The Social Value of Asymmetric Information

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Abstract

A welfare analysis of a simple noisy rational expectations model is carried out. It is shown that the more information prices convey, the worse off everybody is. However, the equilibrium where everybody is uninformed may not be Pareto optimal: imposing a tax on information gathering which finances a lump sum grant may allow everybody to be better off when some people are informed. A corresponding result holds when the model is used to consider the release of information by firms: all shareholders may be better off if information is released to a group of insiders as a form of compensation.

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1. Introduction

The question of when information has social value, in the sense that it allows everybody to be made better off, has received considerable attention in recent years. It is widely recognized that if the information is about productive opportunities then this can be socially valuable. In contrast, Hirshleifer (1971) has argued that in exchange economies improved information about the returns to assets would not be socially valuable. He was able to give a simple example where improved public information actually made everybody worse off than with no information because of a reduction in risk sharing opportunities: if people know the realization of a random variable, it is not possible to share the risks associated with the variable. A number of subsequent authors have found similar results (see, e.g., Fama and Laffer (1971), Marshall (1974), Ng (1975) and Wilson (1975)). However, Marshall, Ng and Jaffe (1975) gave examples of exchange economies where the opposite is true: improved public information does have social value. In a recent paper, Hakansson, Kunkel and Ohlson (1982) have reconciled these results by showing that sufficient conditions for better public information not to have social value are that risk-sharing possibilities are equivalent to those that can be attained with complete markets and investors have homogeneous beliefs. If either of these conditions is not satisfied, as in the Marshall, Ng and Jaffe examples, better public information may lead to a Pareto improvement.

For the case of asymmetric information, Hirshleifer considered a model where the number of people who are informed is determined exogenously and the uninformed do not deduce the information conveyed by prices. In this context, he argued that any gain by the informed must be at the expense of the uninformed so that the market allocation will be Pareto noncomparable to the no-information case. It can be further argued that if government intervention is possible the private acquisition of costly information is Pareto inefficient. This is because, if no information were gathered, the uninformed could compensate the informed and the resources previously expended on gathering information could be used to make everybody better off. The Pareto optimal allocation is thus where nobody is informed.

The purpose of this paper is to reconsider the social value of asymmetric information in exchange economies, using Grossman and Stiglitz's (1980) noisy rational expectations model. In this the number of people who are informed in equilibrium is such that everybody is indifferent between either expending resources and becoming fully informed or observing prices and deducing the information imperfectly. The assumptions of the model are such that Hakansson, Kunkel and and Ohlson's results are applicable and improved public information has no social value. It is shown below that, as in Hirshleifer's original example, a stronger result holds: everybody is in fact strictly worse off with improved public information because of a reduction in risk sharing opportunities. However, Hirshleifer's results concerning the asymmetric information case do not hold in this model. Rather than being Pareto noncomparable, it is shown that market equilibria where some people are informed and some are uninformed are Pareto worse than the no-information case: moreover, the greater the proportion who are informed the Pareto worse is the equilibrium. The reason for this is similar to the public information results: the more people that are informed the more information prices convey and the less risk sharing there is.

The main result of the paper is to show that the no-information equilibrium may not be Pareto optimal. If the government can tax the acquisition of information and use the proceeds to finance a uniform lump sum grant, examples can be constructed where everybody can be made better off in the equilibrium where some people are informed than in the equilibrium where nobody is informed. The reason for this is that in the no-information equilibrium there is still a risk due to the random supply of the risky asset: taxing information gathering and distributing the proceeds allows this risk to be shared. Thus information can be socially valuable even when improved public information cannot make everybody better off. In considering the value of information, it is therefore not possible to restrict attention to differing states of public information.

Diamond (1983) has used a very similar model to consider the release of information by firms, by interpreting the risky asset as a firm. He showed that the optimal policy of the firm is to release information whose precision is such that nobody finds it worthwhile to privately acquire further information. The result that if information acquisition is taxed everybody can be better off in the equilibrium where some people are informed has a counterpart in this context. The tax on information can be thought of as a net payment by the informed to the uninformed for the privilege of acquiring information. It is therefore equivalent to the firm releasing information to a group of employees as a form of compensation, so that their wages are lower and hence the payments to uninformed shareholders are higher, than they otherwise would be. It can be shown that if, in addition to the public release policy advocated by Diamond, the firm privately releases information to a group of employees who trade on the basis of it, all shareholders may be better off as a result of this insider trading.

