THE VALUATION OF FLOATING RATE INSTRUMENTS
THEORY AND EVIDENCE

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May 1984

Revised: December 1985

We are grateful to Clifford Ball, Doug Breeden, Robert Litzenberger, Craig MacKinlay, Jay Ritter, Myron Scholes, Clifford Smith and the referee for their comments, and to In Joon Kim and Charles Wolf for their comments and assistance.

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ABSTRACT

A framework for valuing floating rate notes is developed and used to examine the effects of (1) lags in the coupon averaging formula, (2) special contractual features and (3) default risk. Evidence on a sample of U.S. floaters is presented and indicates that these notes sold at significant discounts over the sample period. We find that while the lag structure in the coupon formulas and the special contractual features make these notes more variable, they are unable to account for the magnitude of the observed discounts. Based on numerical analysis of a valuation model with default, we conclude that the fixed default premium embodied in the coupon formula at the time of issuance of a typical note is inadequate to compensate for the time-varying default premiums demanded by investors, who will treat other corporate short-term paper as close substitutes: the observed discounts are most consistent with this hypothesis.

1. Introduction:

The recent experience of rapid and sizable changes in the inflation rate in the 70's has given rise to new debt securities which have coupon yields that vary over time and reflect, by means of a predetermined formula, the prevailing short term or intermediate term interest rate. Since 1974, when the first of these "variable rate" or "floating rate" instruments was issued, this market has grown rapidly. In 1984, the total new issue volume was $39.7 billion, of which U.S. issuers accounted for $10.4 billion.[1] In the mortgage market, adjustable-rate mortgages are now fairly common. Indeed, a similar concept is in force in some regulatory jurisdictions, where the allowed return on utilities' common stock is based on a formula that incorporates current market conditions.

These floating rate instruments' value is more stable than that of fixed rate investments. An investor considering a strategy of rolling over short-term instruments (e.g. Treasury Bills) will find a floater attractive, since it substitutes a one time transactions cost for the repeated transactions costs of the roll-over strategy. An investor considering an investment in fixed rate notes will demand compensation for any additional risk from the fluctuations in the value of the fixed rate note compared to an equal investment in variable rate notes. From the viewpoint of a potential corporate issuer, if the corporation maintains a relatively stable amount of short-term debt which is rolled over at regular intervals, then the issuance of a floating rate note tied to short-term rates avoids the transactions costs of repeatedly rolling over short debt. This is one reason why banks and financing companies have issued floating rate notes.

The value of a floating rate note depends crucially on the specification of its coupon payment rule. Almost all the floaters in the U.S. employ a coupon rule that is "set off" from a Treasury instrument. That is, the coupon is usually defined as the average of previously issued 6 month (or 3 month, suitably adjusted) Treasury Bill yields, plus a premium or markup to reflect the credit risk of the issuer. The value of a floating rate note also depends on indenture provisions and on other contractual features, such as callability, convertibility, and on any maximum or minimum restrictions on the payable coupon.

We employ a continuous time valuation model to study the effects of (1) lags in the coupon rule, (2) special contractual features, and (3) the issuer's credit risk. The price history of a sample of U.S. floating rate notes reveals that while the dynamic properties of these notes follow the corresponding instruments from which these floaters are set off, they have sold at statistically significant discounts subsequent to their issue dates. The results of numerical analysis of the valuation model indicate that neither the lag structure nor the principal contractual features explain the range of observed prices: the level of the default premium expected by investors relative to the fixed markup comes closest to doing so.

Our study complements and extends the work of Cox, Ingersoll and Ross(1980), who placed special emphasis on coupon formulas that can be designed to eliminate basis risk. In the context of a set of examples, they analyzed the behavior of consol floating rate notes that have call, conversion, and other features. Our study differs from theirs in that (i) we examine formulas which make the coupon rate an average of past rates, as is the case in practice; (ii) we incorporate some of the commonly encountered design features of finitely-lived floating rate notes; and (iii) we examine the effect of default risk on these floaters.

The plan of the paper is as follows. In section 2, we describe the salient features of corporate issues of floating rate notes, and present evidence on their price behavior. In section 3 we develop a continuous time valuation model for these notes to examine the lag structure and other contractual features, and report the results of numerical solutions for these cases. In section 4 we study the valuation of corporate floaters subject to default, ignoring the effects of the lag structure and other contractual features. Section 5 contains some concluding comments.

2. Floating Rate Notes: Description and Price Behavior

The specification of the coupon formula is substantially the same across most U.S. floaters, although there is considerable variation in the type of base rate (3 or 6 month, Treasury Bill Yields or LIBOR (London InterBank Offered Rate) rates) and in the nature and magnitude of the markup over the base rate that reflects the issuer's credit risk. The next (usually semi-annual) coupon payable is calculated from the quoted yields of the base-rate instrument from two or three dates immediately prior to the previous coupon payment date, plus the markup. For example, the Citicorp floater maturing in May 2004 pays coupons on the 1st of May and November. The coupon rate on May 1 is the simple average of 6-month bond equivalent yields on Treasury Bills in the auctions conducted during October 8 through October 21 of the previous year (the coupon rate on November 1 is the simple average of 6-month bond equivalent yields on Treasury Bills in the auctions conducted April 7 through April 20 of the same year) plus a predetermined markup. In most cases the markup declines over time: in the Citicorp example, the markup is 105 basis points for the first five years of the floater's life, 100 basis points for the next five years, and 75 basis points thereafter. Almost all issues employ markups that are predetermined but a few define a "variable" markup as a ratio of the spread between the rate on short-term private issuer obligations and Treasury Bill rates. For example, the Citicorp floater maturing May 1, 2010 defines the markup as 110% of the differential between 3 month CDs and 3 month Treasury Bills (this ratio declined to 102.5% of that differential after May, 1982).

For the majority of floaters, the coupon payment dates and the dates of coupon reset (when the formula is applied to determine the next coupon) are 6 months apart: there are a few floaters whose coupons are paid semi-annually, but whose reset intervals are monthly, and others whose base-rate instrument is an intermediate-term or long-term Treasury index.

The other contractual features of floaters can be quite complex and are generally issue-specific. We list below the most commonly encountered contractual features:

(A) <u>Floor Rate and Ceiling Rate Features</u>: The coupon is found from an average of observable past yields of the base-rate instrument plus a markup, but the coupon is subject to a minimum and a maximum rate (the "collars").

(B) <u>Put Features</u>: The holder has the right to redeem his investment (usually at par) at coupon payment dates, on or after some prespecified date. Notice of redemption typically must be given at least 30 days prior to the coupon payment date.

(C) Drop-Lock Features: These instruments cease to "float" when the base rate "drops" to a prespecified minimum value, whereupon they automatically become fixed rate notes (the "lock") with a prespecified coupon and maturity.

(D) Call Features: They permit the issuer to redeem the issue on or after a prespecified date at a prespecified set of prices.

(E) Conversion Features: They permit the holder to exchange his investment for a fixed rate note with a prespecified coupon and maturity.

The majority of floating rate issues to date incorporate put features, floor and ceiling features. In addition, the floating rate issues are subject to covenants which define default and the rights of the holders of the floating rate issues on default. The covenants associated with these notes are similar to those discussed by Smith and Warner(1979). An interesting and unique contractual provision defines the manner in which the coupon will be reset, if at the time of reset, publication of the base rate (for example, Treasury Bill yields from Federal Reserve Bulletins) are unavailable: this provision defines alternate base rates or a flat rate of interest if no timely information is publicly available.

All the corporate issues are subject to the normal tax laws: coupon income is taxed at the ordinary rate and gains and losses are subject to capital gain rules. There is an influx of municipal issues of floaters, where the coupon is exempt from federal income tax. There has also been a surge of adjustable rate preferred stock issues, which are floaters in the sense defined above, except that the "dividend" that is paid is subject to the exclusion rule for intercompany dividends.

Table 1 summarizes the features of six issues that are representative of the sample of floating rate issues for which we have obtained weekly price data. The Gulf Oil issue is the only one in our sample which has the drop-lock feature, and it is also the only issue whose averaging formula uses the 30 year Treasury yield as the base rate. None of the issues has a ceiling coupon rate, but all of them have a minimum (floor) rate. Three of the issues shown have conversion features, and all but one have sinking funds which apply either to the floater or the debentures into which they convert. None of the issues in our sample has a put feature. The put feature will be employed by the holders if the floater sold below par on coupon payment dates, and the market prices of floaters with put features reflect this.

