THE EFFECTS OF DIFFERENT TAXES ON RISKY AND RISKFREE INVESTMENT AND ON THE COST OF CAPITAL

by

Yu Zhu and Irwin Friend

6-85

1. Introduction

The effects of changes in different types of tax rates on the economy, especially on the personal and corporate savings rates, business investment, the required rate of return on capital assets and the composition of investor portfolios, are matters of obvious interest to both economists and policy makers. Unfortunately, frequently under uncertainty these effects are quite different from those obtained under certainty assumptions and as a result are often counter-intuitive. Thus a reduction in personal or corporate taxes with a symmetric tax treatment of gains and losses would increase the after-tax cash flow of return from stock for individual investors, but at the same time the variance of the return and therefore the required rate of return would also increase, so that the effects of tax change may be ambiguous. In recent studies, Friend and Hasbrouck (1982) and Blume and Friend (1984) used a continuous time model of utility maximization to acertain the effects of a constant percentage reduction in personal and corporate taxes respectively on investment in risky and riskless assets.

These earlier papers found that under plausible assumptions about the parameters of the model they used, the immediate effects of lower corporate tax rates would be a reduction in the firm's before-tax cost of capital and an increase in the after-corporate-tax return required by investors. In the long run, the reduction of corporate taxes would lead to an increase in the proportion of wealth devoted to investment in risky assets. The reduction in personal tax rates, however, would be associated with lower stock prices and an increase in the before-corporate-tax cost of capital. These papers do not allow for the existence of non-marketable human wealth nor for different personal tax rates applicable to income from risky and riskfree assets, though they do allow for different tax rates for different wealth or income levels. It is interesting to see to what extent the above-mentioned findings would hold when these more realistic complications are introduced, and this constitutes the purpose of this paper. In Section 2, the optimization model of demand for risky assets is derived with the existence of n risky assets, one riskfree asset, and human wealth. Sections 3 and 4 analyze the effects of changes in corporate and personal tax rates respectively. Section 5 discusses a model with different personal tax rates on incomes from risky and riskfree assets. Section 6 employs some realistic data to illustrate the results of the extended models and their implications.

2. The Market Price of Risk With Allowance for Human Wealth

Assume that there are three sectors in the economy: government, firms and investors. The investors are characterized by multiperiod utility functions which exhibit constant proportional risk aversion, though not necessarily with the same risk aversion for each investor. The investors can invest their wealth in n risky marketable assets and one riskfree asset. The risky assets have before-corporate-tax rates of return $r_1,\ldots,r_n$ which are governed by a Wiener process

$$(1) \qquad dr_j = Er_j dt + \sigma_j y_j \sqrt{dt} \quad ; \quad y_j \sim N(0,1) \quad , \quad j = 1,\ldots,n \quad .$$

The riskfree rate is r_f. The investor k also has non-marketable human wealth H_k with rate of return r_{kh} [1]

$$(2) \qquad dr_{kh} = Er_{kh} dt + \sigma_{kh} y_{kh} \sqrt{dt} \quad ; \quad y_{kh} \sim N(0.,1) \quad .$$

The random disturbances are correlated in the following way:

$$(3) \qquad \text{Cov}\ (y_i, y_j) = \rho_{ij} \quad ; \quad \text{Cov}\ (y_j, y_{kh}) = \rho_{j,kh} \quad .$$

The government taxes the investors at personal tax rate t_{pk} and taxes firms at corporate tax rate t_c. All the corporate taxes are borne by the investors. Both the corporate and personal taxes are symmetric, i.e. the government would give credits for losses. The personal tax rate is assumed to be determined only by an investor's wealth or income.

Under the above assumptions, the kth investor's wealth dynamics at time t can be written as

$$dW_{k,t+dt} =$$

(4) $$W_{kt}\{(1-t_k)[(1-h_k)\big(r_f dt + \sum_j \alpha_{jk}[\big(Er_j(1-t_c)-r_f\big)dt+(1-t_c)\sigma_j y_j\sqrt{dt}$$
$$+ h_k\big(Er_{kh}dt + \sigma_{kh}y_{kh}\sqrt{dt}\big)]\} \quad ,$$

where W_{kt} is the kth investor's wealth level at time t, $W_{kt} > 0$. α_{jk} is the proportion of the kth investor's total marketable assets invested in the jth asset. h_k is the proportion of the kth investor's total human wealth in his total wealth.

Let $u(W_{kt})$ be the kth investor's utility function. Assume $u'(W_{kt}) > 0$, $u''(W_{kt}) < 0$, and the Arrow-Pratt measure of relative risk aversion

(5) $$C_k = - W_{kt}\left(\frac{u''(W_{kt})}{u'(W_{kt})}\right)$$

is independent of time t. The investor is to maximize his expected utility in the next period by choosing an optimal portfolio $\{\alpha_{1k},\ldots,\alpha_{nk}\}$,[2] i.e.

$$\underset{\{\alpha_{ik}\}}{\text{Max}} \; Eu\big(W_{k,t+dt}\big) \quad .$$

The first-order conditions are

(6) $$Eu'\big(W_{k,t+dt}\big)\big[\big(Er_i(1-t_c) - r_f\big)dt + (1-t_c)\sigma_i y_i\sqrt{dt}\big] = 0$$
$$i = 1,\ldots,n \quad .$$

Expanding $u'(\quad_{,t+dt})$ in Taylor series about W_{kt}, we obtain

$$u'(W_{k,t+dt}) = u'(W_{kt}) + u''(W_{kt})W_{kt}\Big[(1-t_c)(1-t_k)(1-h_k)\sum_j \alpha_{jk}\sigma_j y_j\sqrt{dt} + (1-t_k)h_k\sigma_{kh}y_{kh}\sqrt{dt}\Big] + o(\sqrt{dt}) \quad . \tag{7}$$

Here we have used the wealth dynamics equation (4). Substituting (7) into (6), taking expectations and eliminating terms of order higher than dt, we obtain the following equation:

$$u'(W_{kt})[Er_i(1-t_c) - r_f] + W_{kt}u''(W_{kt})(1-t_k)\Big[(1-t_c)^2(1-h_k)\sum_j \alpha_{jk}\sigma_{ij} + (1-t_k)h_k\sigma_{i,kh}\Big] = 0 \quad .$$

The above equation can be simplified by using equation (5), the definition of the coefficient of relative risk aversion. The result is

$$Er_i(1-t_c) - r_f = C_k(1-t_k)\Big[(1-h_k)(1-t_c)^2\sum_j \alpha_{jk}\sigma_{ij} + h_k(1-t_c)\sigma_{i,kh}\Big] \qquad i = 1,\ldots,n \quad . \tag{8}$$

There are n equations in (8) to determine the optimal portfolio for investor k. Rearranging (8), we have

$$\sum_j \alpha_{jk}\sigma_{ij} = \frac{Er_i(1-t_c) - r_f}{C_k(1-t_c)^2(1-t_k)(1-h_k)} - \frac{h_k\sigma_{i,kh}}{(1-t_c)(1-h_k)} \quad .$$