The paper proceeds as follows: Section 2 describes the model and Section 3 considers equilibrium. Section 4 compares equilibria with different costs of information. In Section 5 the effect of taxing information acquisition is investigated. In Section 6 the model is interpreted in the context of firms releasing information and the desirability of insider trading is considered. Finally Section 7 contains concluding remarks.

2. The Model

The model used is that of Grossman and Stiglitz (1980). There are two assets, which are traded in competitive markets: one safe, yielding R, and the other risky, with a return u where

$$u = \theta + \varepsilon \tag{1}$$

and θ and ε are independent jointly distributed normal variables with mean and variance $(E\theta, \sigma_\theta^2)$ and $(0, \sigma_\varepsilon^2)$ respectively (with $\sigma_\theta^2, \sigma_\varepsilon^2 > 0$). Hence u has mean and variance $(E\theta, \sigma_u^2)$ where

$$\sigma_u^2 = \sigma_\theta^2 + \sigma_\varepsilon^2 . \tag{2}$$

By incurring a cost c it is possible to observe θ. Realizations of ε cannot be observed at any cost.

The i^{th} trader is assumed to be endowed with stocks of the two types of securities: he originally has $\overline{M}_i$ of the riskless asset and $\overline{X}_i$ of the risky asset. The aggregate per capita supply of the risky asset x, which is unobservable, is taken to be a normally distributed random variable with mean Ex and variance σ_x^2 which is independent of both θ, ε and $\overline{X}_i$.

There are a number of possible explanations for the randomness of x. This assumption is often justified on the grounds that it is a result of liquidity (life cycle) motivated trades. In a welfare context this justification would require consideration of this group of people in addition to those who still actively trade. Another possibility is that each agent's endowment of the risky asset is random. However, if the number of agents is finite then, as in Diamond and Verrechia (1981), each individual endowment $\overline{X}_i$

is not independent of x. If the number of agents is infinite then either the per capita endowment is a fixed nonrandom constant by the law of large numbers or, individual endowments are correlated, but then $\overline{X}_i$ and x are again not independent. In the context of these explanations the Grossman Stiglitz model can be regarded as an approximation to a well-specified model.

A more satisfactory justification for the assumptions above is that because of a stochastic birth rate the population each period is a random variable. Taken together with a fixed total endowment of the risky asset, this implies that the per capita endowment x is random. In addition, if there is a continuum of consumers, each individual's endowment $\overline{X}_i$ will be independent of x.

The sequence of events in the model is as follows. Values of x and θ are realized; people who have paid c observe θ; the safe and risky assets are traded and finally the returns to the assets are received and consumed. Utilities are evaluated taking expectations over both x and θ.

Each trader buys M_i of the safe asset and X_i of the risky asset. The price of the safe asset is normalized at unity and the price of the risky asset is P. The i^{th} person's budget constraint is therefore

$$W_{0i} = \overline{M}_i + P\overline{X}_i = M_i + PX_i \ , \qquad (3)$$

where W_{0i} is the i^{th} person's initial wealth. At the end of the period when the asset returns are received the i^{th} trader's wealth will be

$$W_{1i} = RM_i + uX_i \ . \qquad (4)$$

All individuals have identical exponential utility functions:

$$V(W_{1i}) = -\exp[-aW_{1i}] \tag{5}$$

where a is the degree of absolute risk aversion.

3. Equilibrium

A proportion λ of the population becomes informed at cost c; they observe θ. The uninformed just observe P. Now the constant absolute risk aversion assumption implies that individuals' demands will depend only on their information and not on their endowment. Thus equilibrium requires

$$\lambda X_I + (1 - \lambda)X_U = x , \tag{6}$$

where X_I and X_U are the demands of the informed and uninformed respectively.

