A DYNAMIC MODEL OF OPTIMAL INVESTMENT AND FINANCIAL POLICIES WITH COSTS OF ADJUSTMENT AND LEVERAGE

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ABSTRACT

In this study, a dynamic-optimizing framework is developed in which the interaction between the firm's real and financial decisions may be examined. The derivation of the objective function is based explicitly on the maximization of shareholder wealth subject to the firm's cashflow and capital accumulation constraints. By incorporating the financial aspects of investment into a model of optimal capital accumulation, it is shown that changes in "q" may affect the firm's capital structure as well as its investment policies. Although an increase in q generally implies an increase in investment, its impact upon capital structure is shown to depend upon how the marginal costs of leverage vary with investment. It is also demonstrated that marginal q equals a particular tax-adjusted average q which renders the relation between q and capital structure a testable hypothesis.

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I. Introduction.

It is a well-known proposition that in a world with no taxes, perfect capital markets, unrestricted borrowing and lending at the same riskless rate, and no bankruptcy or agency costs, the firm is indifferent between financing its investments with debt or equity. This result, first proposed by Modigliani and Miller (1958), is an extremely powerful and quite general result which essentially allows the firm's investment and financing policies to be considered separately. Capital structure is said to be irrelevant or indeterminate. The criterion for optimality in investment is the simple net present value rule which yields the "correct" answer independent of financing arrangements. If corporate taxes are then introduced, along with the tax-deductibility of interest payments, the firm should borrow as much as possible to take advantage of the tax shields. Although this implies an optimal capital structure of almost all debt, a separation between investment and financing decisions still obtains.

However, both the preference for debt and the investment/financing separation do not seem to be borne out by even casual empirical observations. The natural question to consider then is: how are investment and financing decisions interrelated? Myers (1977) and Green (1984) point out that, in an uncertain environment, there may be perverse investment incentives which arise from the firm's particular financing arrangements. The magnitude of these distortions depend critically on the contractual obligations of the different claims issued and also on the characteristics of the investment project undertaken.

Until recently, however, the investment literature has ignored these financing issues, concentrating only on the optimal investment problem. It should be noted that the term "investment" here does not describe the undertaking of a discrete "project" but rather the continuous accumulation of homogenous depreciating capital. The perfect-foresight models of investment behavior by Eisner and Strotz (1963), Lucas (1967), Gould (1968), and Treadway (1969) solved the firm's optimal capital accumulation problem without any financial considerations since the M&M proposition clearly applied. In a different but related context, Tobin (1969) argued that investment is an increasing function of the ratio of the firm's market-value and replacement cost, known also as q. It has since been shown by Abel (1978) that Tobin's q is in fact an average valuation of capital whereas the relevant variable for investment is the marginal valuation or shadow price of capital. Nevertheless in the context of these q-models of investment, financing decisions are ignored as well.

In this paper, we present a q-model of optimal investment which includes financing considerations under perfect-foresight. Without uncertainty, it is difficult imagine a situation in which financing considerations are nontrivial. However, before examining the much more complicated stochastic case, we first construct a formal framework in which we may isolate particular aspects of interest and discard peripheral issues.

To obtain a non-degenerate financing decision, we relax both the M&M assumptions of no taxes and no leverage costs. Specifically, we introduce corporate taxation with tax-deductibility of interest payments and individual income and capital gains taxation. Costs of leverage are then introduced in an *ad hoc* manner since it clearly cannot be derived rigorously in a deterministic model. We simply assert that the level of debt outstanding reduces the firm's real output via an implicit cost function in the production technology, paralleling Lucas' (1967) formulation of implicit costs of adjustment. The motivation behind this formulation is the agency and monitoring costs argument of Jensen and Meckling (1976), and the perverse incentives problem of Myers (1977) and Green (1984) discussed above. It is assumed that, in order for the firm's debt to be held, the stockholders must bear additional costs of monitoring and signalling which take the convenient form of reduced output. No attempt is made to show just how such a representation might capture the gaming which may occur between bondholders and stockholders; we leave this for the uncertainty case.

In this framework then, the optimal physical investment and financial policies of the firm are derived. In determining the appropriate objective function for the firm's managers, we follow Auerbach's (1979) approach closely, extending it to the continuous-time case. It can be shown that most of his results obtain in continuous-time and we include several of the more relevant results in this chapter, such as the derivation of the weighted-average cost of capital. It is shown that only under certain conditions is the weighted-average cost of capital well-defined and the appropriate hurdle rate for investments.

Turning to the question of how the real and financial aspects of the firm's decisions interact, we focus on the specific question of whether or not changes in the shadow value of capital q affect the firm's capital structure. As might be expected, this depends on the properties of the implicit leverage cost function. It will be seen that how it depends on leverage costs suggests a possible line of research in the stochastic case.

II. The Model.

Consider a perfectly competitive infinitely-lived firm in continuous time which produces a single homogeneous output using capital and labor as inputs. Labor supply is exogenous and upward-sloping and physical capital accumulation is the only form of investment the firm may undertake. The firm finances its investment through retained earnings, new issues of common stock, and the sale of short-term riskless debt. It is assumed that the firm seeks to maximize the wealth of existing shareholders and for clarity of exposition we may partition its decisions into two classes: real or production policies and financial policies.

