EFFECTS OF NON-ASSUMABLE MORTGAGE FINANCE ON HOUSING DEMAND AND RELOCATION DECISIONS

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This analysis seeks to provide clarification of the effect that an unexpected increase in inflation would have on the demand for housing services and housing relocation choice when mortgages are of the standard fixed rate variety with a non-assumability provision. A mortgage contract with a non-assumability restriction is equivalent to requiring that the outstanding mortgage principal becomes due upon the sale of the mortgagor's house. In June of 1982, the legality of bank enforcement of non-assumability provisions in mortgage contracts was upheld by a Supreme Court decision.[1] This ruling could have important consequences, as during 1981, the year prior to this decision, assumed mortgages accounted for 42 percent of all home purchases.[2]

The model used in this paper is one of an optimizing consumer who maximizes utility over his life cycle. Except for a one-time unexpected jump in inflation, perfect foresight behavior is assumed. After inflation makes its one-time increase, the consumer faces the decision of remaining in his current home, collecting the capital gain on his mortgage, or moving to a new house. The consumer's problem is then to decide at which point in time to forego the remaining unrealized capital gain on his mortgage to move to another house.

The reason for wanting to move is modeled in two different ways. One way is by assuming that the quantity of housing services consumed while living in the original house is fixed, i.e., the homeowner cannot increase or decrease his housing services by, for example, building an addition or renting out a room. The consumer's desire to move is then for the purpose of attaining a more desired stock of housing.

The alternative way is to simply assume that while the quantity of housing services can be changed without moving, at some point in time if the consumer doesn't move, he suffers a utility loss, ℓ, per unit time. This way of modeling the incentive to move is designed to capture non-housing reasons for moving, such as the need to move to a different geographic area because of job opportunity changes or a change to retirement status.

The following analysis abstracts from reality in certain respects. Because of the perfect foresight assumption, future inflation uncertainty is not dealt with in that agents expect inflation to remain unchanged at its new higher level. Housing demand is also studied from a partial equilibrium point of view in that the real price of housing stock is assumed fixed. Another assumption made is that consumers bear no costs to adjusting their quantity of housing services. Moving costs are not explicitly modeled. While this is a strong assumption if one is interested in a realistic description of individual housing selection, neglecting costs of adjustment are probably not an essential factor in comparing the qualitative effects of assumable versus non-assumable mortgages, though these costs probably affect the magnitude of the difference.

In spite of these caveats, some qualitative results found in this paper would likely remain in a more complete analysis. It is found that the demand for housing services by consumers with non-assumable mortgages will fall relative to consumers with assumable mortgages following an unexpected increase in inflation. An increase in inflation may even lower the demand for housing by non-assumable mortgagors to a level less than that demand had no inflation occurred at all.

Evidence is also presented that indicates that the time delay in moving to a new home is an increasing function of the magnitude of the inflation change for low levels of inflation but is a decreasing function at higher levels of inflation. Also, ceteris paribus, high tax bracket individuals will tend to move sooner than low tax bracket ones. Finally it is shown that following an unexpected increase in inflation, the holder of the non-assumable mortgage (mortgagee) can minimize his capital loss by offering incentives to the mortgagor to pay off the mortgage early.

I. The Model

A. Assumable Mortgage

The model used in this analysis is derived from those described by Artle and Varaiya [1] and Wheaton [4]. A standard fixed-rate mortgage contract is agreed to at time 0, when inflation equals zero and the nominal rate of interest = real rate of interest = r. The consumer is assumed to start off life with zero marketable assets and therefore must borrow to cover the full value of his initial stock of housing, this quantity being $h(t = 0) = h_0$. Maturity of the mortgage is T years which is assumed to coincide with the consumer's lifespan. The fixed continuous mortgage payment per unit time is then given by:

$$p = \frac{h_0 r}{1 - e^{-rT}} .$$

Shortly after the mortgage is written, the inflation rate jumps unexpectedly from 0 to i and is now expected to persist at rate i forever.

The consumer's problem is then to maximize lifetime utility subject to a budget constraint. Utility depends on consumption of housing, h(t), and consumption of all other goods, c(t). The consumer is also assumed to receive a constant real income payment of y per unit time. After the inflation increase, consumers may borrow or save at the before tax rate of i + r (i.e., the nominal interest rate is assumed to rise one for one with inflation via the Fisher effect), but may only borrow up to the amount of their tangible or "marketable" assets, i.e., they cannot borrow against future income. This may be described as a "liquidity constraint." It is also assumed that consumers can deduct nominal interest expense from income and are taxed on nominal interest income, at the income tax rate ϕ, $0 < \phi < 1$.

In the case of an assumable mortgage, the consumer's problem is:

$$\max_{c(t),h(t)} \int_0^T e^{-\delta t} u\big(c(t),\ h(t)\big)dt$$

Subject to

$$\dot{a}(t) = \big(a(t) - h(t)\big)\big(r(1 - \phi) - \phi i\big) + y - c(t)$$

$$a(0) = \frac{ip}{r(i + r)}\left(1 - e^{-(i + r)(1 - \phi)T}\right) + \frac{ip(1 - \phi)\left(e^{-(i+r)(1 - \phi)T} - e^{-rT}\right)}{r\big(i(1 - \phi) - \phi r\big)}$$

$$a(T) = 0$$

$$a(t) \geq 0 \qquad 0 \leq t \leq T$$

where $a(t)$ is the real value of tangible assets at time t. $a(t) \geq 0$ denotes the above mentioned liquidity constraint, and $a(0)$ equals the instantaneous capital gain made on the mortgage contract from the unexpected increase in inflation which is assumed to occur just after the mortgage is written. δ is the rate of time preference.

Since the mortgage is assumable, when the consumer decides to move, he can sell his mortgage to the buyer of his old home. Therefore, he is assured of collecting the entire capital gain on the mortgage from the inflation increase. This capital gain, $a(0)$, can be re-written as the difference between the present value of after tax mortgage payments using the before inflation interest rate, r, and the present value using the after inflation interest rate, r + i;

$$a(0)=\int_0^T \left(p(1-\phi)+\phi p e^{-(T-t)r}\right)e^{-r(1-\phi)t}dt-\int_0^T \left(p(1-\phi)+\phi p e^{-(T-t)r}\right)e^{-(i+r)(1-\phi)t}dt$$

The Hamiltonian for this problem is

$$H=u\left(c(t),\ h(t)\right)e^{-\delta t}+\mu(t)\left[y-c(t)+\left(a(t)-h(t)\right)\left(r(1-\phi)-\phi i\right)\right] + \lambda(t)a(t)$$

which leads to the first order conditions

$$\frac{\partial u}{\partial c}e^{-\delta t} = \mu(t) \qquad \frac{\partial u}{\partial h}e^{-\delta t} = \left(r(1-\phi)-\phi i\right)\mu(t)$$

$$\dot{\mu}(t) = -\mu(t)\left(r(1-\phi)-\phi i\right)-\lambda(t) \qquad \dot{a}(t)=y-c(t)+\left(a(t)-h(t)\right)\left(r(1-\phi)-\phi i\right)$$

If the constraint $a(t) \geqslant 0$ is not binding, then $\lambda(t) = 0$ and $\mu(t) = \mu_0 e^{-Dt}$ where $D = r(1-\phi) - \phi i$ is the real after-tax interest rate faced by the consumer. If we then assume utility to be of the form $U\left(c(t),\ h(t)\right) = \alpha \ln\left(c(t)\right) + \beta \ln\left(h(t)\right)$, we can explicitly solve for optimal paths of consumption and housing:

$$c(t) = \frac{\alpha\delta\left(\frac{y}{D}\ (1 - e^{-DT}) + a(0)\right)e^{-(\delta-D)t}}{(\alpha+\beta)\ (1 - e^{\delta T})} \tag{1}$$

$$h(t) = \frac{\beta\delta\left(\frac{y}{D}(1 - e^{-DT}) + a(0)\right)e^{-(\delta-D)t}}{(\alpha+\beta)(1 - e^{-\delta T})D} \tag{2}$$

For the case where $a(0) = 0$, Wheaton [4] shows that the liquidity constraint will never be binding if $\delta < D$. This continues to hold when $a(0) > 0$, and therefore (1) and (2) will be optimal when $\delta < D$.