Using the matrix form, the above equations can be rewritten as

$$\sum a_k = \frac{1}{C_k(1-t_c)^2(1-t_k)(1-h_k)}\left[E(r)(1-t_c) - er_f\right] - \frac{h_k}{(1-t_c)(1-h_k)}G_{kh}$$

or

$$a_k = \frac{1}{C\,(1-t_c)^2(1-t_k)(1-h_k)} \sum^{-1} \left[E(r)(1-t_c) - er_f\right] \tag{8'}$$
$$- \frac{h_k}{(1-t_c)(1-h_k)} \sum^{-1} G_{kh} \quad ,$$

where $a_k = (\alpha_{1k},\ldots,\alpha_{nk})'$, $\sum = \{\sigma_{ij}\}$, r is the vector of asset returns $(r_1,\ldots,r_n)'$, e is the vector of unity, and G_{kh} is the covariance vector $(\sigma_{1,kh},\ldots,\sigma_{n,kh})$. Since h_k and G_{kh} are unique for each investor, each holds a portfolio of marketable assets that solve his personal and possibly unique portfolio problem. In other words, because of the nonmarketability of human wealth, the separation theorem no longer holds. But we are not going to pursue the portfolio problem for individual investors in this paper. Instead, since we are interested in the aggregate characteristics of investors' behavior and the market place, we shall aggregate equation (8) in the following two steps.

First, we aggregate (8) over all assets for investor k:

$$\sum_i g_{im} Er_i (1 - t_c) - r_f =$$
$$C_k(1-t_k)\left[(1-h_k)(1-t_c)^2 \sum_i \sum_j g_{im}\alpha_{jk}\sigma_{ij} + (1-t_c)h_k \sum_i g_{im}\sigma_{i,kh}\right] \tag{9}$$

where g_{im} is the proportion of asset i in all marketable risky assets. It is obvious that

$$Er_m = \sum_i g_{im} Er_i \quad .$$

r_m is the rate of return of the market portfolio. We have another two relations to simplify expression (9):

$$\sum_i \sum_j g_{im}\alpha_{jk}\sigma_{ij} = \alpha_k \,\mathrm{Cov}\,(r_m, r_{mk}) \tag{10}$$
$$= \alpha_k \beta_k \sigma_m^2 \quad ,$$

$$\sum_i g_{im}\sigma_{i,kh} = \text{Cov}\,(r_m, r_{kh}) \tag{11}$$
$$= \beta_{kh}\sigma_m^2 \quad ,$$

where α_k is the proportion of the kth investor's risky marketable assets in his total marketable assets. σ_m^2 is the variance of the market portfolio return. r_{mk} is the kth investor's rate of return from his investment in the marketable risky assets. Now (9) can be written as

$$E(r_m)(1 - t_c) - r_f = \tag{12}$$
$$C_k(1-t_k)\left[(1-h_k)(1-t_c)^2\alpha_k \text{Cov}(r_m, r_{mk}) + h_k(1-t_c)\text{Cov}(r_m, r_{kh})\right] \quad .$$

(12) reflects the kth investor's demand for marketable risky assets. The second step is aggregating (12) all over investors, from which we can obtain a macro relationship of demand for risky assets.

If γ_k is the proportion of the kth investor's total assets in the total assets of the economy, then we have

$$\sum_k \frac{\gamma_k}{C_k(1 - t_k)}\left[E(r_m)(1 - t_c) - r_f\right] = \tag{13}$$
$$(1-t_c)^2 \sum_k (1-h_k)\alpha_k\gamma_k \text{Cov}(r_m, r_{mk}) + (1-t_c)\sum_k h_k\gamma_k \text{Cov}(r_m, r_{kh}) \quad .$$

In order to simplify (13), we assume[3]

$$1/C_k = (1/C) + \varepsilon_{ck} \quad , \tag{14}$$

where ε_{ck} is a disturbance with zero mean and is independent of t_k and γ_k. C is the harmonic mean of all individuals' relative risk aversion. Under this assumption, the left-hand-side of (13) can be written as

$$\frac{1}{C(1 - t_p)}\left[E(r_m)(1 - t_c) - r_f\right] \quad ,$$

where $1-t_p$ is the weighted harmonic mean of $1-t_k$. On the right hand side of equation (13), we notice that[4]

$$(15) \qquad \sum_k (1 - h_k)\alpha_k \gamma_k \mathrm{Cov}(r_m, r_{mk}) = (1 - h)\alpha\sigma_m^2 .$$

α is the ratio of total risky marketable assets to the total marketable assets, h is the proportion of total human wealth in the economy. The second term on the right hand side of (13) is simply

$$(1 - t_c)h\,\mathrm{Cov}(r_m, r_h) = (1 - t_c)h\beta_{hm}\sigma_m^2 ,$$

where r_h is the arithmetic weighted average rate of return on human wealth. Using these relationships, (13) can be simplified as

$$(16) \qquad \frac{E(r_m)(1 - t_c) - r_f}{\sigma_m^2(1 - t_c)(1 - t_p)} = \left[(1 - h)(1 - t_c)\alpha + h\beta_{hm}\right]C .$$

(16) is a macro-level equilibrium relationship between the market price of risk and the demand for risky assets. This equation shows that although individual investors hold different portfolios from each other as a result of the existence of human wealth, the equilibrium market price of risk is independent of individuals' unique characteristics. (16) is the basic equation to be used to analyze the effects of reduction of taxes on investment decisions in the following sections. We can see clearly from (16) that the market price of risk would increase if the correlation between the rate of return on human wealth and the rate of return on the market portfolio increases.

3. The Effect of Corporate Tax Reduction

In order to study the tax effects, we shall make some further simplifying assumptions about the firm (corporate) sector. We assume that corporations are fully equity financed and their liabilities constitute the only risky assets in the economy. Firms in turn own productive machines which produce single consumption goods. We also assume that the machines have no further productive capability and are therefore valueless after a fixed amount of time, but up to that point their productivity remains unchanged. Thus the scale of firms can be adjusted over time either upward or downward. Let V_m be the total market value of existing marketable risky assets, V_f the total value of existing riskless assets, and H the total value of human wealth. In the above model, V_m is interpreted as the total market value of corporate capital. Suppose in the period considered the firms' physical capital is fixed, but some exogenous changes such as tax reform can alter the valuation of the existing capital in the short run. In the long run, the value of all productive machines will be their replacement value, but the in the short run there is no guarantee that the market value of these machines will be the same as their replacement value.

