BUDGET DEFICIT, EXTERNAL OFFICIAL BORROWING, AND STERILIZED INTERVENTION POLICY IN FOREIGN EXCHANGE MARKETS

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Abstract

I study the effect of a temporary budget deficit, which is financed in the international capital market, on the exchange rate. First, I show that the exchange rate depreciates both in the short and in the long run if the government finances the deficit by selling debt denominated in foreign currencies to nonresidents. Secondly, I show that the government can prevent an immediate depreciation of the exchange rate by adopting a policy of sterilized intervention; however, the achievement of this short-run exchange rate target implies a long-run depreciation of the real exchange rate.

Models of exchange rate determination have traditionally focussed on the effects of balanced-budget government spending or of budget deficits that are financed with debt denominated in domestic currencies. As Table 1 indicates, however, the governments of many small industrial countries have had substantial recourse to the international capital market in order to fund increases in their expenditures since the mid-1970s. The table also indicates that these governments have pursued, perhaps unintentionally, active sterilized intervention policies, thereby sharply changing the proportion of their overall debt that is denominated in foreign currencies.

In this paper, a portfolio model is used to study the implications that external official borrowing and sterilized intervention policies have on the exchange rate. It is shown that even a temporary expansionary fiscal policy, which is financed by selling foreign currency denominated debt to nonresidents, depreciates the real exchange rate both in the short run and in the long run because of the need to service a larger stock of foreign debt.

In the model, governments can prevent the immediate depreciation of the exchange rate by adopting a policy of sterilized intervention, that is, they borrow in the international capital market in order to withdraw domestic currency denominated debt. It is shown, however, that the achievement of this short-run exchange rate target implies a long-run depreciation of the real exchange rate. The issue that has typically been debated is whether intervention is a feasible and effective policy, which is ultimately an empirical question (Dooley and Isard 1982, Frankel 1982, and Rogoff 1984). This paper adds to the debate from a different angle: it is shown that if governments can successfully neutralize the immediate impact of shocks on the exchange rate, then they have to face a trade-off between short- and long-run exchange stabilization, an important element that has not received adequate consideration in the literature (Branson 1984, Girton and Henderson 1977, Henderson 1984, and Kenen 1982).

It is worthwhile stressing that the paper is not normative, that is, it does not aim at determining the optimal level of official external debt and of exchange market intervention. Rather, given that a government wants to maintain the exchange rate in the face of a budget deficit and assuming that the private sector does not perfectly discount all future tax liabilities associated with the government debt, the paper examines whether debt management policies, such as external borrowing and sterilized intervention, can be adopted to attain exchange rate targets.[1]

I. The Model

Portfolio models in which government debt is net wealth is the framework that has been utilized to support sterilized intervention policies; consequently, a standard portfolio model is developed in this section. Clearly, if the private sector internalized the government's intertemporal budget constraint, debt management policies would not affect the exchange rate and sterilized intervention would be a meaningless policy (Stockman 1983).

The economy is fully described by three equilibrium conditions: one for the financial market, one for the traded goods market, and one for the nontraded goods market.

I.A The financial market

In the domestic financial market there are three assets: bonds denominated in domestic currency (B^T), which are held by both residents (B) and nonresidents (B*), bonds denominated in foreign currency (A) and domestic money (M), which are held by residents. I assume that the economy is not growing, so that only the government issues bonds denominated in domestic currency; and that the country is small, so that the world interest rate is exogenous and issues of domestic currency bonds leave foreigners' wealth practically unchanged. For convenience, I also assume that the private sector of the country has a positive gross foreign asset position, that is, A is positive, and I neglect the capital gains and losses stemming from interest rates movements.[2] The equilibrium in the financial market can be described by the following set of equations that characterizes portfolio models in the open economy:

(1) $M = m(i, i^*, \dot{e})$ $m_i, m_{i^*}, m_{\dot{e}} < 0$

(2) $B = b(i, i^*, \dot{e})W$ $b_i > 0\ b_{i^*}, b_{\dot{e}} < 0$

(3) $B^* = eb^*(i, i^*, \dot{e})W^*$ $b^*_i > 0\ b^*_{i^*}, b^*_{\dot{e}} < 0$

(4) $eA = a(i, i^*, \dot{e})W$ $a_i < 0\ a_{i^*}, a_{\dot{e}} > 0$

(5) $W = M + B + eA$

(6) $B^T = B + B^*$

where W is the domestic nominal wealth, W* the foreign nominal wealth, e the domestic price of a unit of foreign currency, i the domestic interest rate, i* the interest rate prevailing in the world capital market, and $\dot{e}$ the expected change in the exchange rate which is equal to the actual change if market participants' expectations are rational.[3]