When $\delta > D$ and $a(0) > 0$, the paths of housing and consumption will be declining over time. There is an initial period $0 \leqslant t < t_1$ when solutions will be of the form $c(t) = c_0 e^{-(\delta-D)t}$ and $h(t) = h_0 e^{-(\delta-D)t}$ during which time assets decline until $a(t_1) = 0$. From $t_1 \leq t \leq T$ the consumer is liquidity constrained so that the paths of housing and consumption are

$$c(t) = \frac{\alpha}{\alpha + \beta} y \tag{3}$$

$$h(t) = \frac{\beta y}{(\alpha + \beta)D} \tag{4}$$

While t_1 cannot be solved for explicitly, the first order condition that t_1 must satisfy is derived in Appendix 1. See Figure 1 for the paths of consumption and housing for both the $\delta < D$ and $\delta > D$ cases.

B. Non-Assumable Mortgage - Housing Stock Fixed Before Moving

In this case, the consumer is unable to adjust the housing services he consumes prior to moving. The consumer's problem is then

$$\max_{c(t),h(t),\tau} \int_0^{\tau} e^{-\delta t} U\big(c(t),\ h_0\big)dt + \int_{\tau}^{T} e^{-\delta t}\ U\big(c(t),\ h(t)\big)dt$$

subject to

$$\mathring{a}(t) = \big(a(t) - h_0\big)\big(r(1-\phi) - \phi i\big) + i(1-\phi)\frac{p}{r}\big(1 - e^{-r(T-t)}\big)e^{-it} + y - c(t), \qquad 0 < t \leq \tau \tag{5}$$

$$\mathring{a}(t) = \big(a(t) - h(t)\big)\big(r(1 - \phi) - \phi i\big) + y - c(t), \qquad \tau < t \leq T \tag{6}$$

$$a(0) = 0$$

$$a(T) = 0$$

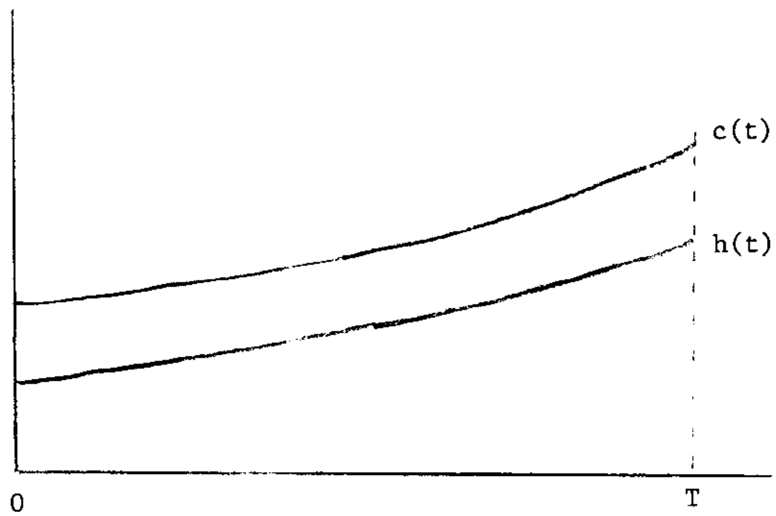
$$a(t) \geq 0 \quad \forall t$$

τ denotes the time when the consumer decides to pay off the remaining principle of his initial mortgage and move and change the desired quantity of housing services. Note that during $0 < t < \tau$, the additional income per unit

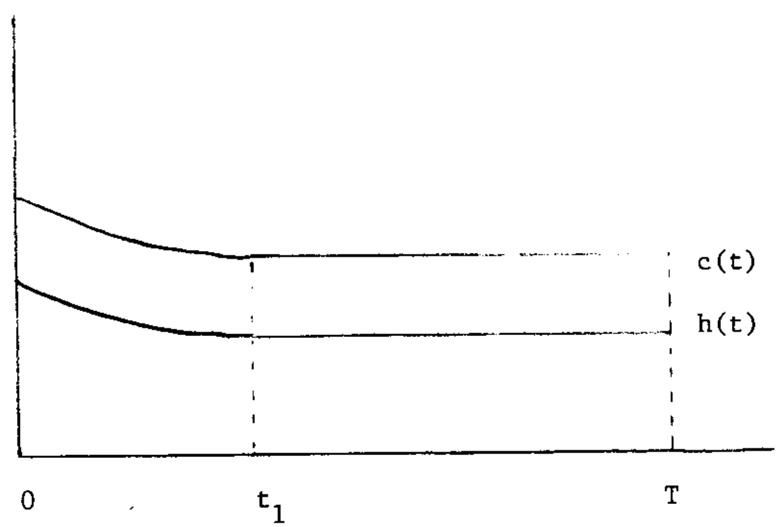
Figure 1

Assumable Mortgage

$\delta < D$



$\delta > D$



time from the non-assumable low interest rate mortgage is equal to $i(1 - \phi)$ times the real remaining principle of the mortgage.

One can, in principle, solve for the optimal paths of consumption for $t \leq \tau$ and housing and consumption for $t > \tau$. This is done below for the unconstrained case with $U\left(c(t), h(t)\right) = \alpha \ln\left(c(t)\right) + \beta \ln\left(h(t)\right)$. The liquidity constraint will not be binding for all t when $\delta < D$, but may also hold in some cases for $\delta > D$.

In the unconstrained case, from first order conditions, c(t) and h(t) will be of the form

$$c(t) = \frac{\alpha}{\mu} e^{-(\delta - D)t} \quad \forall t$$

$$h(t) = \frac{\beta}{\mu D} e^{-(\delta - D)t} \quad \text{for } t > \tau$$

Plugging the value of c(t) into (5) and solving this first order differential equation subject to the boundary conditions $a(0) = 0$ and $a(\tau) = a_\tau$ we get a value for μ, so that

$$c^*_1(t) = \frac{(\gamma - a_\tau e^{-D\tau})\delta e^{-(\delta - D)t}}{(1 - e^{-\delta\tau})} \tag{7}$$

where $\gamma = \int_0^\tau \left(y - h_0 D + \frac{ip}{r}(1 - \phi)e^{-it}\left(1 - e^{-r(T-t)}\right)\right)e^{-Dt}\, dt$ is the present discounted value of income after mortgage payments during the period when the homeowner retains his initial mortgage.

