VALUATION OF CURRENCY DENOMINATION IN LONG-TERM DEBT FINANCING AND DEBT REFINANCING: A PORTFOLIO MODEL

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1. Introduction

As economic agents (public and corporate entities alike) gain a more global outlook, foreign currency denominated debt instruments become an integral part of their financing options. One may argue that in a multi-currency world of efficient and integrated capital markets the selection of the optimal borrowing source should be a matter of indifference, since nominal interest rates reflect inflation rates expectations which, in turn, determine the pattern of the future spot exchange rate adjustment path.[1] However, capital markets segmentation, heterogenous corporate tax rates among different national jurisdictions, asymmetrical tax treatment of capital exchange gains and losses, unpredictable central banks' intervention in exchange market and an ever sprawling web of sporadic capital exchange controls render this hypothesis of market efficiency of dubious operational value in the selection process of the optimal financing option.[2]

How, then, should foreign debt financing decisions be made? The pattern of lower foreign nominal interest rate is generally not enough of an informational basis upon which to make a foreign debt financing decision. Because they ignore the exchange rate variable, nominal interest rates tend to prove of poor discriminating value.

Historically, from 1958 onwards, so-called "strong" currencies, such as the Deutsche mark and the Swiss franc, commanded relatively lower interest rates on long-term debt instruments as compared with similar rates prevailing in the United Kingdom or the United States. Many U.S. or U.K.-based companies were thus misled by lower nominal interest rates and borrowed heavily in Deutsche marks and Swiss francs. Substantial interest costs savings were at first realized, but after 1971, the steep appreciation of the Deutsche mark and the Swiss franc resulted in considerable exchange losses on both interest payments and principal redemption. The transaction exposure incurred through foreign currency denominated debt financing thereby resulted in severe exchange losses and agonizing debt refunding decisions. Conversely, since 1980 the combination of a strong dollar and double-digit U.S. interest rates should have convinced U.S. borrowers that long term borrowing in Swiss franc, Deutsche mark or Japanese yen had indeed become a true "bargain". Thus incurring a long-term foreign currency-denominated liability position, through foreign debt financing, is a perplexing decision to make.

Given that it is generally not meaningful to rely on point estimate forecasts of exchange rates over periods of time encompassing the life of long-term debt instruments, should foreign debt financing be ruled out for risk-averse borrowers? Our contention is that it need not, if the significant risk management aspects of such decisions are properly addressed.

Only scant attention has been devoted to the multi-currency debt financing issue; preliminary answers were provided by Jacque [6] who developed a break-even analysis framework based upon expected net present cost formulae (thus implicitly implying risk neutrality on the part of the decision-maker). Using a similar framework Giddy [4] derived a number of simplifying formulae expressing the effective cost of foreign currency denominated debts financing as a function of its nominal cost; Shapiro refined the approach by including tax factors and floatations costs [14]. All these approaches however failed to recognize the probabilistic dependency of the "ex ante" vector of exchange rates, which determine future interest payments and principal repayments, as well as the risk-aversion of decision-makers.

This article is an attempt at bridging this conceptual gap by setting forth an expected utility framework which allows for the ranking, in a multi-period, multi-currency world, of alternative long-term debt financing options denominated in different currencies (section 2). A comparison of financing in a two-currency world is detailed in section 3. In section 4, the valuation of artificial currency unit debt instrument is derived within our previous framework. Section 5 briefly considers the design of a debt portfolio for the case of public borrowers with constrained access to single national capital market ("crowding out" conditions) in both a static and a dynamic setting. The model, as it will become more apparent as we proceed, is especially relevant for public borrowers such as municipalities, state-owned utilities, and sovereign states which typically have huge long-term financing needs that may not be entirely satisfied from domestic capital markets.[3] Accordingly a total risk paradigm is used in this paper as International Capital Asset Pricing models are not relevant to the problem addressed here.[4]

Several interesting conclusions emerge from this paper: (1) the debt financing decision in a multi-currency world cannot be divorced from the borrowing agent's revenue function; (2) under certain conditions, foreign debt financing is to be preferred to domestic financing for strictly domestic borrowers as foreign portfolio debt financing unveils a cross-hedging effect; (3) the framework can be operationalized to evaluate the use (or misuse) of emerging techniques in the realm of international financing such as artificial currency denominated debt and currency swaps as well as to review debt refunding decisions or additional financing policies when access to a single national capital market is constrained.

2. A Model for Foreign-Currency Denominated Debt Valuation

We now turn to a general formulation of the multi-currency long-term debt portfolio problem which goes beyond the traditional approach in the finance literature of collapsing the multi-period nature of the problem into a "cosy" one-period model; the following formulation captures the intertemporal nature of the multi-currency debt portfolio valuation problem.

Foreign currency denominated debts entail cash streams that vary over time, as a result of uneven interest service and debt redemption schedules, but also of their translation into the borrower's reference (domestic) currency via fluctuating exchange rates. Not only is the lack of stationarity exhibited by exchange rates significant, but so is the uncertainty about their future path, which may be viewed as increasing with the forecasting horizon. It is thus important to recognize such dynamic aspects of the borrower's net financial position in an explicit fashion, a task taken up in the next section.

Uncertainty concerning financing options clearly affects their value to a risk-averse borrower. In a second section, an operational risk-pricing approach is proposed, based on a simple utility-theoretic framework that provides a reasonable approximation to the borrower's risk preference structure.

2.1 Exchange Rates and Cash Streams as Random Walks

We consider a borrowing agent holding a portfolio of long-term debts in N different currency denominations in addition to those in the domestic currency, which by convention will be denoted as currency 0. The agent's net cash flow in year t, $\tilde{\pi}(t)$, is:

$$\tilde{\Pi}(t) = \tilde{R}_0(t) - d_0(t) - \sum_{i=1}^{N} \tilde{S}_i(t) d_i(t) \qquad (1)$$

where $\tilde{R}_0(t)$ is the borrower's domestic currency (currency 0) net cash inflow in year t before debt servicing

$\tilde{S}_i(t)$ is the domestic currency spot price of one unit of currency i at time t and $d_i(t)$ is the net contractual currency i-denominated debt servicing cash outflow in year t with $d_0(t)$ denoting the domestic debt servicing burden

It is assumed in equation (1) that revenues accruing to the borrower are denominated in the domestic currency, not to say, however, that they are independent of foreign currency fluctuations.[5]

To specify our model in more detail, some additional notations will be useful. Let us define the N-dimensional vectors

$$D(t) \equiv \begin{bmatrix} d_1(t) \\ \vdots \\ d_i(t) \\ \vdots \\ d_N(t) \end{bmatrix}, \quad \tilde{S}(t) \equiv \begin{bmatrix} \tilde{S}_1(t) \\ \vdots \\ \tilde{S}_i(t) \\ \vdots \\ \tilde{S}_N(t) \end{bmatrix}$$

and the (N+1) dimensional vectors

$$a(t) \equiv \begin{pmatrix} 1 \\ -D(t) \end{pmatrix}, \quad \tilde{x}(t) \equiv \begin{pmatrix} \tilde{R}(t) \\ \tilde{S}(t) \end{pmatrix}$$

so that the net cash flow in period t can be rewritten:

$$\tilde{\Pi}(t) = a'(t)\tilde{x}(t) - d_0(t) .$$

Our model assumes that the family of (N+1)-dimensional random variates $\{\tilde{x}(t)\}$ is generated by a random walk process of the form:

$$\tilde{x}(t) = P\tilde{x}(t - 1) + \tilde{\varepsilon}(t)$$

where x(0) is known, $\{\tilde{\varepsilon}(t)\}$ is an independent, homoscedasticity family of (N+1)-dimensional normal variates with means and covariances:

$$\mu(t) \equiv E\left[\tilde{\varepsilon}(t)\right] = \begin{bmatrix} \nu(t) \\ \phi(t) \end{bmatrix} \qquad \nu(t) \in R \text{ and } \phi(t) \in R^N$$

and

$$\Omega \equiv \mathrm{Cov}\left[\tilde{\varepsilon}(t), \tilde{\varepsilon}(t)'\right] = \begin{bmatrix} \omega^2 & \eta' \\ \eta & \Sigma \end{bmatrix}$$

with $\omega^2 \in R^+$, $\eta \in R^N$ and $\Sigma \in R^{N \times N}$ and, by serial independence, $\mathrm{Cov}\left(\tilde{\varepsilon}(t), \tilde{\varepsilon}(k)\right) = 0$ for all $t \neq k$.