It follows from the budget constraint (3) and the moment generating function for the normal distribution that

$$E(V_I(W_{1i})|\theta, x) = -\exp[-a(R\bar{M}_i + RP\bar{X}_i - c + (\theta - RP)X_I - \frac{a}{2}\sigma_\varepsilon^2 X_I^2)] . \tag{7}$$

Choosing X_I to maximize this gives

$$X_I = \frac{\theta - RP}{a\sigma_\varepsilon^2} . \tag{8}$$

Substituting this into (6) and rearranging

$$P = \frac{1}{R}\left[\theta - \frac{a\sigma_\varepsilon^2}{\lambda} x + \frac{(1 - \lambda)}{\lambda} a\sigma_\varepsilon^2 X_U\right] . \tag{9}$$

In a rational expectations equilibrium the uninformed will effectively know the parameters λ, a and the means and variances of θ, ε and x. They also know their own demand X_U and R; what they don't know is θ or x. Thus observing P is equivalent to observing

$$w_\lambda = \theta - \frac{a\sigma_\varepsilon^2}{\lambda}(x - Ex) \tag{10}$$

and they should therefore condition their demands on w_λ.

What Grossman and Stiglitz are able to show is that the ratio of the utility of the informed $EV_I(W_{1I})$ to that of the uninformed $EV_U(W_{1i})$ is given by

$$\frac{EV_I(W_{1i})}{EV_U(W_{1i})} = e^{ac}\beta(\lambda)^{1/2} \tag{11}$$

where

$$\beta(\lambda) = \frac{\sigma_\varepsilon^2}{\mathrm{Var}(u|w_\lambda)}, \tag{12}$$

$$\mathrm{Var}(u|w_\lambda) = \sigma_u^2 - \sigma_\theta^2\eta(\lambda), \tag{13}$$

$$\eta(\lambda) = \frac{\sigma_\theta^2}{\mathrm{Var}\ w_\lambda}, \tag{14}$$

$$\mathrm{Var}\ w_\lambda = \sigma_\theta^2 + \frac{a^2\sigma_\varepsilon^4}{\lambda^2}\sigma_x^2. \tag{15}$$

Properties of $\eta(\lambda)$ and $\beta(\lambda)$ which are important for subsequent proofs are

$$\eta(0) = 0 \quad ; \quad 0 \leq \eta(\lambda) < 1, \tag{16}$$

$$\eta'(\lambda) = 2\frac{a^2\sigma_\varepsilon^4}{\lambda^3}\sigma_x^2\frac{\eta(\lambda)}{\mathrm{Var}\ w_\lambda} \begin{cases} > 0 \text{ for } 0 < \lambda \leq 1 \\ = 0 \text{ for } \lambda = 0 \end{cases}, \tag{17}$$

$$0 < \beta(\lambda) < 1, \tag{18}$$

$$\beta'(\lambda) = \frac{\beta(\lambda)}{\mathrm{Var}(u|w_\lambda)}\sigma_\theta^2\,\eta'(\lambda) \begin{cases} > 0 \text{ for } 0 < \lambda \leq 1 \\ = 0 \text{ for } \lambda = 0 \end{cases}. \tag{19}$$

There are then three types of equilibrium.

(i) $\underline{e^{ac}\,\beta(0)^{1/2} > 1}$

In this case even if everybody is uninformed $EV_{Ii} < EV_{Ui}$ (since utility is negative) and so it does not pay for anybody to become informed.

(ii) $\underline{e^{ac}\,\beta(\lambda)^{1/2} = 1}$

In this equilibrium there are both informed and uninformed people whose utilities are the same. It follows from (19) that the equilibria of this type are stable. If $\lambda = \lambda^{e} - \delta\lambda$, where λ^{e} is an equilibrium value of λ, then $e^{ac}\beta(\lambda^{e} - \delta\lambda)^{1/2} < 1$ and $EV_{Ii} > EV_{Ui}$; thus more people will become informed until $e^{ac}\beta(\lambda^{e})^{1/2} = 1$. Similarly if $\lambda = \lambda^{e} + \delta\lambda$. The number of people who demand information and become informed adjusts until in equilibrium people are indifferent between paying c and observing θ directly or observing P and hence w_{λ}, and deducing θ imperfectly.

(iii) $\underline{e^{ac}\beta(1)^{1/2} < 1}$

In this case it pays for everybody to become informed.