To see the impact of change in corporate taxes in the short run, we rewrite (16) as

$$(17) \qquad E(R) = \frac{r_f V_m}{1 - t_c} + \sigma_R^2 (1 - t_p)\, C\left[\frac{(1 - t_c) + H\beta_{hR}}{V_m + V_f + H}\right] ,$$

where R is the total output of existing capital with mean and variance $E(R)$ and σ_R^2. Clearly

$$E(R) = E(r_m)V_m$$

$$\sigma_R^2 = \sigma_m^2 V_m^2$$

β_{hR} is the beta coefficient of total output and the rate of return on human wealth

$$\beta_{hR} = \frac{\mathrm{Cov}(R, r_h)}{\sigma_R^2} = \frac{\beta_{hm}}{V_m} .$$

In the short run, we can assume that exogenous changes will not alter the distribution of R. To see the effect of a corporate tax rate change, we implicilty differentiate V_m in expression (17) with respect to t_c, keeping E(R), σ_R^2 and β_{hR} constant. We then have

$$\frac{\partial V_m}{\partial t_c} = - \frac{\dfrac{r_f V_m}{(1 - t_c)^2} - \dfrac{\sigma_R^2(1 - t_p)C}{V_m + V_f + H}}{\dfrac{r_f}{1 - t_c} - \dfrac{C\sigma_R^2(1 - t_p)[(1 - t_c) + H\beta_{hm}]}{(V_m + V_f + H)^2}} \quad . \tag{18}$$

Let $\alpha_m = V_m/(V_m+V_f+H)$, i.e. α_m is the proportion of the sum of firms' equities in the total wealth of the economy. We have

$$\frac{\partial V_m}{\partial t_c} = - \frac{\dfrac{r_f}{1 - t_c} - C\sigma_m^2(1 - t_p)(1 - t_c)\alpha_m}{\dfrac{r_f}{1 - t_c} - C\sigma_m^2(1 - t_p)(1 - t_c)\alpha_m^2\left[1 + \dfrac{H\beta_{hm}}{V_m(1 - t_c)}\right]} \cdot \frac{V_m}{1 - t} \quad . \tag{19}$$

(19) represents the effect of a corporate tax change on the valuation of existing capital. If $\partial V_m/\partial t_c$ is positive, then the reduction in corporate taxes would reduce the value of the existing capital, so that the investment would shift away from risky assets. If $\partial V_m/\partial t_c$ is negative, the impact would be reversed.

Two sets of sufficient conditions for $\partial V_m/\partial t_c$ to be negative are as follows:

I. (20a)
$$\frac{r_f}{1 - t_c} > \sigma_m^2\, C(1 - t_p)(1 - t_c)\alpha_m$$

and

(20b)
$$\delta \equiv \alpha_m\left[1 + \frac{H\beta_{hm}}{V_m(1 - t_c)}\right] < 1$$

or

II. (21a) $$\frac{r_f}{1 - t_c} < \sigma_m^2 C(1 - t_p)(1 - t_c)\alpha_m$$

and

(21b) $$\delta \equiv \alpha_m\left[1 + \frac{H\beta_{hm}}{V_m(1 - t_c)}\right] > 1 \quad .$$

(20a) says that the numerator on the right hand side of (19) is positive. (20b) indicates that the denominator on the right hand side is also positive. The other set of conditions can be explained in a similar way. In the absence of human wealth, $\alpha_m = \alpha, h = 0$. (19) becomes

$$\frac{\partial V_m}{\partial t_c} = -\frac{\frac{r_f}{1 - t_c} - C\sigma_m^2(1 - t_p)(1 - t_c)\alpha}{\frac{r_f}{1 - t_c} - C\sigma_m^2(1 - t_p)(1 - t_c)\alpha^2} \cdot \frac{V_m}{1 - t_c} \quad .$$

This expression is identical with equation (6) in Blume and Friend (1984). Recall that $\alpha = V_m/(V_m+V_f)$, so $\alpha_m = (1-h)\alpha$. Incorporating human wealth into the model will not change the direction of the inequality (20a) or (21a).[5] But (20b) could be violated if the covariance between the rate of return of human wealth and the rate of return of the market portfolio is positive and relatively large. We will discuss this problem further using some plausible data in Section 6.

The investors' required rate of return after corporate taxes is $(1-t_c)E(r_m)$. In our model, we assume that $E(R)$ is constant in the short run. Hence the effect of tax rate change on the required rate of return can be shown as follows:

(22) $$\frac{\partial}{\partial t_c}\left[(1-t_c)\frac{E(R)}{V_m}\right] = E(r_m)\left[-\frac{1 - t_c}{V_m}\frac{dV_m}{dt_c} - 1\right]$$

$$= E(r_m)\left[\frac{\left(r_f/(1-t_c)\right) - C\sigma_m^2(1-t_p)(1-t_c)\alpha_m}{\left(r_f/(1-t_c)\right) - C\sigma_m^2(1-t_p)(1-t_c)\alpha_m \cdot \delta} - 1\right] \quad .$$

The sign of a change in the required rate of return after corporate taxes when that tax rate increases is the same as the sign of the product of

$$(\delta - 1) \quad \text{and} \quad \left[\frac{r_f}{1 - t_c} - C\sigma_m^2(1-t_p)(1-t_c)\alpha_m \cdot \delta\right] .$$

If the first set of sufficient conditions (20a) and (20b) is satisfied, then (22) is negative. This means that a reduction in the corporate tax rate would increase the after-tax required rate of return.

In the long run, the amount of physical capital can no longer be assumed constant. The value of the capital should be equal to its replacement cost. To determine the efect of tax rate change in the long run, we assume that $E(r_m)$, σ_m^2, and β_{hm} (but obviously not α) are constant. From (16), we have

$$(23) \qquad E(r_m) = \frac{r_f}{1 - t_c} + \sigma_m^2 C(1-t_p)\left[(1-h)(1-t_c)\alpha + h\beta_{hm}\right] .$$

Implicitly differentiating (23) with respect to t_c, we obtain

$$(24) \qquad \frac{\partial \alpha}{\partial t_c} = - \frac{\left(r_f/(1-t_c)\right) - \sigma_m^2 C(1-t_p)(1-t_c)\alpha_m}{\sigma_m^2 C(1 - t_p)(1 - t_c)(1 - h)} .$$

Expression (24) shows that if (20a) is satisfied, then $d\alpha/dt_c$ will be negative. This means that a reduction in the corporate tax rate would increase the proportion of investment in marketable risky assets. It is interesting to notice that in the long run, the existence of human wealth does not change the impact on the investment decision caused by tax reduction, whereas it may in the short run.[6]

4. The Effect of Change in Personal Income Tax Rates

Using the same methodology as in the last section, we can discuss the impact of changes in personal income tax rates. The immediate effect is the change in the market value of existing capital:

$$\text{(25)} \qquad \frac{\partial V_m}{\partial t_p} = \frac{C(1 - t_c)\sigma_m^2[1 + \frac{H\beta_{hm}}{V_m(1 - t_c)}]\ V_m}{\frac{r_f}{1 - t_c} - \sigma_m^2 C(1 - t_p)(1 - t_c)\alpha_m^2[1 + \frac{H\beta_{hm}}{V_m(1 - t_c)}]} .$$

Therefore, if inequalities (20a) and (20b) hold, (25) is positive. This means that a higher personal income tax rate would be associated with a rise in stock prices assuming the symmetry of tax effects. Setting $H = 0$ in (25), we get an expression reflecting the effect of the change in income tax rates when only non-human wealth (and the associated income) is taken into consideration:

$$\text{(26)} \qquad \partial V_m/\partial t_p = \frac{(1 - t_c)\sigma_m^2 C V_m}{\left(r_f/(1-t_c)\right) - (1 - t_p)(1 - t_c)\sigma_m^2\ C\alpha^2} ,$$

which is identical to equation (8) in Friend-Hasbrouck (1982).