In order to solve the model, first, the money market equilibrium condition (1) is inverted to obtain an expression for the domestic nominal interest rate; second, (2) and (3) are substituted into (6) and, together with the expression for the interest rate, they are used to obtain an equation summarizing the equilibrium condition in the financial market; and third, this equation is inverted to yield:

(7) $\dot{e} = \gamma(B^T, i^*, M, W, W^*, e) \qquad \gamma_B, \gamma_{i^*}, \gamma_M < 0 \qquad \gamma_W, \gamma_{W^*}, \gamma_e > 0$

I.B The goods markets

It is assumed that the budget deficit is caused by a temporary increase in government expenditure on nontraded goods and is financed by an issue of debt denominated in foreign currency. Given the small country assumption, this expansionary fiscal policy does not affect the behavior of economic agents abroad and, because the deficit is not expected to persist, does not cause the private sector to revise its expectations about the future course of monetary policy. There are two reasons for focussing on a temporary change in government consumption: first, one objective of the paper is to show that even a temporary change in the government demand for domestic goods has a permanent impact on the real exchange rate. Clearly, if the change were permanent, relative prices would have to change permanently in order to restore goods market equilibrium. Second, as Bailey (1971) and Barro (1981) pointed out, permanent changes in government expenditures may cause an immediate decline in private consumption because these expenditures have a high degree of substitutability for private consumption and increase the present value of future taxes.

The simplest way to model a transitory expansionary fiscal policy is to assume that the initial increase in government expenditure, which occurs at times t_0, declines over time until it reaches its initial level at time τ. It is also assumed that the private sector perfectly anticipates the time path of government expenditure, that is, when the budget deficit, which is caused by the rise in expenditure, unexpectedly occurs, the private sector immediately anticipates that it will disappear in the periods ahead. If G(t) is the level of nominal government expenditure expressed in foreign currency, the expansionary fiscal policy can be described as follows:[4]

(8) $$G(t) = b(t - \tau) \qquad t_0 \leqslant t \leqslant \tau;\ b < 0$$

$$G(t) = 0 \qquad t > \tau \, .$$

It is reasonable to assume that government expenditure declines to its initial level, and thus the budget deficit disappears, before the economy reaches its steady state equilibrium. Thus, in the neighborhood of steady state equilibrium $G(t) = 0$. Let F be the stock government debt denominated in foreign currency; F is equal to:

(9) $$F(t) = \int_{t_0}^{t} G(s)\, ds \qquad t_0 \leqslant t \leqslant \tau$$

$$F(t) = \bar{F} \qquad t > \tau \, .$$

Given the assumptions made before, as soon as the government increases its expenditures, the private sector anticipates the amount of official foreign debt that the country will eventually accumulate as a result of the expansionary fiscal policy. Finally, it is assumed that the government increases income taxes (T) to be able to service the stocks of both foreign and domestic debts. The budget constraint of the government can be thus expressed as

$$e\dot{F} = eG - T + iB^T + i^*eF$$

as $G \to 0$ and $\dot{F} \to 0$, interest payments are equal to tax revenues,

(10) $$T = iB^T + i^*eF \, .$$

In the real sector, two goods are demanded and produced: traded goods and nontraded goods. It is assumed that wage flexibility ensures full employment at any point in time. The excess demand for traded goods, H(. .), is a function of the relative price of traded to nontraded goods (P_T/P_N) the stock of real wealth (W/P) and taxes (T/P_N); P is the aggregate demand deflator which is a geometric average of the prices of traded and nontraded goods, that is, $P = P_T^{\alpha} P_N^{1-\alpha}$. Arbitrage in the market for traded goods ensures that the law of one price always holds in this market, so that $P_T = eP_T^*$. It is further assumed that P_T^* is constant--and set equal to 1--so that the relative price of traded goods can then be written as e/P_N. The current account (CA) is equal to the trade account plus the service account:[5]

$$(11) \quad CA = -\dot{B}^* + e\dot{A} = -e\left[H(e/P_N, W/P, T/P_N) + i^*(F - A) - iB^*/e\right] \quad H_{e/P_N}, H_{T/P} < 0 \quad H_{W/P} > 0$$

or

$$(12) \quad CA = \theta(e, P_N, W, F, A, T, B^*, i) \quad \theta_{P_N} \gtrless 0 \quad \theta_A, \theta_e, \theta_T > 0 \quad \theta_{B^*}, \theta_W, \theta_F, \theta_i < 0$$

where $\theta_F = -\theta_A$. The sign of the partial derivative of the price of nontraded goods is ambiguous. An increase P_N, on the one hand, creates an excess demand for traded goods because it reduces its relative price; on the other hand, the increase reduces demand because it pushes the general price level up, thus reducing real wealth. In this paper, it is assumed that θ_{P_N} is negative, which, as it is shown later on, is a sufficient--but not necessary--condition for the stability of the model.

