ON THE OPTIMALITY OF PORTFOLIO INSURANCE

by

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I. Introduction

Portfolio insurance has always had an intuitive appeal to investors, particularly if the cost is not too great. What could be better than the possibility of making substantial sums of money with no chance of loss? Responding to this appeal, commercial organizations began selling insurance against investment loss in the United Kingdom in 1956 and in the United States in 1971. Gatto, Geske, Litzenberger, and Sosin (1980) provide an excellent and insightful analysis of the specific insurance plans that were offered to individuals by Harleysville Mutual Insurance Company and Prudential Insurance Company of America. Nowadays, several firms are marketing to institutions portfolio "insurance" techniques under such names as "Dynamic Asset Allocation/Protector Portfolio Management," "Portfolio Insulation," and "Portfolio Risk Control."

There are a number of ways to define portfolio insurance. Leland (1980) proposes one such definition in the context of a two-date world. Most of the analysis in this paper is consistent with his definition. At the first date, the investor has some wealth to invest so as to maximize his expected utility at the second date. An insured strategy is one in which the investor places part of his wealth in a portfolio of risky assets and uses his remaining wealth to buy an European put on that portfolio. If the value of the portfolio on the second date is less than the striking price of the put, he exercises the put; otherwise, he lets the put expire. Thus, he has insured his portfolio at the striking price of the put.

The purpose of this paper is to explore conditions under which an investor would utilize portfolio insurance as part of an overall strategy. Whether an investor would utilize portfolio insurance hinges, among other things, upon the completeness of the market. In the complete markets that

Black and Scholes postulate in deriving their option pricing model, the utility function that an investor would need would have such bizarre characteristics that it is highly unlikely that any investor would purchase insurance. However, in some types of less complete markets, investors with plausible utility functions may find it desirable to buy insurance. The desirability of insurance thus hinges upon the way in which incompleteness is introduced into the markets of Black and Scholes. The paper concludes with an examination of an alternative definition of portfolio insurance in a multi-date world that appears similar to the usual insurance problem but is quite different in reality.

II. Portfolio Insurance in a Two-Date Model

The development of the Black-Scholes option pricing model assumes that the returns on risky assets conform to Weiner processes and that there is continuous trading. Under their assumptions, there exists a trading strategy by which an investor can create an asset whose value will be indistinguishable from that of an European put. The investor can then use this so-called pseudo-put to insure a risky portfolio.

In the first part of the section, we define the required notation and show how to insure a portfolio. In the second part, we develop some propositions as to the investment strategy implicit in an insured portfolio. In the final part, we utilize these propositions to infer some characteristics of the von Neumann-Morgenstern utility function that would result in an investor insuring his risky portfolio.

A. Constructing the Two-Date Portfolio Insurance Policy[1]

As already pointed out, an investor can insure a portfolio of risky assets by purchasing a put on that portfolio with a striking price equal to the desired insurance level. Specifically, if the investor at time 0 has wealth W_0, he could insure his portfolio at K by using part of his wealth to purchase stock and the remainder to purchase an European put with striking price K. Let S_0 represent the investment in the stock, and P_0 the price of the put. In conformity with Black-Scholes, assume that the share price follows a Wiener process with expected return α and standard deviation σ, the risk-free rate is fixed at r, there are no dividends, and trading can take place continuously.

It is well known that a put can be replicated through the continuous revision of a portfolio with a long position in the risk-free asset and a short position in the stock. More specifically, the put can be duplicated by holding, for any $0 \leq t \leq 1$,

$P_t + N(-h)S_t$ of a risk-free discount bond maturing at $t = 1$,

$-N(-h)S_t$ of the stock,

where

$$h = \log(S_t/Ke^{-r(1-t)})/\sigma\sqrt{1-t} + \tfrac{1}{2}\sigma\sqrt{1-t}$$

$N(\cdot)$ is the standard cumulative normal distribution

$$P_t = -[S_t N(-h) - Ke^{-r(1-t)} N(\sigma\sqrt{1-t} - h)] .$$

This portfolio is "self-financing" and exactly replicates the value of the put over time.

Thus, even if there is no traded put on the stock, there exists a portfolio that replicates the put and thus provides the desired insurance policy. At any time t, the composition of an insured portfolio will be

$S_t(1 - N(-h))$ of the stock,

$P_t + N(-h)S_t$ of the riskfree asset.