In a world of perfect certainty, frictionless capital markets, and no bankruptcy or agency costs, the Modigliani-Miller theorem clearly applies and an actual separation between real and financial decisions obtains. However, since one of the goals of this chapter is to demonstrate the mechanism through which real shocks affect financial variables, we relax the assumption of no agency or monitoring costs by introducing ad hoc costs of leverage. In particular, it is assumed that output is produced according to the linearly homogeneous production function F with both investment I and debt B as arguments indicating implicit costs of adjustment and leverage respectively. In this model, debt is assumed to be very short-term or "instantaneous" bonds hence interest payments on outstanding bonds (Bt) is always r(t)B(t) where r(t) is the interest rate at time t, i.e., debt is continually rolled-over. Denoting the capital stock by K and labor input by L, we have F(K, L, I, B), F is assumed to satisfy the following conditions:

$$F_L > 0 \quad F_K > 0 \quad F_I < 0 \quad F_B < 0 \tag{1a}$$

$$F_{II} < 0 \quad F_{BB} < 0 \quad F_{LL} < 0 \quad F_{KK} < 0 \tag{1b}$$

$$F_{IL} = F_{BL} = 0 \tag{1c}$$

$$F(K_o, L_o, L, B) \text{ concave for any fixed } K_o \text{ and } L_o \tag{1d}$$

Although the costs of investment and leverage are expressed within F for notational convenience, it should be noted that condition (1c) states that these costs are in fact separable, i.e., $F(K, L, I, B) = f(K, L) + C(K, I, B)$. Note also that the adjustment and leverage costs are convex, i.e., the marginal costs of adjustment and leverage are increasing functions of investment and debt respectively. In this paper we do not attempt to show just how the costs of leverage arise but leave its micro-foundations to the uncertainty case instead and simply assert here that agency and monitoring costs associated with leverage exist and increase with the level of debt outstanding. For convenience, these costs are assumed to be embodied in the function C. The production decisions then involve choosing the time path of investment, labor, and debt issues which maximizes the current wealth of existing shareholders. Note that it is the inclusion of B in the production function which links financial considerations with real variables.

The firm's financial decisions are simply to choose at initial time t the time path of debt and equity B and S which, together with optimal production policies, maximize existing shareholder wealth. It is clear that, with positive costs of leverage and no benefits, it is optimal to hold no debt. The introduction of a corporate income tax τ_c, however, and the tax-deductibility of interest payments provides the firm with positive returns to financing through debt. The tax advantage of debt is assumed to be large enough at the margin to induce a strictly positive debt-equity ratio. Note that $\dot{B}$ and $\dot{S}$ are not constrained to be nonnegative, i.e., share repurchases and debt retirement are feasible options. Clearly both B and S must be constrained to be nonnegative.

Given the optimal time paths of I, L, B, and S, the firm's dividend stream D is uniquely determined through the cash-flow accounting identity. It is assumed that the dividend streams are taxed at the personal income tax rate η, and capital gains are taxed upon accrual at the rate c, with $\eta > c$. We now define the following variables:

$E(t) = p_s(t)S(t)$ = market value of equity

$B(t)$ = market value of debt

$X(t) = (1 - \tau_c)\left[pF(K, L, I, B) - \omega L - p_k I\right]$ = after-tax cash-flow[1]

$D(t)$ = dividend payment

The cash-flow identity for the firm then is:

$$X(t) + p_s(t)\dot{S}(t) + \dot{B}(t) = D(t) + (1 - \tau_c)rB(t) \tag{2}$$

where $(1 - \tau_c)rB$ is the firm's after-tax interest payments on its outstanding debt B. Since F is linearly homogeneous we may express after-tax cash flows and the cash-flow identity as:

$$X = (1 - \tau_c)\left[p(F_K K + F_I I + F_B B) - p_k I\right] \tag{3}$$

Let X^* denote after-tax profits excluding debt costs, i.e.

$$X^* = (1 - \tau_c)\left[p(F_K K + F_I I) - p_k I\right] \tag{4}$$

Then we have the relations:

$$X = X^*(t) + (1 - \tau_c)pF_B B \tag{5a}$$

$$X^* + p_s\dot{S} + \dot{B} = D + (1 - \tau_c)(\gamma - pF_B)B \tag{5b}$$

Let $r_B = -pF_B \frac{B}{E}$. Since $F_{BL} = 0$, we have $r_B = r_B\left(\frac{B}{E}, \frac{B}{K}, \frac{I}{K}, p\right)$. The cash-flow identity may then be rewritten as:

$$X^* + p_s\dot{S} + \dot{B} = D + (1 - \tau_c)rB + (1 - \tau_c)r_B E \tag{6a}$$

$$D = \left[X^* + p_S\dot{S} + \dot{B} - (1 - \tau_C)rB\right] - (1 - \tau_C)r_B E \qquad (6b)$$

Letting $D^* = X^* + p_S\dot{S} + \dot{B} - (1 - \tau_C)rB$ gives the relation:

$$D = D^* - (1 - \tau_C)r_B E \qquad (7)$$

These identities will be of use in relating our results to other commonly cited results in the finance literature.

II.1 The Objective Functional.

It has already been asserted that the objective of the firm is to maximize the current wealth of existing shareholders and, as a partial-equilibrium model of firm behvior, this criterion is not unreasonable. However, it is well known that even in a general equilibrium model with no uncertainty, this criterion is consistent with utility-maximization for any set of arbitrary well-behaved utility functions hence the fact that our analysis is partial-equilibrium in nature does not affect the choice of the firm's objective function.