In the same manner, we can solve differential equation (6) and get a value for μ for the period $t > \tau$, so that

$$c^*_2(t) = \frac{\alpha}{(\alpha + \beta)} \frac{\left(a_\tau + y/D\left(1 - e^{-D(T-\tau)}\right)\right)\delta e^{-(\delta - D)(t-\tau)}}{\left(1 - e^{-\delta(T-\tau)}\right)} \tag{8}$$

$$(9) \qquad h_2^*(t) = \frac{\beta}{(\alpha + \beta)D} \frac{\left(a_\tau + y/D\left(1 - e^{-D(T-\tau)}\right)\right)\delta e^{-(\delta-D)(t-\tau)}}{\left(1 - e^{-\delta(T-\tau)}\right)}$$

For a given τ, we can now solve for the optimal value of a_τ by

$$\max_{a_\tau} \int_0^\tau e^{-\delta t} U\left(c_1^*(t), h_0\right)dt + \int_\tau^T e^{-\delta t} U\left(c_2^*(t), h_2^*(t)\right)dt$$

for $U = \alpha \ln c(t) + \beta \ln h(t)$ this turns out to imply:

$$(10) \qquad \frac{\alpha}{(\alpha + \beta)} \frac{\left(a_\tau + y/D\left(1 - e^{-D(T-\tau)}\right)\right)e^{(\delta-D)\tau}}{\left(1 - e^{-\delta(T-\tau)}\right)} = \frac{(\gamma - a_\tau e^{-D\tau})}{(1 - e^{-\delta t})},$$

or

$$(11) \quad a_\tau = \frac{(\alpha + \beta)e^{-(\delta-D)\tau}\left(1 - e^{-\delta(T-\tau)}\right)\gamma - y/D\left(1 - e^{-D(T-\tau)}\right)\alpha(1 - e^{-\delta\tau})}{\alpha(1 - e^{-\delta T}) + \beta(e^{-\delta\tau} - e^{-\delta T})}$$

for $a_\tau > 0$

= 0 otherwise

Substituting (11) into (7), (8), and (9), and using equality (10) we see that

$$(12) \qquad c^*(t) = \frac{\alpha\left(\gamma + y/D(e^{-D\tau} - e^{-DT})\right)\delta e^{-(\delta-D)t}}{\alpha(1 - e^{-\delta T}) + \beta(e^{-\delta\tau} - e^{-\delta T})}, \quad 0 \leqslant t \leqslant T$$

$$(13) \qquad h^*(t) = \frac{\beta\left(\gamma + y/D(e^{-D\tau} - e^{-DT})\right)\delta e^{-(\delta-D)t}}{D\left(\alpha(1 - e^{-\delta T}) + \beta(e^{-\delta\tau} - e^{-\delta T}\right)}, \quad \tau \leqslant t \leqslant T$$

$$= h_0 = \frac{\beta\delta \frac{y}{r(1-\phi)}\left(1 - e^{-r(1-\phi)T}\right)}{(\alpha + \beta)(1 - e^{-\delta T})r(1 - \phi)}, \quad 0 \leqslant t < \tau$$

Thus, in the unconstrained case, c(t) is continuous, even at the time of moving, τ. It can also be shown that $h_0 < h^*(\tau^*)$ for $r(1 - \phi) > \delta$. Thus in the unconstrained case, desired housing takes a discrete increase when the consumer moves at time τ. (See Appendix 2.) Therefore expenditure also "jumps" at τ.

When δ is much greater than D, the liquidity constraint will be binding in spite of the income from capital gains of

$$i(1-\phi)\frac{p}{r}\left(1-e^{-r(T-t)}\right)e^{-it} \quad \text{for } t < \tau .$$

Therefore, the solution is given by:

$$(14) \quad c_1^*(t)=y+i(1-\phi)\frac{p}{r}\left(1-e^{-r(T-t)}\right)e^{-it} - h_0\left(r(1-\phi)-\phi i\right), \quad 0 < t < \tau$$

$$(15) \quad c_2^* = \frac{\alpha y}{\alpha+\beta}, \quad \tau \leq t < T$$

$$(16) \quad h_2^* = \frac{\beta y}{(\alpha+\beta)D}, \quad \tau \leq t < T$$

As in the unconstrained case, it can be shown (see appendix 2) that $h_0 < h_2^*$ so that the demand for housing again increases at time τ. Since at τ, total expenditure decreases by $i(1-\phi)\frac{p}{r}\left(1-e^{-r(T-\tau)}\right)e^{-i\tau}$, consumption must then take a discrete fall at τ. See Figure 2 for the paths of consumption and housing for both the unconstrained and liquidity constrained cases.

C. Non-Assumable Mortgage - Housing Stock Flexible Before Moving

In this case, the consumer incurs a loss in utility of ℓ per unit time prior to moving. The consumers problem is then

$$\max_{c(t),h(t),\tau} \int_0^\tau e^{-\delta t}\left(U(c(t),\, h(t)) - \ell\right)dt + \int_\tau^T e^{-\delta t}U\left(c(t),\, h(t)\right)dt$$

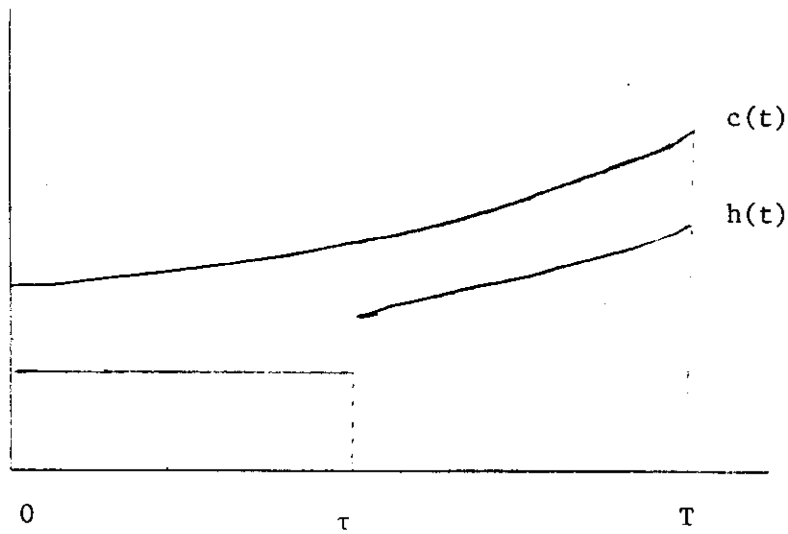
subject to

$$(17) \quad \dot{a}(t)=\left(a(t)-h(t)\right)\left(r(1-\phi)-\phi i\right)+i(1-\phi)\frac{p}{r}\left(1-e^{-r(T-t)}\right)e^{-it}+y-c(t), \quad 0<t<\tau$$

