CAPITAL STRUCTURE AND IMPERFECT COMPETITION IN PRODUCT MARKETS

by

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Revised version

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Abstract

A linear duopoly model is used to consider investment and financing decisions. Bankruptcy is assumed to cause a delay in investment which is not costly in itself. However, the imperfect competition in the product market means this delay puts the bankrupt firm at a strategic disadvantage which forces it to either reduce its size or, in most cases, to liquidate. This is costly because the firm loses the profits it would otherwise have obtained. As a result firms use only a limited amount of debt despite the corporate tax advantage it enjoys.

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1. Introduction

It is often acknowledged that the question of how firms choose their capital structure has not been fully answered yet: current theories seem to be unable to satisfactorily explain the financing decisions firms actually make. The most widely taught theory of capital structure is still perhaps the Modigliani and Miller (1958; 1963) theory extended to include corporate taxes and bankruptcy costs. This argues that firms trade-off the corporate tax advantage of debt that arises from interest being tax deductible against the costs of bankruptcy.

The debate about the plausibility of this theory has revolved around the question of whether the marginal tax advantage of debt can be of a similar magnitude to the marginal costs of bankruptcy. The problem is that many firms paying corporate taxes use only a small amount of debt: debt ratios are typically around twenty to thirty percent. In the versions of the theory which just take account of the effect of corporate taxes (see e.g., Kraus and Litzenberger (1973), Scott (1976), Kim (1978), and Turnbull (1979)), the marginal benefit for such firms from using more debt is large. The marginal costs of bankruptcy consist of the product of the change in the probability of bankruptcy and the costs of bankruptcy. For firms with low debt ratios the former is usually small. Thus to be plausible these versions of the theory require the costs of bankruptcy to be large. A crucial question is therefore what these include.

First there are the direct costs such as lawyers' fees and so on. However there is evidence these are small (Warner (1977); Ang, Chua and McConnell (1982)). Second there are the indirect costs like the difficulty in operating firms' plants while in bankruptcy. These are difficult to measure and there is no consensus on their magnitude. Third there are agency costs that result from the use of debt contracts when managers' actions are unobservable (see, e.g., Jensen and Meckling (1976), and Myers (1977)). Again empirical evidence on the importance of these is lacking. Although these factors are probably important at high debt ratios, it seems less likely that for firms with low debt ratios they offset the corporate tax advantage of debt more than a small amount.

This leaves a fourth type of cost, namely, liquidation costs. It is usually argued these are the costs of actually dismantling and breaking up the firm. As Titman (1984) has pointed out they also include search and retooling costs for workers and suppliers with job specific capital and increased expenses for customers. In many papers liquidation costs are apparently the main costs of bankruptcy that the authors have in mind. A possible reason for this is that bankruptcy is often followed by liquidation. For example, Stanley and Girth (1971) found that in their sample only around twelve and a half percent of corporate bankrupts attempted to reorganize and of these, sixty percent were ultimately unsuccessful. Liquidation costs are potentially sufficiently large that if included in bankruptcy costs the theory might provide a plausible explanation of firms' decisions. However, Haugen and Senbet (1978) have pointed out that liquidation is a capital budgeting decision whereas bankruptcy is a transfer of assets: in standard models there is no reason why bankruptcy should cause liquidation. Thus liquidation costs should not be included in bankruptcy costs. Given this it is not clear that for most firms marginal bankruptcy costs are sufficient to offset the marginal corporate tax advantage of debt.

This and other difficulties have led to the development of theories which take account of personal taxes as well as corporate taxes (see e.g., Stiglitz (1973), King (1974), Miller (1977) and DeAngelo and Masulis (1980)). These argue that by retaining earnings and repurchasing shares, for example, firms can ensure that the personal tax on equity income is largely avoided. Thus the corporate tax advantage to debt is mitigated by the personal tax advantage to equity. An important feature of theories of this type is that they do not require bankruptcy costs to be large to be consistent with the fact that many firms paying taxes have low debt ratios. However the basic assumption of this type of theory is difficult to reconcile with empirical evidence on dividends. Firms pay substantial amounts of dividends directly to individuals and an analysis of tax returns presented by Peterson, Peterson and Ang (1985), for example, suggests that these individuals pay substantially more taxes on their dividends than they would on capital gains.

A theory of capital structure is presented below where bankruptcy causes partial or complete liquidation. This is costly because firms lose the future profits they would otherwise have obtained. The theory is based on imperfections in firms' product markets: most corporations, particularly large ones, operate in product markets with only a few firms. For simplicity a duopolistic industry with a Cournot market structure and linear demand and cost functions is analyzed.

The other main assumption is that the effect of bankruptcy is to delay investment. The justification for this is first that the time spent in bankruptcy court is lengthy. For example, Stanley and Girth (1971) found that a typical chapter XI reorganization took around eighteen months between filing and closing. Second, while in bankruptcy court undertaking investments is a much more drawn out process than otherwise since the approval of the court must be obtained and this is time consuming. Hence a bankrupt firm is delayed in comparison to a firm that is solvent.

