theta_r} = 0$ for the low private benefits case. ◇

Proposition 4 *Both p^* and the expected shareholder value in lowest price equilibrium, $p^*\theta_r + \bar{p}(1-\theta_r)$, are (weakly) increasing functions of both the incumbent's initial holdings, f_0 and private benefit of control, q_0.*

Proof: The numerator of the partial derivative $\frac{\partial \hat{p}}{\partial f_0}$ is

$$-(1-\theta_r)\bar{p}[1-\gamma-f_0(1-\theta_r)] + (1-f_0)[(1-\gamma)p_0 - f_0(1-\theta_r)\bar{p} + q_0]$$

Note that the second term in the numerator is $(1-\theta_r)\hat{p}[1-\gamma-f_0(1-\theta_r)]$. Therefore, first term is less than the second term since $\hat{p} > \bar{p}$ whenever $p^* = \hat{p}$. Therefore, $\frac{\partial \hat{p}}{\partial f_0} > 0$. Consequently, $\frac{\partial \hat{p}\theta_r + \bar{p}(1-\theta_r)}{\partial f_0} > 0$. The second part of the proof is immediate since q_0 appears in the numerator with a positive coefficient in $\hat{p}$. For the low private benefits case, we have $\frac{\partial \bar{p}}{\partial f_0} = \frac{\partial \bar{p}}{\partial q_0} = 0$. ◇

Lemma 5 *The most profitable equilibrium tender offer for type b involves the lowest possible fraction.*

Proof: We first analyze the case with high private benefits. Any offer $(\hat{p}, \theta_r)$ has a value of $(p_b - \hat{p})\theta_r + q_b$ for type b. The partial derivative $\frac{\partial[(p_b-\hat{p})\theta_r + q_b]}{\partial \theta_r}$ is

$$p_b - \hat{p} - \theta_r \frac{f_0\bar{p}[1-\gamma-f_0(1-\theta_r)] - f_0[(1-\gamma)p_0 - f_0(1-\theta_r)\bar{p} + q_0]}{[1-\gamma-f_0(1-\theta_r)]^2}$$

Simplifying the last term and arranging the identity as a single fraction we have

$$\frac{(p_b - \hat{p})[1-\gamma-f_0(1-\theta_r)] - \theta_r(\bar{p} - \bar{p})}{[1-\gamma-f_0(1-\theta_r)]}$$

Given that $p_b < \bar{p}$ and $1-\gamma > f_0$, $\frac{\partial[(p_b-\hat{p})\theta_r + q_0]}{\partial \theta_r}$ is negative. For the low private benefits case we use $\frac{\partial \bar{p}}{\partial \theta_r} = 0$ and follow the same steps. ◇

Proposition 5 *If the incumbent has high private benefits of control, $p^* = \hat{p}$, then there is no voluntary disclosure of information. Otherwise, there is no voluntary disclosure of information only if $\frac{(1-\gamma)p_0 - f_0(1-\gamma)p_b}{(1-\gamma)(1-f_0)} > \bar{p}$.*

Proof: Let $\tilde{p}$ be the tender offer price following such a disclosure. From Lemma 5, we know that the most profitable tender offer for type b involves lowest possible fraction. Hence, without loss of generality we can say that the offer should be in the form of $(\tilde{p}, \gamma)$. The incumbent will be indifferent between making a counter offer and tendering his share(s) when the below equation holds.

$$f_0[\gamma\tilde{p} + (1-\gamma)p_b] = -(1-\gamma-f_0)\tilde{p} + (1-\gamma)p_0 + q_0$$

Therefore, the lowest price, $\tilde{p}$, the type b has to pay in order to buy γ fraction of the shares (following a voluntary disclosure of her type) is $\frac{(1-\gamma)[p_0 - f_0 p_b] + q_0}{(1-\gamma)(1-f_0)}$.

For the high private benefit case, we need to compare $\tilde{p}$ with $\hat{p}$. Recall that $\hat{p} = \frac{(1-\gamma)p_0 - f_0(1-\theta_r)\bar{p} + q_0}{1-\gamma-f_0+f_0\theta_r}$. If replace θ_r with γ the only difference between $\hat{p}$ and $\tilde{p}$ remains in the second term of the numerator. Clearly $p_b < \bar{p}$ and thus $\hat{p}(\theta_r = \gamma) < \tilde{p}$. Hence, type b will not prefer to disclose information and pay a higher price to buy γ fraction of the shares.