Price Behavior of Floaters

Consider a hypothetical floating rate note issued by the Treasury: it pays coupons semi-annually, its coupon reset date and coupon payment dates coincide and are exactly 26 weeks apart, and it uses the yield on the newly issued 26 week T-Bill auctioned during the coupon payment week as the base rate. Because of its default-free status, no markup is provided. Cox, Ingersoll and Ross(1980) show, in a fairly general context, that the market price of this floater must be $100 (the face amount) on the ex-coupon dates, and that the dynamic properties of the floater will be identical to that of a Treasury Bill that matures on the next coupon payment date. That these two properties apply to this "perfectly-indexed" floater is easy to see: the payoff to this floater is identical to the payoff generated by investing $100 in 26 week bills on the coupon payment date, and reinvesting \$100 from the proceeds after 26 weeks in new issues of 26 week bills, ignoring transactions costs. The price of the floater will deviate from \$100 between coupon payment dates reflecting the basis risk that remains.

In a completely analogous way, we can show that these properties extend to corporate floaters. Consider a corporate floater whose coupon payment dates and coupon reset dates coincide, whose coupon is computed on the reset date as the yield on the corporation's newly issued commercial paper with maturity equal to the time until the next coupon, and whose seniority and other indenture provisions are identical to the floater's. Then, again ignoring transactions costs, investors will receive the same payoff from a rolling investment strategy in commercial paper. It follows that the corporate floater should sell at par on the coupon payment dates, and its dynamic properties should correspond closely[3] to those of the corporation's commercial paper issue maturing on the next coupon payment date.

To see whether these implications are confirmed in the market, we obtained weekly closing prices (offered prices) on a sample of 18 floaters from their issue dates (the earliest being June 1978) to June 1983. This sample was chosen to include only those floaters which had no put features attached, because the put feature would bias the ex-coupon day prices upwards; and except for the Gulf Oil issue, the floaters all employed the 6 month Treasury Bill as the base instrument. From this data we selected, for every issue and associated coupon date (except the Gulf Oil issue), the nearest Friday subsequent to the coupon payment date. There were a total of 86 Friday quotes nearest to the coupon date: the average price (cum accrued coupon, for the days since the coupon) was \$93.55, the standard deviation of this mean was \$0.395. These prices ranged from \$86.83 to \$100.56, and there was no tendency for the longer maturity floaters to sell at greater discounts. The mean value of the ex-coupon price was thus significantly below the hypothesized value of \$100.

To check whether the dynamic properties of these floaters are consistent with those of short-term obligations of the same corporation, we need data on prices of these obligations of fixed maturity. Lacking these data, we are unable to provide this comparison: we show in Figure 1 a plot of the prices of the Beneficial floater maturing in 1987 together with the prices of the 6 month Treasury Bill maturing on the Thursday closest to the next coupon payment date. Note that the Treasury Bill's price per \$100 face value is plotted, and the Beneficial issue's price includes the coupon accrued since the immediately previous ex-date which, in turn, includes a default premium. Therefore, the curve corresponding to the floater price should lie everywhere above the Treasury Bill price curve, if the stated default premium were adequate. It is clear that while the initial prices (at issue and through the first coupon) were consistent with these implications, the subsequent prices went to considerable discounts -- and the graph indicates that these two instruments were positively correlated. This pattern is also borne out for the other floaters in our sample.

Theoretical considerations lead one to suspect that the potential explanations for these discounts lie in the lag structure of the coupon formulas, in other contractual features, in inadequate default premiums in the coupon formula relative to time-varying default premium demanded in the market. We examine these explanations separately in the next two sections because the simultaneous study of these effects is complicated, and because floating rate issues have widely varying features.

3. A Continuous Time Valuation Model for Floating Rate Notes

The value of a floating rate note depends on the dynamics of the term structure of interest rates, on the coupon payment formula, its contractual provisions and on the creditworthiness of the issuer. In this section we ignore credit risk and model the movements in the term structure in order to examine contractual provisions and the coupon payment formula.

A typical coupon payment formula defines the coupon rate as an average of past LIBOR rates or Treasury Bill rates. This feature causes the value of the floater to depend on the path of the interest rates to which the coupons are linked. The way in which the coupon is computed has important implications for the intertemporal price behavior of the floater. To see this, let $\{t = 0,1,2,\ldots\}$ be dates at weekly intervals and let y_t be the yield on newly issued Treasury Bills maturing at t+26. Consider a hypothetical default-free floater which pays semi-annual coupons x_s at date s given by the coupon payment formula:

$$x_s = \begin{cases} y_{s-26}, & s = 26,\ 52,\ 78,\ \ldots. \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

This coupon payment scheme corresponds to the "perfectly-indexed" floater discussed in Section 2, and such a floater will always sell at par in each ex-coupon date. By contrast, a typical floater has a semi-annual coupon x_s which is an average of past yields:

$$x_s = \begin{cases} (y_{s-27} + y_{s-28} + y_{s-29})/3, & s = 26, 52, 78, \ldots \\ 0 & \text{otherwise} \end{cases} \qquad (2)$$

In this case the coupon at date s is reset during the interval [s-29,s-27] thereby making the valuation problem path-dependent, at least during that interval. If yields on newly-issued Treasury Bills were a random walk, then in the absence of liquidity premiums the average ex-coupon value of a typical floater (with a coupon formula as in (2)) would also be equal to the face value. If, on the other hand, these yields displayed a secular trend or a mean-reversion, then the typical floater would sell at a predictable discount or premium on the ex-coupon dates.

In a discrete time setting, the valuation of a floater with a coupon formula as in (2) may be carried out in two steps: first, we can value a hypothetical, "elementary" default-free floater whose current coupon rate is the T-Bill yield that prevailed (say) k weeks ago.[4] The second step is to recognize that the value of a floater whose coupon is the simple (linear) average of rates that prevailed at several values of k is simply the weighted average of the values of "elementary" floaters, using the same weights as are used in the averaging formula. This procedure is feasible but computationally cumbersome even for small lags and short maturity floaters: this is because the path of rates (or sufficient information) must be carried along at each stage of the valuation problem. Given our desire to examine the special contractual features of floaters we posed the valuation problem in a continuous time setting where the lag structure as well as these features can be studied with relative ease.

The following assumptions are employed:

(A1) Trading takes place continuously in frictionless markets; there are no taxes.

(A2) The term structure is fully specified by the instantaneously riskless rate r(t). Its dynamics are given by

$$dr = \kappa(\mu - r)dt + \sigma\sqrt{r}dz \tag{3}$$

where μ, κ, $\sigma^2 > 0$ and where $\{z(t), t > 0\}$ represents a standard Wiener process.

(A3) The floater is assumed to pay a coupon continuously at the rate x(t), given by:

$$x(t) = \beta \int_{\infty}^{0} e^{-\beta s} r(t - s)ds, \quad \beta > 0 . \tag{4}$$

(A4) All bonds are priced according to the Local Expectations Hypothesis (LEH). That is, the floater price $F(r(t),x(t),\tau)$ satisfies at each instant t,

$$E_t[dF] + x(t)dt = Fr(t)dt, \tag{5}$$

where $\tau \equiv T - t$ denotes the time to maturity of the floater and $E_t[\cdot]$ is the expectations operator.

Market assumptions are embodied in (A1).[5] The term structure assumptions made in (A2) and (A4) help to keep the problem tractable: specifying the term structure in terms of the short rate may not be unrealistic because most floaters have their coupon formulas based on short rates. The specification used for the evolution of the interest rate implies that the current rate r(t) is pulled towards its long-run mean value μ with a speed of adjustment κ, and the instantaneous variance of the change in the rate is proportional to its level. The properties of this process are provided in Cox, Ingersoll and Ross(1985). While alternative specifications of the stochastic process for r can be accommodated quite easily, our choice of the autoregressive square-root diffusion in (3) was motivated by the fact that solutions for discount bond prices were available, which simplified the computations considerably. The choice of the formula for the coupon rate in (A3) was, however, quite deliberate. More general coupon payment formulas, perhaps dependent on a finite history of rates, lead to descriptions of the state of the system that become unmanageably large. The exponential average formula in relation (4) permits the transition of the continuous coupon rate to be written

$$dx = \beta(r - x)dt \ . \tag{6}$$

Therefore, the levels of r(t) and x(t) at time t (together with the knowledge of their evolution in equations (3) and (6)) completely describe the system. Note that a large value for β implies that greater weight is placed on recent rates of interest in determining the current coupon, and indicates that the coupon rate will never be far from the current interest rate. The local expectations hypothesis assumed in (A4) helps to avoid incorporating preferences into the valuation explicitly: a discussion of this hypothesis is provided by Cox, Ingersoll and Ross(1981).