The $(N+1) \times (N+1)$ matrix P is assumed to be of the form:

$$P = \begin{bmatrix} \delta & 0 \\ 0 & I \end{bmatrix}$$

with $0 \leqslant \delta \leqslant 1$.

Thus, exchange rates are assumed to follow a simple additive pattern. The process governing the borrower's revenue function takes on a more general form, with the polar cases of $\delta = 0$ and $\delta = 1$ corresponding respectively to serially independent revenues and to revenues following an additive random walk, while the intermediate case $0 < \delta < 1$ allows for some serial dependence of revenues.

It should be noted that the random walk envisioned here will usually entail a "drift" in the sense that the disturbance terms $\tilde{\varepsilon}(t)$ may have non-zero, non-stationary expectations. By contrast, the matrix Ω is assumed to be time-invariant. This matrix simply represents the conditional covariance of the various unknown elements which will determine the net cash flow one period ahead: Σ is the variance-covariance matrix of period $t + 1$ exchange rates, η the period $t+1$ covariance vector between each individual exchange rate and the domestic cash flow stream, ω^2 the variance of the latter, as seen upon observing exchange rates and revenues in (any) period t.

Under these assumptions, $\{\tilde{x}(t)\}$ is also a family of normal variates:

$$\tilde{x}(t) = P^t \tilde{x}(0) + \Sigma_{k=1}^{t} P^{t-k} \tilde{\varepsilon}(k)$$

with conditional mean and variances

$$E[x(t) | x(0)] = P^t x(0) + \Sigma_{k=1}^{t} P^{t-k} \mu(k)$$

and

$$\mathrm{Cov}[x(t),\ x(k) | x(0)] \equiv E[(x(t) - E[x(t)]) \cdot (x(k) - E[x(k)])' | x(0)]$$

$$= \Sigma_{\ell=1}^{\min\{t,\ k\}} P^{t-\ell} \Omega P^{k-\ell}, \qquad \forall t,\ k \geq 1 \ .$$

where

$$P^t = \begin{bmatrix} \delta^t & 0 \\ 0 & I \end{bmatrix}, \quad P^0 \equiv I \ .$$

Given a discount factor $\alpha \in [0,\ 1]$, the present value $\tilde{z}$ of the stream of net cash flows over T periods is simply:

$$\tilde{Z} = \Sigma_{t=1}^{T} \alpha^{t-1} a'(t)\tilde{x}(t) - \overline{D}_0$$

where $\overline{D}_0 \equiv \Sigma_{t=1}^{T} \alpha^{t-1} d_0(t)$ is the present value of the domestic debt service, so that $\tilde{Z}$ is also a normal variate with conditional moments written as:

$$E[\tilde{Z}] = \Sigma_{t=1}^{T} \alpha^{t-1} a'(t) E[x(t)] - \overline{D}_0$$

$$= \Sigma_{t=1}^{T} \alpha^{t-1} a'(t) [P^t x(0) + \Sigma_{k=1}^{t} P^{t-k} \mu(k)] - \overline{D}_0 \qquad (2)$$

and $$\mathrm{Var}(\tilde{Z}) = \Sigma_{t=1}^{T} \Sigma_{k=1}^{T} \alpha^{t+k-2} \mathrm{Cov}(a'(t)x(t),\ a'(k)x(k))$$

$$= \Sigma_{t=1}^{T} \Sigma_{k=1}^{T} \alpha^{t+k-2} \Sigma_{\ell=1}^{\min\{t,k\}} a'(t) P^{t-\ell} \Omega P^{(k-\ell)} a(k) \ . \qquad (3)$$

2.2 Valuation of the Debt Portfolio

The general framework adopted here for pricing the debt portfolio is that of expected utility theory, whereby each possible cash stream is assigned a scalar value via a utility function $U(\cdot)$, and the borrower seeks to maximize

$$E\{\hat{U}[(\tilde{\Pi}(1), \tilde{\Pi}(2), \ldots, \tilde{\Pi}(T)]\}$$

If the borrower is risk-averse, as will be assumed henceforth, the function $\hat{U}$ is concave, and conversely.

This approach clearly contrasts with capital asset pricing models which maintain that the firm should maximize its market value (a function of nondiversifiable rather than total risk). When bankruptcy risks are significant, the firm is privately held or the borrower is a public agency (which is our primary concern here) the total risk paradigm used in this paper is justified.

A reasonable approximation to the form of the borrower's utility function can be developed as follows. On one hand, it can be assumed that under proper discounting, net cash flows in different periods with equal expectations and degrees of riskiness are substitutable to the borrower--i.e., that his utility map is a function merely of the present value of net cash streams:

$$\hat{U}(\tilde{\Pi}(1), \tilde{\Pi}(2), \ldots, \tilde{\Pi}(T)) = U(\tilde{Z})$$

where $U(\cdot)$ is increasing and concave.

Given this form of the utility function a "certainty equivalent" to the risky prospect $\tilde{Z}$ is classically defined as a certain income $CE(\tilde{Z})$ satisfying

$$U[CE(\tilde{Z})] = E[U(\tilde{Z})]$$

so that the borrower's objective can be equivalently stated as maximizing $CE(\tilde{Z})$.

A second simplification is achieved by assuming that the borrower's absolute risk aversion

$$r(x) \equiv - U''(x)/U'(x)$$

is approximately constant in the relevant range. Hammond [5] has shown the validity of this approximation for discriminating among certain classes of so-called "simply related" risky prospects. More generally, constant absolute risk aversion can be shown to provide a second-order approximation to certainty equivalents associated with more general utility functions. Clearly, the classical assumptions of non satiation, non-increasing absolute risk aversion and non-decreasing relative risk aversion are satisfied within our framework.

Under constant risk-aversion r and normally distributed present value of cash streams $\tilde{Z}$, the certainty equivalent of the latter is

$$CE(\tilde{Z}) = E[\tilde{Z}] - \frac{1}{2} r \cdot Var(\tilde{Z})$$

whereby it is seen that our valuation model is equivalent to a mean-variance framework.

Applying previous results (2) and (3) the certainty equivalent can be expressed as an explicit function of the debt portfolio:

$$CE(\tilde{Z}) = \sum_{t=1}^{T} \alpha^{t-1} \left[\delta^{t} R(0) + \Sigma_{k=1}^{t} \delta^{t-k} \nu(k) - D(t)' \left(S(0) + \sum_{k=1}^{t} \phi(k) \right) \right] - \bar{D}_0$$

$$- \tfrac{1}{2} r \left\{ \sum_{t=1}^{T} \sum_{k=1}^{T} \alpha^{t+k-2} \sum_{\ell=1}^{\min\{t,k\}} [1, -D(t)'] \begin{bmatrix} \delta^{t-\ell} & 0 \\ 0 & I \end{bmatrix} \right. .$$

$$\begin{bmatrix} \omega^2 & \eta' \\ \eta & \Sigma \end{bmatrix} \begin{bmatrix} \delta^{k-\ell} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} 1 \\ -D(k) \end{bmatrix} \tag{4}$$

Let us define:

$$K = \Sigma_{t=1}^{T} \alpha^{t-1}\left[\delta^t R(0) + \Sigma_{k=1}^{t} \delta^{t-k} \nu(k)\right]$$

$$\psi = \omega^2 \Sigma_{t=1}^{T} \Sigma_{k=1}^{T} \alpha^{t+k-2} \Sigma_{\ell=1}^{\min\{t,k\}} \delta^{t+k-2\ell} > 0$$

$$Q(t) = \Sigma_{k=1}^{T} \alpha^{t+k-2} \Sigma_{\ell=1}^{\min\{t,k\}} \delta^{k-\ell} > 0$$

$$\beta(t,k) = \alpha^{t+k-2} \min\{t, k\} > 0$$

Rearranging the variance term in (4) and isolating the terms K and ψ which are independent of any debt-related parameters, the certainty equivalent reduces to:

$$\begin{aligned} CE(\tilde{Z}) = K - \Sigma_{t=1}^{T} \alpha^{t-1} D(t)'\left[S(0) + \Sigma_{k=1}^{t} \phi(k)\right] - \bar{D}_0 \\ - \frac{1}{2} r\left\{\Sigma_{t=1}^{T} \Sigma_{k=1}^{T} \beta(t, k) D(t)'\Sigma D(k) - 2 \Sigma_{t=1}^{T} Q(t)\eta' D(t) + \psi\right\} \end{aligned} \tag{5}$$

Interestingly the variance term, which generally (but not always) increases the cost of foreign debt is made up of three components: a domestic and foreign currency volatility component as well as a covariance component.