Since the current wealth of existing shareholders is simply the market value of the equity they own, the firm's objective function is the current share price. In order to make this operational, some method of pricing the firm's equity is required. We use the standard asset-pricing assumption for deterministic models: the share price is, in equilibrium, equal to the present discounted stream of net distributions to which the shareholder is entitled. It will be shown later that this is equivalent to the discounted dividend approach.

Denote by N(t) the net distributions received at time t by shareholders exising at time t. Then we have:

$$N(t) = (1 - \eta)D(t) - c\dot{p}_s S(t) \tag{8a}$$

$$n(t) = (1 - \eta)d(t) - cp_s \qquad \text{where } n \equiv \frac{N}{S},\ d \equiv \frac{D}{S} \tag{8b}$$

The equilibrium asset-pricing assumption is then:[2]

$$E(t) = \int_t^\infty \frac{S(t)}{S(\tau)} N(\tau) e^{-\int_t^\tau \rho dv} d\tau \tag{9a}$$

$$p_s(t) = \int_t^\infty n(\tau) e^{-\int_t^\tau \rho dv} d\tau \tag{9b}$$

where $\rho(\tau)$ is the shareholders' discount rate. Differentiating (9b) gives an alternate equilibrium relation which may be interpreted as an arbitrage equation:[3]

$$\rho E = \dot{E} + N - p_s \dot{S} \tag{10}$$

Equation (10) states that the required return to equity ρE must in equilibrium be equal to the net distributions N plus the change in the value of equity $\dot{E}$ net of dilution $p_s \dot{S}$. Using the definition of N and equation (10), we have:

$$\rho E = (1 - \eta)\left[D^* - (1 - \tau_c)r_B E\right] + (1 - c)\dot{p}_s S \tag{11a}$$

$$\left[\rho + (1 - \eta)(1 - \tau_c)r_B\right]E = (1 - \eta)D^* + (1 - c)\dot{p}_s S \tag{11b}$$

Equation (11b) shows that the cost of leverage may be viewed as increasing the required return on equity as the debt-equity ratio increases. Solving for E explicitly, we have:[4]

$$E = \int_t^\infty (\xi D^* - p_s \dot{S}) e^{-\int_t^\tau \left(\frac{\rho}{1 - c} + \xi(1 - \tau_c)r_B\right) dv} d\tau \quad \text{where } \xi = \frac{1 - \eta}{1 - c} \tag{12}$$

From equation (5b) it may seem that the cost of leverage may also be interpreted as a premium on the return to debt. While this interpretation is algebraically accurate, it it not consistent with the assumption that the debt is riskless. The usual rationale for specifying the return on debt as an increasing function of leverage is that as leverage increases, the probability of bankuptcy also increases ceteris paribus, hence bondholders require a higher yield to compensate for the additional risk. However, in this model there is no possibility of bankruptcy since debt may be continually rolled over thus the required return on bonds should not vary with leverage. On the other hand, the shareholders' effective discount rate should increase with leverage since additional debt decreases the firm's after tax cash-flow due to higher agency and monitoring costs. The interpretation then of r_B as a premium to the effective return on equity is more consistent with the model. A general equilibrium analysis which includes bankruptcy risk would no doubt yield both r and ρ as increasing functions of leverage, as Auerbach (1979) suggests.

Differentiating (8b) results in an equivalent arbitrage relation in return form:

$$\frac{\dot{p}_s}{p_s} + \frac{n}{p_s} = \rho \tag{13a}$$

$$\frac{\dot{p}_s}{p_s} + \xi\frac{d^*}{p_s} = \frac{\rho}{1 - c} + \xi(1 - \tau_c)\gamma_B \quad \text{where } d^* \equiv \frac{D^*}{S} \tag{13b}$$

We now proceed to extend Auerbach's (1979) results for the optimal equity policy and cost of capital to our continuous-time framework and derive the necessary first-order conditions for the optimizing firm.

III. Optimal Investment and Financial Policies.

Given the objective functional (9a), the firm's optimization problem is now well-posed and may be solved subject to the capital accumulation and cash-flow constraints:

$$\dot{K} = I - \delta K \tag{14a}$$

$$\dot{B} + p_s\dot{S} = D - (1 - \tau_c)r_B - X \tag{14b}$$

Upon closer inspection, however, it becomes clear that the objective functional is in implicit form. Holding shares $S(\tau)$ constant for all $\tau > t$, we see that maximizing the current share prices $p_s(t)$ requires knowledge of the entire time path of future share prices $p_s(\tau)$ which is yet to be determined. A time-varying $S(\tau)$ may further complicate the problem. With sufficient regularity conditions imposed on the appropriate functions, this simultaneity problem may be resolved (Derzko and Sethi (1982)) but renders the optimization problem intractable. In order to circumvent this technical difficulty, we maintain that the firm issues no new equity, i.e., $S(\tau) = 0$ for all $\tau > t$. It is in fact possible to derive such a result from optimizing behavior without any additional simplifying assumptions and we do so in the next section.

III.1 Optimal Equity Policy.