In the market of nontraded goods, demand is a positive function of the relative price of traded goods (e/P_N), as well as of real government expenditure (eG/P_N), real wealth (W/P), and taxes (T/P_N). By using the definition for the price level, it is possible to write the equilibrium condition in the nontraded goods market as

$$(13) \quad X(e, P_N, G, W, T) = 0 \qquad X_e \gtrless 0 \quad X_{P_N}, X_T < 0 \quad X_G, X_W > 0 \; .$$

The ambiguity of the sign of X_e mirrors that of θ_{P_N}. Once more, it is assumed that X_e is negative, an assumption that is justified by stability conditions.[6]

Summing up, the entire model is composed of three equations, (7), (12), and (13), which determine three endogenous variables, the exchange rate, the stock of nominal wealth, the price of nontraded goods and, consequently, the aggregate price level.

II. The Case of No Intervention

This section analyzes the case in which the government acquires foreign exchange by selling government debt denominated in foreign currency to nonresidents; then, it exchanges domestic currency for the foreign exchange with the central bank; and finally it purchases nontraded goods and services from the private sector. Because the paper focusses on a "pure" fiscal policy, it is assumed that the monetary authorities can prevent the increase in official international reserves from having an expansionary effect on the monetary base, perhaps by inducing a change in bank reserves or by swapping the foreign exchange with another central bank. As a result, the budget deficit initially affects neither the currency composition of the domestic residents' portfolio nor their wealth. Because the currency composition does not change, this case is called the case of no intervention.

In the no intervention case, the stock of domestic currency bonds (B^T) remains constant so that $\dot{B} = -\dot{B}^*$. Consequently, the balance of payments equilibrium can be rewritten as

$$CA = \dot{B} + e\dot{A} \; .$$

As it will become clear in the next section, which considers the case of sterilized intervention, it is convenient to use the stock of wealth evaluated at the initial long-run exchange rate (w) as one of the state variables, together with the exchange rate.[7] Thus,

$$d\bar{W} = d\bar{w} + \bar{A}d\bar{e} \tag{14}$$

where a bar above a variable indicates its value in the steady state. In the neighborhood of long-run equilibrium

$$\dot{w} = CA$$

because portfolio equilibrium is achieved in the long run.

In order to find the steady state response of the endogenous variables to an increase in external official borrowing, CA and $\dot{e}$ are set equal to zero in equations (7) and (12); second, (7), (12) and (13) are totally differentiated and the resulting expressions are used to solve for $d\bar{P}_N$;[8] and, third, $d\bar{T}$ is obtained by differentiating totally (10).[9] The steady state equilibrium in the financial market and in the goods market can now be described by the two equations,

$$0 = \gamma_W d\bar{w} + (\gamma_W \bar{A} + \gamma_e)d\bar{e} \tag{15}$$

and

$$0 = [Z_W\bar{A} + Z_e + Z_T(i^*\bar{F} + \bar{B}^T\phi_W\bar{A}) - \phi_W\bar{B}^*\bar{A}]d\bar{e} + (\theta_F + Z_T i^*\bar{e})d\bar{F} \tag{16}$$

$$+ (Z_W + \bar{B}^T\phi_W Z_T + \theta_A/\bar{e} - \phi_W\bar{B}^*)d\bar{w}$$

where

$$Z_W = \theta_W - \theta_{P_N}(X_W/X_{P_N}) < 0$$

$$Z_e = \theta_e - \theta_{P_N}(X_e/X_{P_N}) > 0$$

$$Z_T = \theta_T - \theta_{P_M}(X_T/X_{P_N}) > 0$$

and $\theta_F + Z_T i^*\bar{e}$ is always negative if the private sector marginal propensity to import out of disposable income is less than one. Equation (16) can be rewritten more compactly as

$$(17) \qquad 0 = \Delta_e d\bar{e} + \Delta_w d\bar{w} + (\theta_F + Z_T i^*\bar{e})d\bar{F}$$

In general, the signs of both Δ_e and Δ_w are ambiguous; however, it is easily shown that $\Delta_e > 0$ and $\Delta_w < 0$ are sufficient conditions for the stability of the model. The ambiguity of Δ_e is caused by the capital gains and losses on financial assets that a depreciation generates. In general, a depreciation improves the current account by increasing the relative price of traded goods and by reducing real wealth (Z_e is positive). However, a depreciation increases both wealth, by generating capital gains for the domestic holders of foreign assets ($\bar{A}Z_W$ is negative), and interest payments to the foreign holders of domestic assets, given the positive correlation between wealth and domestic interest rates ($\phi_W\bar{B}^*\bar{A}$ is positive). The ambiguity of Δ_w depends on the uncertain effect of wealth on the current account. On the one hand, a decline in wealth reduces spending thus improving the current account. On the other hand, the decline drives the interest rate down, thus reducing the amount of taxes that have to be levied to service the government debt, which, in turn, boost spending.