2.3 An illustration: the two-currencies, two-periods case.

A simplified example will help illustrate the point that when the time structures of debt-related cash streams and exchange rates are not stationary and multiperiod formulation is not reducible to a one-period model.

Let us consider a portfolio composed of one foreign currency debt, in addition to the domestic debt, over two periods. Generally, the foreign debt cash streams will be denoted $D = \big(d(1),\ d(2)\big)$. Exchange-rate related parameters $S(0)$, $\phi(1)$, $\phi(2)$, $\sigma^2 = \Sigma$, and y now become scalars, and the foregoing expression (5) reduces itself to:

$$CE(\tilde{Z}|D) = K - D_0 - \big[S(0) + \phi(1)\big]d(1) - \alpha\big[S(0) + \phi(1) + \phi(2)\big]d(2)$$

$$- \tfrac{1}{2}\, r\{\sigma^2\big[\big(d(1) + \alpha d(2)\big)^2 + \alpha^2 d(2)^2\big] - 2y\big[(1 + \alpha S)\big(d(1) + \alpha d(2)\big)$$

$$+ \alpha^2 d(2)\big] + \psi\} \;.$$

Let us contrast this case with an analogue of a one-period model, involving a constant annuity in each period $\hat{D} = (\hat{d},\ \hat{d})$. For (illustrative) comparison purposes, assume that this alternative schedule has the same nominal expected present value (or equivalently the same expected present value under initial exchange rates) as the former schedule, namely

$$\hat{d} = \frac{d(1) + \alpha d(2)}{1 + \alpha} \;.$$

It is straightforward to verify that

$$CE(\tilde{Z}|D) - CE(\tilde{Z}|\hat{D}) = -\frac{\alpha}{1 + \alpha}\big[d(2) - d(1)\big]\{\phi(2) + \frac{\alpha r}{2}\big[\sigma^2\big(d(2) + \hat{d}\big) - 2y\big]\}$$

i.e., the risk-adjusted values of the two schedules generally differ because the process governing exchange rates is additive. Other values for the constant annuity schedule will lead to the same qualitative conclusion that a one-period formulation cannot capture the important dynamic aspects of the multiperiod multicurrency debt financing problem.

3. Analysis of Financing Options

The general framework developed in equation 5 can be applied in a number of different modes to the financing decisions as faced by borrowers in the international capital market. The approach taken in this section as well as the balance of this paper is deliberately operational and pragmatic. This section will consider the use of the model as a ranking algorithm, the sketching of an optimal portfolio path as well as the partial equilibrium properties of the model.

3.1 Ranking Composite Debt Portfolios.

Given single currency debt instrument or composite debt portfolio, equation 5 offers a single ranking algorithm whereby the borrowing agent can compare on a cardinal scale alternative financing options. Section 5.1 will raise the related problem of devising an optimal portfolio of debt instruments denominated in different currencies. Any ranking, however, must be as of necessity, stated as a function of the level of risk aversion r.

3.2 Risk Profile Debt Curves and the Financing Decision

Risk fundamentally is a subjective concept and the reader will notice that so far we have carefully avoided saying much about the risk aversion parameter r. Instead we suggest to sketch the risk-adjusted cost (certainty equivalent) of foreign debt against the risk aversion level r (see Fig. 1). This graph portrays the cost of risk for all decision-makers (as characterized by different levels of risk aversion). It is an objective statement of the risk involved in the sense that all different decision-makers, who agree with each other on the joint probability distributions representing the underlying source of risk, will obtain the same risk profile curve.

There are some general properties of risk profile debt curves which are useful to comment upon:

(a) The height of risk profile debt curves above $r = 0$ (where decision-makers are assumed to be risk-neutral) is simply the expected value of the objective function.

(b) The risk profile debt curves will generally intersect at a point corresponding to a critical level of risk aversion r_c. Clearly, the decision-maker, only needs to decide whether the risk preferences of the borrower lie below (domestic financing) or above that critical level (foreign financing). Our framework, however, does not indicate what risk aversion level is appropriate but it offers a conceptually sound yet operationally manageable framework for measuring risk-cost tradeoffs.

(c) The financing implications of simple changes in parameter estimates can be easily sketched graphically. Referring to the comparative statics results derived in the following section, a lower expected cost for foreign debt financing will result in a vertical shift of the foreign financing debt Π^* curve which will increase the critical level of risk aversion r_c^* beyond which domestic financing becomes the preferred option (cf Figure 1 a). A higher anticipated volatility for any currency in foreign debt financing portfolio will increase the risk premium term by rotating outward the foreign financing debt Π^* curve thereby lowering the critical level of risk aversion beyond which domestic financing becomes the preferred option (cf Figure 1 c). Conversely a higher covariance η between the domestic stream of cash inflows and any exchange rate $S_i(t)$ would reduce the risk premium term associated with a foreign debt financing portfolio thereby rotating downward the foreign debt financing Π^* curve and lowering the critical level of risk aversion (cf Figure 1b).

3.3 Comparative Statics and Sensitivity Analysis

Our analytical valuation formula (5) makes it easy to compare incremental changes in the portfolio. We provide two illustrations of such comparisons.