In the model this delay is assumed not to be costly in itself. However, given the imperfectly competitive nature of the product market it puts the bankrupt firm at a strategic disadvantage which results in it being forced to contract in size. Except when the fixed costs of installing capacity are small this disadvantage is so severe that the bankrupt firm is forced to liquidate. This is costly for the firm because it loses the profits it would have obtained if it had remained solvent and not been put at a strategic disadvantage. In choosing the amount of debt to use firms therefore trade off the corporate tax advantage arising from interest being tax deductible against these costs.

It is shown that the type of equilibrium that results depends crucially on the fixed costs of capacity. If these are low the equilibrium is symmetric so both firms have the same capacity, use the same amount of debt and so on. If they are high the equilibrium is asymmetric: one firm goes for the tax advantage of debt and other goes for the strategic advantage that results from using equity finance.

The paper proceeds as follows. Section 2 outlines the model. In Section 3, equilibrium is analyzed. Finally, Section 4 contains concluding remarks.

2. The Model

There are two identical firms denoted j and $-j$. The market structure is Cournot: the firms produce quantities q_j and q_{-j}, place them on the market and together these determine the market clearing price p.

Demand for the industry is stochastic. The demand curve has the form

$$p = \alpha - \beta(q_j + q_{-j}) \tag{1}$$

where the intercept α is a random variable but the slope β is a constant. The density function of α is linear:

$$f(\alpha) = \alpha - \alpha_\ell \qquad \text{for } \alpha_\ell \leqslant \alpha \leqslant \bar{\alpha} \tag{2a}$$

$$= \alpha_\ell + 2 - \alpha \qquad \text{for } \bar{\alpha} \leqslant \alpha \leqslant \alpha_\ell + 2 \tag{2b}$$

where

$$\bar{\alpha} = \alpha_\ell + 1 \ . \tag{3}$$

It is assumed that α_ℓ is sufficiently large that in equilibrium

$$p > 0 \tag{4}$$

for all possible realizations of α.

A firm's output is equal to its capacity which is determined by its initial investment I_j. The cost of capacity is a linear function:

$$I_j = \phi + \mu q_j \tag{5}$$

where ϕ is the fixed cost and μ is the constant marginal cost.

There are two periods t = 0, 1, 2. The sequence of events in each period is the following.

(i) Initially firms make their investment decisions which determine their capacities. At the same time they make their financing decisions. The capital markets meet, the firms issue the securities corresponding to their decisions and use the proceeds to pay the costs of their investments. All this is done before α is realized.

(ii) Output is produced; α is realized.

(iii) The two firms' quantities, together with α, determine the price. Revenue pq_j is received. This is used to pay the corporate income tax and make payments on the securities issued at stage (i).

Capital markets are perfectly competitive and there are no transaction costs or other frictions. Everybody is risk neutral and securities sell for their expected value. The interest rate is zero.

Firms have two financing instruments: debt and equity. Debt is a promise to repay D_j at the end of the period or the firm goes bankrupt and the bondholders acquire it. Equity receives the profits remaining at the end of the period after taxes and debt repayment.

At the start of each period, before firms make their investment and financing decisions, they are entirely owned by equityholders. When firms finance their investment they issue debt with nominal value D_j. If the debt is risky, its market value D_j^M will be less than D_j. The remainder required to finance the investment, $I_j - D_j^M$, is raised by issuing equity. Since the capital markets are perfect, the original shareholders total expected return is the expected value of their firm. Hence their objective when they make the investment and financing decisions is to maximize the total expected value of the firm.

The corporate tax rate is τ. Investment costs are tax deductible. Nominal debt D_j is also deductible so that there is a tax bias in favor of debt finance. Hence the total corporate taxes paid by a firm, T_j, are:

$$T_j = \tau\left[pq_j - (\phi + \mu q_j) - D_j\right] . \tag{6}$$

Taxes are paid when the tax base is positive and subsidies are received when it is negative. To prevent firms from issuing an infinite amount of nominal debt D_j, and claiming the infinite subsidy this would give rise to, it is assumed that the amount of debt that can be tax deducted is limited to D_{max}:

$$D_j \leqslant D_{max} . \tag{7}$$

There are no personal taxes on either debt or equity income or equivalently the marginal personal rate on equity income is the same as the marginal personal rate on debt income.

Bankruptcy occurs when the total receipts at the end of the period from selling the goods less any tax payments (or plus any subsidies) are insufficient to repay the debt:

$$pq_j - T_j \leqslant D_j \ . \tag{8}$$

This condition assumes that equityholders do not have the opportunity to raise funds against future profits and use them to repay the debt and possibly prevent bankruptcy. This is the simplest case to consider: if future profits were included in the bankruptcy condition, there would still be a determinate bankruptcy point and a similar analysis to that below would be possible. The condition also assumes that the equityholders cannot repurchase the debt at its current market value just before bankruptcy as suggested by Haugen and Senbet (1978). One justification for this is that it will be difficult for a firm to repurchase its debt securities if it is near bankruptcy because, similarly to Grossman and Hart (1980), there is a free-rider problem. Given some people accept the repurchase offer from the firm, anybody who holds out and refuses the offer can expect the value of their securities to increase. This is because if the firm were then to go bankrupt its assets would be divided between fewer bonds and so the payoff on each would be increased (see Allen (1985)).