In order to find the steady state changes in the exchange rate and in the stock of wealth that are caused by a temporary increase in the budget deficit, the equations (15) and (17) are solved for $d\bar{e}$ and $d\bar{w}$:

$$d\bar{e}/d\bar{F} = D^{-1}[\gamma_W(\theta_F + Z_T i^*\bar{e})] > 0$$

$$d\bar{w}/d\bar{F} = -D^{-1}[(\gamma_W\bar{A} + \gamma_e)(\theta_F + Z_T i^*\bar{e})] < 0$$

where D is the determinant of the matrix of the coefficients of the two equations of the model and is always negative

$$D = (\gamma_W \bar{A} + \gamma_e)\Delta_w - \gamma_W \Delta_e < 0 .$$

The change in nominal wealth, which can be found by substituting the expressions for $d\bar{e}$ and $d\bar{w}$ into the definition of dW, is equal to

$$d\bar{W}/d\bar{F} = -D^{-1}\gamma_e(\theta_F + Z_T i^* \bar{e}) < 0 .$$

Finally, the price of nontraded goods, declines in the steady state;

$$d\bar{P}_N/d\bar{F} = -(X_W/X_{P_N} + \bar{B}^T \phi_W)d\bar{W}/d\bar{F} - (X_e/X_{P_N} + X_T i^* \bar{F}/X_{P_N})d\bar{e}/d\bar{F} - X_T i^* \bar{e}/X_{P_N} < 0 .$$

Because the exchange rate depreciates and the price of nontraded goods declines, the real exchange rate (e/P_N) depreciates in the steady state.

The equilibrium conditions (equations (15) and (17)) are illustrated in Figure 1a. The budget deficit shifts the $\dot{w} = 0$ schedule to the left ($\dot{w}' = 0$) without affecting the $\dot{e} = 0$ schedule. Thus, in the long run, the budget deficit depreciates the exchange rate (from Z to X in the figure) and reduces wealth. This occurs because the policy increases the stock of net official foreign debt. The larger stock of debt implies that the country must develop a trade surplus sufficient to service the external debt in order to meet the long-run constraint of a balanced current account. The trade surplus is induced by a depreciation of the nominal exchange rate that reduces the consumption of traded goods by raising their relative price and by causing, other things being equal, a decline in real wealth.

This also explains why a temporary fiscal shock causes a permanent depreciation of the real exchange rate. Because the expansionary fiscal policy is temporary, the excess demand for nontraded goods, which is induced by the government, disappears in the steady state. However, the increase in taxes and the decline in wealth tend to create an excess supply of nontraded goods. A real exchange rate depreciation, which permanently reduces the relative price of nontraded goods, is thus needed to restore equilibrium in that market. It is interesting to note that the depreciation is the result of the long-run budget constraint that the economy faces, an element emphasized, among others, by Rodriguez (1979) and Obstfeld (1981) in their analyses of fiscal policy in the open economy.

To study the dynamic paths that the exchange rate and w follow after market participants learn about the changes in fiscal policy. The system of equations (7), (12) and (13) is linearized in the neighborhood of the steady state. The system can then be solved for $\dot{e}$ and $\dot{w}$ by following the same steps as in the previous paragraphs

$$(18) \qquad \begin{vmatrix} \dot{e} \\ \dot{w} \end{vmatrix} = \begin{vmatrix} \gamma_w \bar{A} + \gamma_e & \gamma_e \\ \Delta_e & \Delta_w \end{vmatrix} \begin{vmatrix} e - \bar{e} \\ w - \bar{w} \end{vmatrix} + \begin{vmatrix} 0 \\ \theta_F + Z_T i^* \bar{e} \end{vmatrix} (F - \bar{F})$$

Because, the determinant of the matrix, which is equal to D, is always negative, the model, like virtually every exchange rate model, is characterized by saddle path stability. In the Appendix, it is shown that the stable arm of the saddle path can be expressed as:

$$(19) \qquad e(t) - \bar{e} = \mu_1\big(w(t) - \bar{w}\big) + k\bar{F}/\lambda_2 + kQ(t)$$

where

$$k = \mu_1(\theta_F + Z_T i^* \bar{e}) > 0$$

$$Q(t) = 0 \qquad t > \tau$$

$$Q(t) \neq 0, \quad \dot{Q}(t) < 0 \quad t \leq \tau .$$

Because μ_1 is negative, a depreciation exchange rate is accompanied by declining w in the transition to the steady state. As it is shown in (19), the relationship between e(t) and w(t) becomes linear when the budget deficit disappears at time τ (after time τ, Q(t) is equal to zero). Before that, the stable arm of the saddle path is convex in w ($\dot{Q}(t)$ is negative).