These assumptions permit the derivation of a valuation equation for the floating rate note. By taking a suitable short position in a discount bond and a long position in the floater and continuously rebalancing this portfolio, one can construct a locally riskless position. In order to preclude arbitrage this portfolio must earn the risk-free rate at each instant. Using this condition and the local expectation hypothesis, it can be shown that the floater price $F(r,x,\tau)$ must satisfy:[6]

$$\tfrac{1}{2}\sigma^2 r F_{rr} + \kappa(\mu - r)F_r + \beta(r - x)F_x + x - rF = F_\tau \qquad (7)$$

where the subscripts denote partial derivatives. The terminal condition which all floaters must satisfy is given by:

$$F(r,x,0) = 1. \qquad (8)$$

The other boundary conditions to the valuation equation will depend on the particular contractual features of the floating rate note.

There are no known analytical solutions to the valuation equation, even for floaters without the complicated features. We have employed the numerical method of alternating directions (see Brennan and Schwartz(1980)) to solve the equation for a chosen set of contract features.[7] In the rest of this section we present the results of these valuations which apply to floaters which have no default risk. The parameters for the interest rate process are $\kappa = 2$, $\sigma^2 = 0.006$, and unless otherwise noted, $\mu = 9\%$. These represent estimates for the period covering our sample of floaters, consistent with the procedures described in Cox, Ingersoll and Ross(1979), and derived from Treasury Bill prices. The parameter values for β, in the coupon payment formula, are 0.7 and 7; at these values, the difference between the current coupon rate and the current interest rate is expected to be halved in approximately 360 days and 36 days respectively.[8]

Straight Floaters

The term "straight" floater is used to mean a floating rate note that has no contractual feature beyond the coupon rate formula in (4).

Figure 2 provides a plot of floating rate values versus interest rate values for values of $\beta = 0.7$ and 7; for each value of β, plots are provided for coupon rates (x) of 4%, 10% and 16%.[9] Note that floater values are declining functions of the interest rate r and increasing functions of the coupon rate x. Floaters sell at a premium for coupon values of $x = 10\%$ and $x = 16\%$: this behavior, especially at high interest rates, follows because the coupon averaging scheme places less weight on the current interest rate with $\beta = 0.7$, and because interest rates are reverting to 9%. As β increases to 7, the straight floater values are bunched together, regardless of the substantial differences in the coupon values. The range of floater values with $\beta = 7$ is 0.993 to 1.010, in contrast to the range of floater values (0.925 to 1.10) with a beta value of 0.7. This is consistent with one's intuition: as β increases the floater tends to become a perfectly indexed instrument, with future coupons tending to the prevailing interest rate. As a result one would expect the floater to sell close to its face value.

Floaters with Ceiling and Floor Coupons

Consider the imposition of a floor and a ceiling on the coupon rate. Let x^f represent the floor rate and x^c represent the ceiling rate. The state of the system is still $\{r(t),x(t)\}$ but the coupon payments are bounded by the floor and ceiling rates. In the region where the state variable $x(t) > x^c$, the coupon will be x^c, although the difference $x(t) - x^c$ will surely affect the value of the floater. In other words, if the past rates are such that the ceiling constraint will be binding in the foreseeable future, the floater will at a discount relative to a floater whose coupon "floats" without a ceiling. Similar comments apply for the floor rate.

The floater values are obtained by solving the valuation equation (7) subject to the terminal condition (8) and relevant boundary conditions.[10] Figure 3 displays the results for floaters with 5 years to maturity and coupons which are subject to a ceiling rate of 15% and a floor rate of 5%.

At the high β value of 7, the unconstrained coupon rate is very close to the current interest rate and the floater is relatively inelastic to interest rates, except in regions where the interest rate stays above the ceiling coupon or below the floor coupon rate -- for in this case the floater behaves like a fixed rate note in that its sensitivity to interest rates is high. At the lower β value of 0.7, the unconstrained coupon value can deviate considerably from the current interest rate; and the floater again displays a higher sensitivity to interest rates, especially below 5% and above 15%. A comparison of Figures 2 and 3 confirms that the imposition of floor and ceiling coupons increases the sensitivity of floater values to interest rates.

<u>Floaters with "Drop-Lock" Feature</u>

Floating rate notes with a "drop-lock" feature cease to float when the rate of interest reaches a prespecified minimum. We assume that the issue ceases to "float" when the rate of interest falls to a prespecified floor of r^f and becomes a fixed rate instrument for its remaining life, and that the fixed coupon rate is simply r^f. The Drop-Lock feature imposes a lower boundary condition on the partial differential equation, given by

$$\lim_{r \downarrow r^f} F(r,x,\tau) = \int_{s=t}^{T} r^f P(r^f,T-s)ds + P(r^f,T-t)$$

where $P(r,\tau)$ is the value of a default free discount bond paying \$1 in τ periods. For the assumed stochastic process (5), the discount bond pricing function $P(r,\tau)$ has been derived by Cox, Ingersoll and Ross(1985). This permits a straightforward numerical solution to (7) subject to the terminal condition (8) and relevant boundary conditions to obtain values of a floater with a drop-lock feature. In Figures 4 and 5 results are shown for a floater whose "lock" becomes effective at $r = r^f = 8\%$. In Figure 4 we have used a long-run mean rate of 10% and in Figure 5 we have used a long-run mean rate of 6%. When the interest rate is below 8% the floater behaves like a fixed rate note with a continuous coupon of 8% and sells at a discount (Figure 4), reflecting the fact that the interest rate is expected to revert to 10%. As the interest rate moves up from 8%, floater prices increase dramatically, regardless of their current coupon levels. This can be attributed to two effects: first, the interest rate is pulled towards its long run mean, reflecting higher future coupons for floaters whose current coupons are 8%. Second, as the rate increases, the probability of "drop-lock" decreases, and as a result future cash flows are expected to be higher.

The results in Figure 5 are quite different, however. The differences come about because of the location of the "drop-lock" rate of 8% relative to the long-run mean rate of 6%. This implies that the interest rates will be pulled towards 6%: consequently, the floaters sell at a premium for the entire range of interest rates. In the range of interest rates from 0 to 8%, the floater becomes a fixed rate note paying a continuous coupon of 8% and sells at a premium: beyond 8%, there are two effects, as discussed earlier, except that they tend to counterbalance each other in this case. In other words, increases in interest rates increase the future coupon flows, but the tendency is for the interest rate to revert to 6% which is below the "drop-lock" rate of 8%.

We have examined the effects of put features within this framework. If the investor has the right to put the floater to the issuer (at par) at any time, then the floater will always sell at a premium, and the premium will increase as the current interest rate falls below the coupon rate. For the parameter values chosen above, a floater with 5 years to maturity and a coupon rate of 10% will be put to the issuer only if the interest rate rises above 25%. The put feature will clearly not contribute to discounts at the ex-coupon dates even in a discrete time setting, because the investors will exercise the put feature in that case.[11]

The results reported in this section confirm that floaters will tend to sell at discounts or premiums when the current coupon rate is below or above the current interest rate, and that these deviations form par will shrink if the coupon formula places greater weight on more recent rates.[12] Distant lags in the coupon formula, and the contractual features examined here tend to increase the range of price fluctuations that are predicted: however, there will be no systematic bias towards the prediction of discounts, unless the current coupon rate is well below the current interest rate. Indeed, for the straight floaters examined, the discounts predicted would approach those reported in Section 2 only at a coupon rate of 4% and at an interest rate in excess of 10%, with $\beta = 0.7$. Because the deviations of coupon rates from market rates is never that large and because even moderate premiums are not observed, it seems unlikely that lags or contractual features will serve to explain the reported prices.