The proportion of the investor's wealth held in the risky asset at time t, ω_t, is given by

$$\omega_t = \frac{S_t(1 - N(-h))}{S_t(1 - N(-h)) + (P_t + N(-h)S_t)}$$

$$= \frac{S_t(1 - N(-h))}{S_t + P_t} \tag{1}$$

$$= \frac{S_t(1 - N(-h))}{S_t(1 - N(-h)) + Ke^{-r(1-t)}N(\sigma\sqrt{1-t} - h)} .$$

B. The Two-Date Model: The Implicit Portfolio Strategy

The only decision variable in this problem is the proportion of wealth to be placed in the risky asset, ω_t. Thus, the relationship of ω_t to time and wealth characterizes the implicit investment strategy, and the following remarks address this relationship. Proofs, where not obvious, are included in footnotes.

First, if an investor wishes to insure a portfolio at the striking price K, his initial wealth must be no less than the discounted value of K or Ke^{-r}. If it were less than Ke^{-r}, the investor would be assured of a minimum rate of return in excess of the riskfree rate, creating an arbitrage opportunity that is inconsistent wiht the Black-Scholes model. Moreover, for any t, $0 \leq t \leq 1$, the investor's wealth can never be less than $Ke^{-r(1-t)}$. Thus, for any feasible investment strategy, the investor's wealth must always be on or above the line in Figure 1 that portrays the striking price K, discounted at the riskfree rate.

Second, if the investor's wealth at any time t is equal to $Ke^{-r(1-t)}$, the only feasible insurance strategy is to invest everything in the riskfree asset in order to be certain of receiving K. Thus, in this case, $\omega_t = 0$.

Third, at any time t, the proportion invested in the risky asset ω_t increases with increases in wealth.[2] Thus, in Figure 1, portfolio B would have a greater proportion in risky assets than portfolio A. Morever, it can be shown that, whenever $S_t \geq K$, $\omega_t > 1/2$.[3]

Fourth, as t approaches one, the proportion invested in risky assets will with probability one approach either zero or one.[4] Intuitively, if the stock price is greater than K, the "smoothness" of the assumed Wiener process implies that the probability that the stock price will ever fall below K approaches zero as t approaches one, so that the investor can allow ω_t to approach one and be assured of always obtaining K. The intuitive reason that ω_t approaches zero if the stock price is less than K is less clear, but is undoubtedly related to the fact above that as the investor's wealth approaches the discounted value of the insurance level, the investor must place a greater proportion of his assets in the riskfree asset to be assured of obtaining K.

C. The Implied Characteristics of the Investor's Utility Function

Whether the insurance policy analyzed above is optimal depends, of course, upon the investor's utility function. The purpose of this subsection is to determine what characteristics of the investor's utility function would dictate an insurance strategy. The logic will be to assume that an investor has solved the usual expected utility maximization problem and that the solution is an insurance strategy. We shall then ask what properties the original utility function must have had for an insurance strategy to have been optimal.

Consider an investor who at each time t, $0 \leq t \leq 1$, chooses a portfolio composed of a proportion ω_t of a single risky asset and a proportion $1 - \omega_t$ of a riskless asset with return $r > 0$ so as to maximize his expected utility of end-of-period wealth. The investor's utility maximization problem may be written:

$$\max EU[W_1]$$

s.t.

$$W_t \geq 0, \quad 0 \leq t \leq 1 \quad ,$$

$$W_0 = \bar{W}_0 > 0$$

$$dW = W_t\{[\omega_t(\alpha - r) + r]dt + \omega_t\sigma(t)dZ\} \quad ,$$

where dZ is a Weiner process. We make the usual assumptions that $U' > 0$ and $U'' < 0$.