Setting $\hat{F} = D - p_s\dot{S}$ as in Auerbach (1979) and using the relation $\dot{E} = \dot{p}_sS + p_s\dot{S}$, the arbitrage equation (II.1.5) may be rewritten as:

$$\dot{E} - \frac{\rho}{1 - c}E = -\left(\xi\hat{F} - \frac{n - c}{1 - c}\,p_s\dot{S}\right) \tag{15}$$

Solving for E gives:

$$E = \int_t^{\infty} \left(\xi \hat{F} - \frac{\eta - c}{1 - c} p_s \dot{S}\right) e^{-\int_t^{\tau} \rho dv} d\tau \qquad (16)$$

Given particular investment and debt policies X and B, $\hat{F}$ is determined. Since $\eta > c$, it can be seen from equation (16) that any future issues of equity only decrease the market value of equity outstanding at time t. In fact, as long as dividend payments D are positive, the firm may increase the current wealth of existing shareholders by simultaneously decreasing dividends and new share issues by the same amount without changing its debt and investment policies. Indeed, $\dot{S}$ may be negative as Auerbach (1979) points out, which corresponds to the distribution of profits to shareholders through share repurchases in place of dividend payments. Since the tax code prohibits such tax-arbitrages we rule this out and assume for the remainder of the paper that $\dot{S} = 0$. Without loss of generality, set $S(\tau) = 1$ for all $\tau > t$. The objective functional then becomes:

$$E = \int_t^{\infty} \xi D e^{-\int_t^{\tau} \rho dv} d\tau \qquad (17)$$

which is simply the after-tax discounted dividend valuation model.

III.2 The Cost of Capital.

Using equation (11b) it is possible to derive the weighted average cost of capital, i.e., that convex combination of the rate of return on debt and equity which, when used to discount net cash flows, yields the correct market price of equity. Recalling that $\dot{S} = 0$ hence $\dot{E} = \dot{p}_s S$, equation (11b) becomes:

$$[\rho + (1-\eta)(1-\tau_c)r_B]E = (1-\eta)X^* + (1-\eta)\dot{B} + (1-c)\dot{E} - (1-\eta)(1-\tau_c)r_B \qquad (18)$$

Let $V = (1-\eta)B + (1-c)E$. Then (18) may be re-expressed as:

$$[\rho + (1-\eta)(1-\tau_c)\tau_B]E = (1-\eta)X^* + \dot{V} - (1-\eta)(1-\tau_c)\gamma_B \qquad (19)$$

In terms of an "adjusted" debt-to-value ratio α, $\alpha = \frac{(1-\eta)B}{(1-\eta)B + (1-c)E}$, we have:

$$\alpha(1-\tau_c)\gamma + (1-\alpha)\left[\frac{\rho}{1-c} + \xi(1-\tau_c)r_B\right] = \frac{(1-\eta)X^* + \dot{V}}{V} . \qquad (20)$$

Substituting $-pF_B \frac{B}{E}$ for r_B and simplifying further yields:

$$\dot{V} - RV = -(1-\eta)X^* \quad \text{where} \qquad (21a)$$

$$R = \alpha(1-\tau_c)(r - pF_B) + (1-\alpha)\frac{\rho}{1-c} . \qquad (21b)$$

Finally, solving (21a) for V and E explicitly gives us:

$$V(t) = \int_t^{\infty} (1-\eta)X^* e^{-\int_t^{\tau} R dv} d\tau \qquad (22a)$$

$$E(t) = \int_t^{\infty} \xi X^* e^{-\int_t^{\tau} R dv} d\tau + \xi B(t) . \qquad (22b)$$

From (22b) we see that the appropriate cost of capital is R(t), which is simply the convex combination of the required return to equity $\frac{\rho}{1-c}$ and the effective cost of debt $(1 - \tau_c)(r - pF_B)$. Defining the adjusted debt-equity ratio as $\beta = \frac{(1-\eta)B}{(1-c)E}$, the cost of capital may be rewritten as:

$$R = \frac{1}{1+\beta}\left[\frac{\rho}{1-c} + \beta(1-\tau_c)(r - pF_B)\right] . \qquad (23)$$

III.3 Optimal Debt and Investment Policies.

Using the valuation equation (9a) the firm's optimization problem may now be stated completely in explicit form as:

$$\underset{\{I,B,L\}}{\text{Max}} \int_t^{\infty} \left[X + B - (1 - \tau_c)rB\right] e^{-\int_t^{\tau} \rho dv} d\tau \tag{24a}$$

subject to:

$$\dot{K} = I - \delta K \tag{24b}$$

where the cash-flow constraint has been substituted into the objective functional. Applying the calculus of variations to (24) yields the following Euler equations:

$$pF_L = \omega \tag{25a}$$

$$p\dot{F}_L + \left(\frac{\rho}{1 - c} + \delta\right)(p_k - pF_I) - pF_k = 0 \tag{25b}$$

$$pF_B = \gamma - \frac{\rho}{(1 - \tau_c)(1 - c)} . \tag{25c}$$

Equation (25a) is the familiar marginal productivity of labor condition. It is shown below that equation (25b) may be thought of as an arbitrage relation in the shadow value of capital q. Finally, (25c) when rewritten as:

$$(1 - \tau_c)(r - pF_B) = \frac{\rho}{1 - c} \tag{26}$$

is just the condition that the after-tax effective cost of debt is equal to the opportunity cost of funds in terms of the equityholders discount rate. These three equations characterize the firm's optimal investment, debt, and labor demands. Together with the optimal equity policy $\dot{S} = 0$, the complete solution to (24) is obtained. Note that the dividend policy is then uniquely determined by the cash-flow constraint.