As explained in the introduction, the effect of bankruptcy at the end of a period is to delay the investment decision at the start of the next period. If both firms are in the same position at the end of period 1 so that they are either both solvent or both bankrupt, they make their decisions simultaneously at the start of period 2 and so play a Nash game in that period. However, if one firm is solvent and the other is bankrupt so that its investment decision is delayed, they play a Stackelberg game with the solvent firm as leader and the bankrupt one as follower. Hence if one firm becomes bankrupt, it is put at a strategic disadvantage.

3. Equilibrium

The equilibrium of the model depends crucially on the level of the fixed cost of capacity ϕ. In subsection (a) the case where $\phi = 0$ is analyzed; the opposite extreme where the fixed costs are large is looked at in (b), and (c) deals with the intermediate case. Finally (d) contains a brief summary.

(a) No fixed costs of capacity

In this subsection it is assumed there is only a marginal cost of capacity:

$$\phi = 0 \ . \tag{9}$$

A rational expectations perfect Nash equilibrium concept is used. The solution procedure involves solving backwards by first looking at the possibilities at t = 1 and then analyzing the decisions at t = 0 given that the firms play equilibrium strategies in the subgames at t = 1.

First, consider decisions at t = 1. Since the only effect of bankruptcy is to delay investment decisions in the subsequent period, it follows that bankruptcy in period 2 has no effect so that firms will always use as much debt as possible. For simplicity it is simplest to assume that in period 2

$$D_{max} = 0 \ . \tag{10}$$

At t = 1 the only decision firms are then concerned with is the level of investment I_j or equivalently capacity q_j.

There are two possibilities at $t = 1$. Either both firms are solvent or both are bankrupt in which case they play a Nash game. Alternatively one firm is solvent and the other is bankrupt in which case they play a Stackelberg game. Consider the Nash game first. Each firm chooses its capacity to maximize its expected profit taking the other firm's capacity as given:

$$\underset{q_j}{\text{Max}} \ (1 - \tau)[\bar{\alpha} - \beta(q_j + q_{-j}) - \mu]q_j \tag{11}$$

taking q_{-j} as given.

The first order condition implies

$$q_j = \frac{\bar{\alpha} - \mu}{2\beta} - \frac{q_{-j}}{2} . \tag{12}$$

Similarly for $-j$. Solving these simultaneously gives

$$q_j = \frac{\bar{\alpha} - \mu}{3\beta} \tag{13}$$

$$E\pi_{2j} = (1 - \tau)\frac{Z}{9} \tag{14}$$

where $E\pi_{2j}$ is the expected profit of j in period 2 and

$$Z = \frac{(\bar{\alpha} - \mu)^2}{\beta} . \tag{15}$$

The other possibility is that one firm, say firm j, is solvent and the other, $-j$, is bankrupt. Firm j acts as leader and makes its investment decision while $-j$ is tied up in bankruptcy court. When $-j$ finally makes its investment decision it takes j's as given. Firm j takes this into account when it makes its decision initially in the usual Stackelberg way. Hence $-j$'s decision as follower is

$$\underset{q_{-j}}{\text{Max}} \ (1 - \tau)[\bar{\alpha} - \beta(q_j + q_{-j}) - \mu]q_{-j} \tag{16}$$

taking q_j as given. Hence

$$q_{-j} = \frac{\bar{\alpha} - \mu}{\beta} - \frac{q_j}{2} \; . \tag{17}$$

Then j's decision as leader is

$$\max_{q_j} \; (1 - \tau)\{\bar{\alpha} - \beta[q_j + (\frac{\bar{\alpha} - \mu}{\beta} - \frac{q_j}{2})] - \mu\}q_j \; . \tag{18}$$

It follows

$$q_j = \frac{\bar{\alpha} - \mu}{2\beta} \tag{19}$$

$$E\pi_{2j} = (1 - \tau)\frac{Z}{8} \tag{20}$$

$$q_{-j} = \frac{\bar{\alpha} - \mu}{4\beta} \tag{21}$$

$$E\pi_{2-j} = (1 - \tau)\frac{Z}{16} \; . \tag{22}$$

The effect of a single firm going bankrupt can be seen by comparing (13) and (14) with (19) - (22). The leader expands its capacity compared to the Nash case. The follower is at a strategic disadvantage, and as a result has a lower capacity than before. The bankrupt firm is worse off since its profits are reduced from $(1 - \tau)Z/9$ to $(1 - \tau)Z/16$ but the solvent firm is made better off: its profits are increased from $(1 - \tau)Z/9$ to $(1 - \tau)Z/4$. Hence the short delay in a firm's investment decision, although not costly in itself, means bankruptcy is undesirable.