Merton (1969), Friend and Blume (1975), and Ross (1975) show that an investor's coefficient of relative risk aversion C_t in an optimal strategy must satisfy

$$\omega_t = \frac{(\alpha - r)}{\sigma^2} \cdot \frac{1}{C_t} \quad . \tag{2}$$

This relationship shows how the coefficient of relative risk aversion of an investor who chooses the two-date portfolio insurance strategy will vary over time and as a function of wealth. Namely, the coefficient of relative risk aversion will be given by

$$C_t = \frac{1}{\omega_t}\frac{(\alpha-r)}{\sigma^2} = \frac{S_t\big(1-N(-h)\big) + Ke^{-r(1-t)}N(\sigma\sqrt{1-t} - h)}{S_t\big(1-N(-h)\big)} \cdot \frac{\alpha - r}{\sigma^2} \quad . \tag{3}$$

Since the proportion invested in risky assets varies both as a function of time and wealth, one-period insurance strategies can in general be optimal only for utility maximizers whose utility functions display non-constant coefficients of relative risk aversion. In the limit the coefficient of relative risk aversion will approach infinity when as $t \to 1$, $S_t < K$. More generally, for given K, $t < 1$, and other parameters, C_t decreases with increases in wealth. Thus, an investor must have decreasing proportional risk aversion function up to the exercise date of the put. This argument can be summarized as follows:

Theorem 1: Under the assumptions made in the Black-Scholes model, the investor's utility function must have an unbounded coefficient of proportional risk aversion at some level of wealth if the investor finds it optimal to insure his total portfolio at some positive level and decreasing proportional risk aversion above that level.

III. Less Complete Markets

Under the assumptions of the Black-Scholes model, the end-of-period utility function of an investor who insured his portfolio at some level would have to exhibit an unbounded coefficient of relative risk aversion below the insurance level and decreasing relative risk aversion above that level. The available empirical evidence is not consistent with this type of utility function, but rather points towards constant relative risk aversion over wide ranges of wealth.[5] Moreover, most of the empirical evidence suggests a minimum value of the coefficient of relative risk aversion of around two. If all of the Black-Scholes assumptions are made, an investor with a utility function displaying constant relative risk aversion would not insure a portfolio. However, in a world that is less complete than that of

Black-Scholes, an investor might decide to insure a portfolio.

A. An Extreme Example

In this section, we examine this possibility. Throughout we assume that markets are segmented in the following way: One set of investors have available to them the Black-Scholes markets and can avail themselves of continuous trading opportunities. These investors price the assets in the market and specifically set the prices of puts according to the Black-Scholes formula. Another group of investors can trade only at the beginning of the time period and face an incomplete market. We shall further assume that a typical investor in this latter group has a utility function displaying constant proportional risk aversion.

Let us begin with an extremely incomplete market. For the moment, assume that the investor is prohibited, perhaps for some institutional reason, from investing in riskfree assets and that there is only one risky asset available to the investor. Although the investor cannot invest in riskfree assets, he is permitted to purchase a put on the risky asset with any striking price that he sets.

In this highly stylized world, the only decision that the investor faces is the striking price of the put, and this decision must be made at the start of the period. Subsequently, we shall consider more realistic scenarios.

If the investor's initial wealth and the price of one share are each one dollar, he could buy one share of the risky asset with no insurance. Alternatively, he could buy a fraction of a share and a put on that asset with striking price K. If the price of a put on one share with striking price K is P, the investor would buy $1/(1 + P)$ share and the same fraction of the put. Thus, the investor's wealth at the end of the period,

W_1, will be

$$W_1 = \max(S_1, K)/(1 + P) \quad , \tag{4}$$

where S_1 is the value of one share at the end of the period.

Now, assume that the investor's utility function is of the constant relative risk aversion form

$$U(W_1) = \frac{1}{1 - \gamma} W_1^{1-\gamma} \tag{5}$$

with the coefficient of relative risk aversion γ taking on the values of 2, 4, 8, and 16. For any specific set of return assumptions, it is possible to determine numerically the optimal striking price for any coefficient of relative risk aversion.

We use two sets of assumptions for the underlying parameters. The first set assumes that the Weiner process for the risky asset has a drift term of 0.15 and a standard deviation of 0.20. Together, these imply that the expected one-period return is 18.5 percent.[6] The drift term, or equivalently the continuously compounded rate of return, for the risk-free asset is 0.10 for an effective rate of 10.5 percent.