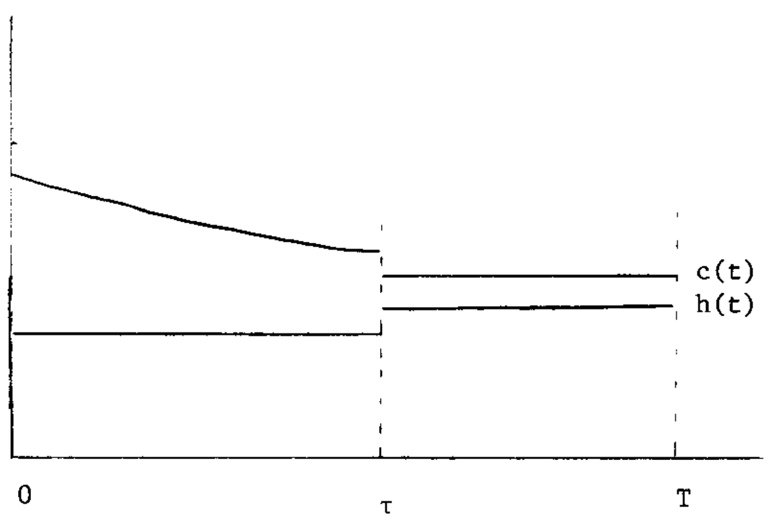
Figure 2

Non-Assumable Mortgage
Housing Stock Fixed Prior to Moving

Unconstrained



Constrained



$$(18)\qquad \dot{a}(t) = \left(a(t) - h(t)\right)\left(r(1-\phi)-\phi i\right) + y - c(t)\ , \quad \tau < t \leq T$$

$$a(0) = 0$$

$$a(T) = 0$$

$$a(t) \geq 0 \quad \forall t$$

If the liquidity constraint, $a(t) \geq 0$, is not binding $\forall t$, ($\delta < D$ is a sufficient condition for this), then using similar methods to those in part B, the solution to the consumer's problem is then;

$$(19)\qquad c(t) = \frac{\alpha\delta\psi e^{-(\delta-D)t}}{(\alpha+\beta)(1-e^{-\delta T})} \qquad 0 \leq t \leq T$$

$$(20)\qquad h(t) = \frac{\beta\delta\psi e^{-(\delta-D)t}}{(\alpha+\beta)D(1-e^{-\delta T})} \qquad 0 \leq t \leq T$$

where $\psi = \int_0^T ye^{-Dt}dt + \int_0^\tau i(1-\phi)\frac{p}{r}\left(1 - e^{-r(T-t)}\right)e^{-it}e^{-Dt}dt$

is the present discounted value of income and realized capital gains over the consumer's lifetime. Here we see that both consumption and housing demand are continuous over all t, $0 \leq t \leq T$.

The solution to the constrained problem (which will be the case if δ is sufficiently greater than D) is simply

$$(21)\qquad c(t) = \frac{\alpha}{\alpha+\beta}\left(y + i(1-\phi)\frac{p}{r}\left(1 - e^{-r(T-t)}\right)e^{-it}\right)\ , \quad 0 \leq t < \tau$$

$$(22)\qquad h(t) = \frac{\beta}{(\alpha+\beta)D}\left(y + i(1-\phi)\frac{p}{r}\left(1 - e^{-r(T-t)}\right)e^{-it}\right)\ , \quad 0 \leq t < \tau$$

along with (15) and (16) which are the optimal values of c and h for

$\tau \leq t \leq T$. In this case both consumption and the demand for housing take a discrete fall at τ.

A comment should be made concerning the assumption of the mortgage contract being written just prior to the unexpected increase in inflation at time 0. Had we assumed the mortgage was written prior to time 0, the problem would be essentially the same if $r(1 - \phi) < \delta$ since prior to time 0 the consumer would have been liquidity constrained so that $a(0) = 0$. If $r(1 - \phi) > \delta$ and $r(1 - \phi) - \phi i > \delta$ then the solutions in part B and C above are still the same except we replace γ with $\gamma + a(0)$ and ψ with $\psi + a(0)$.

If $r(1 - \phi) > \delta$ but $r(1 - \phi) - \phi i < \delta$ we then have the possibility that there may be an initial period where the unconstrained solutions will now hold, since $a(0) > 0$, where in part B and C above constrained solutions held when $a(0) = 0$.

II. Comparison of Housing Demand with Assumable and Non-Assumable Mortgages After an Unexpected Increase in Inflation

In this section it is shown that for the model where the quantity of housing services consumed prior to moving is fixed, housing demand under a non-assumable mortgage is always less than housing demand under the same conditions with an assumable mortgage. Recall that for an assumable mortgage;

$$h^A(t) = \frac{\beta\delta}{(\alpha + \beta)D} \frac{\left[y/D(1 - e^{-Dt_1^*}) + a(0)\right]}{(1 - e^{-\delta t_1^*})} e^{-(\delta - D)t}, \quad 0 < t < t_1^* \leq T$$

$$= \frac{\beta y}{(\alpha + \beta)D}, \qquad t_1^* \leq t \leq T$$

$$\text{where } a(0) = \frac{ih_0^{N.A.}}{r} \frac{\left(1 - e^{-(i+r)(1-\phi)T}\right)}{\left(1 - e^{-(i+r)T}\right)} + \frac{ih_0^{NA}(i+r)(1-\phi)\left(e^{-(i+r)(1-\phi)T} - e^{-rT}\right)}{r\left(i(1 - \phi) - \phi r\right)}$$

$$a(t_1^*) = 0$$

$$t_1^* = T \quad \text{if } \delta < D$$

$$t_1^* < T \quad \text{if } \delta > D$$

and for a non-assumable mortgage

$$h_0^{N.A.} = \frac{\beta Y}{(\alpha + \beta) r(1 - \phi)} \quad , \qquad \delta > r(1 - \phi)$$

$$= \frac{\beta\delta\, Y\left(1 - e^{-r(1-\phi)T}\right)}{(\alpha + \beta)(1 - e^{-\delta T})\left(r(1 - \phi)\right)^2} \quad , \quad \delta < r(1 - \phi)$$

We wish to show that $h_0^{N.A.} < h^A(t)$ for all t, $0 \leq \tau$.

Three different cases must be considered;

<u>Case 1</u> $\delta > r(1 - \phi) > D$

$$\Rightarrow h_0^{N.A.} = \frac{\beta Y}{(\alpha + \beta) r(1 - \phi)} \quad ,$$

$h^A(t)$ is decreasing over time, its minimum is $= \dfrac{\beta Y}{(\alpha + \beta)D} > h_0^{N.A.}$

<u>Case 2</u> $D < \delta < r(1 - \phi)$ (High inflation case)

$$\Rightarrow h_0^{N.A.} = \frac{\beta\delta Y\left(1 - e^{-r(1 - \phi)T}\right)}{(\alpha+\beta)(1 - e^{-\delta T})\left(r(1 - \phi)\right)^2} = \frac{\beta Y}{r(1-\phi)(\alpha+\beta)} \; \frac{\delta}{r(1-\phi)} \; \frac{\left(1-e^{-r(1-\phi)T}\right)}{(1 - e^{-\delta T})}$$

$h^A(t)$ is falling over time, its minimum is $\dfrac{\beta Y}{D(\alpha + \beta)} > h_0^{N.A.}$

Since $\quad \dfrac{\beta Y}{D(\alpha+\beta)} > \dfrac{\beta Y}{r(1 - \phi)(\alpha + \beta)}$ and $\dfrac{\delta}{r(1 - \phi)} \; \dfrac{\left(1 - e^{-r(1-\phi)T}\right)}{(1 - e^{-\delta T})} < 1$

<u>Case 3</u> $\delta < D < r(1 - \phi)$ (Low inflation case)

$$\Rightarrow h_0^{N.A.} \quad \frac{\beta\delta Y\left(1 - e^{-r(1 - \phi)T}\right)}{(\alpha + \beta)(1 - e^{-\delta T})\left(r(1 - \phi)\right)^2}$$

$$\min h^A(t)=h^A(0)=\beta\delta\left[\frac{y/D(1-e^{-DT})+a(0)}{(\alpha+\beta)(1-e^{-\delta T})D}\right]=\frac{\beta\delta y}{(\alpha+\beta)(1-e^{-\delta T})}\left[\frac{1-e^{-DT}}{D^2}+\frac{a(0)}{Dy}\right]$$

Now

$$h^A(t) > h^{N.A.} \quad \text{since} \quad \frac{(1-e^{-DT})}{D^2} > \frac{\left(1-e^{-r(1-\phi)T}\right)}{\left(r(1-\phi)\right)^2}$$

III. Optimal Time of Relocation

This section presents conditions that determine the optimal time of relocation, τ, and analyzes how τ varies with changes in inflation, i, and changes in the tax rate, ϕ. This is done for both models of non-assumable mortgages, when the liquidity constraint is binding and when it is not.