Since F is linearly homogeneous, (F_L, F_K, F_I) are all functions of only $(\frac{L}{K}, \frac{I}{K}, \frac{B}{K})$. The first-order necessary conditions (25a) and (25c) may then be inverted for the labor and debt demand functions.

$$L = KL^d(\frac{\omega}{p}, \frac{I}{K}) \qquad \text{since } F_{LB} = 0 \qquad (27a)$$

$$B = KB^d(r, p, \frac{\rho}{1 - c}, \tau_c, \frac{I}{K}) \qquad \text{since } F_{BL} = 0 \qquad (27b)$$

The demand for capital may be obtained similarly by solving the remaining Euler equation (25b). However, equation (25b) lends itself more readily to economic and econometric analysis when re-expressed in terms of the shadow value of capital q. It is straightforward to show, using the Pontryagin maximum principle, that the shadow value of capital in this model is:

$$q = \xi\left[1 - (\frac{p}{p_k})F_I\right] . \qquad (28)$$

This states that along the optimal policy trajectory, the marginal value of an additional unit of capital must equal its after-tax marginal cost. Note that in the absence of taxes, with $p = p_k$ and a separable adjustment cost function $IC(\frac{I}{K})$, equation (28) reduces to the familiar relation:

$$q = 1 + C" + \frac{I}{K} C" . \qquad (29)$$

Using (28), equation (25b) then becomes:

$$\dot{q} = (\frac{\rho}{1 - c} + \delta)q + \xi(\frac{p}{p_k})F_K = 0 . \qquad (30)$$

IV. The Equivalence of Marginal and Average q.

Recent papers in the investment and public finance literature have pointed out the important distinction between marginal and average valuations of incremental additions to the capital stock, concluding that the relevant variable for investment demand should be marginal q. Although this variable is unobservable, Hayashi (1982) has demonstrated that under certain linear-homogeneity restrictions, marginal and average q coincide. It is no surprise then that, given our homogeneity assumptions, a formal equivalence obtains in our model. Denote average and marginal q by q_a and q_m respectively. Then by evaluating the share price $p_s(t)$ for the optimal (I, B, S, L) trajectories it is shown in the appendix that:

$$q_a = \frac{B + \xi E}{(1 - \tau_c)p_k K} = \xi(1 - \frac{p}{p_k} F_I) = q_m \ . \tag{31}$$

Note that the above definition of q_a differs from the original "market-value over replacement cost" definition proposed by Tobin (1965) only through the tax parameter ξ. Furthermore q will be less than unity in equilibrium due to the presence of tax effects. This is the usual "q-less-than-one" phenomenon.

V. Investment and Costs of Leverage.

Having solved the firm's optimization problem, we may now consider the effect of changes in real variables on financial variables. In particular, attention will be focused on how changes in the shadow value q affect investment and debt policies. It will become clear that the critical and, indeed, only link between q and debt policy is F_{BI}. Recall that the first-order necessary conditions of the firm's problem are:

$$\dot{q} = (\frac{\rho}{1 - c} + \delta)q - \xi(\frac{p}{p_k})F_K \tag{32a}$$

$$q = \xi\left[1 - (\frac{p}{p_k})F_I\right] \tag{32b}$$

$$pF_B = r - \frac{\rho}{(1 - \tau_c)(1 - c)} \ . \tag{32c}$$

Equations (32b) and (32c) may be solved simultaneously for $\frac{I}{K}$ and $\frac{B}{K}$ which, given the initial condition $K(t) = K_0$ and the capital accumulation equation, yields the optimal investment and debt demands.

Alternatively, we may express (32b) and (32c) in "intensive" form by first substituting optimal labor demand into the production function and then dividing F by K:

$$F\left(K,\ KL^d\left[\frac{\omega}{p},\ \frac{I}{K}\right],\ I,\ B\right) = F_0(K,\ I,\ B)$$

$$\text{hence} \qquad F = KF_0\left(1,\ \frac{I}{K},\ \frac{B}{K}\right) = Kf(i,\ b) \text{ for a given } \frac{\omega}{p}$$

where $i = \frac{I}{K}$ and $b = \frac{B}{K}$ are said to be the investment and debt intensities respectively. Since $F_I = f_I$ and $F_B = f_B$, the last two Euler equations (25b) and (25c) become:

$$q = \xi\left[1 - \left(\frac{p}{p_k}\right)f_i(i,\ b)\right] \qquad \text{TT locus} \qquad (33a)$$

$$\frac{\rho}{(1 - \tau_c)(1 - c)} = r - f_b(i,\ b) \qquad \text{CC locus} \qquad (33b)$$

It can easily be shown that:

$$f_{ii} = KF_{II} < 0 \qquad (34a)$$

$$f_{bi} \gtreqless f_{ib} \gtreqless 0 \text{ as } KF_{BI} \gtreqless 0 \qquad (34b)$$

$$f_{bb} = KF_{BB} < 0\ . \qquad (34c)$$

Clearly the only source of indeterminancy is F_{BI}. Suppose that $F_{BI} = 0$. This corresponds to a situation where the marginal cost of leverage does not vary with investment intensity. The equilibrium is determined by the intersection of the TT and CC loci and is depicted in Figure 1.