The second set assumes for the risky asset a drift term of 0.04 and a standard deviation of 0.20 for an expected one-period return of 6.2 percent. The continuously compounded rate of return for the risk-free asset is .01 for an effective rate of just over 1 percent. Some might view the first set as roughly approximating the nominal rates of today, and the second set as roughly approximating the real rates since the turn of the century.[7]

Using the first set of assumptions, an investor with a relative risk coefficient of 2.0 would realize his highest level of expected utility by purchasing a put with a striking price of 0.70 dollars.[8] Since the investor's total wealth is one dollar, the same as the price of one share of the risky asset, and since the investor must pay a premium for the put, he can only buy a fraction of a full share. In this case, his optimal strategy is to buy 99.9 percent of a share of stock and 99.9 percent of a put with an exercise price of 0.70 dollars. Since the investor purchases less than a full put, the insurance level is somewhat less than 0.70 dollars.

In deriving these numbers, the computer program only considered striking prices in the range of zero to one dollar. The theoretical results from the previous section indicate that it is possible to insure a portfolio at a somewhat greater amount than the investor's current wealth, providing the riskfree rate is positive. Nonetheless, the assumption that the maximum exercise price for a put can never exceed one dollar is a reasonable one for our purposes. First, purchasing some fraction of a put with a maximum striking price of one dollar is always feasible for any positive risk-free rate. Second, it seems in the spirit of an insurance policy not to insure one's wealth for more than its current value.

Thus, an investor in this form of an incomplete market would choose to buy a put. An important question, especially for public policy, is how valuable is the availability of such a put to an investor. A measure of how valuable the put is to the investor is the amount that the investor would be willing to pay to have such a put available.

Following Goldman (1974), an answer to this question is obtainable as follows: Let $E\{U(W_1)\}$ be the expected utility of the end of period wealth associated with some arbitrary and generally non-optimal policy. For the moment, let this policy be an investment without insurance. If puts are valuable to an investor, an investor would be willing to pay some fraction of his end of period wealth to be able to purchase a put. Let π be the fraction of his wealth that he would pay. After paying the proportion π, the investor's expected utility with a put having an optimal striking price is $E\{U^*[(1-\pi)W_1]\}$, where the superscript "*" indicates an optimal strategy. The value of π that equates the expected utility without a put to that with an optimal put is the maximum that an investor would pay to have the option of buying the put.

Equating these two expected utilities yields

$$E\{U^*[(1-\pi)W_1]\} = E\{U(W_1)\}$$

or

$$E\{\frac{1}{1-\gamma}[(1-\pi)W_1]^{1-\gamma}\} = E\{\frac{1}{1-\gamma}W_1^{1-\gamma}\} \qquad (6)$$

$$(1-\pi)^{1-\gamma}E[U^*(W_1)] = E[U(W_1)]$$

Finally solving for π gives the desired formula:

$$\pi = 1 - \left[\frac{E\{U(W_1)\}}{E\{U^*(W_1)\}}\right]^{\frac{1}{1-\gamma}} . \qquad (9)$$

An investor with a coefficient of relative risk aversion of 2.0 and facing the set of return assumptions corresponding roughly to today's nominal interest rates would be willing to pay up to 0.001 percent of his wealth in order to be able to buy an optimal put. Table 1 contains the results of these and similar calculations for values of the coefficient of relative risk ranging from two to sixteen for both sets of assumed return characteristics. With a coefficient of relative risk aversion of two, an investor would not find puts very valuable, but, as his coefficient of relative risk aversion increases to four and beyond, the investor would find puts increasingly valuable.

B. A More Complete World

In the incomplete market assumed above, an investor with a reasonable utility function and under plausible return assumptions would want to buy a put. Let us now examine what happens in a somewhat more complete world. Specifically, assume that the investor can, in addition to buying a put, lend money at the risk-free rate. As before, the investor cannot trade after the start of the period. The investor's decisions are what fraction of his wealth to invest in risk-free assets, the striking price of any put that he buys, and the proportion of his wealth to place in the risky asset.

Table 2 contains the optimal investment strategies for the various utility functions and for the two sets of return assumptions.[9] None of these optimal strategies involves the purchase of a put. The investor can achieve an optimal investment strategy using just the risky and risk-free asset. In fact, if he could, the investor would like to short the put.[10]

Moreover, given a choice as between adding a put or a risk-free investment, an investor would prefer the addition of the risk-free asset. From Table 1, an investor with a coefficient of relative risk aversion of 4 and facing the higher return assumptions would be willing to pay up to 1.47 percent of his wealth to have a put included in the market place, but 2.49 percent to have a risk-free asset included. The same qualitative conclusion applies to the lower return assumptions.