4. Pricing of Corporate Floating Rate Notes

Corporate floating rate notes are subject to default risk, and rational investors will take this risk into account when valuing these notes. A corporate floater should sell at a discount relative to a government floater if they both have identical coupon payments. The magnitude of this price discount will depend on the probability of default, on the contractual provisions that define payoffs contingent upon default, and on the premium demanded in the market for similar instruments. Since the coupons to a corporate floater contain a markup to reflect the credit risk, the floater's price should be closer to par and any deviations will depend on the relationship between the markup stated in the floater and the premium demanded in the market.

Ideally, default risk should be modelled simultaneously with the coupon averaging formula. Such an approach, however, renders the valuation problem virtually intractable because floater values will depend on three state variables: the interest rate, the coupon rate and a variable which proxies for credit risk. In order to keep the problem computationally manageable, we assume that the continuous coupon rate on the floater will be the current interest rate plus a fixed premium π. This permits us to describe the state of the system with two state variables -- the interest rate and a variable that captures default risk. Furthermore, it allows us to abstract from the effects of coupon averaging (examined in Section 3) and enables us to focus on the impact of default risk on the floater value.

The traditional approach to modelling default risk, pioneered by Merton(1974) and extended by Black and Cox(1976) assumes that the market value of the issuing firm follows an exogenously specified stochastic process and that the term structure is deterministic (and flat). The payoffs to the creditors, both over time and contingent on bankruptcy are then specified in detail. The latter depends on the economic events in bankruptcy and the way in which the reorganization boundary is specified. In the context of corporate zero-coupon debt, the lower reorganization boundary is usually defined such that whenever the value of the issuing firm falls to a prespecified level, the bondholders take over the firm. The empirical application of this approach to corporate fixed rate debt has yielded disappointing results; Jones, Mason and Rosenfeld(1984) found that the observed spreads between corporate fixed rate debt and government debt are too large to be explained with reasonable parameter values. The application of this method to floating rate notes requires an important modification: the coupon stream associated with a floating rate note will become known if a deterministic term structure is assumed. This makes the valuation of floating rate debt trivial and indistinguishable from the valuation of fixed rate debt. Hence, it becomes essential to model uncertainty in the term structure. We applied the traditional approach to the floater valuation problem, retaining the assumptions on the term structure spelled out in Section 3 and assuming that the value of the firm follows a lognormal diffusion. We find that, regardless of the choice of the reorganization boundary, this approach is simply unable to account for the magnitude of the discounts that are reported in Section 2.[13]

These findings lead us to consider an alternative approach which employs an instrumental variable to account for default risk. This variable is the default premium that is expected by investors on newly issued short-term obligations, which are close substitutes. Short-term obligations represent the natural instrument in this context because coupons are readjusted to short-term rates repeatedly; for floaters whose coupons are adjusted at longer intervals and tied to intermediate term rates, the intermediate term default premium is appropriate. This choice of the instrument has important implications. First, it delinks the value of the issuing firm and the risk premium demanded by investors on its debt obligation, and as a result it avoids specifying the economic events that occur upon bankruptcy -- although it should be clear that the default premium and the value of the firm should be inversely related. Second, because the premium will differ across firms with differing credit risk, this choice focuses attention on floaters issued by firms in the same risk "class." As a result, issues related to business risk and leverage are not treated explicitly. Finally, because this default premium is time-varying and because we are modeling floaters with fixed markups in their coupons, this choice of an explanatory variable is not self-referential -- rather it rests on the assumption that these floaters and newly issued short term obligations are close substitutes.

In this framework, the floater value depends on two variables in addition to τ: (a) the current rate of interest $r(t)$ which evolves according to equation (3) and (b) the instrumental variable, $p(t)$, which is the expected market premium on newly issued short term obligations. The floater is assumed to pay a coupon continuously at the rate $r(t) + \pi$, where π is set at the time of issuance. We assume that the floater is valued at each t to provide the investor with the instantaneous cum-coupon return $r(t) + p(t)$, which is the return promised on newly issued short-term obligations of similar risk. This assumes a pricing condition which is similar to the Local Expectations Hypothesis, modified to reflect credit risk:

$$E_t[dF] + (r(t) + \pi)dt = F\{r(t) + p(t)\}dt \tag{9}$$

where $F(r,p,\tau;\pi)$ is the price of a floater with a time to maturity τ paying a continuous coupon at a rate $r(t) + \pi$. It is noteworthy that for extremely general processes on r and p the floater will sell at par, provided that the coupon rate is time-varying and set at $r(t) + p(t)$ for all t. Therefore, the price behavior of the floater will depend, in part, on the location of $p(t)$ relative to the fixed premium π. Our specification in (9) only defines the expected return over an infinitesimal holding period. For any finite holding period, the default premiums will be maturity specific and therefore will differ from p. In order to complete the description of the pricing problem we need to specify the dynamics for $p(t)$. We rely here on the empirical findings of Fama(1984). In his analysis of default premiums in money market instruments, Fama reports that these are related to the stage of the business cycle. This regularity is captured in our choice of the mean reverting stochastic process for $p(t)$:

$$dp = \kappa_p(\mu_p - p)dt + \sigma_p\sqrt{p}dz_p \tag{10}$$

In (10) $\{z_p(t), t > 0\}$ is a standard Wiener process which may be correlated with the process $\{z(t), t > 0\}$; μ_p is the long run mean of the premium, $p\sigma_p^2$ is the variance of the changes in the premium and κ_p is the speed of adjustment. One implication of (10) is that the volatility of the changes in the default premiums is higher at higher levels of default premiums. Using (3), (9) and (10), one can derive the valuation equation that the floater's value must satisfy:

$$\tfrac{1}{2}F_{rr}\sigma^2 r + \tfrac{1}{2}F_{pp}\sigma_p^2 p + F_{pr}\sigma_{rp}\sqrt{r}\sqrt{p} + F_r\kappa(\mu - r) + F_p\kappa_p(\mu_p - p) + (r + \pi) - F(r + p) = F_\tau \qquad (11)$$

Note that when the continuous coupon flow rate is set at $(r + p)$ the floater always sell at par, and this serves as a natural benchmark to evaluate coupon rules[14] of the form $(r + \pi)$. We expect the floater to sell at a discount when $p(t) > \pi$; this discount structure could persist for a long time if the speed of adjustment factor κ_p is small and $p(t) > \pi$. In an analogous manner if $\mu_p > \pi$, then we expect the floater to sell almost always at a discount, especially for large values of κ_p.

The valuation equation (11) has a terminal condition given by

$$F(r,p,0;\pi) = 1.$$

We provide below the results which are obtained by solving equation (11) using the numerical method of alternating directions.[15] We maintain the parameter values of the interest rate process, and for the process on the market premium we assume $\kappa_p = 1$ and $\sigma_p^2 = 0.002$. The low value of κ_p was chosen so that the default premium p will exhibit a random walk pattern documented by Fama(1984). We use different values for the long run mean market premium μ_p, ranging from 0.0075 to 0.025. The stated premium π was held fixed at 0.01. We compute the prices of floaters with maturities of 5 and 10 years. Figure 6 contains the results corresponding to $\mu_p = 0.025$ for floaters maturing in 10 years: even when $p(t) = \pi = 1\%$, this floater sells at a discount of about 8% relative to its par value. This result is due to the fact that in the long run, the stated premium is inadequate compensation for the risk borne by investors.[16] As p(t) increases, the discount become larger at all levels of interest rates. Holding p(t) fixed, as the interest rates increase, the discounts decrease: at high rates of interest, the required premium is relatively a minor component of the total return from a floater where the coupon is indexed to the interest rate. We examined these issues for a floater maturing in 5 years. The results were quite similar and the discounts ranged from 4% to about 6.5%. We further investigated these issues for different values of σ_p^2. The results were fairly robust, indicating that for the specification used here, the volatility of the changes in default premiums did not significantly affect the valuation of floaters. It should be clear that, as in the case examined here, if the markup in the formula is fixed, then a put feature appended to the contractual terms will have considerable value.