For the low private benefit case, we need to compare $\tilde{p}$ with $\bar{p}$. Then, there will be no disclosure of information as long as $\tilde{p} = \frac{(1-\gamma)p_0 - f_0(1-\gamma)p_b}{(1-\gamma)(1-f_0)} > \bar{p}$. $\diamond$

Proposition 6 *For any θ_r, p^* is a (weakly) increasing function of the supermajority rule, γ.*

Proof: We first analyze the case with high private benefits. The partial derivative $\frac{\partial \bar{p}}{\partial \gamma}$ is

$$\frac{-p_0[1-\gamma-f_0(1-\theta_r)] + [(1-\gamma)p_0 - f_0(1-\theta_r)\bar{p} + q_0]}{[1-\gamma-f_0(1-\theta_r)]^2}$$

Dividing both numerator and denumerator by $1-\gamma-f_0(1-\theta_r)$ we get

$$\frac{-p_0 + \hat{p}}{1-\gamma-f_0(1-\theta_r)}$$

Given $\hat{p} > p_0$, we have $\frac{\partial \hat{p}}{\partial \gamma} > 0$. For low private benefits case, we have $\frac{\partial \bar{p}}{\partial \gamma} = 0$ $\diamond$

Proposition 7 *For a given f_0, there exists an interval of supermajority rules $(\frac{1}{2}, \bar{\gamma}(f_0))$ such that for all $\gamma \in (\frac{1}{2}, \bar{\gamma}(f_0))$, there exists a socially optimal equilibrium outcome. Furthermore, for any $\gamma > \bar{\gamma}(f_0)$, there is no socially optimal equilibrium. Similarly, for a given γ there exists a $\bar{f}_0(\gamma)$ such that all $f_0 < \bar{f}_0(\gamma)$ are socially optimal and all $f_0 > f_0\bar{(}\gamma)$ are socially sub-optimal.*

Proof: The proof follows immediately from Propositions 2 and 4. $\diamond$

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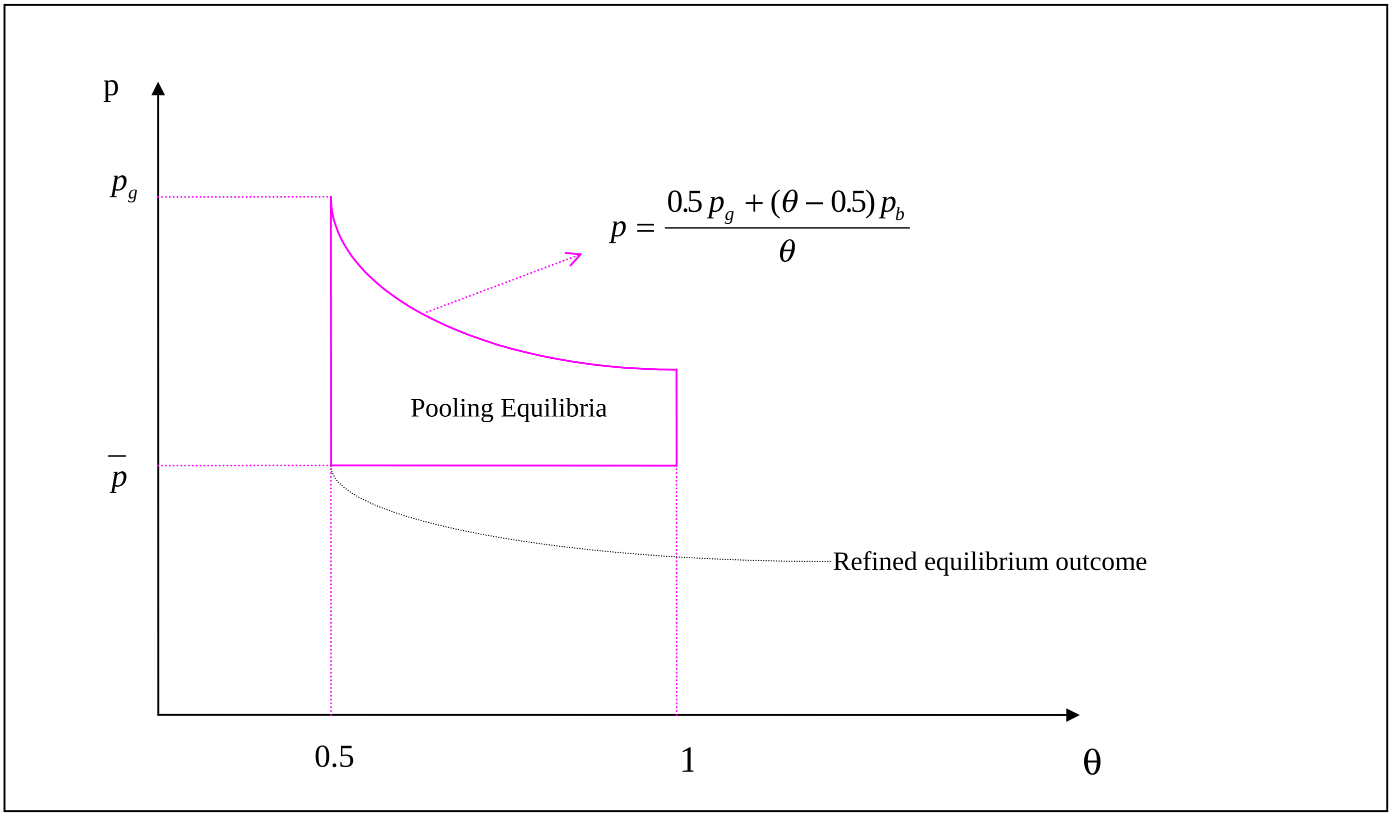