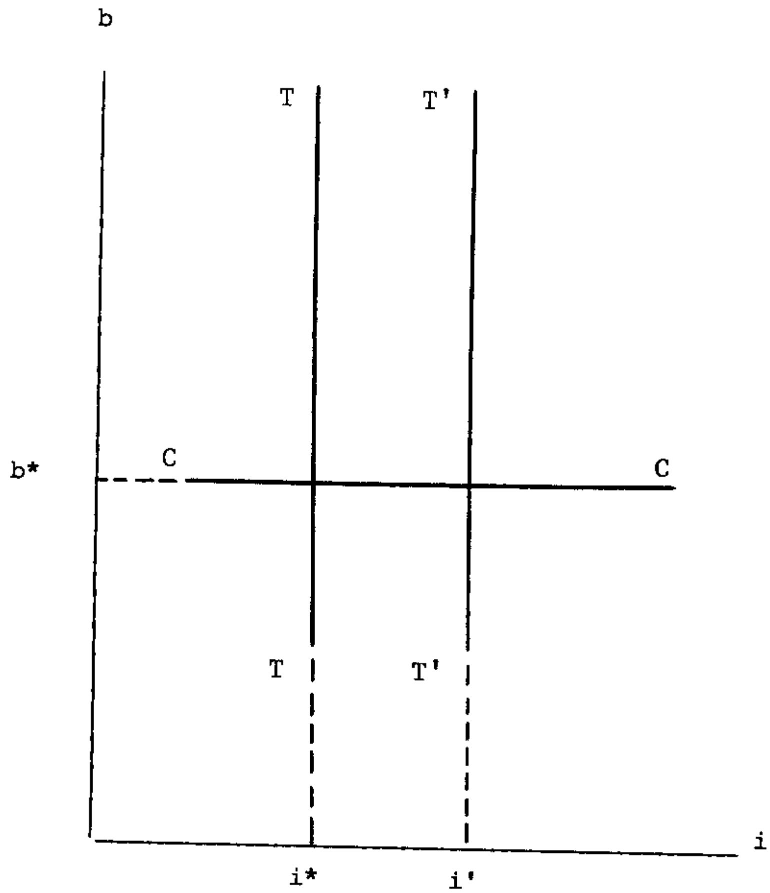

Figure 1.

Since the marginal cost of debt is constant, the debt intensity is independent of the investment intensity as expected. At the optimal debt intensity the marginal value of debt is equal to its marginal cost of leverage. If the cost of leverage is not a function of $\frac{I}{K}$ then the debt intensity will be constant over all possible values of $\frac{I}{K}$.

Next, consider the case in which $F_{BI} > 0$. Together with assumptions (1), this implies the following relative slopes for the TT and CC loci:

$$\frac{db}{di}\bigg|_{TT} = -\frac{f_{ii}}{f_{bi}} > -\frac{f_{ib}}{f_{bb}} = \frac{db}{di}\bigg|_{CC} > 0 \qquad (35)$$

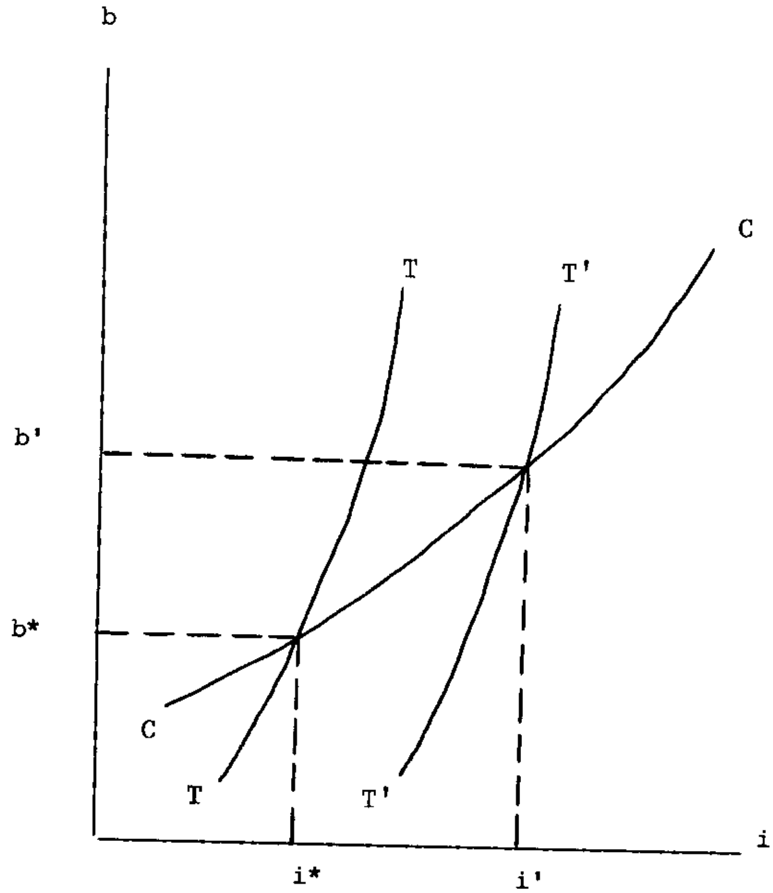

Figure 2.

From Figure 2, we see that a higher value of capital increases the optimal debt intensity as well as the investment intensity. Since the marginal cost of debt decreases with investment intensity (recall $F_B < 0$), this result is not surprising.

Finally, let $F_{BI} < 0$. By a similar analysis, we conclude in this case that:

$$\left.\frac{db}{di}\right|_{TT} = -\frac{f_{ii}}{f_{bi}} < -\frac{f_{ib}}{f_{bb}} = \left.\frac{db}{di}\right|_{CC} < 0 \; . \qquad (36)$$

Figure 3 exhibits the expected result: an increase in q decreases the optimal debt intensity due increases in the marginal cost of leverage from higher investment.