Next consider the decisions of the firms at t = 0. In period 1 it is assumed that

$$D_{max} > 0 \; . \tag{23}$$

The analysis will be concerned with interior solutions such that (7) does not bind.

In period 1 firm j's expected profits are

$$E\pi_{1j} = (1 - \tau)[\bar{\alpha} - \beta(q_j + q_{-j}) - \mu]q_j + \tau D_j \ . \tag{24}$$

The realization of α in period 1 determines whether or not the firms go bankrupt and hence the type of game played and the expected profits in period 2. In order to derive an expression for j's expected profits in period 2 evaluated at $t = 0$, it is necessary to define the level of demand α_j^* such that firm j goes bankrupt. Using (1), (6) and (8)

$$\alpha_j^* = \frac{D_j}{q_j} + \beta(q_j + q_{-j}) - \frac{\tau}{1 - \tau}\mu \ . \tag{25}$$

For values of α above α_j^*, firm j is solvent; for values below it is bankrupt. Similarly for $-j$. If $\alpha_j^* \geqslant \alpha_{-j}^*$ then

$$E\pi_{2j} = (1 - \tau)\left[\int_{\alpha_\ell}^{\alpha_{-j}^*} \frac{Z}{9} f(\alpha)d\alpha + \int_{\alpha_{-j}^*}^{\alpha_j^*} \frac{Z}{16} f(\alpha)d\alpha + \int_{\alpha_j^*}^{\alpha_\ell+2} \frac{Z}{9} f(\alpha)d\alpha\right] \ . \tag{26}$$

For α between α_ℓ and α_{-j}^* both firms go bankrupt and in period 2 they play a Nash game so j's expected profits are $(1 - \tau)Z/9$. From α_{-j}^* to α_j^* firm j is bankrupt but $-j$ is not: they play a Stackelberg game with j as a follower so its expected profits are $(1 - \tau)Z/16$. For α above α_j^* they again play a Nash game.

For $\alpha_j^* \leqslant \alpha_{-j}^*$,

$$E\pi_{2j} = (1 - \tau)\left[\int_{\alpha_\ell}^{\alpha_j^*} \frac{Z}{9} f(\alpha)d\alpha + \int_{\alpha_j^*}^{\alpha_{-j}^*} \frac{Z}{8} f(\alpha)d\alpha + \int_{\alpha_{-j}^*}^{\alpha_\ell+2} \frac{Z}{9} f(\alpha)d\alpha\right] \ . \tag{27}$$

This case is the same as when $\alpha_j^* \geqslant \alpha_{-j}^*$ except that between α_j^* and α_{-j}^*, j is the leader so its expected profits are $(1 - \tau)Z/8$.

Since the objective of the initial shareholders is to maximize the total value of the firm as explained in Section 2, firm j's decision problem at $t = 0$ is

$$\max_{q_j, D_j} E\pi_j = E\pi_{1j} + E\pi_{2j} \tag{28}$$

taking q_{-j} and D_{-j} as given.

Two types of equilibrium are possible. The first is symmetric with $\alpha_j^* = \alpha_{-j}^*$ so both have the same capacity and debt. In the other $\alpha_j^* > \alpha_{-j}^*$ (say) so that one goes for the tax advantage of debt and the other goes for the equity advantage of being leader if demand is low.

Consider the symmetric equilibrium first and suppose initially $\alpha_j^* = \alpha_{-j}^*$. Now

$$\frac{\partial \alpha_j^*}{\partial D_j} = \frac{1}{q_j} \tag{29}$$

$$\frac{\partial \alpha_{-j}^*}{\partial D_j} = 0 \ . \tag{30}$$

Hence if j increases its debt a small amount then $\alpha_j^* > \alpha_{-j}^*$ and so (26) is the relevant expression for period 2's profits. Thus

$$\frac{\partial E\pi_j^+}{\partial D_j} = \tau - m_F \frac{(\alpha_j^* - \alpha_\ell)}{q_j} \tag{31}$$

where

$$m_F = (1 - \tau)\frac{7}{144} Z \ . \tag{32}$$

The first term τ is the tax benefit in period 1 from increasing debt. The second term is the loss in expected profits in period 2 arising from the fact that j will go bankrupt in more states. In these states, instead of having expected profits of $(1 - \tau)Z/9$ when playing a Nash game, as Stackelberg follower j only has $(1 - \tau)Z/16$ so that the difference between them is m_F. The term $(\alpha_j^* - \alpha_\ell)/q_j$ is $f(\alpha)\partial\alpha_j^*/\partial D_j$: in order for there to be an interior solution, $f(\alpha)$ must be given by (2a) since otherwise $E\pi_j$ would be convex in D_j.

In contrast if j reduces its debt then $\alpha_j^* < \alpha_{-j}^*$ and (27) is the relevant expression for period 2 profits. Thus

$$\frac{\partial E\pi_j^-}{\partial D_j} = \tau - m_L \frac{(\alpha_j^* - \alpha_\ell)}{q_j} \tag{33}$$

where

$$m_L = (1 - \tau)\frac{2}{144} Z . \tag{34}$$

In this case the change in expected profits resulting from bankruptcy, m_L, is the difference between those from being a Stackelberg leader $(1 - \tau)Z/8$ and those in the ordinary Nash game of $(1 - \tau)Z/9$.