The long-run effect can be expressed in the following equation:

$$\text{(27)} \qquad \partial\alpha/\partial t_p = \frac{(1 - h)(1 - t_c)\alpha + h\beta_{hm}}{(1 - h)(1 - t_p)(1 - t_c)} .$$

(27) is always positive if β_{hm} is positive or zero.

5. Model with Different Personal Income Tax Rates and Human Wealth

In order to make the model more realistic, we now introduce personal income tax rates which differ as between risky marketable assets as a whole and all other assets, reflecting the different tax rates and incidence of capital gains and ordinary income. Assume for investor k the tax rate on the returns from marketable risky assets is t_{sk} and the tax rate on the returns from riskfree assets and human wealth is t_{fk}. Both tax rates are assumed to be symmetric. Under the same set of assumptions as in Section 2, the wealth dynamics can be written as follows

$$dW_{k,t+dt} = W_{kt}\Big\{(1-h_k)\Big[\sum_j \alpha_{jk}\big(Er_j dt + \sigma_j y_j \sqrt{dt}\big)(1 - t_c)(1 - t_{sk}) + \Big(1 - \sum_j \alpha_{jk}\Big) r_f dt (1 - t_{fk})\Big] + h_k(Er_{kh} dt + \sigma_{kh} y_{kh} \sqrt{dt})(1 - t_{fk})\Big\} \quad . \tag{28}$$

The investor maximizes his next period expected utility of wealth by choosing optimal coefficients of his investment portfolio α_{jk}, $j = 1,\ldots,n$. The first-order conditios for the maximization are

$$\begin{aligned} & Er_i(1 - t_c)(1 - t_{sk}) - r_f(1 - t_{fk}) \\ & = C_k(1-t_c)(1-t_{sk})\Big[(1-h_k)(1-t_c)(1-t_{sk}) \sum_j \alpha_{jk}\sigma_{ij} + h_k(1-t_{fk})\sigma_{i,kh}\Big] \\ & \qquad i = 1,\ldots,n \quad . \end{aligned} \tag{29}$$

(29) can be aggregated over all risky marketable assets, which leads to the following equation:

$$\begin{aligned} & Er_m(1-t_c)(1-t_{sk}) - r_f(1-t_{fk}) \\ & = C_k(1-t_c)(1-t_{sk})\big[(1-h_k)(1-t_c)(1-t_{sk})\alpha_k \mathrm{Cov}(r_m,r_{mk}) \\ & \quad + h_k(1-t_{fk})\mathrm{Cov}(r_m,r_{kh})\big] \quad . \end{aligned} \tag{30}$$

The second step is to aggregate over all investors. Rewriting (30), multiplying both sides by γ_k which is the proportion of wealth of investor k to the total wealth of the economy, and then making the summation over k, we obtain an aggregate equation which relates investors' demand for risky assets to various economic determinants:

$$\begin{aligned} & \frac{Er_m}{1 - t_c} \sum_k \frac{\gamma_k}{C_k(1 - t_{sk})} - \sum_k \gamma_k(1 - h_k)\alpha_k \mathrm{Cov}(r_m,r_{mk}) \\ & = \frac{r_f}{(1 - t_c)^2} \sum_k \frac{(1 - t_{fk})\gamma_k}{C_k(1 - t_{sk})^2} + \frac{1}{1 - t_c} \sum \frac{1 - t_{fk}}{1 - t_{sk}} \gamma_k h_k \mathrm{Cov}(r_m,r_{mk}) \quad . \end{aligned} \tag{31}$$

The left-hand side of equation (31) can be easily simplified by using equations (14) and (15). The result is

$$(32) \qquad \frac{Er_m}{C(1 - t_c)(1 - t_s)} - (1 - h)\alpha\sigma_m^2 \quad ,$$

where $(1-t_s)$ is the harmonic mean of $(1-t_{sk})$. All other notations in (32) are the same as in Section 2. The simplification of the right-hand side of (31) causes some difficulties. We assume that the following expression can be taken as the first approximation of the right-hand side of equation (31):

$$(33) \qquad \frac{r_f(1 - t_f)}{C(1 - t_c)^2(1 - t_s)^2} + \frac{1 - t_f}{(1 - t_s)(1 - t_c)} h \operatorname{Cov}(r_m, r_h) \quad ,$$

where t_f is the arithmetic mean of t_{fk}. The purpose of making this strong assumption is to obtain a simple form of aggregate relationship for facilitating the following analysis. We will discuss this approximation problem in some detail in Appendix 2.

Substituting (32) and (33) into (31) and collecting terms, we get the following macro relationship between the demand for risky assets and the market price of risk and the different tax rates:

$$(34) \quad \frac{Er_m(1 - t_c)(1 - t_s) - r_f(1 - t_f)}{C\sigma_m^2(1 - t_c)^2(1 - t_s)^2} = \alpha(1 - h) + \frac{(1 - t_f)h\beta_{hm}}{(1 - t_s)(1 - t_c)} \quad .$$

In the subsequent analysis, we will make the same assumptions as in Sections 3 and 4. In particular, we assume that in the short run the probability distribution of the total market return R does not change. Since $R = V_m r_m$, $\sigma_R^2 = V_m^2 \sigma_m^2$ and $\beta_{hR} = \beta_{hm}/V_m$, we can rewrite (34) in terms of the parameters of the distribution of R. This leads to equation (35):

$$(35) \qquad ER = \frac{r_f V_m(1 - t_f)}{(1 - t_c)(1 - t_s)} + C\sigma_R^2 \frac{(1 - t_c)(1 - t_s) + (1 - t_f)H\beta_{hR}}{V_m + V_f + H} \quad .$$

When tax rates change, the market value of existing risky marketable assets will change while leaving ER, σ_R^2 and β_{hR} constant. It can be readily shown by implicit differentiation that

$$(36)\qquad \frac{\partial V_m}{\partial t_c} = - \frac{\dfrac{r_f(1 - t_f)}{(1 - t_c)(1 - t_s)} - C\sigma_m^2(1 - t_s)(1 - t_c)\alpha_m}{\dfrac{r_f(1-t_f)}{(1-t_c)(1-t_s)} - C\sigma_m^2\left[(1-t_c)(1-t_s) + \dfrac{H}{V_m}(1-t_f)\beta_{hm}\right]\alpha_m^2} \cdot \frac{V_m}{1-t_c} .$$