A. Model with Housing Fixed Prior to Moving

The optimal value of τ is that value which maximizes

$$\max_{\tau} \int_0^{\tau} e^{-\delta t}U\left(c_1(t;\tau),\, h_0\right)dt + \int_{\tau}^{T} U\left(c_2(t;\tau),\, h_2(t;\tau)\right)dt$$

By Liebnitz's rule, this will be the value of τ that satisfies

$$(23) \quad \int_0^{\tau} e^{-\delta t}\frac{\partial U\left(c_1(t;\tau),\, h_0\right)}{\partial\tau}dt + \int_{\tau}^{T} e^{-\delta t}\frac{\partial U\left(c_2(t;\tau),\, h_2(t;\tau)\right)}{\partial\tau}dt$$

$$+\, U\left(c_1(\tau;\tau),\, h_0\right)e^{-\delta\tau} - U\left(c_2(\tau;\tau),\, h_2(\tau;\tau)\right)e^{-\delta\tau} = 0$$

The case where the liquidity constraint binds over all t is analytically the more simple one. It's more instructive to analyze this case first and then return to interpreting the unconstrained case. Recall that the constrained case holds when $\delta - D$ is sufficiently large. Substituting (14), (15), and (16) into (23) we have

$$f=\alpha\ \ln\left(y+i(1-\phi)\frac{p}{r}\,(1-e^{-r(T-t)})e^{-i\tau}-h_0D\right)-\alpha\ \ln\frac{(\alpha y)}{\alpha+\beta}-\beta\ \ln\left(\frac{\beta y}{(\alpha+\beta)h_0D}\right)=0$$

i.e., the point in time when the household decides to relocate is when

$U\left(c_1(t),\ h_0\right) = U(c_2,\ h_2)$. With use of the implicit function theorem, we can calculate

$$(24)\quad \frac{d\tau}{di} = -\frac{\frac{df}{di}}{\frac{df}{d\tau}} = -\frac{\frac{\alpha}{c_1^*(\tau)}\left[(1-\phi)\frac{p}{r}\left(1-e^{-r(T-\tau)}\right)e^{-i\tau}(1-\tau i) + h_0\phi\right] - \frac{\beta}{D}\phi}{\frac{\alpha}{c_1^*(\tau)}\, i(1-\phi)\frac{p}{r}\, e^{-i\tau}\left(-i-(r-i)e^{-r(T-\tau)}\right)}$$

Note that the denominator of (24) is negative which is also the second order condition that τ is a maximum. Therefore the sign of $\frac{d\tau}{di}$ depends on the sign of the numerator, $\frac{df}{di}$. If $\phi = 0$, i.e., we ignore the effects from taxes, then the sign of $\frac{d\tau}{di}$ depends on $1 - \tau i$. This implies that with increases in inflation, the optimal relocation time increases up to the point where the increase in instantaneous real capital gains income at τ is positive, i.e., where $\frac{\partial\, \frac{ip}{r}\left(1-e^{-r(T-\tau)}\right)e^{-i\tau}}{d\tau} > 0$. Thus for low levels of inflation, $\frac{\partial\tau}{\partial i}$ is positive, but $\frac{\partial\tau}{\partial i}$ is negative for higher values of inflation.

Since in this constrained case, the first order condition implies that the consumer doesn't move until utility falls to $U(c_2,\ h_2)$ which is fixed, τ will increase with inflation as long as the instantaneous marginal utility in collecting more of a capital gain is positive. Because all capital gains received are spent instantly, affecting only current utility, this is equivalent to saying τ will increase with inflation while the change in the real capital gain at τ is positive. Intuitively, it makes sense that very high rates of inflation enable the great proportion of capital gains to be collected quicker. Think of the polar case where inflation jumps to infinity. There is no additional capital gain to be had after the first instant, and the consumer will choose to move immediately.

The additional term in the numerator from the effect of taxes is $\phi\left(\frac{\alpha h_0}{c_1^*(\tau)} - \frac{\beta}{D}\right)$. Note that after τ, $\frac{h_2^*}{c_2^*} = \frac{\beta}{\alpha D}$, and prior to τ, h_0 is lower while $c(t)$ is higher, thus $\frac{\alpha h_0}{c_1^*(\tau)} < \frac{\beta}{D}$, so the effect of taxes is to decrease $\frac{\partial\tau}{\partial i}$.

In a similar manner, $\frac{\partial\tau}{\partial\phi}$ is calculated,

$$\frac{d\tau}{d\phi} = \frac{\frac{\alpha}{c_1^*(\tau)}\left(i\frac{p}{r}\left(1 - e^{-r(T-\tau)}\right)e^{-i\tau} + i\left(\frac{\alpha h_0}{c^*(t)} - \frac{\beta}{D}\right)\right.}{\frac{\alpha}{c_1^*(\tau)}\, i(1 - \phi)\frac{p}{r}\, e^{-1\tau}\left(i + (r - i)e^{-r(T-\tau)}\right)} < 0$$

Thus higher tax rates tend to reduce the optimal time to relocate. This is, in part, because higher tax rates reduce the realized capital gain per period. The higher tax rate also reduces the relative after tax price of housing, D, and creates an incentive to move earlier so that the level of housing services can be increased sooner. This is reflected in the term $i\left(\frac{\alpha h_0}{c^*(\tau)} - \frac{\beta}{D}\right) < 0$.

For the unconstrained case, we substitute (12) and (13) into (23), carry out the differentiation and integration and obtain:

$$(25)\quad \frac{\left(\alpha(1 - e^{-\delta T}) + \beta(e^{-\delta\tau} - e^{-\delta T})\right)\left[i\frac{p}{r}(1 - \phi)\left(1 - e^{-r(T-\tau)}\right)e^{-(i+D)\tau} - h_0 De^{-D\tau}\right]}{\delta\left(\gamma + y/D(e^{-D\tau} - e^{-DT})\right)}$$

$$+ \beta e^{-\delta\tau}\left(1 + \ln\left(\frac{h_0}{h_2(\tau)}\right)\right) = 0$$

Unfortunately, to use this condition to do comparative statics produces expressions whose signs are extremely difficult to interpret.

The complication in interpreting how changes in inflation and tax rates affect the optimal relocation time for this unconstrained case is that capital gains collected by deferring relocation will be used to increase both first period and second period expenditure. In addition, expenditure (and utility) take a discrete jump at τ, reflecting a move to a more optimal level of housing. However, the intuition gained from the analysis of the constrained case would seem to carry over here. For rather high increases in the inflation rate, the vast proportion of the capital gain could be collected quickly, so one would expect $\frac{\partial \tau}{\partial i}$ to be negative in this range.