It can be seen

$$m_F > m_L \tag{35}$$

and

$$\frac{\partial E\pi_j^+}{\partial D_j} - \frac{\partial E\pi_j^-}{\partial D_j} = (m_L - m_F)\frac{(\alpha_j^* - \alpha_\ell)}{q_j} < 0 \tag{36}$$

so that $E\pi_j$ is kinked at $\alpha_j^* = \alpha_{-j}^*$. Moreover, the kink is concave so that it can correspond to a maximum.

Next consider changes in capacity q_j.

$$\frac{\partial\alpha_j^*}{\partial q_j} = -\frac{D_j}{q_j^2} + \beta \tag{37}$$

$$\frac{\partial\alpha_{-j}^*}{\partial q_j} = \beta . \tag{38}$$

These imply

$$\frac{\partial \alpha_j^*}{\partial q_j} < \frac{\partial \alpha_{-j}^*}{\partial q_j} \; . \tag{39}$$

For increases in capacity (27) is therefore the relevant expression for $E\pi_{2j}$. Using the fact that $\alpha_j^* = \alpha_{-j}^*$ it can be shown:

$$\frac{\partial E\pi_j^+}{\partial q_j} = (1 - \tau)[\bar{\alpha} - \mu - \beta(2q_j + q_{-j})] + m_L(\alpha_j^* - \alpha_\ell)\,\frac{D_j}{q_j^2} \; . \tag{40}$$

For reductions in capacity (26) is relevant, so

$$\frac{\partial E\pi_j^-}{\partial q_j} = (1 - \tau)[\bar{\alpha} - \mu - \beta(2q_j + q_{-j})] + m_F(\alpha_j^* - \alpha_\ell)\,\frac{D_j}{q_j^2} \; . \tag{41}$$

Hence

$$\frac{\partial E\pi_j^+}{\partial q_j} - \frac{\partial E\pi_j^-}{\partial q_j} = (m_L - m_F)(\alpha_j^* - \alpha_\ell)\,\frac{D_j}{q_j^2} < 0 \; . \tag{42}$$

There is again a kink which is concave so that it can correspond to a maximum.

Similarly for $-j$.

Given $\alpha_j^* = \alpha_{-j}^*$, $q_j = q_{-j}$ and the derivatives are independent, linear equations, a symmetric equilibrium always exists. It follows from there being kinks at each firm's optimum that in fact multiple equilibria exist. The presence of kinks also implies that at any particular equilibrium, a range of tax rates is consistent with that particular equilibrium level of capacity and debt.

The other type of equilibrium that might exist is asymmetric with $\alpha_j^* \neq \alpha_{-j}^*$. Suppose

$$\alpha_j^* > \alpha_{-j}^* \; . \tag{43}$$

The first order conditions for j and $-j$ are then

$$\frac{\partial E\pi_j}{\partial D_j} = \tau - m_F \frac{(\alpha_j^* - \alpha_\ell)}{q_j} = 0 \tag{44}$$

$$\frac{\partial E\pi_j}{\partial q_j} = (1 - \tau)[\bar{\alpha} - \mu - \beta(2q_j + q_{-j})]$$

$$+ \beta m_F(\alpha_{-j}^* - \alpha_\ell) - \left(\beta - \frac{D_j}{q_j^2}\right)m_F(\alpha_j^* - \alpha_\ell) = 0 \tag{45}$$

$$\frac{\partial E\pi_{-j}}{\partial D_{-j}} = \tau - m_L \frac{(\alpha_{-j}^* - \alpha_\ell)}{q_{-j}} = 0 \tag{46}$$

$$\frac{\partial E\pi_{-j}}{\partial q_{-j}} = (1 - \tau)[\bar{\alpha} - \mu - \beta(q_j + 2q_{-j})]$$

$$+ \beta m_L(\alpha_j^* - \alpha_\ell) - \left(\beta - \frac{D_{-j}}{q_{-j}^2}\right)m_L(\alpha_{-j}^* - \alpha_\ell) = 0 . \tag{47}$$

Solving these simultaneously it can be shown

$$\alpha_j^* - \alpha_\ell = C\left(\beta m_F + \frac{\beta m_L}{\tau} - \tau\right) \tag{48}$$

$$\alpha_{-j}^* - \alpha_\ell = C\left(\beta m_L + \frac{\beta m_F}{\tau} - \tau\right) \tag{49}$$

where

$$C = \{(1 - \tau)\bar{\alpha} - [1 - \tau - \tau^2/(1 - \tau)]\mu + \tau\alpha_\ell\}/$$
$$[(\tau - 2\beta m_F/\tau)(\tau - 2\beta m_L/\tau) - \beta^2(m_F - m_L/\tau)(m_L - m_F/\tau)] . \tag{50}$$

The capacities q_j and q_{-j} can then be found directly from these together with (44) and (46).