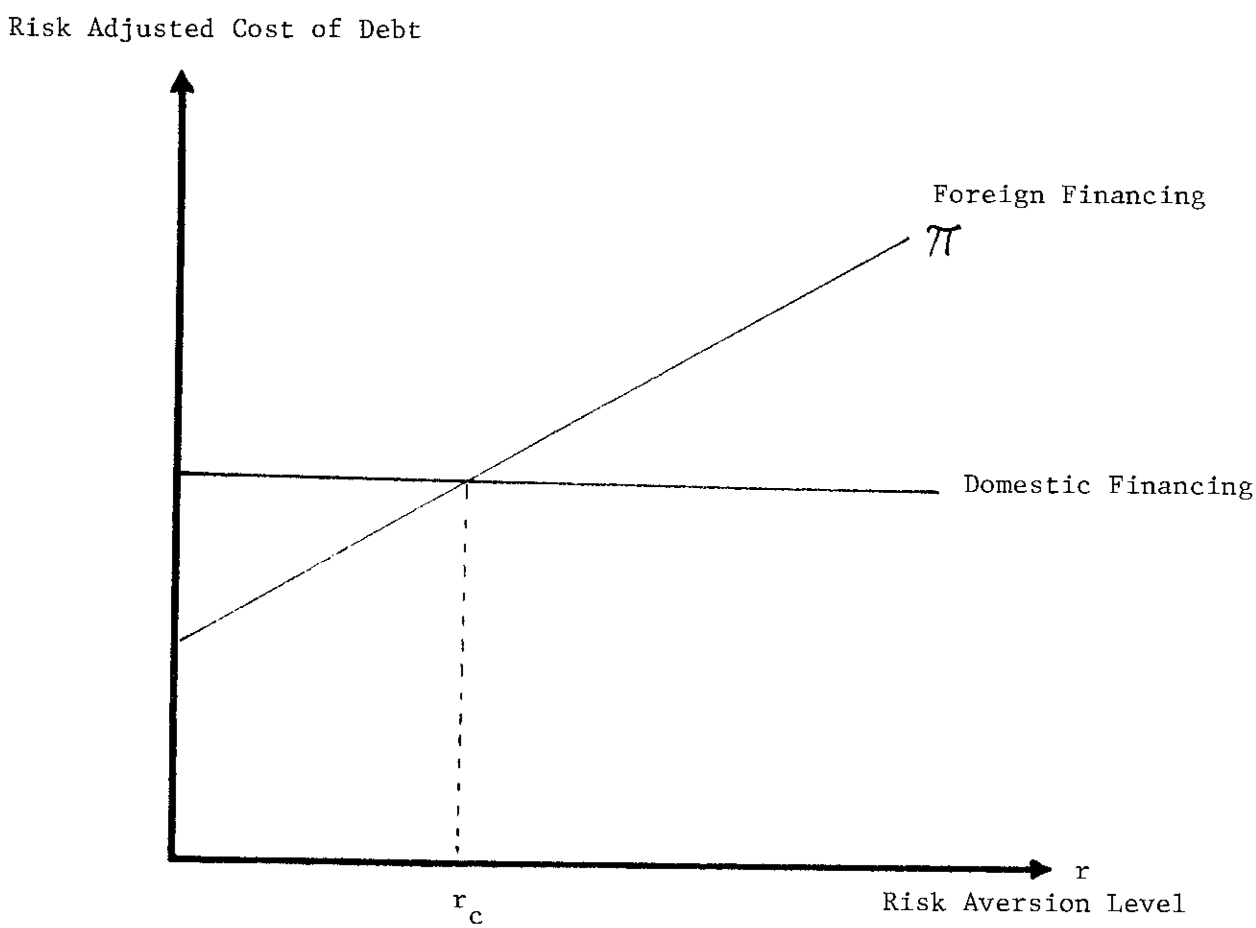

Figure 1 : Risk Profile Curves

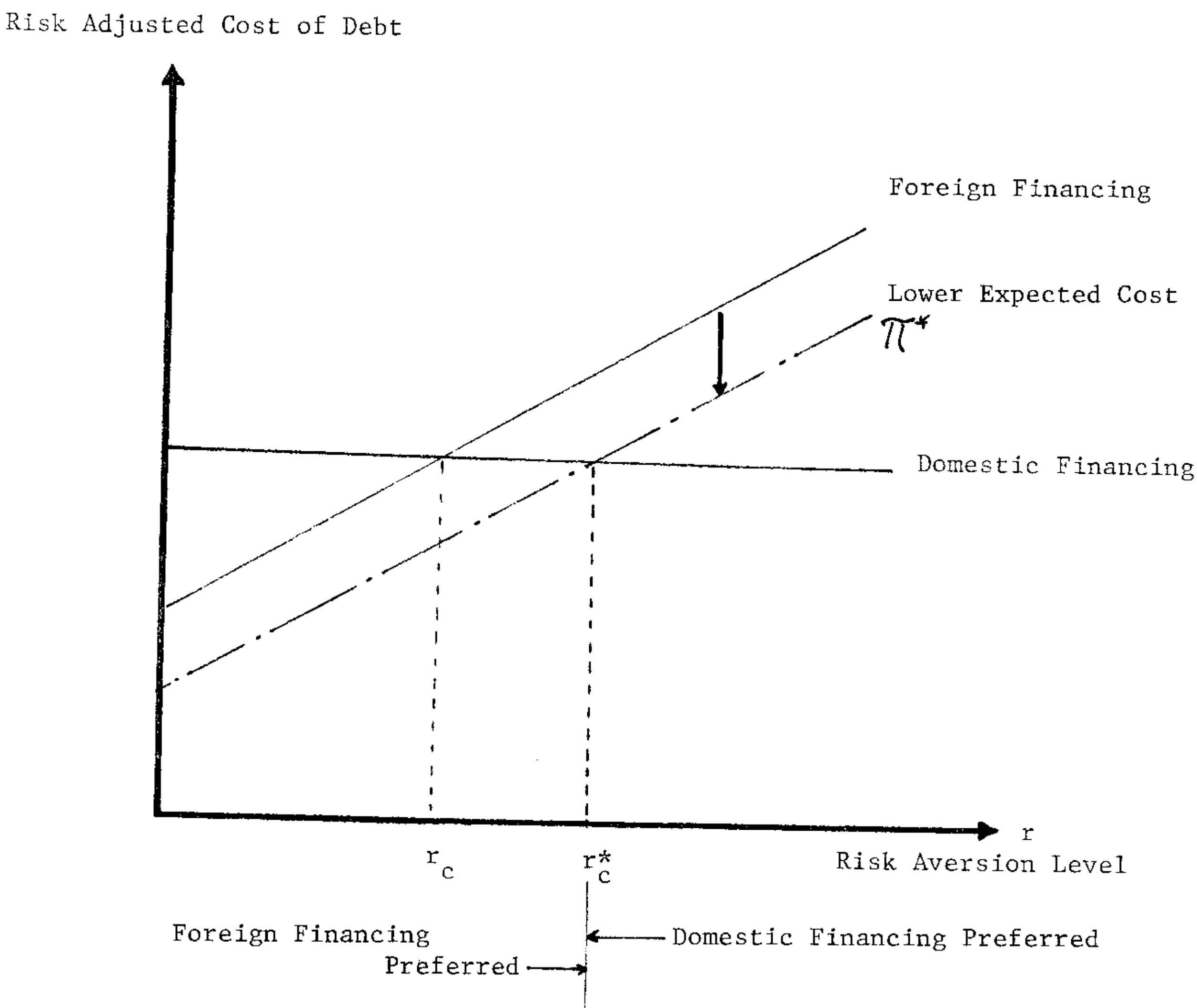

Figure 1a : Break-Even Risk- Aversion Level with Lower Expected Cost of Foreign Financing