To see whether an increase in the required default premium could have explained the floater discounts reported in Section 2, we plot the spread between Certificates of Deposit of 6 month maturity and 6 month Treasury Bills from January 1978 to June 1983 (Figure 7). We have chosen to plot the spread for this instrument because it is most appropriate to short term issues of financial corporations: the data are from Salomon Brothers' Analytical Record of Yields and Yield Spreads. The spread shows a marked increase over this period, and the level of the spread in June 1979 is consistent with the 50 basis point markup provided in the Beneficial floater at that time. The subsequent discounts exhibited by the floaters in our sample are broadly consistent with the increase in the spread over this period: it is possible to interpret these data as indicating an upward shift in the long-run mean premium

μ_p. Further empirical work needs to be conducted to estimate the parameters of the process on interest rates and the default premium in this context.

5. Conclusion

We have developed models for the valuation of floating rate notes, incorporating into the theoretical framework coupon averaging formulae, and several contractual features that are observed in issues of corporate floaters. The solutions presented for plausible parameter values indicate that ignoring default risk, these features taken singly cause these floaters to fluctuate in value more than one would find desirable, given the motivation for their issuance. For a plausible set of parameters it appears that the floater will sell at a discount whenever the stated default premium in the coupon formula is less than the long-run mean of the default premium expected by the market. We conclude that the observed discounts can be rationalized only if the stated premiums are much less than the premiums demanded by the investors.

FOOTNOTES

[1]See Hanna and Parente(1985).

[2]Chance(1983) also observes that floating-rate notes would have lower transactions costs than a series of short-term loans. Santomero(1983) has studied issues related to the comparison of fixed and floating rate instruments.

[3]The comparison between these strategies will not be justified for floaters issued by banks, if there are any de facto Federal guarantees on the CDs issued by these institutions.

[4]In a discrete-time setting elementary floaters may be valued by assuming that the short-term interest rate follows a two state branching process and that bonds are priced according to the local expectations hypothesis (see Cox, Ingersoll and Ross(1981)). Equivalently, one could apply the arguments of Cox, Ross and Rubinstein(1979) in conjunction with the local expectations hypothesis. We have evaluated floaters using this procedure: the path-dependent nature of the problem increases the computational complexity, and this limits its usefulness considerably.

[5]If the coupon formula is close to "perfect-indexation" and investors view these floaters as a substitute for a short-term paper, then the effects of differential taxation of capital gains and of coupon income on the value of the floater are likely to be minor.

[6]This equation can be derived in one of two equivalent ways: either by applying the Local Expectations Hypothesis directly to the floater's value or by usual hedging and continuous rebalancing arguments in conjunction with the LEH.

[7]For the lower boundary conditions with respect to r and x (at $r = 0$ and $x = 0\%$), and for the upper boundary condition with respect to x (at $x = 30\%$), we have employed the partial differential equation (7): for the interest rate process (3), zero is an accessible boundary, provided $\sigma^2 > 2\kappa\mu$. The value of the floater for a given x as r increases should decline to zero. We have employed $F_{rr} = 0$ at the upper bound for r, which was 40%. In all these cases we employed 200 mesh points in the r direction and 150 mesh points in the x direction. We evaluated floaters for maturities up to 10 years by this method, using 100 mesh points along the time axis.

[8]Because we are passing from a realistic setting with coupons found from average bill yields 27, 28 and 29 weeks "ago" to one in which the continuous coupon is an exponentially weighted average of past rates, no "best" choice of β suggests itself. Rather, we were guided by a desire to vary β and hence accommodate varying rates of convergence of x to r. We have also tried a β value of 70, which effectively makes the floater "perfectly-indexed."

[9]The floater values were increasing and nearly linear in the coupon rates (x) for a given value of the interest rate r. We have chosen to show the graphs for x = 4, 10 and 16% to avoid clutter and to indicate the ranges at which sizeable discounts and premiums occur.

[10]The boundary conditions remain as noted earlier. In the partial differential equation, however, the term representing the payout to the security is the floor or the ceiling rate whenever that is in force.

[11]For high values of β, the coupon rate is fairly close to the current short-term rate and therefore the floater tends to sell close to par. Under this setting, the put feature will not have much value. On the other hand, if the coupon averaging rule causes the coupon rate to deviate significantly from the current short-term rate, then the put feature will be of value to the investor.

[12]We computed the floater values for alternate values of σ^2 and found that they were quite insensitive to this parameter. Considerations of computational expense limited the number of evaluations we could conduct -- we varied the β values, and the effects were as reported above.

[13]We employed a fixed lower reorganization boundary as well as a time-varying boundary equal to the discounted face value. In both cases, the floater values were quite insensitive to interest rate levels.

[14]This method can also accommodate coupon formulas in which the stated default premium declines over time in a prespecified way.

[15]Brennan and Schwartz(1980) discuss this procedure. The numerical solution of the valuation equation requires additional boundary conditions. At r = 0 and p = 0, the valuation equation itself served as the lower boundary. $F_{rr} = 0$ and $F_{pp} = 0$ served as upper boundary conditions for r and p respectively. The upper boundary for p was chosen to be 10%, and the covariance σ_{rp} was set to zero.

[16]Puglisi and Cohen(1981) have made a similar observation.

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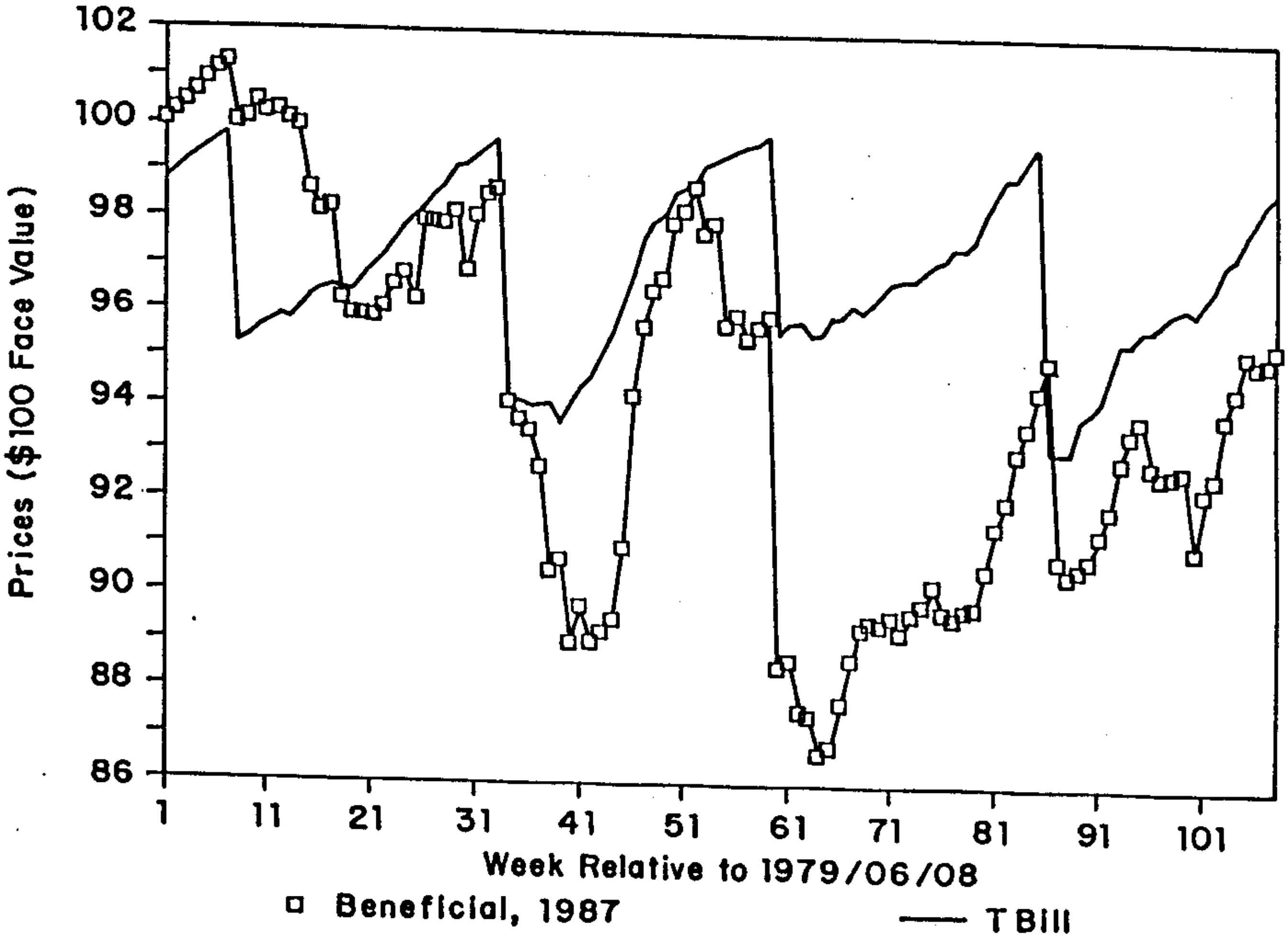

Figure 1. Weekly closing prices of Beneficial 1987 floating rate note and of U.S. Treasury Bill.