Notice in (35) that t_c and t_s are in symmetric positions. So, by interchanging t_c and t_s in equation (36), we get

$$(37)\qquad \frac{\partial V_m}{\partial t_s} = - \frac{\dfrac{r_f(1 - t_f)}{(1 - t_c)(1 - t_s)} - C\sigma_m^2(1 - t_s)(1 - t_c)\alpha_m}{\dfrac{r_f(1-t_f)}{(1-t_c)(1-t_s)} - C\sigma_m^2\left[(1-t_s)(1-t_c) + \dfrac{H}{V_m}(1-t_f)\beta_{hm}\right]\alpha_m^2} \cdot \frac{V_m}{1-t_s} .$$

Finally, the effect of the change in the personal income tax rate on the returns from riskfree assets and human wealth can be written as

$$(38)\qquad \frac{\partial V_m}{\partial t_f} = \frac{\dfrac{r_f V_m}{(1 - t_c)(1 - t_s)} + C\sigma_m^2 H\beta_{hm}\alpha_m}{\dfrac{r_f(1-t_f)}{(1-t_c)(1-t_s)} - C\sigma_m^2\left[(1-t_s)(1-t_c) + \dfrac{H}{V_m}(1-t_f)\beta_{hm}\right]\alpha_m^2} .$$

6. Numerical Illustrations

In this section, we use some realistic data to illustrate the results we obtained in the previous sections on the effects of tax rate changes. The focus is on the short-run effect, i.e. the change in the current market value of capital with respect to the changes in tax rates.

First, let us examine the model in Sections 3 and 4 which does not distinguish between the different personal tax rates on risky and riskfree incomes. Assume the average corporate tax rate $t_c = .45$, the (harmonic) average personal income tax rate t = .25, and the standard deviation of market return $\sigma_m = .15$ -- all rough estimates but at least reasonable orders of magnitude. Let the ratio of marketable risky assets to total marketable assets $\alpha = V_m/(V_m+V_f) = .85$. All of these numbers seem to be reasonable approximations of reality. We know that

$$\alpha_m = \frac{V_m}{V_m + V_f + H} = \alpha(1 - h)$$

and

$$\frac{V_m}{h} = \frac{\alpha_m}{h} = \frac{\alpha(1 - h)}{h} \quad .$$

So given the riskfree rate, the coefficient of relative risk aversion of investors, the ratio of human wealth to the total wealth in the economy and the beta coefficient of the return on human wealth on the market return, we can estimate all the derivatives which reflect the effects of tax rate changes on the investment decisions. The calculation shows that if the investors' (harmonic) average relative risk aversion $C = 6$ with human wealth (see Friend and Blume (1975) and Friend and Hasbrouck (1981)), then for $r_f > .01$, $0 < \beta_{hm} < .5$, both conditions (20a) and (20b) are satisfied.[7] Hence, for a fairly wide range of plausible data, $\partial V_m/\partial t_c$ is negative, while $\partial V_m/\partial t_p$ is positive.

Defining the corporate tax rate elasticity of the value of capital as

$$e_{tc} = \frac{t_c}{V_m}\frac{\partial V_m}{\partial t_c}$$

and the personal income tax rate elasticity as

$$e_{tp} = \frac{t_p}{V_m}\frac{\partial V_m}{\partial t_p} \quad ,$$

Table 1 shows that the absolute values of e_{tc} and e_{tp} increase with β_{hm}. For a given riskfree rate, this means that the higher the covariance between the returns on human wealth and on the market, the more sensitive the market value of capital with respect to the tax rate changes. From Table 1, it seems that introducing human wealth into the model does not change the sign of the elasticities. It is interesting to note that for a given set of parameters the sensitivity of corporate tax rate change increases with the riskfree rate while the sensitivity of personal tax rate change decreases with the riskfree rate.

In the model with different personal tax rates on incomes from risky and riskfree assets, we assume that the personal tax rate on the income from stocks is $t_s = .20$ and the personal tax rate on riskfree assets is $t_f = .30$.[8] By equations (36)-(38), we can calculate the elasticities of the market value of capital with respect to t_c, t_s and t_f. The results are shown in Table 2. We can see that e_{tc} and e_{ts} have the same sign which is negative if r_f is greater than .0105. For the given set of parameters, e_{tf} is always positive. Thus, under the parameters assumed, capital gain taxation or the lower taxation associated with risky assets has a qualitatively different effect on the prices of stocks (and other risky assets) than the taxation of income from riskfree asset and from human wealth.

A notable feature of the above numerical illustration is that the changes in corporate and personal income tax rates have opposite effects on the market value of capital. The reason for that is basically the asymmetry of institutional tax structure; i.e., the corporate income tax is only imposed on income from risky marketable assets, while the personal income tax is imposed on both risky and riskless income.

To get further insight into the different effects of corporate and personal taxes, let us look at equation (17) again

$$(17) \qquad E(R) = \frac{r_f V_m}{1 - t_c} + C\sigma_R^2(1 - t_p)\left[\frac{(1 - t_c) + H\beta_{hR}}{V_m + V_f + H}\right] .$$

This is in fact a market equilibrium equation. $E(R)$ is the expected total return from the capital capacity of all firms, which is assumed not to be changed in the short run. The right hand side is the required total return (before taxes) from risky marketable assets. The required total return consists of two parts: the certainty return R_c, which is the first item on the right hand side of (17), and the aggregate risk premium π, which is the second item in that equation.

When t_c increases, the required certainty return R_c will increase and the risk premium π will decrease. For the set of parameters we used, particularly assuming r_f is reasonably large, the change in R_c is dominant. Since $E(R)$ is constant in the short run, the increase of required return caused by the increase in the corporate tax rate forces the market value of capital to fall in order to restore the market equilibrium.

When t_p increases, however, the only direct impact is that π will decrease. Under the assumption that the effect of change in certainty return is dominant, the market value V_m has to increase to restore the equilibrium.

From equation (17), we can see clearly that the corporate income tax and personal income tax have different impacts on the aggregate risk premium as well as on the certainty return required by investors. It is worth pointing out that although the magnitudes of both elasticities e_{t_c} and e_{t_p} are increasing in β_{hm} as shown in Table 1, the elasticity of the personal tax rate e_{t_p} is more sensitive to β_{hm} than is e_{t_c}. This can be accounted for by the different impacts of t_c and t_p on the contribution of human wealth to the aggregate risk premium in equation (17).

7. Summary

In this paper, we have discussed the short-run effects of the different tax rate changes on the market value of risk capital and the rate of return required by investors, and the long-run effect on the ratio of risky to total marketable assets, with the model allowing for the existence of human wealth. If tax changes decrease the corporate tax rate or the tax rate on the returns from stocks and correspondingly increase the personal tax rate on returns from risk-free assets and human wealth, then for plausible values of the relevant parameters, this model implies that stock prices would go up in the short run, while in the long run the proportion of wealth of the economy invested in risky assets such as equities would also go up. But it should be pointed out that if in the long run the proportion of wealth devoted to risky assets does increase, it does not necessarily guarantee the increase of the total amount of investment in risky assets. The total amount of investment in risky assets depends not only on the proportion of wealth invested in such assets but also on the total savings available for investment which is subject to change if tax changes are implemented. However, there is substantial evidence that private savings are not very sensitive to changes in the after-tax rate of return (see Friend and Hasbrouck (1983)).