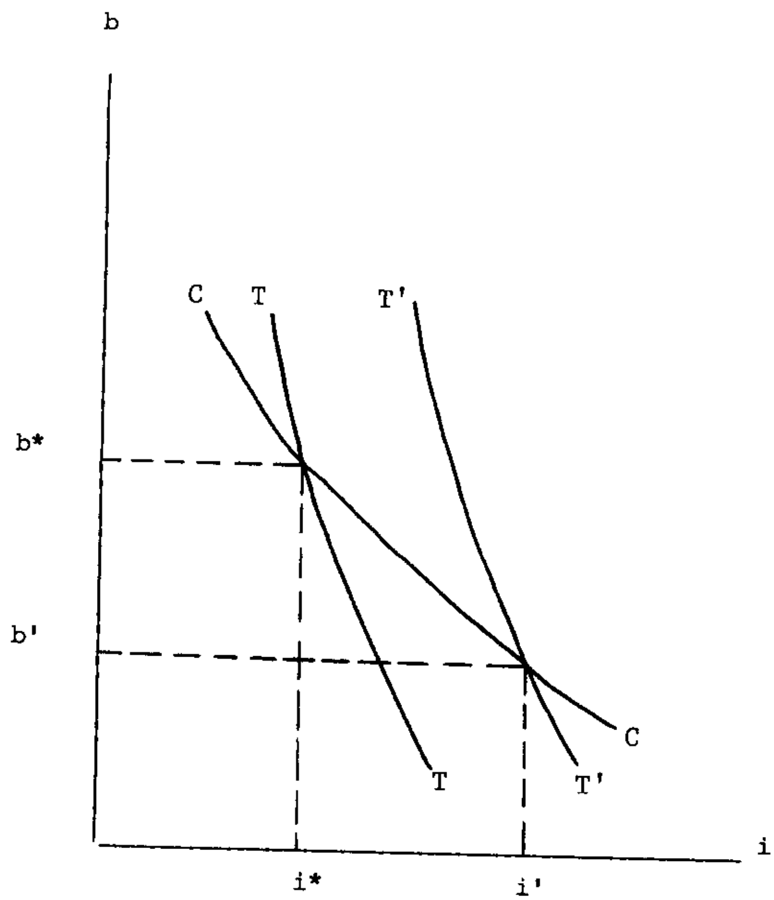

Figure 3.

The three cases examined above clearly demonstrate that F_{BI} is in fact the channel through which changes in q affect the firm's capital structure. For example, if the price of capital p_k declines exogenously then investment demand rises, but how such investment is financed depends on the sign of F_{BI}. If additional investment leaves marginal leverage costs unchanged then the current capital structure is maintained. If, however, marginal investments change the marginal cost of leverage, capital structure also varies until marginal costs are equated with marginal benefits.

One possible interpretation of F_{BI} is that it is a measure of the effect of investment risk on agency and monitoring costs. Myers (1977) has demonstrated that if shareholders issue risky debt with maturity exceeding the life of the investment opportunity, a perverse bias against taking good investment projects is created. Myers points out that "Honesty is the best policy." However, honesty is not costless. Convincing creditors of one's integrity, etc. is usually best done through protective covenants and other contractual arrangements which legally limit the shareholders' actions. In our simple model, the uncomfortable assumption of riskless debt is slightly more palatable with implicit costs of leverage. It is an attempt to summarize the presence of those monitoring costs mentioned above.

This interpretation must, of course, be qualified in several respects. The fact that our model is deterministic is obviously one of its major drawbacks. Actual costs of leverage depend on the availability of alternative financial instruments, probability of default, the type and magnitude of investment risks, and other stochastic factors. The implicit cost of debt in the production function is at best an ad hoc proxy for all these considerations.

VI. Conclusion.

In this study, we have presented a dynamic optimizing framework in which the interaction between the firm's real and financial decisions may be studied. By incorporating the financial aspects of investment into a model of optimal capital accumulation, it has been shown that changes in "q" may affect the firm's capital structure as well as its investment policies. Although an increase in q generally implies an increase in investment, its impact upon capital structure is determined by how the marginal costs of leverage vary with investment. The presence of agency and monitoring costs generated by perverse investment incentives due to debt-financing would indicate that the marginal costs of leverage may be an increasing function of investment ($F_{BI} < 0$). This then implies that q and the debt-intensity b are inversely related. Since it was demonstrated in our model that marginal q equals a tax-adjusted average q, the inverse relation of q and b is a testable hypothesis and should be explored in future research.

FOOTNOTES

[1]Note that this definition of after-tax profits assumes that investment is completely expensed. This is done only for computational convenience; allowing a more realistic depreciation deduction scheme would have greatly complicated the analysis without adding much to the main focus of this paper.

[2]It must be assumed that $\frac{N(\tau)}{S(\tau)}$ grows no faster than the exponential of the discount rate if the right-hand side of equation (9a) is to be a well-defined integral.

[3]In order to differentiate the integral in (9a) the integrand, call it g, must satisfy the following conditions:

(i) g and $\frac{\partial g}{\partial \tau}$ are continuous for all $\tau \geq t$.

(ii) There exists a function $M(\tau)$ independent of t such that for all $\tau \geq t$ and for all $t \in (0, \infty)$:

$$\left|\frac{\partial g}{\partial \tau}\right| \leq M(\tau) \; .$$

(iii) The following integral converges:

$$\int_t^\infty M(\tau)d\tau \; .$$

[4]It is assumed that the integrand vanishes at $\tau = \infty$ which effectively rules out Ponzi-type schemes. Similar conditions will be assumed elsewhere in this chapter without further remark.