Notes: The Beneficial prices include the accrued coupon since the previous coupon payment date. The Treasury Bill is chosen to have a maturity date closest to Beneficial's coupon dates; it had a 6 month maturity at issue and it is plotted assuming a $100 face value.

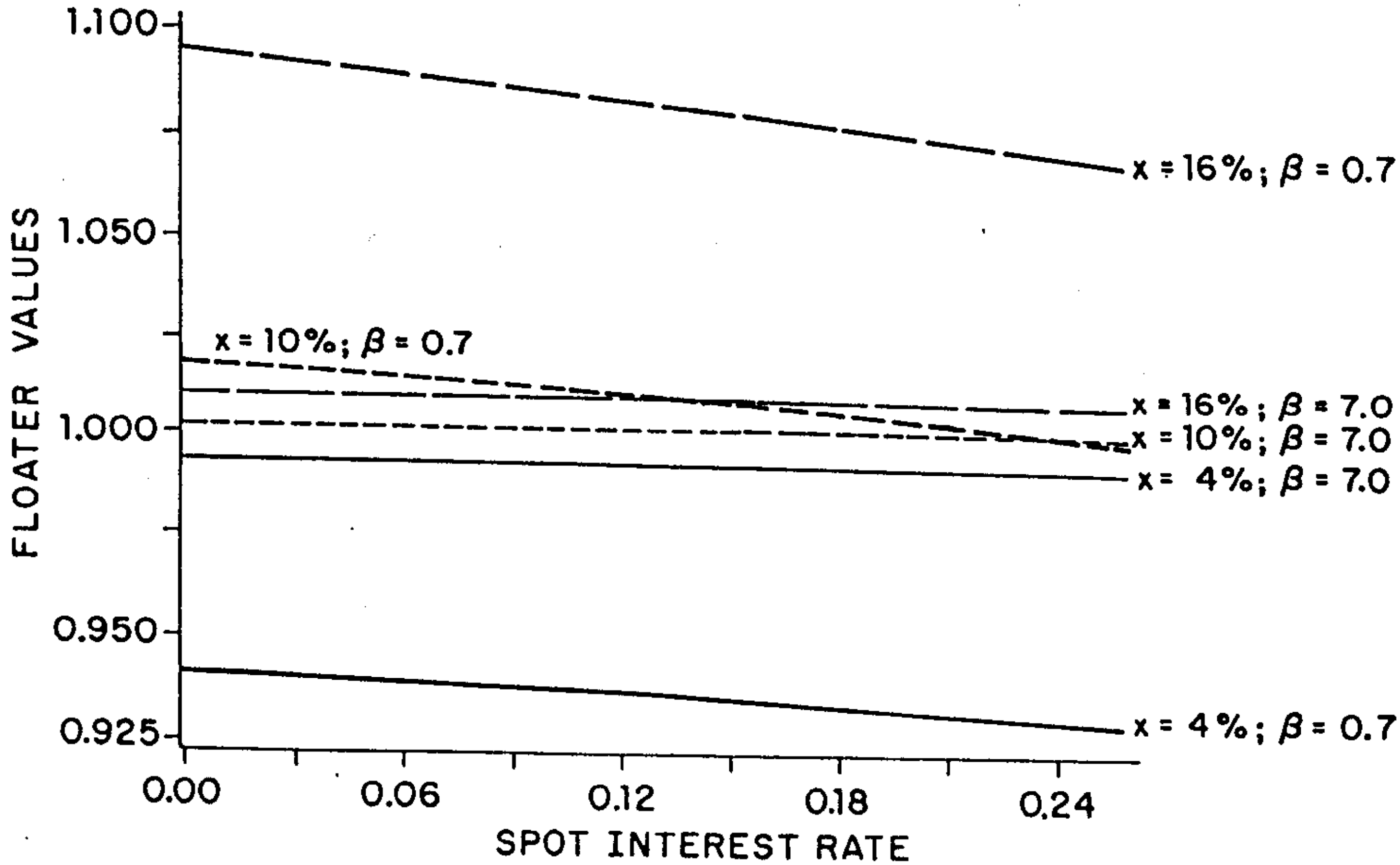

Figure 2. Straight floater values versus interest rates.

Notes: x is the specified coupon rate on the floater and it reverts to the current interest rate with a speed of adjustment, β. The interest rate r evolves as a diffusion

$$dr = \kappa(\mu - r)dt + \sigma\sqrt{r}\, dz,$$

where $\kappa = 2$, $\mu = 0.09$ and $\sigma^2 = 0.006$. The floater has 5 years to maturity.

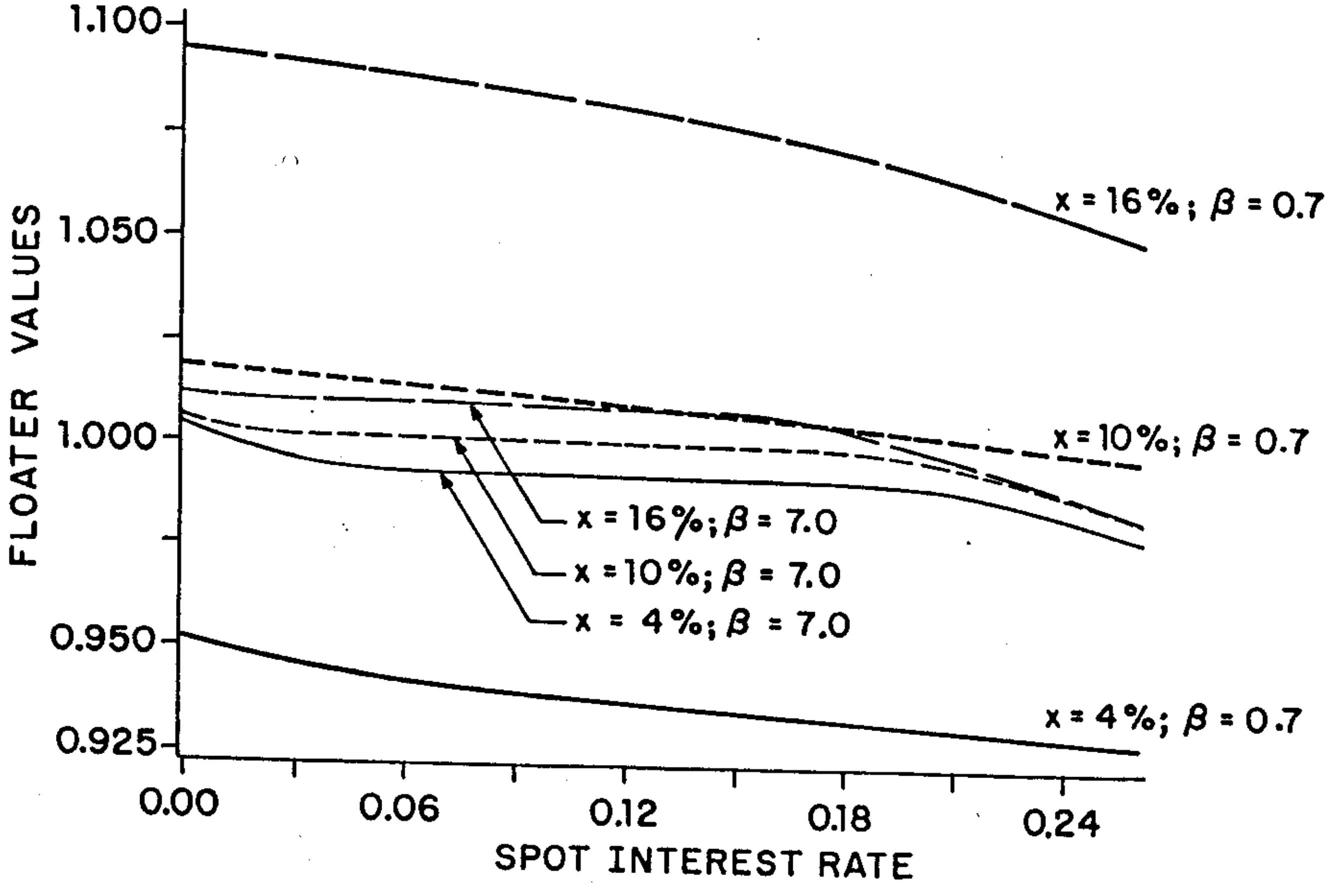

Figure 3. Floater values (with ceiling and floor coupon rate) versus interest rates.