The intuition behind these results is something like the following: In the Black-Scholes world of Weiner processes and continuous trading, an investor with an utility function displaying constant proportional risk aversion would continuously rebalance his portfolio to maintain a constant proportion in the risky asset. Denied continuous trading, the investor may still desire to invest in the riskfree asset, but the proportion at risk would drift over time according to the relative realized returns on risky and riskfree assets. If the expected returns on risky assets were greater than the riskfree rate, the proportion invested in risky assets would tend to increase over time. Selling a put would help to reverse this tendency for the proportion in the risky asset to increase over time, although it could not replicate the result from continuous trading.

Denied both continuous trading and a riskfree rate, the investor might purchase a put as an indirect way of including a position in a riskfree asset. Since the proportion in the risky asset would tend to increase even more with a put than with a direct holding of the riskfree asset, the purchase of a put is inferior to the direct purchase of a riskfree asset.[11]

IV. Psuedo Insurance

One major corporation known to the authors has set as one of its objectives in managing its pension fund that in the next ten years there should be virtually no possibility that the total return on the fund be less than zero. The rationale given for this objective stems from the Chairman's view that it is imprudent to spend principal. At the end of each year, the objective is redefined in terms of the next ten years. There are undoubtedly other investors who think in similar terms.

Put more formally and in a somewhat more general form, the investor has a horizon of a fixed number of years at the end of which he wants to insure against any losses in excess of ϕW_o. As time marches on, the investor continually revises his insurance so as to keep his horizon constant at a fixed number of years and the level of insurance constant at a proportion ϕ of his then current level of wealth. On the surface, this strategy may appear to be similar to the insurance strategy that was examined in the prior two sections. However, this moving horizon problem turns out not to involve an insurance strategy at all.

In this moving horizon problem, the proportion invested in risky assets at the beginning of each period will remain unchanged over time since the length of the horizon is constant. Specifically, if T represents the horizon, Equation (1) can be written as

$$\omega_t = \frac{S_t\big(1 - N(-H)\big)}{S_t\big(1 - N(-H)\big) + Ke^{-rT}N(\sigma\sqrt{T} - H)}$$

where

$$H = \log(S_t/Ke^{-rT})/\sigma\sqrt{T} + {}^1\!/_2\,\sigma\sqrt{T} .$$

The use of H denotes the fact that the only time dependence in this model is caused by the change in the stock price. Note also that in the moving horizon case, the put portfolio is no longer necessarily self-financing--it may produce a profit or a loss.

If the insurance level at time t is a constant proportion of the then current wealth, the implied utility function exhibits constant proportional risk aversion. Thus, an investor with such a utility function can always be viewed as facing a dynamic portfolio insurance strategy, but in an almost trivial sense. Note that the actual portfolio implications for this case are diametrically opposite of those in the one-period model: In an insurance strategy, the proportion of assets invested in the risky asset increases as the price of that asset increases; in the moving horizon case where the insurance level is a constant fraction of current wealth, the proportion of assets invested in the risky asset does not change with changes in the price of that asset.

V. Conclusion

The optimality of an insurance strategy in a portfolio-choice context depends on the completeness of markets. In complete markets with continuous rebalancing of portfolios, the characteristics of the implied utility functions are so peculiar that it is doubtful that any investor would want to follow a two-date insurance strategy. Insurance strategies may, however, be optimal in some types of incomplete markets. However, under at least one set of reasonable assumptions, the paper showed that an investor having the option of investing in the risk-free asset would not purchase a put. In addition, the paper illustrated a technique to measure quantitatively the value to an investor of having the markets more complete.

Interestingly, an insurance strategy with a finite moving horizon was consistent with a constant proportional risk aversion function. Put another way, an investor who has a constant proportional risk aversion function in a continuous time framework with a finite moving horizon can be viewed as facing a type of insurance problem, but it is quite a different insurance problem from the usual two-period problem involving the purchase of a put.

FOOTNOTES

1 The problem discussed in this sub-section resembles that of pricing an equity-linked life insurance policy (see Brennan and Schwartz, 1976).