APPENDIX

In this appendix, the firm's optimization problem is solved using Pontryagin's maximum principle and the relation between marginal and average q is derived. Let $\phi(\tau) = e^{-\int_0^\tau \frac{\rho}{1-c} dz}$ and consider the Hamiltonian corresponding to equation (24):

$$H = \{\xi[(1 - \tau_c)(pF - wL - -p_k I) + B - (1 - \tau_c)rB] + q_0(I - \delta K) + \beta B\}\phi(\tau) \quad (A1)$$

The necessary conditions for optimal investment, debt, and labor demand policies are:

$$\frac{\partial H}{\partial I} = [\xi(1 - \tau_c)(pF_I - p_k) + q_0]\phi(\tau) = 0 \quad (A2a)$$

$$\frac{\partial H}{\partial B} = (\xi + \beta)\phi(\tau) \quad (A2b)$$

$$\frac{\partial H}{\partial L} = \xi(1 - \tau_c)(pF_L - \omega)\phi(\tau) = 0 \quad (A2c)$$

$$-(q_0 - \frac{\rho}{1 - c} q_0)\phi(\tau) = \frac{\partial H}{\partial K} = (\xi pF_K - q_0\delta)\phi(\tau) \quad (A2d)$$

$$-(\beta - \frac{\rho}{1 - c} q_0)\phi(\tau) = \frac{\partial H}{\partial B} = \xi(1 - \tau_c)(pF_B - r)\phi(\tau) \quad (A2e)$$

$$\lim_{\tau\to\infty} q_0 K\phi(\tau) = 0 \quad (A2f)$$

$$\lim_{\tau\to\infty} \beta B\phi(\tau) = 0 \, . \quad (A2g)$$

Simplifying the relations (A2a) through (A2e) yields:

$$q_0 = \xi(1 - \tau_c)(p_k - pF_I) \quad (A3a)$$

$$\beta = -\xi \quad (A3b)$$

$$pF_L = \omega \quad (A3c)$$

$$\dot{q}_0 = (\frac{\rho}{1 - c} + \delta)q_0 - \xi(1 - \tau_c)pF_K \tag{A3d}$$

$$\dot{\beta} = \frac{\rho}{1 - c}\beta + \xi(1 - \tau_c)(r - pF_B) \; . \tag{A3e}$$

Define after-tax normalized marginal q as q_m where:

$$q_m = \frac{q_0}{(1 - \tau_c)p_k} \; . \tag{A4}$$

Using this definition and the fact that (A3b) implies $\dot{\beta} = 0$, we may rewrite (A3a) through (A3e) as:

$$q_m = \xi(1 - \frac{p}{p_k} F_I) \tag{A5a}$$

$$pF_L = \omega \tag{A5b}$$

$$\dot{q}_m = (\frac{\rho}{1 - c} + \delta)q_0 - \xi \frac{p}{p_k} F_K \tag{A5c}$$

$$pF_B = r - \frac{\rho}{(1 - c)(1 - \tau_c)} \; . \tag{A5d}$$

Define after-tax average q as:

$$q_a = \frac{E(t) + \xi B(t)}{(1 - \tau_c)p_k K} \; . \tag{A6}$$

We may then prove the following result:

<u>Proposition</u>: Under the optimal policies for $(I, \dot{B}, \dot{S}, L)$, the relation

$$q_a = q_m$$

obtains for all $t > 0$.

Proof: Following Hayashi's (1982) method of proof, we calculate

$$\frac{d}{d\tau}\left(q_m K\phi(\tau)\right) = \left[\dot{q}_m K + q_m \dot{K} - \frac{\rho}{1-c} qK\right]\phi(\tau) \; . \tag{A7}$$

Using Euler's theorem, the definition of profits X, and equations (A5b), (A5c), and (A5d), relation (A7) may be re-expressed as:

$$\frac{d}{d\tau}\left(q_m K\phi(\tau)\right) = -\frac{\xi}{(1-\tau_c)p_k}\left[X - (1-\tau_c)rB + \frac{\rho}{1-c}B\right]\phi(\tau) \; . \tag{A8}$$

Integrating both sides of (A8) from t to infinity, multiplying both sides by $(1-\tau_c)p_k$, and using (A2f) gives us:

$$q_m(1-\tau_c)p_k(t) = \xi \int_t^\infty \left[X - (1-\tau_c)rB + \frac{\rho}{1-c}B\right]\phi(\tau)d\tau \; . \tag{A9}$$

Integration of $\int_t^\infty \dot{B}\phi(\tau)d\tau$ by parts and the application of (A2f) yields the relation:

$$\int_t^\infty \frac{\rho}{1-c} KB\phi(\tau)d\tau = \int_t^\infty \dot{B}\phi(\tau)d\tau + B(t) \; . \tag{A10}$$

Substituting (A10) into (A9) yields:

$$q_m(1-\tau_c)p_k K(t) = \xi \int_t^\infty \left[X + \dot{B} - (1-\tau_c)rB\right]\phi(\tau)d\tau + \xi B(t) \tag{A11}$$

which is, by definition of E(t), equivalent to the expression:

$$q_m(1-\tau_c)p_k K(t) = E(t) + \xi B(t) \tag{A12}$$

$$\therefore \quad q_m = \frac{E(t) + \xi B(t)}{(1-\tau_c)p_k K(t)} = q_a \tag{A13}$$

Q.E.D.

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