Whether allowance is or is not made for human wealth in our model within a reasonable range of real riskfree interest rates, an increase in corporate tax rates is associated with a decrease in the market value of risky assets (at least in the short run), whereas an increase in personal tax rates which are not dependent on the type of asset is associated with an increase in the market value of risky assets (Table 1). The larger the riskfree rate, the greater the corporate tax effect and the weaker the personal tax effect (again within a reasonable range of interest rates). When human wealth is provided for in the model, the larger the covariance between income from human and non-human wealth, the greater the effect of tax changes, again negative for corporate tax increases and positive for personal tax changes.[9]

Once more, when plausible income tax rates which are different for risky marketable assets than for other assets are introduced, increased corporate tax rates generally retain the same negative effect on the market value of risky assets, while increased personal tax rates on income from riskfree assets and human wealth continue to have a positive effect on market value, which is now higher than under a uniform schedule of personal income taxes for risky marketable and other assets (Table 2). Personal tax changes for income from risky marketable assets have effects which are intermediate between those for corporate taxes and for taxes on income from other personal assets. As noted earlier, however, an increase in personal tax rates on risky marketable assets will generally decrease the market value of such assets.

Obviously, these results depend on the assumptions made in the model including the invariance of the before-tax real riskfree interest rate to personal as well as to corporate income taxes and the validity of the range of values estimated or assumed for such parameters as the actual value of the riskfree rate, the effective tax rates on the three major categories of assets, the ratio of human wealth to total wealth including marketable assets, the covariance of income from human wealth and from marketable assets, the initial ratio of risky marketable assets to total marketable assets, and the measure of relative risk aversion for the market as a whole. The short-run results also depend on the assumption that the supply of physical plant and capacity output cannot be changed in a short period of time, whereas the long-run analysis does permit such changes but does not explicitly introduce the supply conditions. As a result, the short-run results are more satisfactory than the long-run results.

In spite of these serious deficiencies, some of which we hope to remedy in subsequent work, we believe the results are adequate to indicate some of the major factors on which the effect of taxation on the value of risky assets and on the cost of risk capital depends, though with the problem of the determination of the riskfree rate, the results may be more germane to the relative than to the absolute value and cost of risk capital.

APPENDIX 1

Assume that there are n risky marketable assets and one riskfree asset in the economy. The rates of return of risky marketable assets can be described by a vector

$$\underline{r} = (r_1 , \dots , r_n) \quad .$$

The riskfree rate is r_f. Investors can invest in these $n+1$ marketable assets, and they also have human wealth H_k with rate of return r_{hk}, where $k = 1,\dots,N$; N is the number of investors. For simplicity in the following discussion, we omit the subscript k. The marketable and non-marketable risky asset returns are governed by Wiener processes

$$d\underline{r} = E\underline{r}dt + \underline{\sigma dz} \quad \text{and} \quad dr_h = Er_h + \sigma_h dz_h \quad ,$$

where $E\underline{r} = (Er_1 , \dots , Er_n)'$ and $\underline{\sigma dz} = (\sigma_1 dz_1 , \dots , \sigma_n dz_n)'$.
Suppose

$$\underline{\sigma dz} \cdot \underline{\sigma dz}' = \sum dt \quad \text{and} \quad \underline{\sigma dz} \cdot \sigma_h dz_h = \underline{G}_h dt \quad ,$$

where $\sum = \{\sigma_{ij}\}$ is a $n x n$ covariance matrix of returns on risky marketable assets and $\underline{G}_h = (\sigma_{1h},\dots,\sigma_{nh})$ is the covariance vector of the returns on risky marketable assets and human wealth.

The representative investor's objective is to maximize the lifetime expected utility

$$\text{(A1)} \qquad \text{Max } E \int_0^T u\big(C(t),t\big)dt$$

subject to the wealth dynamic constraint

$$dW = W(1-t_p)\{(1-h)[r_f dt + \underline{\alpha}'(E\underline{r}(1-t_c) - er_f)dt + (1-t_c)\underline{\alpha}'\underline{\sigma dz} + h(Er_h + \sigma_h dz_h]\} - C(t)dt \quad , \tag{A2}$$

where $C(t)$ is the investor's consumption rate in time t, $\underline{\alpha}$ is the coefficient vector of investor's portfolios, and $\underline{e}$ is a vector of unity. Notice in (A1) that we have assumed a nonbequest condition.

By stochastic dynamic programming, we can obtain the following Hamilton-Jacobi-Bellman equation:

$$\text{Max}\left[u(C(t),t) + (d/dt)EJ(W(t),t)\right] = 0 \quad . \tag{A3}$$

where $J(W(t),t) = \text{Max } E_t \int_t^T u(C(t),t)dt$. Applying Ito's lemma, we have

$$dJ = \frac{\partial J}{\partial t}dt + \frac{\partial J}{\partial W}dW + \tfrac{1}{2}\frac{\partial^2 J}{\partial W^2}(dW)^2 \quad . \tag{A4}$$

Hence

$$d/dt\ EJ(W(t),t) =$$

$$\frac{\partial J}{\partial t} + \frac{\partial J}{\partial W}\{\{W(1-t_p)(1-h)[r_f + \underline{\alpha}'(Er(1-t_c) - \underline{e}r_f)] + hEr_h\} - C(t)\}$$

$$+ \tfrac{1}{2}W^2\frac{\partial^2 J}{\partial W^2}(1-t_p)^2[(1-t_c)^2(1-h)^2\underline{\alpha}'\textstyle\sum\underline{\alpha} + h^2\sigma_h^2 + 2(1-t_c)(1-h)h\underline{\alpha}'\underline{G}_h] \quad .$$

Substituting (A5) into (A3), we can get the first-order conditions:

$$\partial u/\partial C - \partial J/\partial W = 0 \quad . \tag{A6}$$

This is the intertemporal envelope condition by which the investor decides the optimal consumption. The optimal portfolio condition is

$$W\frac{\partial J}{\partial W}(1-t_p)[(1-h)(E\underline{r}(1-t_c) - \underline{e}r_f)$$

$$+ W^2\frac{\partial^2 J}{\partial W^2}(1-t_p)^2[(1-t_c)^2(1-h)^2\textstyle\sum\underline{\alpha} + (1-h)(1-t_p)h\underline{G}_h] = 0 \quad .$$

The above expression can be rewritten as

$$\text{(A7)} \qquad \frac{\underline{Er}(1-t_c) - \underline{e}r_f}{K(1-t_p)(1-t_c)^2(1-h)} = \sum \underline{\alpha} + \frac{h\underline{G}_h}{(1-t_c)(1-h)}$$

or

$$\text{(A8)} \qquad \underline{\alpha} = \frac{1}{K(1-t_c)^2(1-t_p)(1-h)} \sum^{-1} \left[\underline{Er}(1-t_c) - \underline{e}r_r\right] - \frac{h}{(1-t_c)(1-h)} \sum^{-1} \underline{G}_h \quad ,$$

where

$$K = -W\left(\frac{\partial^2 J}{\partial W^2}\right) / \left(\frac{\partial J}{\partial W}\right) \quad .$$

$J(W(t),t)$ can be viewed as the investor's indirect utility function, so K is the Arrow-Pratt measure of relative risk aversion of the indirect utility function. In this paper, we assume that $u(\cdot)$ is an isoelastic marginal utility function. Merton (1969) has shown that $J(W(t),t)$ is also an isoelastic marginal utility function with the same coefficient of relative risk aversion, i.e. $K = c$. Thus, we have proved that (A8) is identical to equation (8') in Section 2 of the text.