2 As the stock price increases, the value of the put decreases and $N(-h)$ decreases. Thus, from (1), ω_t increases.

3. From (1),

$$\omega_t = \frac{1}{1 + \frac{K}{S} e^{-r\tau} \dfrac{N(\frac{1}{2}\sigma\sqrt{\tau} - r\sqrt{\tau}/\sigma - \log(S/K)/\sigma\sqrt{\tau})}{N(\frac{1}{2}\sigma\sqrt{\tau} + r\sqrt{\tau}/\sigma + \log(S/K)/\sigma\sqrt{\tau})}} ,$$

where $\tau = 1-t$ and the index on S has been dropped. Now, consider the fraction which appears in the denominator. As long as $K \leq S$,

$$\tfrac{1}{2}\sigma\sqrt{\tau} - r\sqrt{\tau}/\sigma - \log(S/K)/\sigma\sqrt{\tau} < \tfrac{1}{2}\sigma\sqrt{\tau} + r\sqrt{\tau}/\sigma + \log(S/K)/\sigma\sqrt{\tau} .$$

Since in this case the whole denominator is less than 2, $\omega_t > \frac{1}{2}$. It is tempting to hypothesize that when $K > S_t$, $\omega_t \leq \frac{1}{2}$. A simple continuity argument shows, however, that this need not be true. Since ω_t is continuous in K and S, a small increase in K/S will still leave $\omega_t > \frac{1}{2}$.

4 We may use equation (1) to calculate the limiting insurance portfolio proportions as $\tau \to 0$, where $\tau = 1-t$. To do this, write

$$h = \frac{\log(S_t/K)}{\sigma\sqrt{\tau}} + \frac{r}{\sigma}\sqrt{\tau} + \tfrac{1}{2}\sqrt{\tau}$$

As $\tau \to 0$, we may differentiate among three cases:

Case 1: $S_t = K$. In this case $h \to 0$ as $\tau \to 0$, and
$N(h) = N(-h) \to \frac{1}{2}$.

Case 2: $S_t > K$. In this case $h \to \infty$ as $\tau \to 0$, and
$N(h) \to 1$, $N(-h) \to 0$.

Case 3: $S_t < K$. In this case $h \to \infty$ as $\tau \to 0$, and
$N(h) \to 0$, $N(-h) \to 1$.

FOOTNOTES (cont.)

The proof of the proposition now follows from (1). The probability of Case 1 is zero, so that with probability one, $\omega_t \rightarrow 0$ or 1.

5 Cf. Friend and Blume (1975) for evidence in the U.S. and Morin and Suarez (1983) for evidence in Canada.

6 The end of period return is log normal. Therefore, one plus the expected one-period return is exp(0.15 + .5 × .04), where exp is the exponential function.

7 Cf., Stocks, Bonds, Bills and Inflation Yearbook (1926-1983).

8 In calculating the expected utility for any specific strategy, the integral was evaluated from the drift term of the risky asset minus 5 standard deviations to the drift term plus five standard deviations. The interval from minus 5 to plus 5 standard deviations was divided into 100 equal subintervals. Increasing the number of subintervals to 500 produced no change in the expected utility values up to six places. The function to be integrated (the utility function times the density) was evaluated at the midpoint of each subinterval and then multiplied by the length of the subinterval. To minimize rounding errors, these resulting products were summed alternately from the extremes, first using the smallest area on the left and then the smallest area on the right and so on.

If the strike price was more than 5 standard deviations below the drift term, the integral was calculated as if the strike price was zero. Thus, in the tables, any strike price of zero should really be interpreted as a strike price of more than 5 standard deviations below the drift term. Using any striking price between these two extremes would produce virtually the same expected utility and the same portfolio allocations up to the precision shown in the tables. Finally, a gradient procedure was used to determine the optimal strike price.

9 The optimal policy was determined by a gradient search.

10 As an example, under the lower return assumptions, an investor with a coefficient of relative risk aversion of 2.0 who was permitted to sell a put would have achieved a maximum expected utility with the following portfolio: 38% in the risk-free asset, 62.002% in the risky asset, and a short sale of .62 of one put with exercise price of .59. The results were similar for other parameters.

FOOTNOTES (cont.)

11 Brennan and Solanki (1981) contains an alternative approach to the optimality of portfolio insurance in segmented markets. As in the analysis of this section, Brennan and Solanki assume that puts are priced in accordance to the Black-Scholes formula, and that the individual purchasing the portfolio insurance maximizes the expected utility of terminal wealth in a two-date framework. They prove that portfolio insurance is optimal only if the utility function is linear below the insurance level and the stock purchased has a zero risk premium -- highly unrealistic conditions.