If $C < 0$ then only corner solutions are possible. This follows from the fact that given (2) and (43) it is necessary for an interior solution that

$$0 \leq \alpha_{-j}^* - \alpha_\ell < \alpha_j^* - \alpha_\ell \leq 1 . \tag{51}$$

This together with (49) implies that if $C < 0$ then

$$\tau^2 > \beta m_F + \tau\beta m_L . \tag{52}$$

Also (44), (46), and (51) imply

$$q_j \leqslant \frac{m_F}{\tau} \tag{53}$$

$$q_{-j} \leqslant \frac{m_L}{\tau} . \tag{54}$$

Given (4), (43) and (52) - (54), (47) cannot be satisfied since all the terms on the left-hand side are positive. Hence for interior asymmetric solutions it is necessary that

$$C > 0 . \tag{55}$$

From (48) and (49)

$$\alpha_j^* - \alpha_{-j}^* = - C(\frac{1}{\tau} - 1)(m_F - m_L) . \tag{56}$$

Using (35), (55) and the fact that $0 < \tau \leqslant 1$ it follows (43) and (56) are inconsistent and no asymmetric equilibrium can exist.

The results of this section are summarized by the following proposition.

Proposition 1

When $\phi = 0$ the only type of equilibrium that exists is symmetric with $\alpha_j^* = \alpha_{-j}^*$. At each firm's optimum in such equilibria, profits are a nondifferentiable function of capacity and debt.

(b) High fixed costs of capacity

In this subsection it is assumed there is a fixed cost of capacity such that

$$\frac{Z}{16} \leqslant \phi \leqslant \frac{Z}{9} . \tag{57}$$

The right-hand inequality is necessary to ensure that both firms are viable in a Nash equilibrium. The significance of the left-hand inequality will be seen below.

As before in order to find the equilibrium it is necessary to first consider investment decisions at t = 1. If both are solvent or both are bankrupt the firms play a Nash game similarly to (11) except with the inclusion of ϕ. Each firm's optimal capacity is again given by (13) and their expected profits are

$$E\pi_{2j} = (1 - \tau)\left[\frac{Z}{9} - \phi\right] . \tag{58}$$

When one firm is solvent (say j) and the other is bankrupt (say -j) they play a Stackelberg game with j as leader and -j as follower. In this case if $q_j = (\bar{\alpha} - \mu)/2\beta$ as in (19) then it follows that the highest profits -j can obtain with a positive output are

$$E\pi_{2-j} = (1 - \tau)\left[\frac{Z}{16} - \phi\right] . \tag{59}$$

The left-hand inequality of (57) implies $E\pi_{2-j} < 0$ and -j's optimal strategy in this case is to liquidate so that

$$q_{-j} \ ; \quad E\pi_{2-j} = 0 . \tag{60}$$

The q_j in (19) was derived on the assumption that $q_{-j} = (\bar{\alpha} - \mu)/4\beta$. However, it turns out even if $q_{-j} = 0$ that j can do no better than $q_j = (\bar{\alpha} - \mu)/2\beta$. This can be seen immediately from the first order condition for j's problem as a monopolist:

$$\underset{q_j}{\text{Max}}\ (1 - \tau)\left[(\bar{\alpha} - \beta q_j - \mu)q_j - \phi\right] . \tag{61}$$

Thus j's profits when it is the Stackelberg leader are

$$E\pi_{2j} = (1 - \tau)\left[\frac{Z}{4} - \phi\right] . \tag{62}$$

The analysis of decisions at t = 0 is then similar to that in (a). Expected profits in the first period $E\pi_{1j}$ are as in (24) except for the inclusion of the fixed cost ϕ. If $\alpha_j^* \geq \alpha_{-j}^*$, expected profits in the second period are

$$E\pi_{2j} = (1 - \tau)\left(\frac{Z}{9} - \phi\right)\left[\int_{\alpha_\ell}^{\alpha_{-j}^*} f(\alpha)d\alpha + \int_{\alpha_j^*}^{\alpha_\ell+2} f(\alpha)d\alpha\right] . \tag{63}$$

Profits are of this form since between α_j^* and α_{-j}^*, j is forced to liquidate and receives nothing in the second period. If $\alpha_j^* \leq \alpha_{-j}^*$, j is the leader between α_j^* and α_{-j}^* and receives $(1 - \tau)(Z/4 - \phi)$:

$$E\pi_{2j} = (1-\tau)\left[\int_{\alpha_\ell}^{\alpha_j^*}\left(\frac{Z}{9} - \phi\right)f(\alpha)d\alpha + \int_{\alpha_j^*}^{\alpha_{-j}^*}\left(\frac{Z}{4} - \phi\right)f(\alpha)d\alpha + \int_{\alpha_{-j}^*}^{\alpha_j^*}\left(\frac{Z}{9} - \phi\right)f(\alpha)d\alpha\right] . \tag{64}$$

For symmetric equilibria the partial derivates of $E\pi_j$ are the same as in (31), (33), (40) and (41) except now

$$m_F = (1 - \tau)\left[\frac{4}{36} Z - \phi\right] \tag{64}$$

$$m_L = (1 - \tau) \frac{5}{36} Z . \tag{65}$$

In contrast to (35)

$$m_F < m_L . \tag{66}$$

Hence there cannot be any symmetric equilibria since from (36) the kinks at firms' optima are now convex and correspond to minima rather than maxima.