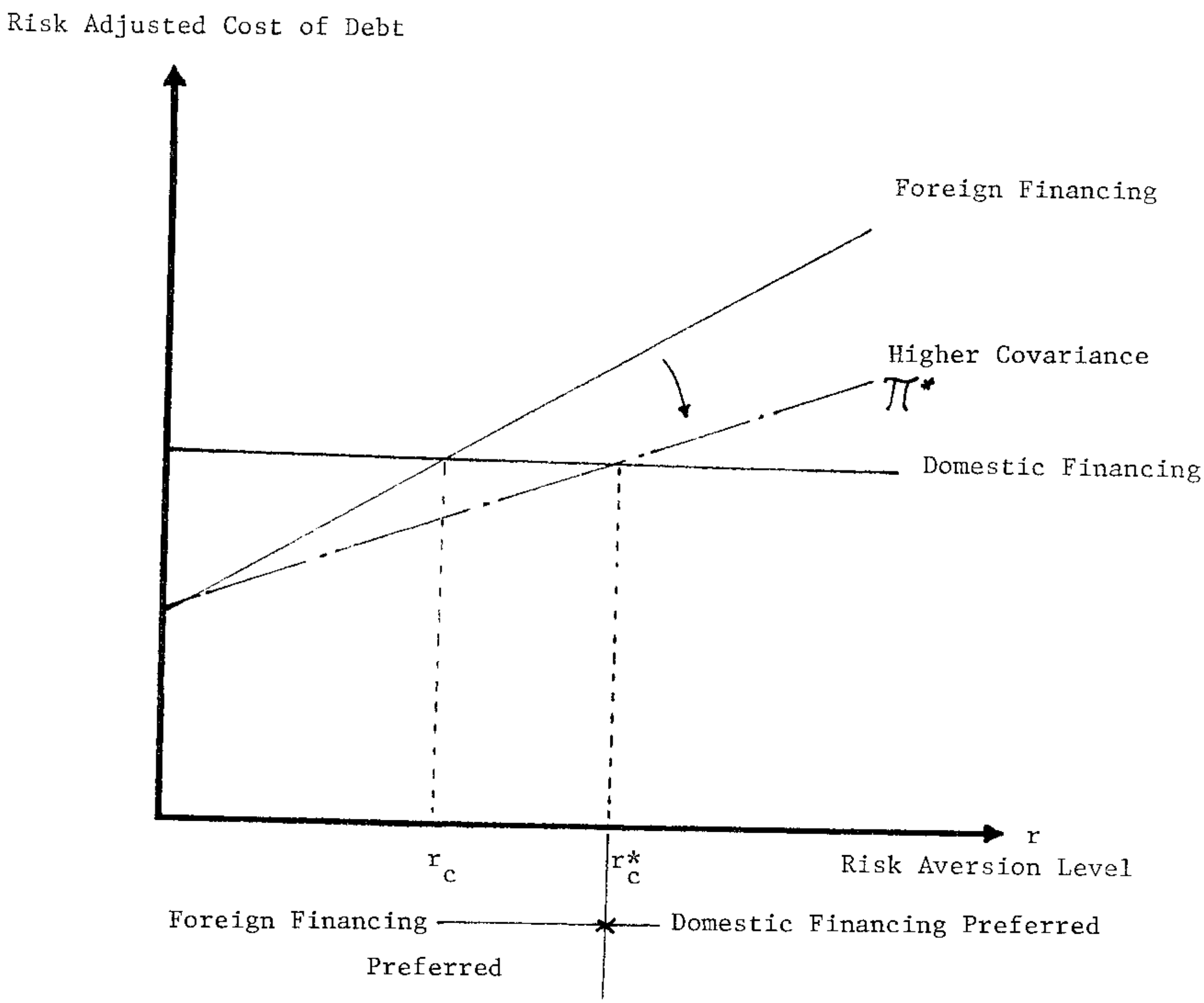

Figure 1b : Break-Even Risk-Aversion Level with Higher Covariance between Domestic Revenues and Exchange Rates

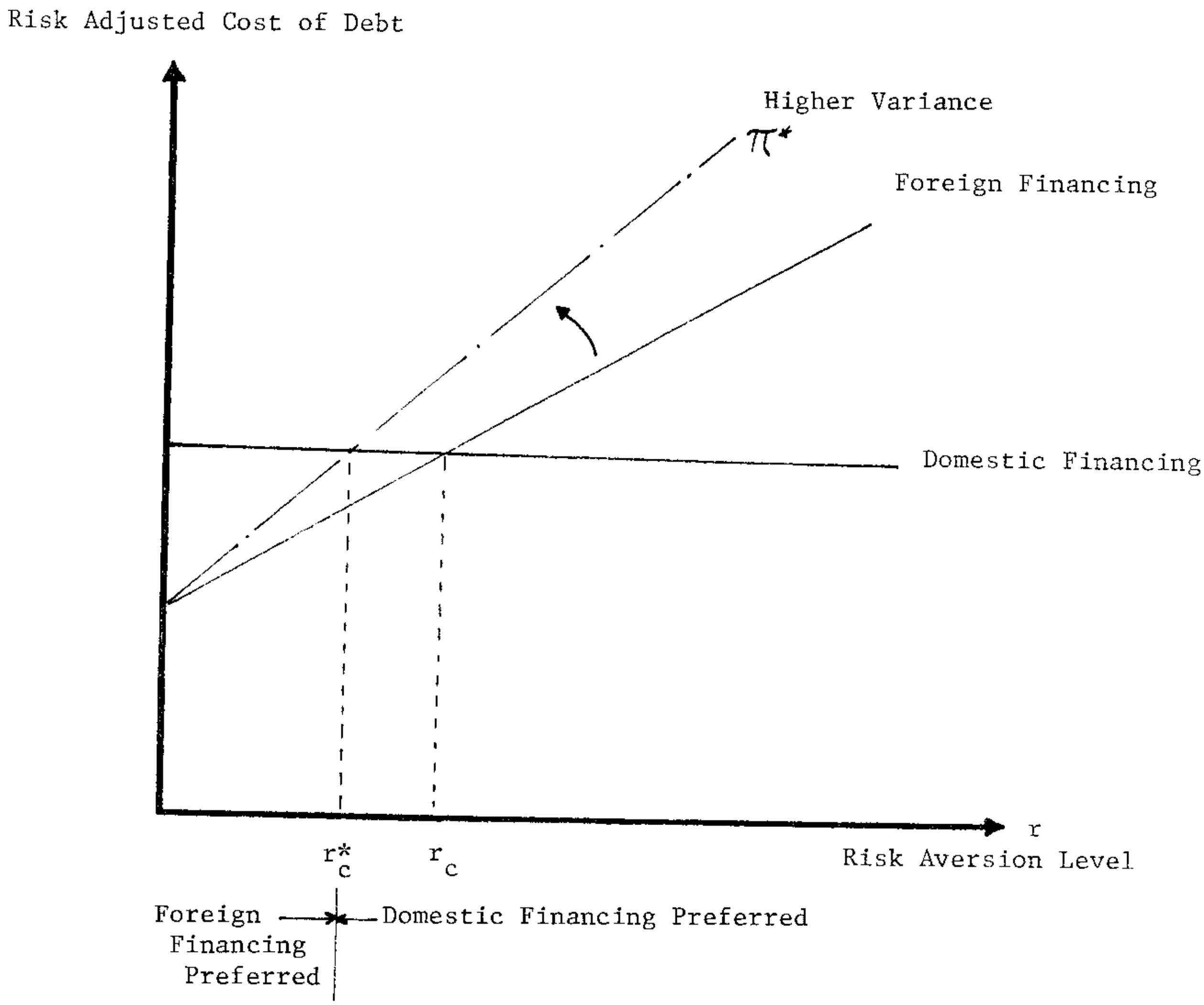

Figure 1c : Break-Even Risk-Aversion Level with Higher Variance of Exchange Rates

Extensions are straightforward. More general-purpose optimization approaches are briefly discussed in section 5.

Assume the borrower has a (new) financing requirement in period 0 that can be totally covered by either one of two "pure" options:

(i) Additional domestic financing, resulting in an increase in domestic debt service over the next T periods of

$$\Delta d_0 = \left(\Delta d_0(1), \ldots, \Delta d_0(T)\right)$$

(ii) Additional financing in currency N, resulting in an increase in that currency's debt service of

$$\Delta d_N = \left(\Delta d_N(1), \ldots, \Delta d_N(T)\right)$$

In general, the borrower may consider a mix of these extreme options of the form $\lambda\Delta d_N + (1 - \lambda)\Delta d_0$, $0 \leq \lambda \leq 1$ ($\lambda = 0$ and $\lambda = 1$ corresponding respectively to pure domestic and foreign financing). His net cash streams and their present value are changed by

$$\Delta\tilde{\Pi}(t) = -(1 - \lambda)\Delta d_0(t) - \lambda\tilde{S}_N\Delta d_N(t) \qquad 1 \leq t \leq T \tag{6a}$$

$$\Delta\tilde{Z} = \Sigma_{t=1}^{T} \alpha^{t-1}\Delta\tilde{\Pi}(t) \tag{6b}$$

Let σ_{ij} denote the (i, j)th element of the matrix Σ. Using the previous developments we have:

$$E[\Delta\tilde{Z}|\lambda = 1] = -\Sigma_{t=1}^{T} \alpha^{t-1}\left[S_N(0) + \Sigma_{k=1}^{t} \Phi_N(k)\right] \cdot \Delta d_N(t) \tag{7a}$$

$$\mathrm{Var}(\tilde{Z} + \Delta\tilde{Z}|\lambda) = \mathrm{Var}(Z) + \lambda^2\mathrm{Var}(\Delta\tilde{Z}|\lambda = 1) + 2\lambda\mathrm{Cov}(\tilde{Z}, \Delta\tilde{Z}|\lambda = 1) \tag{7b}$$

$$\mathrm{Var}(\Delta\tilde{Z}|\lambda = 1) = \sigma_{NN}\Sigma_{t=1}^{T}\ \Sigma_{k=1}^{T}\ \beta(t, k) \cdot \Delta d_N(t) \cdot \Delta d_N(t) \tag{7c}$$

$$\mathrm{Cov}(\tilde{Z},\Delta\tilde{Z}|\lambda=1)=\Sigma_{i=1}^{N}\ \sigma_{iN}\ \Sigma_{t=1}^{T}\ \Sigma_{k=1}^{T}\ \beta(t,k)d_i(t) \cdot \Delta d_N(t)- \eta_N\Sigma_{t=1}^{T}q(t) \cdot \Delta d_N(t) \tag{7d}$$