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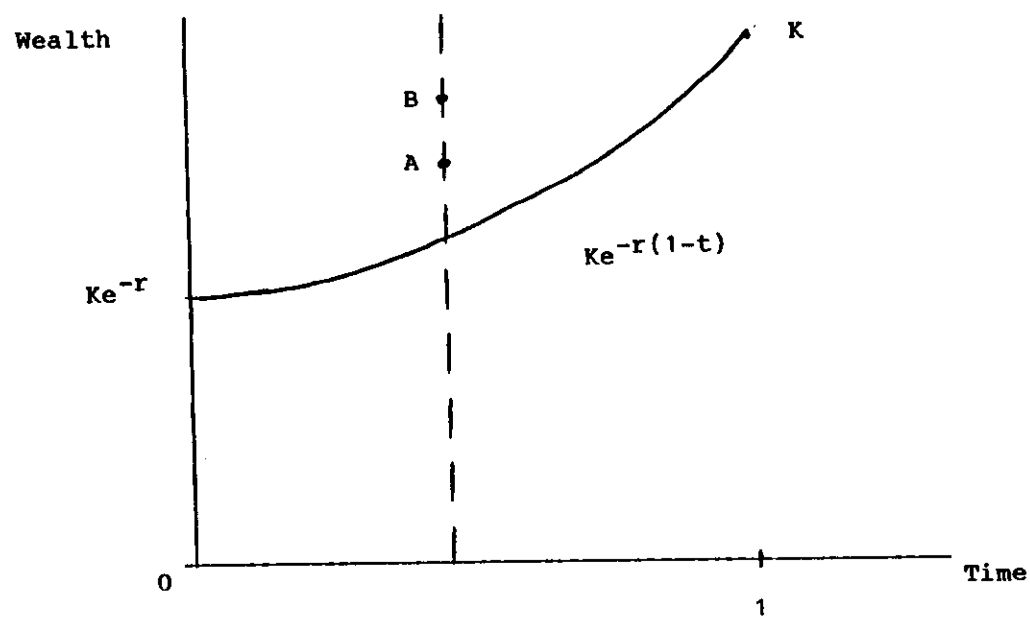
FIGURE 1

Characteristics of Insured Strategy



Characteristics:

Total wealth must always be on or above curved line

Portfolio B has a greater proportion in risky assets than Portfolio A

As wealth approaches $Ke^{-r(1-t)}$, the proportion in risky assets approaches zero.

As t approaches zero, $\text{plim}\ \omega_t = \begin{cases} 1 & \text{if } S_t > K \\ 0 & \text{if } S_t < K \end{cases}$

Table 1

Optimal Strategies Involving Puts and Risky Assets

Return Parameters %			Coefficient of Relative Risk Aversion	Optimal Strike Price	Portfolio Allocation %		Value of Right to Purchase Put (as a Percentage of Wealth)
Risk-free	Risky Drift	Risky Dispersion			Risky	Put	
10	15	20	2	0.70	99.9	0.1	0.001
			4	1.00*	96.4	3.6	1.47
			8	1.00*	96.4	3.6	6.01
			16	1.00*	96.4	3.6	16.74
1	4	20	2	0.92	96.8	3.2	0.25
			4	1.00*	93.1	6.9	2.79
			8	1.00*	93.1	6.9	8.54
			16	1.00*	93.1	6.9	20.31

* represents a constrained maximum.

Table 2

Optimal Strategies Involving Puts, Risky Assets, and Riskfree Assets

Return Parameters %			Coefficient of Relative Risk Aversion	Optimal Strike Price	Portfolio allocation %			Value of Right to Risk-free Asset (as a Percentage of Wealth)
	Risky							
Risk-free	Drift	Dispersion			Risk-free	Risky	Put	
10	15	20	2	0	12	88	0	0.06
			4	0	58	42	0	2.49
			8	0	78	22	0	9.29
			16	0	89	11	0	22.30
1	4	20	2	0	37	63	0	0.56
			4	0	69	31	0	3.70
			8	0	82	18	0	10.75
			16	0	92	8	0	23.69