For the asymmetric equilibrium the first order conditions are again (44) - (47) but m_F and m_L are given by (64) and (65) respectively. In this case (66) together with (56) means that an interior asymmetric equilibrium will exist provided τ is sufficiently small so that $C > 0$ and $\tau \leq \beta(m_L + m_F/\tau)$. For large τ either one or both firms set $D_j = D_{max}$ in equilibrium.

From (44), (46), (48) and (49)

$$q_{-j} - q_j = \frac{C}{\tau}(m_L - m_F)[\beta(m_L + m_F) - \tau] . \tag{67}$$

Capacity in period 1 is larger for $-j$ than j if τ is below $\beta(m_L + m_F)$ but is smaller if τ is above this.

The results of this subsection are summarized by the following proposition.

Proposition 2

If $Z/16 \leq \phi \leq Z/9$ the only type of equilibrium that exists is asymmetric with $\alpha^*_j > \alpha^*_{-j}$ (say). For realizations of α in period 1 such that $\alpha^*_{-j} < \alpha < \alpha^*_j$ the bankrupt firm j liquidates at $t = 1$ and $-j$ becomes a monopolist in period 2. Also:

$$q_{-j} \gtrless q_j \quad \text{as} \quad \tau \lessgtr \beta(m_L + m_F) . \tag{68}$$

(c) Intermediate fixed costs of capacity

This subsection considers the case where

$$0 < \phi < \frac{Z}{16} . \tag{69}$$

With no fixed costs of capacity, a bankrupt firm acting as follower chooses a smaller capacity in the equilibrium at $t = 1$ than it would if it were not bankrupt. With a high fixed cost a bankrupt firm acting as follower liquidates. With intermediate fixed costs it is shown both of these are possible: below a certain level ϕ_1^* the follower stays in the market, above it, it leaves. Similarly with no fixed costs the equilibrium at t = 0 is symmetric, with high fixed costs it is asymmetric. For ϕ satisfying (69) the equilibrium is symmetric below $\phi_2^*(> \phi_1^*)$ and asymmetric above.

Consider the decision of a bankrupt firm, say -j, at t = 1 given that the solvent firm j has already chosen capacity q_j. It can either set $q_{-j} = 0$, or it can set it so that (17) is satisfied and profits are maximized given a positive capacity. Its profits in the latter case are

$$E\pi_{2-j} = (1 - \tau)\left[\beta\left(\frac{\bar{\alpha} - \mu}{2\beta} - \frac{q_j}{2}\right)^2 - \phi\right] . \tag{70}$$

Hence if

$$\left(\frac{\bar{\alpha} - \mu}{\beta} - q_j\right)^2 \leq \frac{4\phi}{\beta} \tag{71}$$

the firm liquidates; otherwise it stays in the market.

In subsection (b) it was shown that if $\phi > Z/16$ and j chooses the monopoly capacity $(\bar{\alpha} - \mu)/2\beta$ the follower liquidates. If $\phi = 0$ j's optimal strategy is also to choose a capacity of $(\bar{\alpha} - \mu)/2\beta$ but now -j remains in the market. For intermediate ϕ, j has two possible courses of action. First it could set q_j so that (71) is satisfied: this pushes -j out of the market and j becomes a monopolist. Alternatively it could allow -j to remain in the market. In order to determine which of these it should do, it is necessary to find the profitability of each.

If j is a monopolist its profits are

$$E\pi_{2j} = (1 - \tau)\left[(\bar{\alpha} - \mu - \beta q_j)q_j - \phi\right] . \tag{72}$$

It follows that for j to be able to make a positive profit $q_j < (\bar{\alpha} - \mu)/\beta$. Given this (71) is satisfied if

$$q_j \geq \frac{\bar{\alpha} - \mu}{\beta} - 2\left(\frac{\phi}{\beta}\right)^{1/2} . \tag{73}$$

When $\phi < Z/16$ the right hand side of this inequality is greater than $(\bar{\alpha} - \mu)/2\beta$. For such q_j, j's profits as a monopolist are a decreasing function of capacity. Hence if j wants to force -j to liquidate it should choose q_j so that (73) is satisfied with an equality. Its expected profits are then

$$E\pi^0_{2j} = (1 - \tau)\left[2\left(\frac{\phi}{\beta}\right)^{1/2}(\bar{\alpha} - \mu) - 5\phi\right] . \tag{74}$$

If j allows -j to remain in the market it can be shown similarly to the case where $\phi = 0$ that its optimal capacity is again $(\bar{\alpha} - \mu)/2\beta$ and its expected profits are

$$E\pi^+_{2j} = (1 - \tau)\left(\frac{Z}{8} - \phi\right) . \tag{75}$$

It follows from these that for ϕ satisfying (69)

$$E\pi^0_{2J} \gtreqless E\pi^+_{2j} \qquad \text{as} \qquad \phi \gtreqless \phi^*_1 \tag{76}$$

where

$$\phi^*_1 = \frac{1}{4}\left(\frac{1}{2} - \frac{1}{\sqrt{8}}\right)^2 Z = 0.0054\ Z . \tag{77}$$

All this implies the following proposition.