B. Model with Housing Flexible Prior to Moving

In this model, the optimal value of τ is that value which satisfies;

$$(26)\quad \int_0^{\tau} e^{-\delta t}\,\frac{\partial U\big(c(t;\,\tau),\, h(t;\,\tau)\big)}{\partial \tau}\,dt + \int_{\tau}^{T} e^{-\delta t}\,\frac{\partial U\big(c(t;\,\tau),\, h(t;\,\tau)\big)}{\partial \tau}\,dt$$

$$e^{-\delta\tau}\Big(U(c_1(\tau;\,\tau),\, h_1(\tau;\,\tau)\Big) - \ell - U\Big(c_2(\tau;\,\tau),\, h_2(\tau;\,\tau)\Big) = 0$$

For the case where the liquidity constraint binds over all t, the consumer's problem is

$$\max_{\tau} \int_0^{\tau} e^{-\delta t}\big(\alpha \ln c_1(t)+\beta \ln h_1(t)-\ell\big)dt+\int_{\tau}^{T} e^{-\delta t}\big(\alpha \ln c_2+\beta \ln h_2\big)dt$$

$$\Rightarrow e^{-\delta\tau}\Big[(\alpha+\beta)\ln\Big(y+i(1-\phi)\frac{p}{r}\big(1-e^{-r(T-\tau)}\big)e^{i\tau}\Big)+\alpha \ln\Big(\frac{\alpha}{\alpha+\beta}\Big)+\beta\ln\Big(\frac{\beta}{(\alpha+\beta)D}\Big) - \ell\Big]$$

$$- e^{-\delta\tau}\Big(\alpha \ln\Big(\frac{\alpha y}{\alpha+\beta}\Big) + \beta \ln\Big(\frac{\beta y}{(\alpha + \beta)D}\Big)\Big) = 0$$

or

$$(27)\quad f = (\alpha + \beta)\ln\Big(y + i(1-\phi)\frac{p}{r}\big(1 - e^{-r(T-\tau)}\big)e^{-i\tau}\Big) - \ell - (\alpha + \beta)\ln y = 0$$

or

$$f = (\alpha + \beta)\ln\big(Y(\tau)\big) - \ell - (\alpha + \beta)\ln y$$

$$\frac{\partial f}{\partial \tau} = -\frac{(\alpha + \beta)}{Y(\tau)}\, i(1 - \phi)\frac{p}{r}\, e^{-i\tau}\big(i + (r - i)e^{-r(T-\tau)}\big) < 0$$

Note that unlike the unconstrained case, the liquidity constrained first order condition, (27), equates utility just before and after τ. This was also the case in the liquidity constrained model with housing fixed prior to moving. Thus, it's not surprising that the derivatives $\frac{\partial f}{\partial i}$ and $\frac{\partial f}{\partial \phi}$ are similar.

$$(28) \qquad \frac{\partial f}{\partial i} = \frac{(\alpha + \beta)}{Y(\tau)}(1 - \phi)\frac{p}{r}\left(1 - e^{-r(T-\tau)}\right)e^{-i\tau}(1 - i\tau)$$

$$(29) \qquad \frac{\partial f}{\partial \phi} = - \frac{(\alpha + \beta)}{Y(t)}\, i\frac{p}{r}\left(1 - e^{-r(T-\tau)}\right)e^{-i\tau} < 0$$

Therefore $\frac{\partial \tau}{\partial i} > 0$ if $i\tau < 1$ and $\frac{\partial \tau}{\partial i} < 0$ if $i\tau > 1$. Also, as before, $\frac{\partial \tau}{\partial \phi} < 0$.

For the unconstrained case, we substitute (19) and (20) into (26) and obtain;

$$\int_0^T e^{-\delta t}\frac{\partial U}{\partial \tau}(c, h)dt - \ell e^{-\delta\tau} = 0$$

$$\int_0^T e^{-\delta t}(\alpha + \beta)\frac{\partial \ln \psi}{\partial \tau}dt - \ell e^{-\delta\tau} = 0$$

$$(30) \quad f = \frac{(1 - e^{-\delta T})}{\delta}\frac{1}{\psi} \cdot i(1 - \phi)\frac{p}{r}\left(1 - e^{-r(T-\tau)}\right)e^{-(i+r)(1-\delta)\tau} - \frac{\ell e^{-\delta\tau}}{(\alpha + \beta} = 0$$

$$\frac{df}{d\tau} = \frac{(1-e^{-\delta T})(1-\phi)ipe^{-(i+r)(1-\phi)\tau}\left(1-e^{-r(T-\tau)}\right)\left[-\left(i(1-\phi)-\phi r\right) - \frac{r}{\left(1-e^{-r(T-\tau)}\right)} - \frac{\frac{d\psi}{d\tau}}{\psi}\right]}{\delta r\psi}$$

$$+ \ell\delta e^{-\delta\tau}/(\alpha + \beta)$$

which can be seen to be < 0 by virtue of using condition (30) and the assumption that $\delta < D$. Clearly $\frac{\partial \psi}{\partial \tau} > 0$ since the longer one puts off moving, the greater the total realized capital gain will be. Hence the sign of $\frac{\partial \tau}{\partial i}$ and $\frac{\partial \tau}{\partial \phi}$ depends on the sign of $\frac{\partial f}{\partial i}$ and $\frac{\partial f}{\partial \phi}$ respectively.

The first order condition, (30), illustrates some economic intuition in that it says that the optimal time to move is when the rate of time preference discounted ratio of instantaneous capital gain to total lifetime income equals the "normalized" instantaneous loss from not moving, $\frac{\ell}{(\alpha + \beta)}$. How this discounted realized capital gain to total discounted income changes is critical in determining how inflation or the tax rate affects the optimal relocation time.

Differentiating (30), we get;

$$(31)\ \frac{\partial f}{\partial i} = \frac{(1-e^{-\delta T})}{\delta}\,(1-\phi)\frac{p}{r}\left(1-e^{-r(T-\tau)}\right)e^{-(i+r)(1-\phi)\tau}\cdot\frac{1}{\psi}\left(1-i(1-\phi)\tau-\frac{i}{\psi}\frac{\partial\psi}{\partial i}\right).$$

$$(32)\ \frac{\partial f}{\partial \phi} = -\frac{(1-e^{-\delta T})}{\delta}\,\frac{ip}{r}\left(1-e^{-r(T-\tau)}\right)e^{-(i+r)(1-\phi)\tau}\cdot\frac{1}{\psi}\left(1-(i+r)(1-\phi)\tau-\frac{(1-\phi)}{\psi}\frac{\partial\psi}{\partial(1-\phi)}\right)$$

We see that the last terms in parenthesis for both of these expression determine their signs, and hence the signs of $\frac{\partial\tau}{\partial i}$ and $\frac{\partial\tau}{\partial\phi}$ respectively.

In (31), while $\frac{i}{\psi}\frac{\partial\psi}{\partial i}$ is > 0 (since an increase in inflation lowers the real discount rate, $D = r(1 - \phi)-\phi i$, and it also increases the realized capital gain from 0 to τ), it doesn't appear that $i(1 - \phi)\tau + \frac{i}{\psi}\frac{\partial\psi}{\partial i}$ is necessarily > 1 so that $\frac{\partial\tau}{\partial i}$ may be < or > 0 depending on the values of other parameters. However, one can surmise that the same qualitative results as those found previously would continue to apply. $\frac{\partial\tau}{\partial i}$ would be negative for large changes in the inflation rate. The same seems to be true of $\frac{\partial\tau}{\partial\phi}$. Here even the sign of $\frac{(1 - \phi)}{\psi}\frac{\partial\psi}{\partial(1 - \phi)}$ depends on other parameters, since an increase in ϕ lowers the real discount rate but it also lowers the realized capital gain. For the unconstrained case, a simulation study would be necessary to determine the sign of $\frac{\partial\tau}{\partial i}$ and $\frac{\partial\tau}{\partial\phi}$ for various values of the other parameters.