The dynamics of the system can be better described with the help of Figure 1a. In the case of no intervention, when government expenditures increase, the private sector immediately reckons the stock of official foreign debt that the country will eventually accumulate as a result of the expansionary fiscal policy. As a result, market participants anticipate the long-run depreciation of the exchange rate and immediately adjust their portfolios by moving into foreign assets, thus depreciating the actual exchange rate. In the figure, an unanticipated budget deficit causes an immediate depreciation of the exchange rate equal to ZD.

From the level reached at D, the exchange rate continues to depreciate towards its long-run equilibrium level at X. However, as equation (19) indicates, the exchange rate initially depreciates at a slow (but increasing) rate because the excess demand for nontraded goods, originating in the government sector, tends to raise the relative price of nontraded goods. After a while (precisely when $t = \tau$) the pressure on the relative price of nontraded goods disappears because the expansionary fiscal policy is temporary. In the figure, this occurs at H. From H, the exchange rate depreciates at a constant rate that is determined by the decline in wealth caused by the current account deficit.

III. The Case of Sterilized Intervention

In the case of no intervention, an expansionary fiscal policy depreciates the exchange rate because it initially affects only the goods market. In terms of Figure 1a, the policy shifts only the $\dot{w} = 0$ schedule. The authorities can therefore maintain the exchange rate if they shift the $\dot{e} = 0$ schedule to the left by adopting a policy of sterilized intervention.

The simplest way to introduce intervention into the model is to assume that the government borrows abroad in excess of its cumulated expenditures; the foreign exchange proceeds are then used to buy domestic currency bonds from residents.[10] Thus, the steady state stock of official external debt in the case of sterilized intervention ($\bar{F}'$) is equal to

$$\bar{F}' = (1 + \beta)\bar{F}$$

where β is a constant greater than zero and $\bar{F}$ is the stock of official external debt when no intervention occurs. The reduction in the stock of domestic currency bonds ($-d\bar{B}^T$) caused by the policy of sterilized intervention is then equal to $\bar{e}\beta d\bar{F}$.[11] The two equations describing the steady state equilibrium in the goods and financial markets, now become[12]

$$0 = (\gamma_e + \gamma_W\bar{A})d\bar{e} + \gamma_W d\bar{w} - \gamma_B\beta\bar{e}d\bar{F}$$

$$0 = \Delta_e d\bar{e} + \Delta_w d\bar{w} + [\theta_F + Z_T(i^* - \bar{i}\beta)\bar{e}]d\bar{F} .$$

The two-equation system is solved once more for $d\bar{e}$ and $d\bar{w}$

$$(20) \qquad d\bar{e}/d\bar{F} = D^{-1}[\Delta_w\gamma_B\beta + \gamma_W(\theta_F + i^*\bar{e}Z_T) - \gamma_W\bar{i}\beta Z_T\bar{e}] \gtrless 0$$

(21) $$d\bar{w}/d\bar{F} = -D^{-1}[(\gamma_W\bar{A} + \gamma_e)(\theta_F + Z_T i^*\bar{e} - \gamma_W\bar{i}\beta Z_T\bar{e}) + \gamma_B\beta\Delta_e] < 0 \; .$$

The change in the long-run exchange rate is uncertain. The first term in the parenthesis of equation (20) shows the amount of the appreciation that is induced by the intervention policy. As in every portfolio model, the appreciation is negatively related to the substitutability between domestic and foreign currency denominated bonds (γ_B). The second term captures the long-run depreciation that puts downward pressure on the exchange rate as soon as the budget deficit and the intervention policy take place. The third term indicates the size of the depreciation that is needed to offset the expansionary effect of the decline in the taxes, which is caused by the reduction in interest payments on domestic bonds.

Equation (20) shows that the authorities can theoretically achieve any exchange rate target by choosing the appropriate level 'β' of sterilized intervention. If they adopt such a policy, however, they face a trade-off between short- and long-run stabilization of the exchange rate. To illustrate this point, it is convenient to turn to the dynamics of the model. In the case of sterilized intervention, the system of differential equations governing the motion of e and w remains (18), with the vector

$$[-\gamma_B\beta \qquad \theta_F + Z_T\bar{e}(i^* - \bar{i}\beta)]'$$

multiplying $(F - \bar{F})$. Thus, the model is still characterized by saddle path stability and, as it is shown in the Appendix, the shape of the stable arm is similar to that of the case of no intervention.[13] If the authorities want to prevent the exchange rate from depreciating immediately, they choose β that exactly accommodates the portfolio adjustment of the private sector triggered by the news of the budget deficit. However, because the stable arm of the saddle path is negatively sloped, the exchange rate soon begins to depreciate. This case is illustrated in Figure 1b. Sterilized intervention maintains the exchange rate at K, even though the budget deficit induces expectations of an exchange rate depreciation. From K, the exchange rate initially depreciates at a slow--but increasing--rate until it reaches J. Then, it depreciates at a constant rate from J to L.[14]