Notes: x is the specified coupon rate on the floater and it reverts to the current interest rate with a speed of adjustment, β; it is subject to a ceiling and floor rate of 15% and 5% respectively. The interest rate r evolves as a diffusion

$$dr = \kappa(\mu - r)dt + \sigma\sqrt{r}\,dz$$

where $\kappa = 2$, $\mu = 0.09$ and $\sigma^2 = 0.006$. The floater has 5 years to maturity.

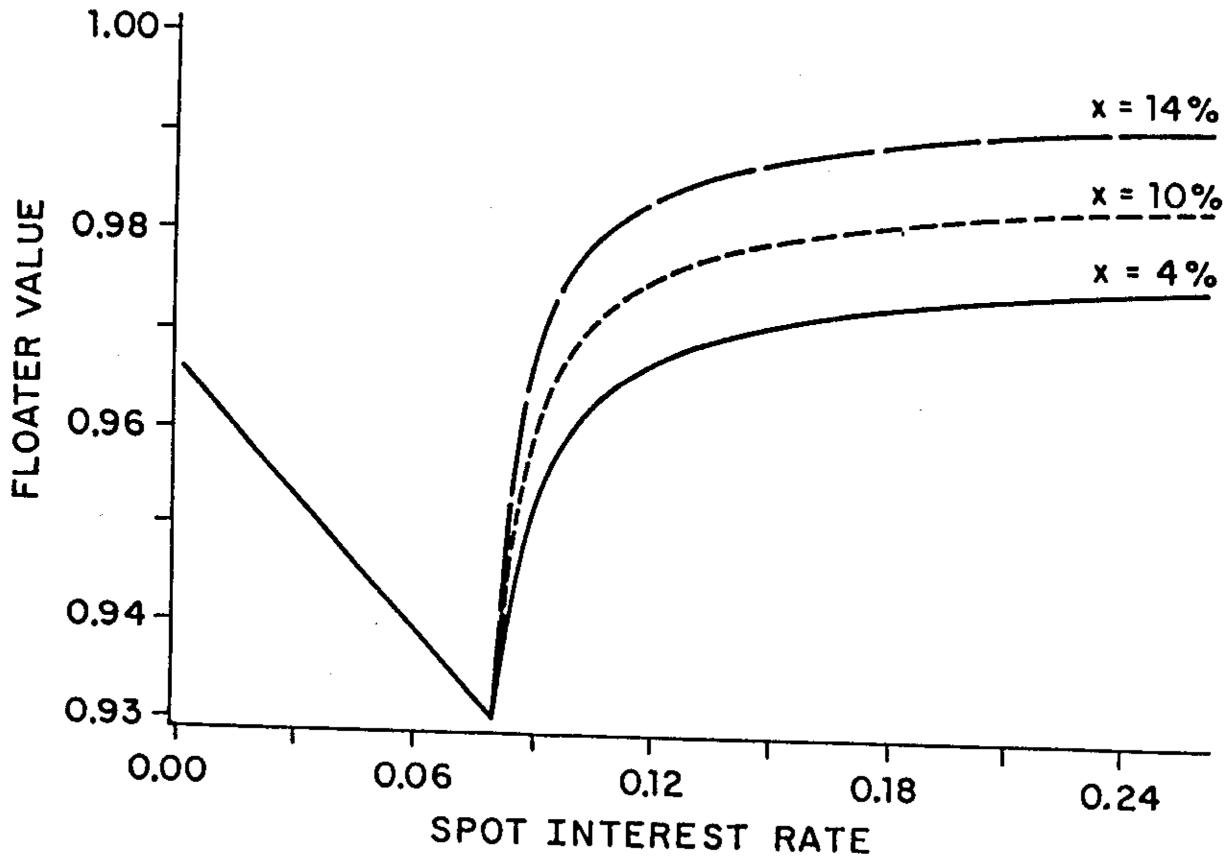

Figure 4. Floater values (with Drop Lock feature) versus interest rates (long-run means interest rate = 10%).

Notes: When the interest rate reaches 8% (drop), the issue becomes a fixed rate note (the lock) at 8% coupon until maturity. x is the specified coupon rate on the floater and it reverts to the current interest rate with a speed of adjustment, β. The interest rate r evolves as a diffusion

$$dr = \kappa(\mu - r)\,dt + \sigma\sqrt{r}\,dz$$

where $\kappa = 2$, $\mu = 0.10$ and $\sigma^2 = 0.006$. The floater has 5 years to maturity.

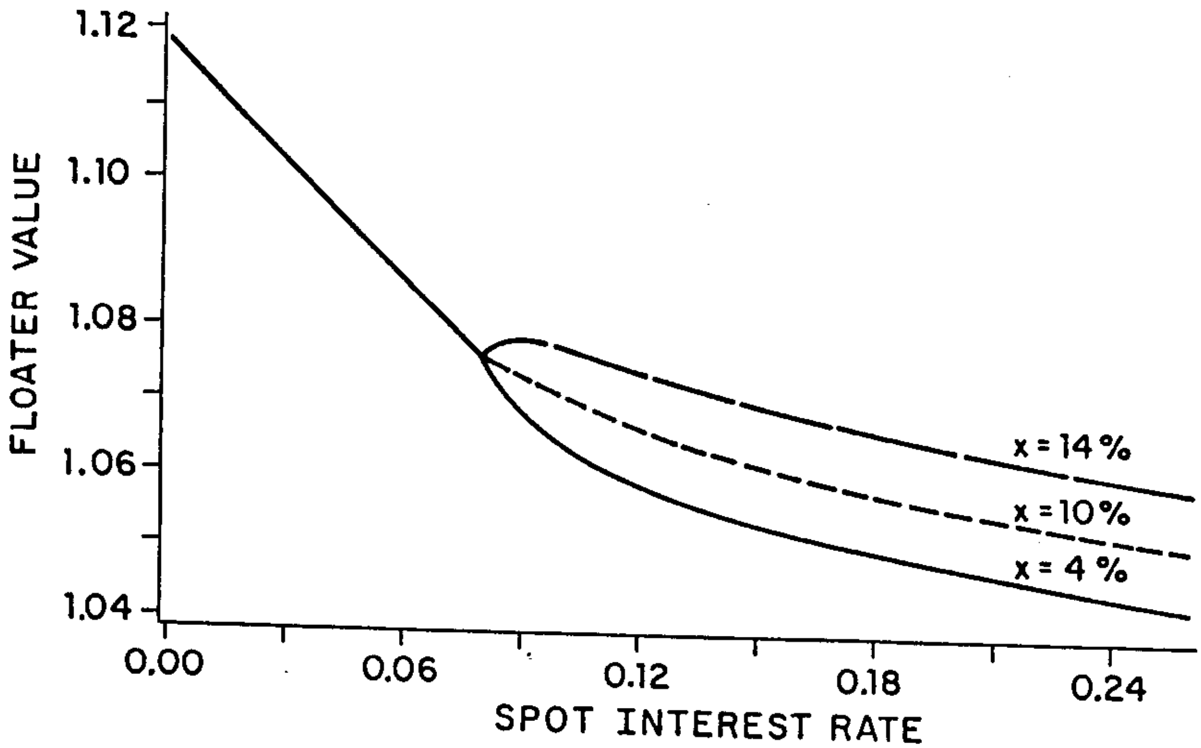

Figure 5. Floater values (with Drop Lock feature) versus interest rates (long-run means interest rate = 6%).