Figure 1: The set of pooling equilibrium outcomes for $\gamma = 0.5$, under the assumption: $p_b + 2\ q_b > p_g$.

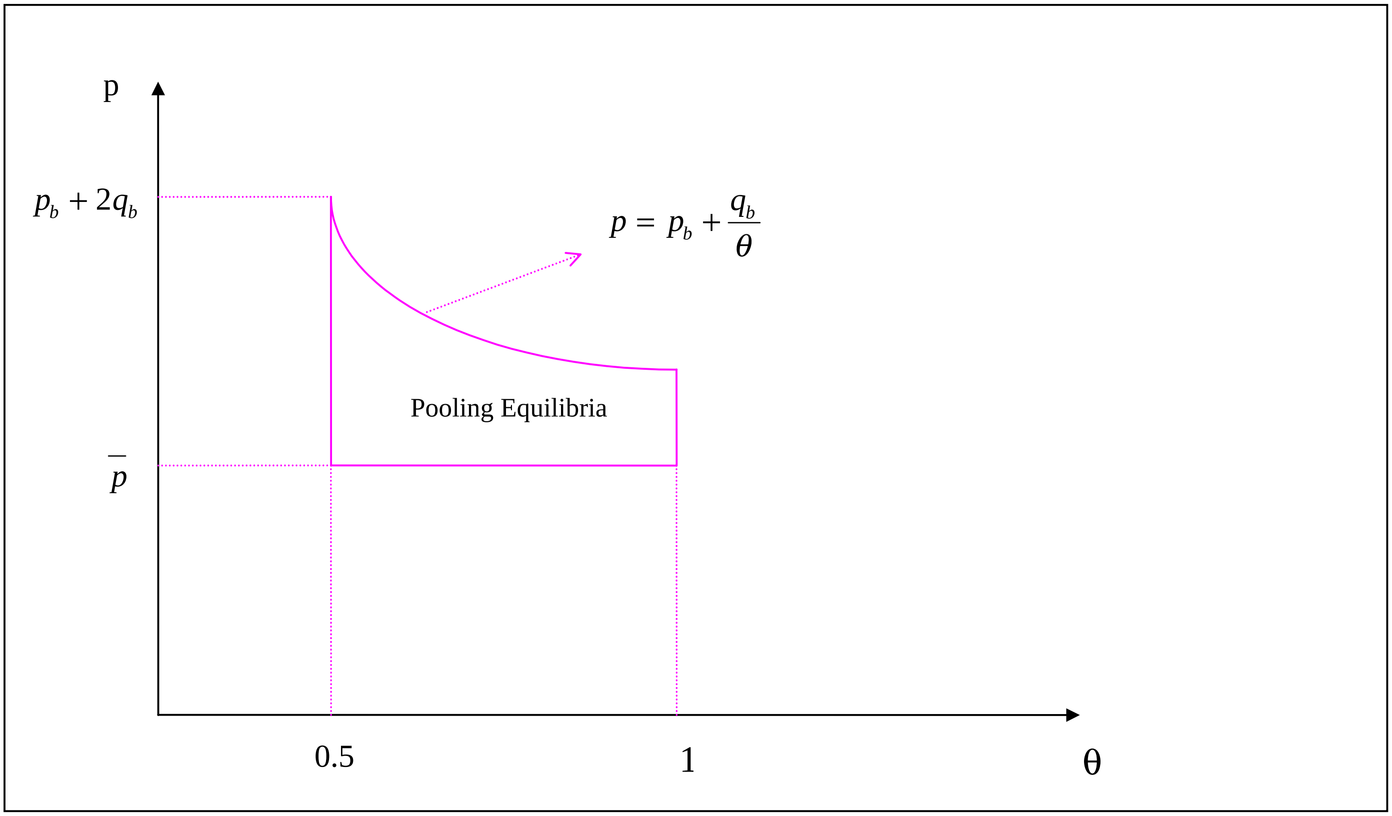

Figure 2: The set of pooling equilibrium outcomes for $\gamma = 0.5$, under the assumption: $p_b + 2\,q_b < p_g$.