APPENDIX 2

After taking into account equation (14), the right-hand side of (31) can be written as

$$\text{(A9)} \qquad \frac{r_f}{C(1-t_c)^2} \sum_k \frac{1-t_{fk}}{(1-t_{sk})^2} \gamma_k + \frac{1}{1-t_c} \sum_k \frac{1-t_{fk}}{1-t_{sk}} h_k \mathrm{Cov}(r_m, r_{hk}) \gamma_k \quad .$$

Because $\gamma_k = W_k/W$, $\sum_k \gamma_k = 1$ and $\gamma_k > 0$, $\{\gamma_k\}$ can be thought of as a set of probabilities corresponding to variable W_k. (A9) now can be rewritten as

$$\text{(A10)} \qquad \int_{W_k} \frac{1-t_{fk}}{1-t_{sk}} \left[\frac{r_f}{C(1-t_c)^2(1-t_{sk})} + \frac{h_k \mathrm{Cov}(r_m, r_{hk})}{1-t_c}\right] g(W_k) dW_k \quad ,$$

where we employ a continuous probability density function $g(W)$ to describe approximately the discrete distribution $\{\gamma_k\}$. If we consider t_{fk}, t_{sk} and r_{hk} as continuous functions of W_k, then by the mean value theorem of integration (A10) becomes

$$\text{(A11)} \qquad \frac{1-t_f(W^*)}{1-t_s(W^*)} \left[\int_{W_k} \frac{r_f}{C(1-t_c)^2(1-t_{sk})} g(W_k) dW_k + \int_{W_k} \frac{h_k \mathrm{Cov}(r_m, r_{hk})}{1-t_c} g(W_k) dW_k \right] \quad ,$$

where W^* is somewhere in the interval $(\underline{W}, \overline{W})$. $\underline{W}$, $\overline{W}$ are the lower and upper bound of W, respectively. It is clear that the first integral in (A11) is just

$$\frac{r_f}{C(1 - t_c)^2(1 - t_s)} \quad ,$$

where $1-t_s$ is the harmonic mean of $(1-t_{sk})$. In calculating the second integral, we go back to the discrete notations:

(A12)
$$1/(1-t_c)\sum_k h_k \mathrm{Cov}(r_m, r_{hk})\gamma_k$$
$$= 1/(1-t_c)\sum_k \left(H_k/W_k\right) \mathrm{Cov}(r_m, r_{hk})\left(W_k/W\right)$$
$$= 1/\left((1-t_c)W\right) \mathrm{Cov}\left(r_m, \textstyle\sum_k H_k r_{hk}\right)$$
$$= \frac{h\ \mathrm{Cov}(r_m, r_h)}{1-t_c} .$$

Now we assume that

(A13)
$$\frac{1-t_f(W^*)}{1-t_s(W^*)} \approx \frac{1-t_f}{1-t_s} ,$$

where t_f is the arithmetic mean of t_{fk}. If (A13) is justified, we directly obtain expression (33) in Section 5. As mentioned earlier, this rather strong assumption is made only for convenience of theoretical analyses. But from the nature of the tax rates on the returns from risky marketable assets and from riskfree assets, perhaps (A13) is not too bad a first approximation. The personal income from risky marketable assets (taken to be common stocks) can be roughly divided into two equal parts: dividends, which are subject to the same tax rate as the income from riskfree asset, and realized capital gains, which are taxed at about one-half of the tax rate on the income from riskfree assets. So the tax rate on the income from the risky marketable assets can be approximated as

(A14)
$$t_{sk} = \mu t_{fk} .$$

If the capital gains are fully realized and half of the income from risky assets come from capital gains which are taxed at half the rate on ordinary income, μ = .75. On the other hand, if no capital gains are realized, $\mu = .5$. Hence $.5 \leq \mu \leq .75$. From a pragmatic point of view, when μ is between .5 and .75 and t_{fk} varies is a plausible range $(1-t_{fk})/(1-t_{sk}) = (1-t_{fk})/(1-\lambda t_{sk})$ as a function of t_{fk} is smooth and pretty flat. Therefore, the approximation equation (A13) is probably valid.

We could have a better approximation if we employed another more accurate expression with an adjustment factor

$$\text{(A15)} \qquad \frac{1 - t_f(W^*)}{1 - t_s(W^*)} \approx \frac{1 - \lambda t_f}{1 - t_s} \quad .$$

Because of the flatness of $(1-t_{fk})/(1-t_{sk})$, λ seems to be a positive number which is close to one. The equations (34)-(38) would remain unaltered if we substitute $t_f' = \lambda t_f$ for t_f. Since $t_f' \approx t_f$ and

$$\frac{\partial V_m}{\partial t_f} = \lambda \frac{\partial V_m}{\partial t_f'} \quad ,$$

no significant changes would be brought about by introducing the adjustment factor λ.

FOOTNOTES

1. H_k is assumed exogenous. We use r_{kh} instead of expected cash flow simply for convenience.

2. Here we follow the methodology used in Friend and Blume (1975). This is a direct and simple way to derive the results. The implication of this approach is that the investor is only concerned with his wealth level in the next period. A more sophisticated derivation is presented in Appendix 1.

3. By analyzing the cross-sectional data, Friend and Blume (1975) have shown that the assumption of constant proportional risk aversion for households is, as a first approximation, a fairly accurate description of the market place. The coefficients of proportional risk aversion for households (abstracting from human wealth) are well in excess of one and probably in excess of two.

4. Since $\gamma_k = W_k/W$ and $W_k(1-h_k)$ represents the total marketable assets of investor k, we have

$$\gamma_k(1 - h_k)\alpha_k = R_k/W \quad ,$$

where R_k is the total risky marketable assets of investor k. Notice that $\sum_k R_k r_{mk}$ is the total return from the market portfolio of the economy, which is equal to Vr_m, where V is the total risky marketable assets of the economy. Hence

$$\sum_k \gamma_k(1 - h_k)\alpha_k r_{mk} = \sum_k (1/W)R_k r_{mk}$$

$$= (1/W)Vr_m$$

$$= (1 - h)\alpha r_m \quad .$$

This justifies (15).

5. Recall that $\alpha = V_m/(V_m+V_f)$. So $C\alpha_m = C(1-h)\alpha$. It should be noted that the base estimate of C(1-h) remains the same in this model. See Friend and Hasbrouck (1981).