The risk-adjusted incremental cost associated with each pure alternative is:

- Domestic financing:

$$C_0 = - \left[\mathrm{CE}(\tilde{Z} + \Delta\tilde{Z}|\lambda = 0) - \mathrm{CE}(z)\right]$$

$$= \Sigma_{t=1}^{T}\ \alpha^{t-1}\Delta d_0(t) \equiv \Delta\overline{D}_0 \ .$$

- Foreign financing:

$$C_1 = -\left[\mathrm{CE}(\tilde{Z} + \Delta\tilde{Z}|\lambda = 1) - \mathrm{CE}(Z)\right]$$

$$= E\left[\Delta\tilde{Z}|\lambda = 1\right] + \frac{1}{2} \cdot r\left\{\mathrm{Var}(\Delta\tilde{Z}|\lambda = 1) + 2\lambda\mathrm{Cov}(\tilde{Z}, \Delta\tilde{Z}|\lambda = 1)\right\}$$

Using the framework of Figure 1 the risk adjusted cost of incremental financing, given an existing foreign debt portfolio is depicted in Figure 2. The slope of the incremental foreign debt financing option is the change in variance $\mathrm{Var}(\tilde{Z} + \Delta\tilde{Z}/\lambda) - \mathrm{Var}(\tilde{Z})$ due to that option.

Given any $\lambda \in [0, 1]$ the comparative marginal attractiveness of foreign financing over domestic financing is measured by

$$\frac{\partial \mathrm{CE}(Z + \Delta Z|\lambda)}{\partial\lambda} = \Delta\overline{D}_0-E[\Delta\tilde{Z}|\lambda=1]-r\left\{\lambda\mathrm{Var}(\Delta\tilde{Z}|\lambda = 1) + \lambda\mathrm{Cov}(\tilde{Z}, \Delta\tilde{Z}|\lambda = 1)\right\} \ .$$

Assuming <u>for instance</u> $d_i(t) \geqslant 0$ and $\Delta d_i(t) \geqslant 0\ \forall\ i$, and t, the effect of parameter changes on this ranking can be immediately traced:

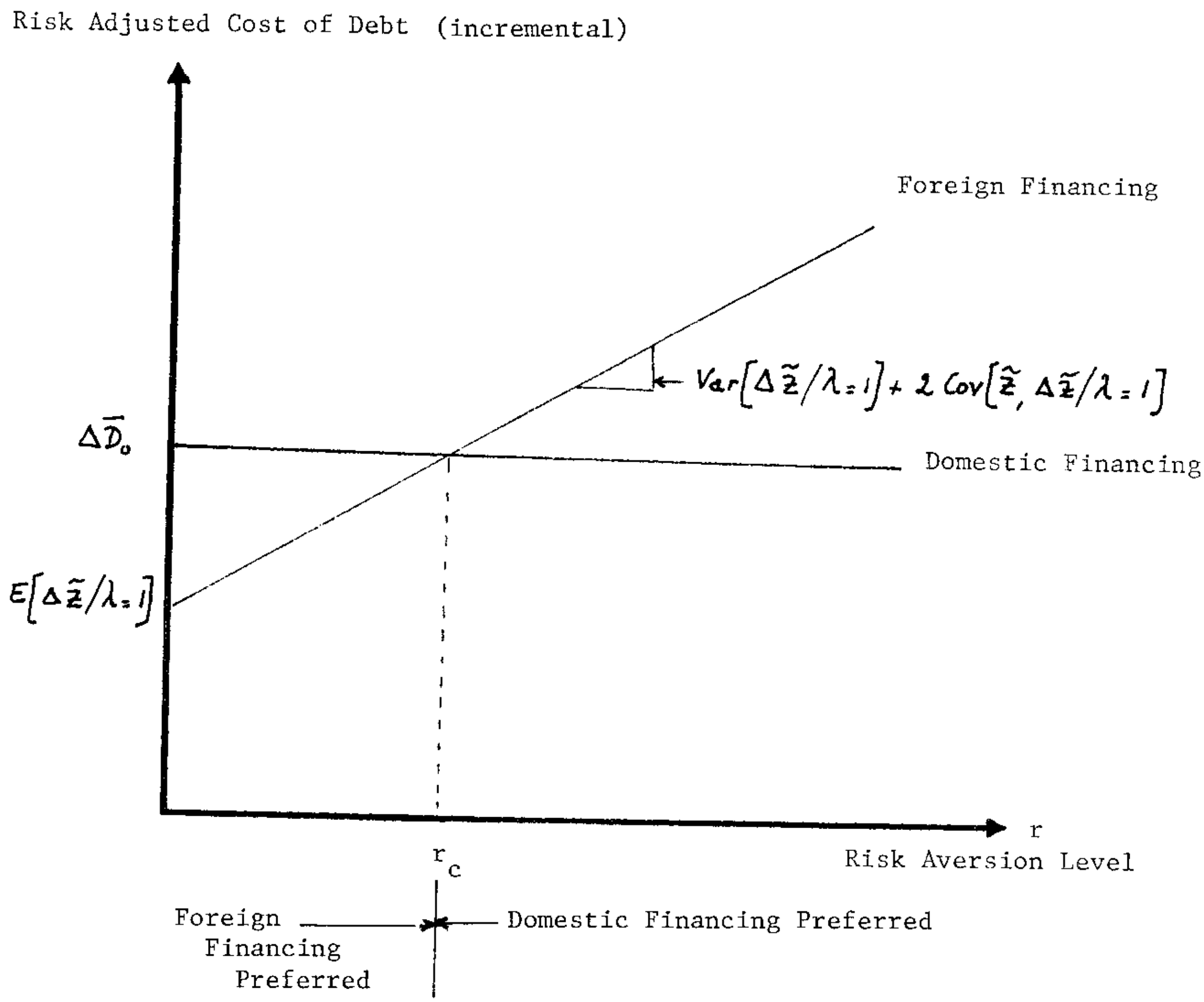

Figure 2 : Risk Profile Curves for Two Financing Options

$\dfrac{\partial^2 CE(\tilde{Z}+\Delta\tilde{Z}\mid\lambda)}{\partial\lambda\partial\Phi_N(t)} < 0$	An expected appreciation of the foreign currency relative to the domestic currency favors domestic financing.
$\dfrac{\partial^2 CE(\tilde{Z}+\Delta\tilde{Z}\mid\lambda)}{\partial\lambda\partial\sigma_{NN}} < 0$	An increase in the one-year-ahead variance of currency N's exchange rate favors domestic financing
$\dfrac{\partial^2 CE(\tilde{Z}+\Delta\tilde{Z}\mid\lambda)}{\partial\lambda\partial\sigma_{iN}} < 0$	The relative attractiveness of foreign financing increases as one-year-ahead covariances between currencies i and N's exchange rates become more negative
$\dfrac{\partial^2 CE(\tilde{Z}+\Delta\tilde{Z}\mid\lambda)}{\partial\lambda\partial\eta_N} > 0$	The relative attractiveness of foreign financing increases with the one-year-ahead covariance between net domestic revenues and currency N's exchange rate

Thus currency N financing provides a hedge to the extent that it is

- negatively correlated with other foreign currencies
- positively correlated with domestic revenues.

3.4. Parameters Estimation

The first informational hurdle to be overcome is the forecasting of the mean vector of future exchange rates over the financing horizon. For bridging this informational gap it is suggested to use two well-known propositions of financial economics i.e., the Expectations Hypothesis of Interest Rates and the International Fisher Effect.

Simply stated, it is first proposed to extract from yield curves of two group of securities (equivalent in all respects except for their currency of denomination) market-based forecasts of future interest rates and, second, to derive through a repeated application of the International Fisher Effect the sequence of period-to-period exchange rate changes measured by the corresponding premium or discount.