A measure of the amount of information conveyed by the price, or in other words the informational efficiency of the price system, is given by the squared correlation coefficient between P and θ which is equal to $\eta(\lambda)$. It follows from (17) that the greater the proportion of the population that are informed, the more informative are prices, or the more efficient in an informational sense, is the market for the risky asset.

It can be shown from the type (ii) equilibrium condition $e^{ac}\beta(\lambda)^{1/2} = 1$ that

$$\frac{d\lambda}{dc} < 0 \qquad (20)$$

so the lower the cost of information the greater the proportion who are informed.

Together (17) and (20) imply that as the cost of information is increased the informational efficiency of the market is reduced. In the welfare analysis below, economies with different costs of information and hence different degrees of informational efficiency will be compared.

4. Comparing equilibria with different costs of information

The two extreme cases of informational efficiency in the model are the equilibrium where nobody is informed and the equilibrium where everybody is informed. The first step in the analysis is to compare these (ignoring the cost of information in the latter).

Proposition 1

For every person, irrespective of their endowments $\overline{M}_i$ and $\overline{X}_i$, utility in the equilibrium where everybody is uninformed $EV_{Ui}\big|_{\lambda=0}$ is greater than utility in the equilibrium where everybody is informed at zero cost $EV_{Ii}\big|_{\lambda=1,c=0}$.

The proof is given in the Appendix.

As in Hirshleifer's original example the result arises because information is revealed before trade and investors are unable to insure against the distributive risk due to fluctuations in θ and hence in the value of their endowments. However when information is not revealed they are effectively insured against this risk. The feature of the model which underlies this result is the fact that there is no trading before θ becomes known. If there were then the information would have no effect: this initial round of trading would allow investors to share the risk of fluctuations in θ. The strict inequality in the proposition is a result of the initial allocation not being a Pareto optimal risk sharing arrangement: otherwise it is well known there will be no trade and hence no change in expected utility when more public information is available.

An important difference between public information and asymmetric information is that in the latter case there cannot be a pre-signal round of trading if equilibrium is to exist. This is because traders could deduce x from the price in the initial pre-signal market and θ from the price in the post-signal market. The Grossman Stiglitz argument for nonexistence when there is costly information, would then hold: if prices reveal information fully nobody will pay anything to discover it; however, if nobody observes it, then there is an incentive for somebody to discover it. Hence, the only case that can be considered with asymmetric information is the one used here where there is no pre-signal trading.

It follows from (17) and (20) that having more private information collected is similar to adding public information, since this makes prices better signals. It is therefore natural to suppose that increasing the proportion of the population that is informed could make everybody worse off. This in fact turns out to always be the case.

Proposition 2

An increase in the cost of information and hence a reduction in the equilibrium proportion of people who are informed and the informational efficiency of the market, always leads to a Pareto improvement in welfare. Moreover, the Pareto optimal equilibrium occurs where nobody is informed and prices convey no information.

Proof

Since along the equilibrium path as c is altered, it is an identity that informed and uninformed agents have the same utility levels, they are equally sensitive to the cost of information. Thus to demonstrate the first part of the proposition, it is sufficient to show that $dEV_{Ui}/d\lambda < 0$ for all $\overline{M}_i$ and $\overline{X}_i$.

The uninformed observe P and hence w_λ. Their prior on u thus has mean and variance $(E(u|w_\lambda), \text{Var}(u|w_\lambda))$ where the latter is given by (13) and

$$E(u|w_\lambda) = E\theta + (w_\lambda - E\theta)\eta(\lambda) \; . \tag{21}$$

It can be shown in the usual way that

$$X_U = \frac{E(u|w_\lambda) - RP}{a\,\text{Var}(u|w_\lambda)} \tag{22}$$

and

$$E(V_U(W_{1i})|w_\lambda) = -\exp\left[-a\left(R\overline{M}_i + RP\overline{X}_i + \frac{(E(u|w_\lambda) - RP)^2}{2a\,\text{Var}(u|w_\lambda)}\right)\right] \; . \tag{23}$$

In order to evaluate this it is necessary to derive the relationship between P and w_λ. Using (6), (8), (10) and (22) it follows that

$$RP = \frac{\lambda w_\lambda + (1-\lambda)\beta\, E(u|w_\lambda) - a\sigma_\varepsilon^2 Ex}{\lambda + (1-\lambda)\beta} \; . \tag{24}$$