<u>Proposition 3</u>

In the Stackelberg equilibrium at $t = 1$, if $0 \leq \phi \leq \phi^*_1$ the solvent firm chooses a capacity of $(\bar{\alpha} - \mu)/2\beta$ and the bankrupt firm stays in the market at a reduced size. For $\phi^*_1 < \phi < Z/16$ it is optimal for the solvent firm to expand capacity above $(\bar{\alpha} - \mu)/2\beta$ and force the bankrupt firm to liquidate.

The other question of interest concerns whether the equilibria at $t = 0$ in this intermediate case are symmetric or asymmetric. It can be seen from the analyses of the previous subsections that this is determined by whether

m_F, the difference between the expected profits in a Nash game and the expected profits as a Stackelberg follower, is above or below, m_L, the difference between the expected profits as a Stackelberg leader and those in a Nash game. If $m_F > m_L$ the equilibrium is symmetric; if $m_F < m_L$ it is asymmetric.

For $\phi \leq \phi_1^*$, m_F and m_L are as in (32) and (34) and the equilibria are symmetric. For $\phi > \phi_1^*$

$$m_F = (1 - \tau)\left(\frac{Z}{9} - \phi\right) \tag{78}$$

$$m_L = (1 - \tau)\left[2\left(\frac{\phi}{\beta}\right)^{1/2}(\bar{\alpha} - \mu) - \frac{Z}{9} - 4\phi\right] . \tag{79}$$

Hence

$$m_F \gtreqless m_L \qquad \text{as} \qquad \phi \lesseqgtr \phi_2^* \tag{80}$$

where ϕ_2^* is the solution to

$$2\left(\frac{\phi}{\beta}\right)^{1/2}(\bar{\alpha} - \mu) - 2\,\frac{Z}{9} - 3\phi = 0 . \tag{81}$$

All this is summarized by the following.

Proposition 4

For $0 \leq \phi \leq \phi_2^*$ the equilibrium at $t = 0$ is symmetric and for $\phi_2^* < \phi \leq Z/16$ it is asymmetric.

(d) Summary

The results of this section are summarized diagrammatically in Figure 1. The crucial determinant of the form of equilibrium is the fixed cost of capacity. For $0 \leq \phi \leq \phi_1^*$ a solvent firm acting as a Stackelberg leader at $t = 1$ chooses a capacity of $(\bar{\alpha} - \mu)/2\beta$. The bankrupt firm acting as follower is forced to reduce its size relative to the equilibrium if it were not bankrupt, but still remains in the market. For $\phi_1^* < \phi < Z/16$ the solvent firm expands its capacity above $(\bar{\alpha} - \mu)/2\beta$ in order to force the bankrupt firm to liquidate. For $Z/16 \leq \phi \leq Z/9$ the solvent firm produces the monopoly output which is again $(\bar{\alpha} - \mu)/2\beta$ and the bankrupt firm's best response to this is to liquidate.

As far as the equilibrium at $t = 0$ is concerned, it is symmetric so that firms have the same capacity, use the same amount of debt, and so on if $0 \leq \phi \leq \phi_2^*$. It is asymmetric if $\phi_2^* < \phi \leq Z/9$: one firm goes for the tax advantage of debt and the other goes for the equity advantage of being able to force the debt firm out of the market if demand turns out to be low.

4. Concluding Remarks

This paper has considered the effect of modeling bankruptcy as causing a delay in investment which in itself is costless. If the firm's product market is imperfectly competitive this delay puts the bankrupt firm at a strategic disadvantage which can be costly. In particular, except when the fixed costs of capacity are very small, bankrupt firms are pushed out of the market and forced to liquidate. In such cases, the total costs of bankruptcy include the costs of liquidation which consist of the foregone future profits. When the fixed costs of capacity are very small, bankruptcy does not lead to liquidation but instead results in a reduced size and so can again be costly. The model is thus capable of providing an explanation of why bankruptcy costs may possibly be large.

ϕ:	0	ϕ_1^*	ϕ_2^*	Z/16 — Z/9
Stackelberg equilibrium at t = 1:				
Solvent firm j:	j's capacity is $(\bar{\alpha} - \mu)/2\beta$	j expands capacity above $(\bar{\alpha} - \mu)/2\beta$		j's capacity is the monopoly level $(\bar{\alpha} - \mu)/2\beta$
Bankrupt firm -j:	j's size reduced relative to Nash equilibrium	-j liquidates		
Equilibrium at t = 0:	Symmetric equilibria		Asymmetric equilibria	

Figure 1

The relationship between fixed costs and equilibrium.

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