IV. Optimal Strategy of Mortgage Holder

The preceding analysis of housing demand with non-assumable mortgages after unexpected inflation assumes that the mortgage holder reacts passively to the unexpected inflation increase. However, it will be shown here that in a perfect certainty model, it will be optimal for a profit maximizing mortgage holder to compensate the homeowner to pay up the principle of the non-assumable mortgage early. The model where housing is assumed to be flexible prior to moving is used. We wish to calculate the compensation to be paid the homeowner that would minimize the mortgage holder's total loss from the non-assumable mortgage. This compensation could be paid in many forms. Currently, some Savings and Loans offer home buyers a "blended" mortgage, averaging the rate on the old mortgage with the current market rate when these home buyers move, i.e, the thrifts offer a subsidy on the new mortgage.[3]

In this analysis, it is assumed that the homeowner is not liquidity constrained and that the form of compensation is simply a direct payment to the homeowner made at time 0 if the homeowner agrees to pay off his old mortgage at time t*. Now define

$$\bar{\psi} = \psi - \int_{t^*}^{\tau} i(1 - \phi)\frac{P}{r}\left(1 - e^{-r(T-t)}\right)e^{-(i+r)(1 - \phi)t}dt + \text{compensation.}$$

$\bar{\psi}$ is now the new present discounted value of all future income assuming the homeowner moves at t* but collects compensation from the mortgage holder. In this case the optimal values of housing and consumption are given by (19) and (20) but with ψ replaced with $\bar{\psi}$. We denote these values of housing and consumption by $\bar{h}(t)$ and $\bar{c}(t)$.

One can then calculate the compensation that would have to be paid to the homeowner to make him just as well off if he pays off his old mortgage at t* instead of τ. This is done below:

$$\int_0^{t^*} e^{-\delta t}\left(U\left(\bar{c}(t),\ \bar{h}(t)\right) - \ell\right)dt + \int_{t^*}^{T} e^{-\delta t}U\left(\bar{c}(t),\ \bar{h}(t)\right)dt$$

$$= \int_0^{\tau} e^{-\delta t}\left(U(c(t),\ h(t)) - \ell\right)dt + \int_{\tau}^{T} e^{-\delta t}U\left(c(t),\ h(t)\right)dt$$

$$\text{or } \int_0^{T} e^{-\delta t}\left(U(\bar{c}(t),\ \bar{h}(t)) - U\left(c(t),\ h(t)\right)\right)dt = \frac{\ell}{\delta}(e^{-\delta\tau} - e^{-\delta t^*})$$

Substituting $U\left(c(t),\ h(t)\right) = \alpha \ln c(t) + \beta \ln h(t)$ we get

$$\int_0^{T} e^{-\delta t}(\alpha + \beta)(\ln \bar{\psi} - \ln \psi)dt = \frac{\ell}{\delta}(e^{-\delta\tau} - e^{-\delta t^*})$$

or

$$\ln\left(\frac{\bar{\psi}}{\psi}\right) = \frac{\ell}{(\alpha + \beta)} \frac{(e^{-\delta\tau} - e^{-\delta t^*})}{(1 - e^{-\delta T})}$$

$$\Rightarrow \bar{\psi} = \psi \exp\left(\frac{\ell}{(\alpha + \beta)} \frac{(e^{-\delta\tau} - e^{-\delta t^*})}{(1 - e^{-\delta T})}\right)$$

Therefore, compensation paid, CP, must be

$$CP = \bar{\psi} - \psi - \int_{c^*}^{\tau} i(1 - \phi)\frac{p}{r}\left(1 - e^{-r(T-\tau)}\right)e^{-(i+r)(1-\phi)t}\,dt$$

Thus the total loss to the mortgage holder is

$$TL = CP + \int_0^{t^*} i(1 - \phi_b)\frac{p}{r}\left(1 - e^{-r(T-\tau)}\right)e^{-(i+r)(1 - \phi_b)t}\,dt$$

where ϕ_b is the tax rate on nominal income of the mortgage holder. Or

$$TL = \psi \exp\left(\frac{\ell}{(\alpha + \beta)} \frac{(e^{-\delta\tau} - e^{-\delta t^*})}{(1 - e^{-\delta T})}\right) - \frac{y}{D}(1 - e^{-DT}) +$$

$$\int_0^{t^*} \frac{ip}{r}\left(1 - e^{-r(T-t)}\right)\left[(1 - \phi_b)e^{-(i+r)(1-\phi_b)t} - (1 - \phi)e^{-(1+r)(1-\phi)t}\right]dt$$

If $\phi_b = \phi$,

$$TL = \psi \exp\left(\frac{\ell}{(\alpha + \beta)} \frac{(e^{-\delta\tau} - e^{-\delta t^*})}{(1 - e^{-\delta T})}\right) - \frac{Y}{D}(1 - e^{-DT})$$

and if we want to find the minimum total loss to the mortgage holder for $0 \leq t^* < \tau$, this implies $\min_{t^*}\left(\frac{\ell}{(\alpha + \beta)} \frac{(e^{-\delta\tau} - e^{-\delta t^*})}{(1 - e^{-\delta T})}\right) \Rightarrow t^* = 0$.

So we see that if the tax rates are equal, it is in the mortgage holder's interest to compensate the homeowner to pay off his old mortgage as soon as possible. At $t^* = 0$,

$$TL = \psi \exp\left(-\frac{\ell}{(\alpha + \beta)} \frac{(1 - e^{-\delta\tau})}{(1 - e^{-\delta T})}\right) - \frac{Y}{D}(1 - e^{-DT})$$

which we can see is always less than $\int_0^\tau i(1 - \phi)\frac{P}{r}\left(1 - e^{-r(T-t)}\right)e^{-it}e^{-Dt}dt$ the total loss if the mortgage holder is passive, since

$$\exp\left(-\frac{\ell}{(\alpha + \beta)} \frac{(1 - e^{-\delta\tau})}{(1 - e^{-\delta T})}\right) < 1$$

The previous analysis is unrealistic in that it assumes the mortgage holder knows with certainty the preferences of the homeowner, i.e. ℓ, α, β, δ. A more refined model would have the values of these parameters varying across homeowners, with the values for individual homeowners unknown to the mortgage holder. The mortgage holder may, however, know the probability distribution of the minimum compensation needed to coerce the homeowner to pay off his mortgage early. Since the mortgage holder couldn't discriminate between homeowners with large values of $\frac{\ell}{(\alpha + \beta)}(1 - e^{-\delta\tau})$ over those with smaller values, its problem would be to offer one rate of compensation set in such a manner as to minimize its total loss from its entire portfolio of mortgages.