Notes: When the interest rate reaches 8%, the issue becomes a fixed rate note at 8% coupon until maturity. x is the specified coupon rate on the floater and it reverts to the current interest rate with a speed of adjustment, β. The interest rate r evolves as a diffusion

$$dr = \kappa(\mu - r)\, dt + \sigma\sqrt{r}\, dz$$

where $\kappa = 2$, $\mu = 0.06$ and $\sigma^2 = 0.006$. The floater has 5 years to maturity.

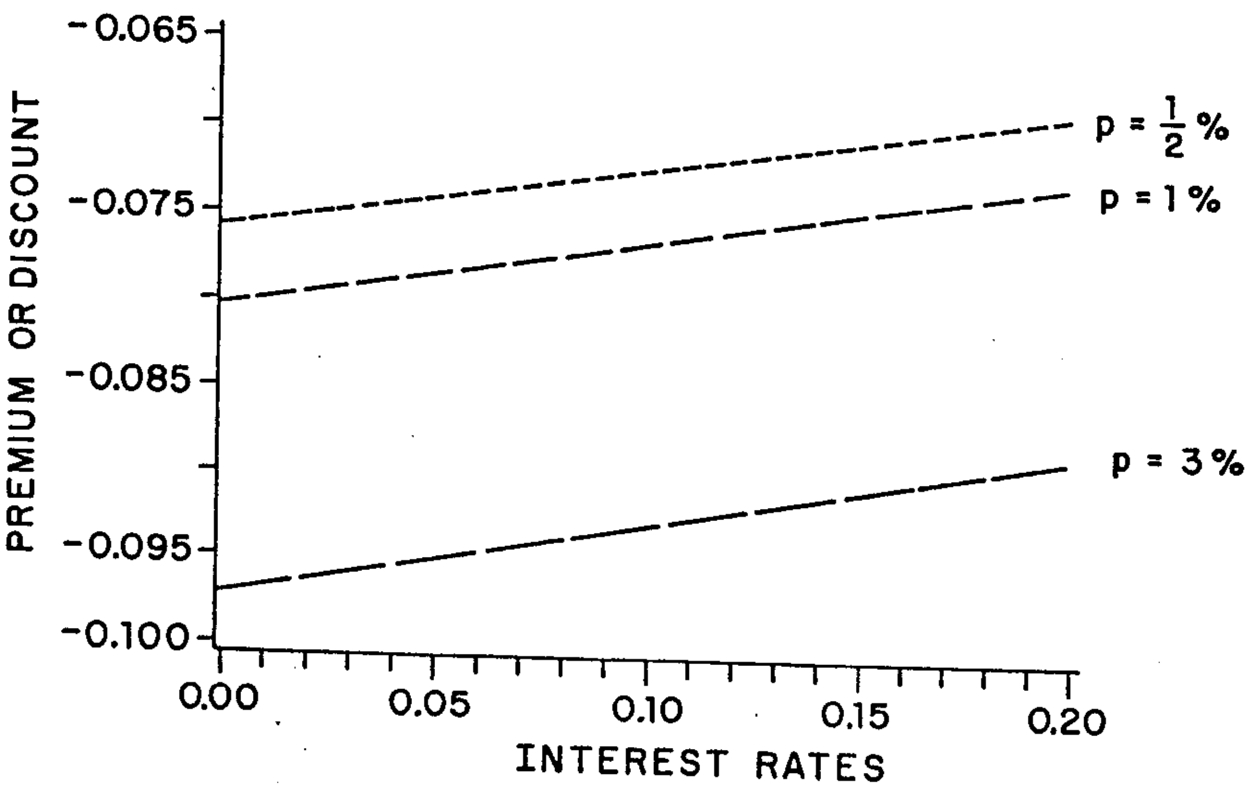

Figure 6. Discounts for corporate floaters versus interest rates.

Notes: p is the default premium demanded of newly issued short-term corporate paper. It evolves as a diffusion

$$dp = \kappa_p(\mu_p - p)\,dt + \sigma_p\sqrt{p}\,dz_p$$

where $\kappa_p = 1$, $\mu_p = 0.025$, $\sigma_p^2 = 0.002$. The interest rate r evolves a diffusion

$$dr = \kappa(\mu - r)\,dt + \sigma\sqrt{r}\,dz$$

where $\kappa = 2$, $\mu = 0.10$, $\sigma^2 = 0.006$, and Cov $\{dz_p, dz\} = 0$. The floater has 10 years to maturity, with a continuously indexed coupon rate $r + \pi$. $\pi = 0.01$ is the fixed default premium.

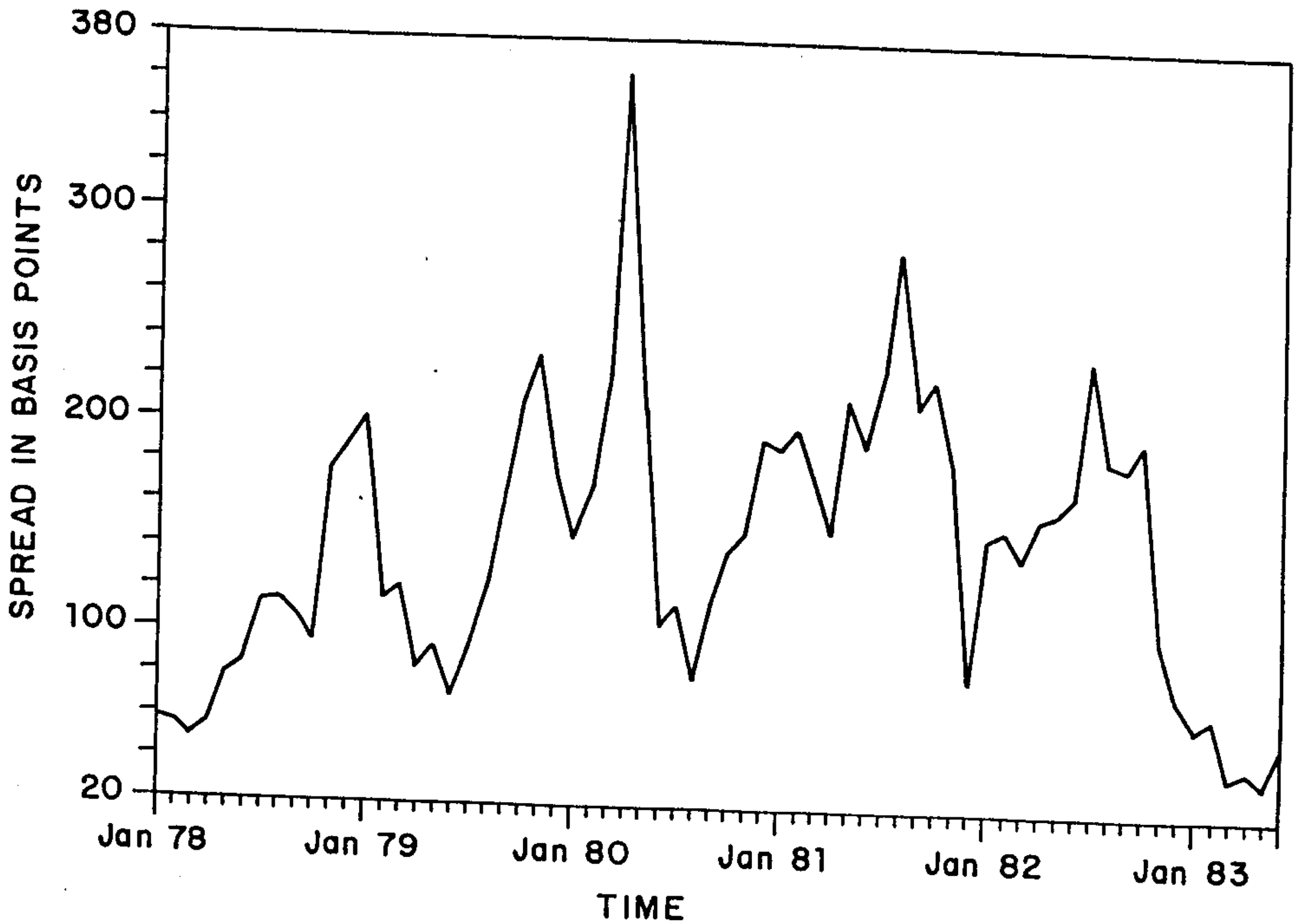

Figure 7. Spread of CD yields over Treasury Bills (6 month maturity).