IV. Conclusions

Since the beginning of the floating period, the governments of many small industrial countries have both financed budget deficits in the international capital market and changed the currency composition of the outstanding debt. In the paper, it is argued that these debt management policies cannot prevent the real exchange rate from depreciating in the long run, even though, theoretically, governments can temporarily achieve their exchange rate targets. In practice, however, the size of official external borrowing and intervention that is needed to maintain the exchange rate even for a short period of time may not be feasible because countries may be rationed in the international capital market when their external debt grows rapidly and expectations of a long-run depreciation are sufficiently strong.

APPENDIX

The Dynamics of the System

In this appendix, the solution of the system of differential equations (18) is derived (Kaplan, 1958). The paths followed by the exchange rate and w as they move towards their long-run equilibria are described by the two equations:

$$(22)\quad e(t) = \bar{e} + \mu_1 e^{\lambda_1 t}\left[C_1 + M\int_0^t F(s)e^{-\lambda_1 s}ds\right] + \mu_2 e^{\lambda_2 t}\left[C_2 + N\int_0^t F(s)e^{-\lambda_2 s}ds\right]$$

$$(23)\quad w(t) = \bar{w} + e^{\lambda_1 t}\left[C_1 + M\int_0^t F(s)e^{-\lambda_1 s}ds\right] + e^{\lambda_2 t}\left[(C_2 + N\int_0^t F(s)e^{-\lambda_2 s}ds\right]$$

where $\lambda_1 < 0$ and $\lambda_2 > 0$ are the two eigenvalues of the system; μ_1 and μ_2 are the eigenvectors associated with λ_1 and λ_2 and they are equal to

$$\mu_1 = \gamma_W/[\lambda_1 - (\gamma_e + \gamma_W\bar{A})] < 0$$

$$\mu_2 = \Delta_e/[\lambda_2 - (\gamma_e + \gamma_W\bar{A})] \gtrless 0 .$$

N and M are two constants equal to

$$M = -\mu_2(\theta_F + Z_T i^*\bar{e})/(\mu_1 - \mu_2)$$

$$N = \mu_1(\theta_F + Z_T i^*\bar{e})/(\mu_1 - \mu_2) .$$

C_1 and C_2 are two constants determined by the initial and terminal conditions respectively.

The case of no intervention is analyzed first. The initial step is to determine the value of the constants in equations (22) and (23). C_1 is found by letting $t \to 0$ in (23):

$$w(0) = C_1 + C_2 .$$

Because λ_2 is the positive root, the system is stable if (Gray and Turnovsky, 1979):

$$\lim_{t \to \infty} \left[C_2 + N \int_0^t F(s) e^{-\lambda_2 s} ds \right] = 0$$

or

$$C_2 = -N \int_0^\infty F(t) e^{-\lambda_2 t} dt \; .$$

It is assumed that this condition is satisfied. Thus,

$$C_1 = w(0) + N \int_0^\infty F(t) e^{-\lambda_2 t} dt \; .$$

Substituting the expressions for C_1 and C_2 into (22) and (23) the solution of the differential equation system becomes:

$$(24) \qquad e(t) = \bar{e} + \mu_1 e^{\lambda_1 t} \alpha(t) - \mu_2 e^{\lambda_2 t} N \int_t^\infty F(s) e^{-\lambda_2 s} ds$$

$$(25) \qquad w(t) = \bar{w} + e^{\lambda_1 t} \alpha(t) - e^{\lambda_2 t} N \int_t^\infty F(s) e^{-\lambda_2 s} ds$$

where

$$\alpha(t) = w(0) + N \int_0^\infty F(t) e^{-\lambda_2 t} dt + M \int_0^t F(s) e^{-\lambda_1 s} ds \; .$$

Equations (24) and (25) are solved for $e^{\lambda_1 t}\alpha(t)$ and then equated:

$$\left(e(t) - \bar{e}\right) = \mu_1\left(w(t) - \bar{w}\right) + \mu_1(\theta_F + Z_t i^* \bar{e})\Gamma(t)$$

where

$$\Gamma(t) = e^{\lambda_2 t} \int_t^\infty F(s) e^{-\lambda_2 s} ds \; .$$

When $t > \tau$, that is, when the budget becomes balanced, the stock of government debt $F(s)$ is a constant equal to $\bar{F}$. The equation for the stable arm is thus a straight line:

$$\left(e(t) - \bar{e}\right) = \mu_1\left(w(t) - \bar{w}\right) + k\bar{F}/\lambda_2$$

where $k = \mu_1(\theta_F + Z_T i^*\bar{e}) > 0$ which is negatively sloped because μ_1 is negative.