6. See Footnote 5.

7. Fama (1975) suggested that the real riskfree interest rate is in the neighborhood of .01, though it clearly has been much higher in the United States for a number of years. Fama and Schwert (1977) presented some evidence showing that the relationship between the return on human capital and that on various marketable assets is weak. On the other hand, Friend and Blume (1975) provided evidence that the covariance between the returns on human wealth and the market is significantly positive.

8. t_s should be the value taken from the harmonic mean of $(1-t_s)$, while t_f is the arithmetic mean. The fact that we use different averages reflects the asymmetry of these two personal taxes.

9. The impact of human wealth on the market is reflected in the change in the market price of risk. Without taxes, the market price of risk can be defined as

$$\eta = \frac{Er_m - r_f}{\sigma_m^2} = C\alpha(1 - h) + Ch\beta_{hm} \quad ,$$

Notice that if $\beta_{hm} = 0$, as one set of estimates implies [4], $C\alpha(1-h)$ is the same with or without human wealth, since an allowance for human wealth requires a corresponding adjustment in the esimate of C. Therefore the allowance of human wealth has an impact on the market price of risk only if the covariance between human and non-human income is not zero. For example, if $\alpha = 0.85$, η is equal to 1.70 in the cases without human wealth or with human wealth but $\beta_{hm} = 0$. η becomes 3.70 if $h = 2/3$ and $\beta_{hm} = 0.50$. It is obvious that the larger the covariance between incomes from human and non-human wealth, the greater the impact on the market price of risk.

When we consider corporate and personal income taxes, the market price of risk can be written as

$$\eta = \frac{Er_m(1 - t_c) - r_f}{\sigma_m^2(1 - t_c)^2(1 - t_p)} = C(1 - h)\alpha + \frac{Ch\beta_{hm}}{1 - t_c} \quad .$$

We can see the situation is quite similar. Suppose $t_p = 0.25$, $t_c = 0.45$, and $\alpha = 0.85$. Without human wealth or with human wealth but $\beta_{hm} = 0$, we have $\eta = 1.7$, while η becomes 5.34 if $h = 2/3$ and $\beta_{hm} = 0.50$.

REFERENCES

1. K.J. Arrow, Essays in the Theory of Risk-Bearing, (Amsterdam: North Holland, 1971).

2. M. Blume & I. Friend, "The Effect of Reduction in Corporate Taxes on Investment in Riskfree and Risky Asset," (1984) Rodney L. White Center for Financial Research Working Paper #3-84, Wharton School, University of Pennsylvania.

3. E.F. Fama "Short-Term Interest Rates as Predictors of Inflation," American Economic Review 65, (1975), 269-282.

4. E.F. Fama & G. Schwert, "Human Capital and Capital Market Equilibrium," Journal of Financial Economics 4, (1977), 95-125.

5. I. Friend & M. Blume, "The Demand for Risky Assets," American Economic Review 65, (1975), 900-922.

6. I. Friend & J. Hasbrouck, "Effect of Inflation on the Profitability and Valuation of U.S. Corporations," in Saving and Investment Incentives in an Inflationary Environment, Marshall Sarnat and Giorgo Szego (eds.), (Cambridge, MA: Ballinger Publishing Company, 1981).

7. I. Friend & J. Hasbrouck, "Inflation and the Stock Market: Comment," American Economic Review 72, (1982), 237-242.

8. I. Friend & J. Hasbrouck, "Saving and After-Tax Rates of Return," Review of Economics and Statistics 65, (1983), 537-543.

9. D. Mayers, "Non-Marketable Assets and Capital Market Equilibrium Under Uncertainty," in Studies in the Theory of Capital Markets, M.C. Jensen (ed.), (New York: Praeger Publishers Inc., 1972).

10. R.C. Merton, "Lifetime Portfolio Selection Under Uncertainty," Review of Economics and Statistics 51, (1969), 247-257.

11. R.C. Merton, "Optimum Consumption and Portfolio Rules in a Continuous Time Model," Journal of Economic Theory 3, (1971), 373-413.

12. R.C. Merton, "An Intertemporal Capital Asset Pricing Model," Econometrica 41, (1973), 867-887.

TABLE 1

Elasticity of Value of Capital With Respect to
Corporate Income Tax Rate and Personal Income Tax Rate

	No Human Wealth		With Human Wealth					
	e_{tc}	e_{tp}	e_{tc}			e_{tp}		
			β_{hm} =			β_{hm} =		
Riskfree rate			0.00	0.25	0.50	0.00	0.25	0.50
0.01	-0.412	1.102	-0.143	-0.220	-0.474	0.384	1.219	3.979
0.02	-0.734	0.229	-0.528	-0.621	-0.754	0.165	0.401	0.739
0.03	-0.771	0.128	-0.633	-0.700	-0.783	0.105	0.240	0.408
0.04	-0.786	0.089	-0.683	-0.734	-0.794	0.077	0.171	0.281
0.05	-0.793	0.068	-0.711	-0.753	-0.800	0.061	0.133	0.215
0.06	-0.798	0.055	-0.730	-0.765	-0.803	0.050	0.109	0.174

$$e_{tc} = (t_c/V_m)(\partial V_m/\partial t_c) \quad ; \quad e_{tp} = (t_p/V_m)(\partial V_m/\partial t_p)$$

Parameters: $\alpha = 0.85$, $t_c = 0.45$, $t_p = 0.25$, $\sigma_m = 0.15$

TABLE 2

Elasticity of Value of Capital With Respect to
Corporate Income Tax Rate and Different Personal Income Tax Rates

Riskfree rate	No Human Wealth			With Human Wealth								
	e_{tc}	e_{ts}	e_{tf}	e_{tc}			e_{ts}			e_{tf}		
				β_{hm} =			β_{hm} =			β_{hm} =		
				0.00	0.25	0.50	0.00	0.25	0.50	0.00	0.25	0.50
0.01	0.470	0.144	4.252	0.068	0.113	0.340	0.021	0.034	0.104	0.612	2.032	9.171
0.02	-0.700	-0.214	0.779	-0.453	-0.543	-0.677	-0.139	-0.166	-0.207	0.504	0.903	1.497
0.03	-0.756	-0.231	0.612	-0.588	-0.657	-0.734	-0.180	-0.201	-0.227	0.476	0.707	0.998
0.04	-0.776	-0.237	0.553	-0.651	-0.704	-0.767	-0.199	-0.215	-0.234	0.463	0.625	0.816
0.05	-0.787	-0.240	0.523	-0.686	-0.730	-0.779	-0.210	-0.233	-0.238	0.456	0.581	0.723
0.06	-0.793	-0.242	0.504	-0.709	-0.746	-0.787	-0.217	-0.228	-0.240	0.451	0.553	0.665

$$e_{tc} = (t_c/V_m)(\partial V_m/\partial t_c) \quad ; \quad e_{ts} = (t_s/V_m)(\partial V_m/\partial t_s) \quad ; \quad e_{tf} = (t_f/V_m)(\partial V_m/\partial t_f)$$

Parameters: $\alpha = 0.85$, $t_c = 0.45$, $t_s = .20$, $t_f = 0.30$, $\sigma_m = 0.15$