The expected value of the exchange rate prevailing at the end of period τ is generally formulated as:

$$E\left[S_h(\tau)/S(0)\right] = S_h(0) \cdot \prod_{t=1}^{\tau-1} \frac{\{1+ I_0[0;\ (t+1)]\}^{t+1}}{\{1 + I_0(0;\ t)\}^t} \Big/ \frac{\{1+I_h[0;\ (t+1)]\}^{t+1}}{\{1 + I_h(0;\ t)\}^t} \tag{8}$$

where $S_h(0)$ is the exchange rate prevailing at the outset of the forecasting horizon, $I_0(0;\ t)$ and $I_h(0;\ t)$ are respectively annual interest rates on current bonds denominated in currency 1 or currency h and maturing in t periods.

Estimating the coefficients of the variance-covariance matrix Ω may be achieved with historical exchange rates time series. The key assumption of stationarity of variance or covariance coefficients remains to be investigated[6] whereas the normality assumption for exchange rate distribution model remains at least controversial but central to any multi-dimensional (in the currency space) modelling effort.[7] An additional concern in estimating exchange rates variances and covariances is the degree of market intervention by central banks: the higher the degree of central banks intervention on the spot exchange rate the lower the volatility of the exchange rate as measured by the variance of the exchange rate and conversely. To the extent that a central bank intervention has been accidental and is perceived not to be repeated in the future, the analyst may want to measure the underlying rather than the actual variance.

4. Debt Financing in Artificial Currency Units (Currency Cocktails).

The era of floating exchange rates has witnessed a multiplication of artificial currency units (commonly known as "Currency Cocktails") which have been used, among other things, for denominating long-term issues. The rationale is simple: by borrowing in a once-for-all fixed portfolio of currencies, the borrower (as well as the lender) expects to hedge--in a somewhat indeterminate way--against excessive single currency fluctuations. The framework introduced in this article lends itself readily to the certainty equivalent valuation of a debt instrument denominated in an artificial currency unit (ACU). Recall that an ACU would be valued in the domestic currency of the borrower as:

$$S_{ACU}(0) = \sum_{k=1}^{K} a_k \cdot S_k(0) \qquad \text{with } K \leqslant N \tag{9}$$

where a_k is the number of units of currency k included in one artificial currency. Let D_{ACU} be the known debt servicing requirement associated with borrowing in ACUs rather than in a homemade portfolio D_p. Borrowing portfolios D_{ACU} versus D_p can readily be compared in the context of our model by substituting in the debt vector D(t) the corresponding number of units of currency k, a_k for the amount borrowed in currency k, $d_k(t)$. Since, typically the interest rate on artificial currency unit-denominated bonds tends to closely follow a currency-weighted interest rate and therefore to ignore the cross hedging effect of such multiple currency denomination our portfolio model will assist the potential borrower in valuing correctly the diversification effect imbedded in such debt instruments. Ultimately the borrower will compare the significant savings in floatation costs resulting from one artificial currency unit debt instrument with the multiple floatation costs associated with a 'homemade' diversified portfolio of single currency denominated debt instrument. Such savings in floatation costs should then be compared with the cost of paying for potential redundant currency denomination.[8] Clearly the flexibility of refunding a single debt instrument D_i would be lost under an exclusive ACU borrowing strategy.

5. Optimal Debt Portfolios:

The discussion of the foreign debt financing decision was limited to the ranking on a risk-adjusted basis of alternative foreign currency denominated debt portfolios. The structure of the general model introduced in section 2 is rich enough to allow for a resolution of the optimal debt portfolio problem where N+1 debt financing instruments are available. Specifically, three questions can be addressed:

(a) Given an additional need for debt financing ΔD_p (in domestic currency terms), how should the financing be allocated among the N+1 debt instruments available (assumption of increasing financing needs)?

(b) Given an existing debt portfolio D_p, how should maturing debt instrument d_i be refinanced (assumption of constant financing needs)?

(c) Given an existing debt portfolio $D_p(t)$ structured at time $\tau < t$ should debt refinancing be considered.

All three cases are highly representative of public borrowers' predicament (at the state, city, municipal, public utility or government-owned corporations level) and amount to a constrained optimization problem with a quadratic objective function. It is also the major justification for the total risk approach adopted throughout this paper.

The constraints to be imposed on the design of the optimal debt portfolio simply reflects the limited capacity of national capital markets for absorbing large financing needs. Typically (fixed) floatation costs will set the lower bound on the amount to be raised in a single issue in a given capital market whereas the higher cost of debt beyond a threshold level of borrowing will effectively set a higher bound. Formally the optimal debt portfolio in an N-currency world would be formulated as :

$$\max_{\{d_i\}} \quad CE(Z) \tag{10}$$

$$\text{subject to } d_i^{min} \leq d_i \leq d_i^{max}$$

5.1 Additional Financing

Given an existing debt portfolio $\{d_i(t)/1 \leq t \leq T,\ 0 \leq i \leq N\}$ and the need for additional financing, assume that the borrower has identified for each currency i M_i "pure" (re)financing options[9] implying (positive or negative) cash stream changes

$$\Delta d_i^J = \Delta d_i^J(1), \ldots, \Delta d_i^J(T) \quad \text{with } J = 1, \ldots, M_i$$
$$\text{and } i = 0, \ldots, N\ .$$

The borrower will, in general, satisfy his additional financing need by considering a mix of these "pure" options of the form $\lambda_i^J\ \Delta_i^J\{J = 1, \ldots, M_i$ and $i = 0, \ldots, N\}$ with $\lambda_i^J \geq 0$ and $\Sigma_{i=0}^{N}\ \Sigma_{J=1}^{M_i}\ \lambda_i^J \leq 1$. By assessing the expected utility of the revised portfolio(s), the borrower will be able to determine the optimal financing mix by optimizing the function:

$$f(\lambda) = CE\left[\tilde{Z} + \Delta Z(\lambda)\right] \tag{11}$$

$$\text{subject to } \lambda_i^{Jmin} \leq \lambda_i^J \leq \lambda_i^{Jmax}$$

where λ is a vector of dimension $\Sigma_{i=0}^{N} M_i$ and the constraint reflect the possible "crowding out" effect of borrowing in single national market and therefore limit the extent to which each individual option is to be exercised.

Constrained access to single national capital markets can increasingly be circumvented through a long-term currency swap. The borrower would source long-term financing from the unconstrained capital market, e.g., U.S. dollar-denominated bonds and swap its dollar liability for the restricted third currency liability. In addition to interest cost savings the cross-hedging benefits of sourcing say Swiss Francs (restricted financing) rather than U.S. dollars (unrestricted financing) would be acheived. In terms of equation (10) the upper bound d_i^{max} would no longer be binding (or binding at a higher level). The framework developed in this paper would clearly assist the borrower in his negotiating the terms of such long-term currency debt swaps.

5.2 Debt Refunding

The problem of debt refunding is somewhat different insofar as prepayment penalty on refunding a given debt instrument d_i at time $t < T$ (where time T refers to the maturity of debt instrument d_i) would have to be incorporated in the above constrained optimization problem.

5.2.1 Straight Debt:

Typically, such debt portfolio revisions will entail significant transaction costs as a function of call premia (as defined in the covenant of the debt instrument to be retired) , floatation costs (associated with the new financing mix) as well as possible higher interest cost during the overlapping period between the time when the new bonds are issued and the old ones refunded. Denoting by $\psi_i^J(\lambda_i^J)$ the transaction costs incurred from financing d_i (resulting from either additional financing need or retiring the same amount) through the financing mix $\lambda_i^J d_i^J(t)$ the borrower will derive the optimal incremental financing portfolio by optimizing the objective function

$$\text{Max}\left\{f(\lambda) - \Sigma_{i=0}^{N} \Sigma_{J=1}^{M_i} \psi_i^J(\lambda_i^J)\right\} \tag{12}$$

$$\text{subject to } \Sigma_{i=0}^{N} \Sigma_{J=0}^{M_i} \lambda_i^J \leq 1$$

$$\text{and } \lambda_i^J \in \{0\} \cup [\lambda_i^{J\min}, \lambda_i^{J\max}]$$

Perold (1984) discusses solution procedures for similarly-structured problems with disjunctive constraints of the above form which provides parametric solutions as a function of the risk-aversion parameter r.