When $t \leqslant \tau$, that is, when government expenditures induce an excess demand for nontraded goods, the function $\Gamma(t)$ can be rewritten as

$$\Gamma(t) = e^{\lambda_2 t}\int_t^{\infty} F(s)e^{-\lambda_2 s}ds = e^{\lambda_2 t}\left[\int_t^{\tau} F(s)e^{-\lambda_2 s}ds + \int_{t>\tau}^{\infty} F(s)e^{-\lambda_2 s}ds\right] =$$

$$= Q(t) + \bar{F}/\lambda_2$$

where

$$Q(t) = e^{\lambda_2 t}\int_t^{\tau} F(s)e^{-\lambda_2 s}ds \ .$$

The equation for the stable arm is thus:

$$\left(e(t) - \bar{e}\right) = \mu_1\left(w(t) - \bar{w}\right) + k\bar{F}/\lambda_2 + kQ(t) \ .$$

Along the stable arm, the rate of depreciation is:

$$\dot{e}(t) = \mu_1\dot{w}(t) + k\dot{Q}(t)$$

which is now a function of $\dot{Q}(t)$. In order to study how the stable arm moves through time, one has to determine the sign of $\dot{Q}(t)$. By integrating by parts twice and using (8) and (9), $\dot{Q}(t)$ can be shown to be equal to:

$$\dot{Q}(t) = -F(t) - G(t)/\lambda_2 + b/\lambda_2^2\left[1 - e^{\lambda_2(t-\tau)}\right] < 0$$

which is always negative because b and $(t-\tau)$ are always negative. As a result, for any $w(t)$, the exchange rate will depreciate at an increasing rate between t_0 and τ.

In the case of sterilized intervention, the equation for the stable arm of the saddle path is:

$$\left(e(t) - \bar{e}\right) = \mu_1\left(w(t) - \bar{w}\right) + h\bar{F}/\lambda_2 + hQ(t)$$

where

$$h = \mu_1\left[\theta_F + Z_T\bar{e}i^* - \overline{ei}\beta Z_T + \gamma_B\beta\right] > 0 \ .$$

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FOOTNOTES

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[1]The governments in many industrial countries have tried to maintain their exchange rates because of the perception that resources are immobile in the short run and thus inelastic with respect to exchange rate movements. It is thought that the primary effect of these movements is to affect the domestic price level, as well as the domestic wage level. The ultimate effect is to put pressure on monetary authorities in order to accomodate the initial shock, thus triggering "vicious" circles. This view is clearly expressed in the Report of the Working Group on Exchange Market Intervention (March 1983). The Group was established at the Versailles Summit in June 1982.

[2]Various authors have argued that exchange rate models may become unstable if the country has a negative foreign asset position. For example, see Masson (1981). By contrast, Henderson and Rogoff (1981) found that the private sector's portfolio composition is not a source of exchange rate instability in portfolio balance models with rational expectations.

[3]A dot above a variable indicates a time derivative.

[4]For simplicity, it is assumed that the initial level of government expenditure is zero. Nothing changes if another initial level of government expenditure is chosen.

[5]The term $e\dot{F}$ is not included in the balance of payments equation because the government and the Central Bank are aggregated: when external official

borrowing increases, reserves increase as well so that the two changes cancel out.

[6]Dornbusch (1976) makes similar assumptions.

[7]The use of this variable is suggestd in Henderson and Rogoff (1982).

[8]In doing so, the assumptions that the country is small, that fiscal policy is independent of monetary policy, that the stock of domestic government debt is unchanged and that the level of government expenditure goes back to its initial level in the steady state, are used, that is, $d\bar{M} = d\bar{i}^* = d\bar{W}^* = d\bar{G} = d\bar{B}^T = 0$. In addition, the expressions (14) and $d\bar{i} = \phi_W d\bar{W}$ are also used; ϕ_W, which is always positive, is obtained from the equilibrium condition in the domestic money market, that is: $i = \phi(i^*, \dot{e}, M, W)$ ϕ_i^*, $\phi_{\dot{e}}$, $\phi_M < 0$, $\phi_W > 0$.

[9]That is $d\bar{T} = i^*\bar{F}d\bar{e} + i^*\bar{e}d\bar{F} + \bar{B}^T d\bar{i}$.

[10]In theory, a government could use its foreign exchange reserves to intervene in the foreign exchange market. In practice, however, countries tend to borrow the foreign exchange that is needed to implement their intervention policies.