V. Summary

Unlike much of the previous research on the effect of inflation on housing demand when homeowners have standard fixed rate mortgages (See Kearl [2], Schwab [3], and Wheaton [4], this analysis has focused on unexpected inflation rather than expected inflation. It was shown that the capital gain on the homeowner's mortgage may not in general be fully realized if mortgages are non-assumable. Also, the initial paths of housing demand are always less when homeowners have non-assumable mortgages than under the same circumstances but with assumable mortgages. In fact, considering the model with the quantity of housing fixed prior to moving, an unexpected increase in inflation, while lowering the relative after tax price of housing and providing the homeowners with more income (via the capital gain) may actually lower housing demand to a level less than what would have resulted had there been no inflation at all. This is the case when $\delta < r(1 - \phi)$, i.e., this effect doesn't rely on a liquidity constraint. Thus non-assumable mortgages may explain some of the negative effect of inflation on housing demand, besides the previously documented "tilt" of real mortgage payments from the standard fixed rate mortgage.

The results of this analysis should be taken with caution. As stated in the introduction, inflation uncertainty is not rigorously modeled and housing market analysis is only done in partial equilibrium. While the nature of the housing market may reasonably permit, at least in the short run, ignoring the effect of flow supply on the price of housing, since the flow is so small relative to the stock, non-assumable mortgages, besides having the above stated effects on demand, will also affect the stock supply of housing. This is because homeowners who are in a position to realize large capital gains on their non-assumable mortgages will have their supply curves for housing shifted upwards, i.e., they reduce their supply of housing. Therefore the impact of non-assumable mortgages on the price of a unit of housing stock may be more complicated than what the analysis of this paper has suggested. Considering non-assumable mortgages within models which incorporate more realistic inflation uncertainty and/or general equilibrium characteristics is probably an area worthy of further research.

APPENDIX 1

The first order conditions for t, the time when the consumer becomes liquidity constrained under the assumable mortgage model of Section IA. is given.

Since $a(0) = \int_0^T i(1-\phi)\frac{p}{r}\left(1 - e^{-r(T-t)}\right)e^{-(i+r)(1-\phi)t}dt$ and $a(t_1) = 0$

we have $c^A(t) = \frac{\alpha\delta}{(\alpha+\beta)} \frac{\left(a(0) + y/D(1 - e^{-Dt_1})\right)e^{-(\delta-D)t}}{(1 - e^{-\delta t_1})}$, $0 \leq t \leq t_1$

$$h^A(t) = \frac{\beta\delta}{(\alpha+\beta)D} \frac{\left(a(0) + y/D(1 - e^{-Dt_1})\right)e^{-(\delta-D)t}}{(1 - e^{-\delta t_1})}, \quad 0 \leq t \leq t_1$$

$$\max_{t_1} \int_0^{t_1} e^{-\delta t}\left[(\alpha+\beta)\ln\left[\frac{\delta}{(\alpha+\beta)} \frac{\left(a(0)+y/D(1-e^{-Dt_1})\right)e^{-(\alpha-D)t}}{(1 - e^{-\delta t_1})}\right] + \alpha\ln\alpha + \beta\ln\left(\frac{\beta}{D}\right)\right]dt$$

$$+ \int_{t_1}^{T} e^{-\delta t}\left[\alpha \ln\left(\frac{\alpha y}{(\alpha+\beta)}\right) + \beta \ln\left(\frac{\beta y}{(\alpha+\beta)D}\right)\right]dt$$

$$= \max_{t_1}\left\{(\alpha+\beta)\ln\left[\frac{\delta}{(\alpha+\beta)} \frac{\left(a(0) + y/D(1 - e^{-Dt_1})\right)}{(1 - e^{-\delta t_1})}\right] + \alpha\ln(\alpha) + \beta\ln(\beta)\right\}(1 - e^{-\delta t_1})$$

$$+ \frac{(\delta - D)}{\delta} e^{-\delta t_1}\left(t_1 + \frac{1}{\delta}\right) + \left(\alpha \ln\left(\frac{\alpha y}{\alpha+\beta}\right) + \beta \ln\left(\frac{\beta y}{(\alpha+\beta)D}\right)\right)e^{-\delta t_1}/\delta$$

differentiating with respect to t_1

$$\{\quad\}\delta e^{-\delta t_1} + \frac{(\alpha+\beta)(1 - e^{-\delta t_1})}{\left(a(0)+y/D(1 - e^{-Dt_1})\right)}\left[ye^{-Dt_1} - \delta \frac{\left(a(0)+y/D(1-e^{-Dt_1})\right)e^{-\delta t_1}}{(1 - e^{-\delta t_1})}\right]$$

$$- (\delta - D)e^{-\delta t_1} t_1 = 0$$

is the first order condition that t_1 will satisfy.

APPENDIX 2

This appendix shows that in the non-assumable mortgage model with the quantity of housing fixed prior to moving, housing demand always takes a discrete increase at τ, the relocation time.

Define $$D = r(1-\phi) - \phi i \,, \quad q = r(1-\phi)$$

Case 1 $\delta > q > D$ (Constrained Case)

$$\text{Then } h(0) = \frac{\beta y}{(\alpha+\beta)q} < h(\tau^+) = \frac{\beta y}{(\alpha+\beta)D}$$

Case 2 $D < \delta < q$ (Constrained After τ)

$$\text{Therefore } h(0) = \frac{\beta\delta y/q(1-e^{-qT})}{(\alpha+\beta)(1-e^{-\delta T})q} < \frac{\beta y}{(\alpha+\beta)q} \qquad \text{since}$$

$$\frac{(1-e^{-qT})}{q}\,\frac{\delta}{(1-e^{-\delta T})} < 1 \quad \text{for } q > \delta$$

Also
$$h(\tau^+) = \frac{\beta y}{(\alpha+\beta)D} > \frac{\beta y}{(\alpha+\beta)q} \qquad \text{so} \qquad h(\tau^+) > h(0)$$

Case 3 $q > D > \delta$ (Unconstrained Case)

$$h(\tau^+) = \frac{\beta\delta}{(\alpha+\beta)D}\,\frac{\left(a_\tau + y/D\left(1-e^{-D(T-\tau)}\right)\right)}{\left(1-e^{-\delta(T-\tau)}\right)} > \frac{\beta\delta}{(\alpha+\beta)D}\,\frac{y/D\left(1-e^{-D(T-\tau)}\right)}{\left(1-e^{-\delta(T-\tau)}\right)}$$

$$> \frac{\beta\delta}{(\alpha+\beta)}\,\frac{y/q\left(1-e^{-q(T-\tau)}\right)}{\left(1-e^{-\delta(T-\tau)}\right)} \qquad \text{since} \quad \frac{\left(1-e^{-D(T-\tau)}\right)}{D} > \frac{\left(1-e^{-q(T-\tau)}\right)}{q}$$

$$> \frac{\beta\delta}{(\alpha+\beta)}\,\frac{y}{q^2}\,\frac{\left(1-e^{-q(T-\tau)}\right)}{\left(1-e^{-\delta(T-\tau)}\right)}$$

$$> \frac{\beta\delta}{(\alpha + \beta)} \,{}^{y}/_{q^2}\, \frac{(1 - e^{-qT})}{(1 - e^{-\delta T})} = h(0)$$

Since $$\frac{\partial\left(\frac{1 - e^{-qx}}{1 - e^{-\delta x}}\right)}{\partial x} < 0 \quad \text{for } q > \delta > 0$$

FOOTNOTES

[1]"Mortgage Shift Can Be Blocked, Top Court Rules," The Wall Street Journal, June 29, 1982.

[2]"Housing Keeps Falling," The New York Times, July 18, 1982.

[3]"Power to Call in Mortgages upon Sale of Home is Seen Being Given All Thrifts," The Wall Street Journal, June 29, 1982.

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