Substituting for RP in (23), taking expectations over w_λ and rearranging it can be shown (see Appendix)

$$EV_{Ui} = -\left(1 + \frac{a^4\sigma_\varepsilon^6\sigma_x^4}{[\lambda + (1-\lambda)\beta]^2[a^2\sigma_\varepsilon^2\sigma_x^2\sigma_u^2 + \lambda^2\sigma_\theta^2]}\right)^{-1/2} \tag{25}$$

$$\exp\left[-a\left(R\overline{M}_i + \overline{X}_i E\theta + \frac{a}{2}\sigma_u^2\left(\frac{(\overline{X}_i - Ex)^2}{1 + a^2\sigma_u^2\sigma_x^2 + \frac{\sigma_\theta^2}{\sigma_\varepsilon^2}\left[1 - (1-\lambda)^2(1-\beta\eta)\right]} - \overline{X}_i^2\right)\right)\right] \; .$$

Differentiating with respect to λ

$$\frac{dEV_{Ui}}{d\lambda} = EV_{Ui}\{(\frac{a^4\sigma_\varepsilon^6\sigma_x^4}{[\lambda + (1 - \lambda)\beta]^2[a^2\sigma_\varepsilon^2\sigma_x^2\sigma_u^2 + \lambda^2\sigma_\theta^2] + a^4\sigma_\varepsilon^6\sigma_x^4})$$

$$(\frac{1 - \beta + (1 - \lambda)\beta'}{\lambda + (1 - \lambda)\beta} + \frac{\lambda\sigma_\theta^2}{a^2\sigma_\varepsilon^2\sigma_x^2\sigma_u^2 + \lambda^2\sigma_\theta^2})$$

$$- \frac{a^2}{2}\sigma_u^2(\frac{(\sigma_\theta^2/\sigma_\varepsilon^2)[2(1 - \lambda)(1 - \beta\eta) + (1 - \lambda)^2(\beta'\eta + \beta\eta')](\bar{X}_i - Ex)^2}{[1 + a^2\sigma_u^2\sigma_x^2 + (\sigma_\theta^2/\sigma_\varepsilon^2)[1 - (1 - \lambda)^2(1 - \beta\eta)]]^2})\} < 0 \ . \qquad (26)$$

This negative sign results from (16) - (19) and the fact that utility is negative. Hence the first part of the proposition is proved.

It follows from Proposition 1 that for every $\bar{M}_i$ and $\bar{X}_i$

$$EV_{Ui}\Big|_{\lambda=0} > EV_{Ii}\Big|_{\lambda=1} \qquad \text{for all } c \geq 0 \ . \qquad (27)$$

Combining this with the fact that EV_{Ui} has its maximum at $\lambda = 0$ the second part of the proposition follows immediately.

The proposition shows that Hirshleifer's result that improved public information can make everybody worse off because of increased distributive risk carries over directly to the asymmetric information case. When the cost of information is lower, so that more agents purchase information, prices are better signals and there are less risk sharing opportunities in the usual way. In addition the uninformed agents now have to trade with more informed agents. Thus it is clear they will be worse off. In equilibrium the informed must have the same utility as the uninformed and hence they are also worse off.

5. Taxing information gathering

The comparison of equilibria in the previous section demonstrates that if the government can impose a tax on information gathering then even if they discard the revenue it will be possible to make everybody better off: the tax increases the cost of information and reduces the informational efficiency of the market. If the government used the revenue to pay a uniform lump sum grant then this would further improve people's welfare. Thus some form of government intervention is always desirable. However, whereas the Pareto optimal market equilibrium involves no information gathering, there is a possibility the government can use the tax to make everybody better off than in the no-information equilibrium. This in fact turns out to be the case for some configurations of parameters.

Proposition 3

A tax that leads to no information collection may be Pareto suboptimal.