[11]I am implicitly assuming that the authorities exchange domestic for foreign currency denominated securities at the exchange rate that prevails after the intervention policy is completed. On this point, see also footnote (13).

[12]To obtain the two equations, the following expressions were used

$$\bar{e}d\bar{A} = d\bar{w} + \beta\bar{e}d\bar{F}$$

and

$$d\bar{T} = (i^*\bar{F} + \bar{B}^T\phi_W\bar{A})d\bar{e} + \bar{B}^T\phi_W d\bar{w} + (i^* - i\beta)\bar{e}d\bar{F} .$$

[13]In analyzing the dynamics of the model, w is held constant immediately after the fiscal disturbance. However, w constant is only an approximation which is of the same order of magnitude as a second term in a Taylor expansion and, therefore, can be neglected (see Henderson and Rogoff 1982). The approximation is exactly equal to the private sector purchase of foreign assets from the authorities multiplied by the immediate change in the exchange rate that is caused by the policy of sterilized intervention.

[14]Although the exchange rate path in the case of intervention is similar to that of no intervention, the rate of depreciation from D to H in Figure 1 differs from the rate of K to J in Figure 1b. The expression for the saddle path in Figure 1b is given in the Appendix.

Table 1. Budget Deficit and External Official Borrowing: Selected Small Industrial Countries

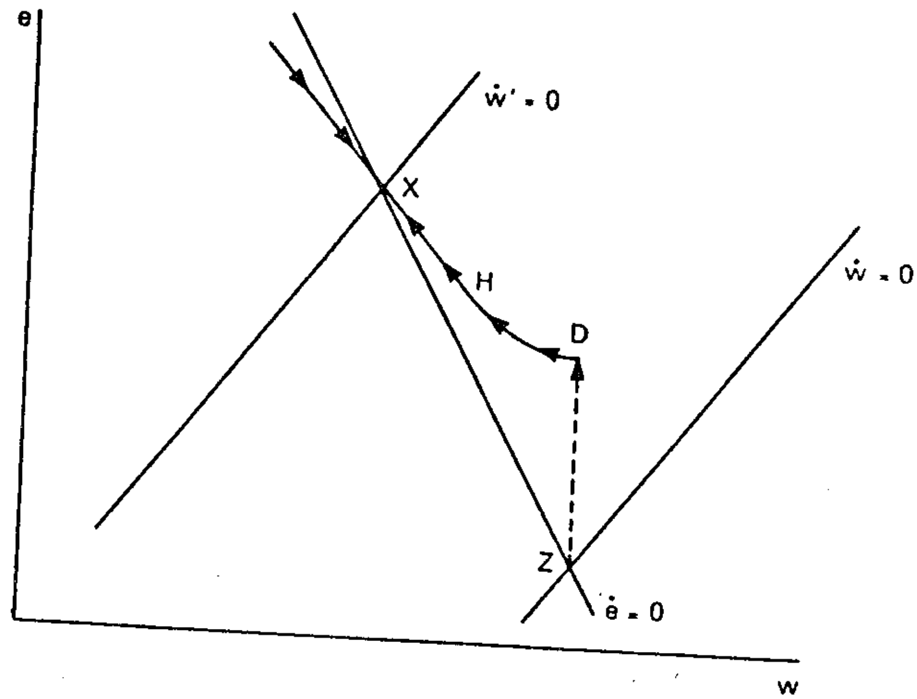
(In per cent)

		Budget deficit/GNP	Foreign currency government debt/ Total government debt (Both expressed in local currency)
Austria	1976	18.5	35.3
	1977	20.8	40.3
	1978	23.8	43.2
Belgium	1979	6.5	3.4
	1980	8.7	7.7
	1981	13.4	15.7
New Zealand	1973	2.6	12.4
	1974	3.9	20.5
	1975	8.8	26.3
Denmark	1979	6.1	55.4
	1980	7.1	52.4
	1981	10.5	46.3
Finland	1976	0.2	54.7
	1977	1.0	59.4
	1978	1.6	63.7
	1979	3.0	60.8
Italy	1979	11.1	5.2
	1980	11.0	6.9
	1981	13.5	9.5
Sweden	1978	5.0	10.6
	1979	7.3	15.0
	1980	8.3	23.3

Sources of data: International Monetary Fund, _International Financial Statistics_; Sveriges Riksbank, _Annual Report_, 1981; Banca d' Italia, _Annual Report_, 1982; Danmarks Nationalbank, _Annual Report_, 1982; OECD, _New Zealand_, _Economic Surveys_, 1982; and OECD, _Austria_, _Economic Surveys_, 1982.

Figure 1

(a) The case of no intervention



(b) The case of sterilized intervention