5.2.2 Currency Swaps:

A first alternative to early debt redemption (perhaps prompted by a higher/lower than expected appreciation of the currency of denomination) is the use of fixed interest rate to fixed interest rate currency swap. By exchanging interest payment and principal repayment obligations at a fixed (once-for-all) exchange rate borrowers can free themselves from a foreign debt obligation without incurring the additional cost of prepayment penalty and floatation cost on a new debt issue. The framework developed in this section would assist the borrower in negotiating terms that would improve his utility position as well as determine an optimal fraction λ_i^{SW} of the original debt to be redeemed through a swap. The early case of IBM and the World Bank swapping long term debt in Swiss franc and Deutsche mark for dollar has been followed by a flurry of large transactions, notably the recent swap of a long term dollar debt for ECU by the French utility company, Gaz de France: for an authoritative study of swap financing, see Antl [3].

5.2.3 Long-Term Forward Contracts

A second alternative to early debt redemption would be to buy the currency forward in amount and maturity matching the debt servicing schedule.

Significant prepayment penalty costs can again be avoided. Given offered long-term forward rate the above framework allows the borrowing agent to derive the optimal percentage λ_i^F to be covered through a forward contract rather than prepaid.

6. Conclusion

Unexpected exchange rate changes and national financial markets segmentation have turned the market efficiency hypothesis as applied to the foreign exchange market into a somewhat dubious operational proposition for international financial management purposes.[10] This article attempted to enlarge the still embryonic body of international financial management theory by developing a multi-currency intertemporal decision analysis framework for assessing the cost of foreign debt financing i.e., the risk neutrality assumption behind the traditional expected cost criterion used for ranking alternative financing options was relaxed and a risk-adjusted cost of foreign debt formulated when exchange rates obey a random walk law.

A significant result in the nature of a cross-hedging effect was that a domestic borrower may, on a risk-adjusted basis, prefer foreign financing to domestic financing. An outstanding concern remains the relevant moments of the probability distribution of exchange rates as well as the estimation of the variance-covariance matrix.

The framework was then applied to some of the practical problems faced by borrowers on the international capital market: specifically the use of artificial currency units, substituted for a portfolio of single national currency, was evaluated from the point of view of mispricing of such debt instruments given the redundant diversification imbedded in such instruments. Similarly the framework was operationalized for debt refunding either directly through actual prepayment of the debt instrument or indirectly through the use of currency swaps and long-term forward contracts.

Finally, this utility-theoretic framework could be directly combined with a dynamic programming algorithm for solving the complex problem of debt refunding in a multi-currency world which has plagued a number of multinational corporate borrowers throughout the first decade of the floating exchange rates era.

FOOTNOTES

[1]For an early intertemporal elaborate testing of this form of Market Efficiency the reader is referred to Michael G. Porter [11] and Solnik and Grall [15].

[2]For a thorough discussion of the segmentation of national financial markets see Robert Z. Aliber [2].

[3]Witness the scope of foreign financing relied upon by Japanese utility companies. In 1982 alone, all eight Japanese electric power companies (with the exception of Hokkaido Electric) issued debt in the Swiss capital market to the tune of 1.25 billion Swiss Francs which is equal to one fifth of all total new bond issues in the Japanese domestic market of electric power companies. Any list of new Eurobond issues would illustrate the same phenomenon, i.e., domestic public entities, crowded out by their own government and relying on foreign financing on a grand scale.

[4]For a masterful review of the literature on International Capital Asset Pricing Models, see Adler and Dumas [1].

[5]This is the case of domestic firms selling to exporters or competing with imports. Consider, for example, the hypothetical case of Hydro-Quebec, the public utility company selling a significant percentage of its power to Alcan which, in turn, exports aluminum ingots and other derivatives to the U.S. market. A weak Canadian dollar would tend to boost Alcan's exports to the U.S. market and indirectly Hydro-Quebec profitability and vice-versa. Thus, for Hydro-Quebec, financing part of its capital requirements in U.S. rather than Canadian dollars should cushion its debt servicing requirements.

[6]For a recent discussion of the numeric stationarity of the variance-covariance matrix of exchange rates, see Madura and Nosari [7] as well as Praagman and Soenen [10].

[7]For a recent empirical investigation of the distribution properties, see McFarland, Pettit and Sung [8]. Earlier work by Westerfield [16] and Rogalski and Vinso [13] are also helpful.

[8]The encouragement by the European economic community to float debt in ECU rather than single currency is a case in point. Such state-owned utility companies as Gaz de France or Electricity de France may be simply overpaying by sourcing long-term financing needs in ECU rather than sterling, DM, DG, etc., . . . It is not clear that these borrowers have explored the true cost of such redundant currency diversification.

[9]We are considering the general case of several financing options in a given currency (either on-shore or off-shore) as call features and/or floatation costs associated with public or private placement will result in different financing costs.

[10]Interestingly enough, this is a problem analogous to computing the <u>real</u> cost of domestic debt (as opposed to the <u>nominal</u> cost of debt) when the <u>price</u> level is assumed to vary randomly. Clearly, the differential in random level

of prices between two numeraire currencies is directly related to the change in the exchange rate through the Purchasing Power Parity theorem.

REFERENCES

[1] Adler, Michael and Bernard Dumas, "International Portfolio Choice and Corporation Finance: A Synthesis," Journal of Finance (June 1983), pp. 925-984.

[2] Aliber, Robert Z., "The Integration of National Financial Markets: A Review of Theory and Findings," Weltwirtschaftliches Archiv (1978), pp. 448-479.

[3] Antl, Boris, Swap Financing Techniques (Euromoney Publications: London, 1983).

[4] Giddy, Ian H., "The Effective Cost of Foreign Currency Borrowing," unpublished manuscript, University of Chicago (1977).

[5] Hammond, John S., "Simplifying the Choice between Uncertain Prospects where Preference is Nonlinear," Management Science 20 (1974), pp. 1047-1072.

[6] Jacque, Laurent L., Management of Foreign Exchange Risk: Theory and Praxis (Lexington, Mass.: D. C. Heath, 1978).

[7] Madura, Jeff and E. Joe Nosari, "Utilizing Portfolios to Mitigate Exchange Rate Risk," Columbia Journal of World Business (Spring 1984), pp. 96-99.

[8] McFarland, J. W., R. R. Pettit and S. K. Sung, "The Distribution of Foreign Exchange Price Changes: Trading Day Effects and Risk Measurement," Journal of Finance (June 1982), pp. 693-716.

[9] Perold André F., "Large Scale Portfolio Optimization," Management Science 30 (1984), pp. 1143-1160.

[10] Praagman, J. and L. A. Soenen, "Stability of Optimal Currency Cocktails," unpublished working paper (University of Technology, Eindhoven - The Netherlands - : 1983).

[11] Porter, Michael C., "A Theoretical and Empirical Framework for Analyzing the Term Structure of Exchange Rate Expectations," IMF Staff Papers (November 1971), pp. 13-42.

[12] Pratt, John W., "Risk Aversion in the Small and the Large," Econometrica (January 1964), pp. 122-135.

[13] Rogalski, R. and J. D. Vinso, "Empirical Properties of Foreign Exchange Rates," Journal of International Business Studies (Fall 1978).

[14] Shapiro, Alan C., "The Impact of Taxation on the Currency-of-Denomination Decision for Long-Term Borrowing and Lending," Journal of International Business Studies (Spring/Summer 1984).

[15] Solnik, Bruno H., and Grall, Jean, "Eurobonds: Determinants of the Demand for Capital and the International Interest Rate Structure," Journal of Bank Research, (Winter 1975).

[16] Westerfield, J. M., "An Examination of Foreign Exchange Risk under Fixed and Floating Rate Regimes," Journal of International Economics (May 1977), pp. 181-200.