Proof

With a tax t on information the type (ii) equilibrium condition changes to

$$e^{a(c+t)}\beta^{1/2} = 1 \ . \tag{28}$$

Rearranging gives

$$t = -\frac{1}{2a}\log\beta - c \ . \tag{29}$$

Hence

$$\frac{dt}{d\lambda} = -\frac{1}{2a}\frac{\beta'}{\beta} \qquad \begin{cases} < 0 \text{ for } 0 < \lambda \leqslant 1 \\ = 0 \text{ for } \lambda = 0 \end{cases} \tag{30}$$

from (19). Thus by choosing a particular level of t the government can determine a particular equilibrium level of λ.

The analysis is the same as in Section 4 except it is now possible for the government to use the revenue from the tax to give a lump sum subsidy of

λt to everybody. The expected utility of the uninformed then becomes

$$EV_{Ui}^{t} = \exp[-a\lambda t]EV_{Ui} \tag{31}$$

where EV_{Ui} is given by (25). Here

$$\frac{dEV_{Ui}^{t}}{d\lambda} = -EV_{Ui}^{t}\ a(t + \lambda \frac{dt}{d\lambda}) + \exp[-a\lambda t]\frac{dEV_{Ui}}{d\lambda}\ . \tag{32}$$

The proposition is concerned with whether the optimal level of t is such that $\lambda = 0$. It can be shown using (16), (17), (19) and (26)

$$\left.\frac{dEV_{Ui}^{t}}{d\lambda}\right|_{\lambda=0} = -EV_{Ui}^{t}\ \{\frac{1}{2}\log(1 + \frac{\sigma_{\theta}^{2}}{\sigma_{\varepsilon}^{2}}) - ac - a^{2}\sigma_{u}^{2}\frac{\sigma_{\theta}^{2}}{\sigma_{\varepsilon}^{2}}[\frac{\sigma_{x}^{2}}{1 + a^{2}\sigma_{u}^{2}\sigma_{x}^{2}} + \frac{(\overline{X}_{i} - Ex)^{2}}{(1 + a^{2}\sigma_{u}^{2}\sigma_{x}^{2})^{2}}]\}\ . \tag{33}$$

It is clear that this cannot be signed in general. Moreover the effect of changing the tax so that $\lambda > 0$ will depend on endowments of $\overline{X}_i$. To see that it can be positive for everybody, consider the case where there is a bounded distribution of wealth with lower bound $\overline{X}_1$ and upper bound $\overline{X}_2$. When a, c, σ_u^2, σ_x^2 and $(\overline{X}_j - Ex)^2$ (j = 1, 2) are sufficiently small it is possible to construct examples where the derivative is positive for everybody. Thus the proposition is demonstrated.

Hirshleifer has argued that in exchange economies the private acquisition of costly information is undesirable since it simply leads to a reallocation of consumption but uses up costly resources: by an appropriate reallocation it would be possible to make everybody better off in the case where no private information is gathered, than in any state in which it is. Proposition 3 shows that this result does not hold in the Grossman Stiglitz model. Taxing the informed group and reallocating to the uninformed allows improved sharing of the risks due to variations in x and everybody can be better off.

Although in the equilibrium with no information transactors are effectively insured against variations in θ, since P still depends on x, they are not insured against variations in x. It can be shown (using (A9) in the Appendix) that

$$E\left(V_U(W_{1i})\,|\,x\right)\Big|_{\lambda=0} = -\exp\left[-a\left(R\overline{M}_i + E\theta\overline{X}_i - \frac{a}{2}\sigma_u^2\overline{X}_i^2 + \frac{a}{2}\sigma_u^2(x - \overline{X}_i)^2\right)\right] . \quad (34)$$

Thus the greater the deviation of x from $\overline{X}_i$ the greater the person's utility. People would be better off if they could share this risk so that for any given realization of x those with high values of $(x - \overline{X}_i)^2$ would make payments to those with low $(x - \overline{X}_i)^2$. The effect of taxing the informed and using the proceeds to finance a uniform lump sum grant can result in a similar redistribution.

To see this consider the effect on the expected return (given θ and x) of an uninformed person's portfolio, when λ is increased from zero. It can be shown

$$\frac{dE\left[(uX_U + RM_i)\,|\,\theta,\ x\right]}{d\lambda}\Bigg|_{\lambda=0} = -\frac{1}{a\sigma_\varepsilon^2}\Big[\frac{\sigma_\theta^2}{\sigma_x^2}(x - \overline{X}_i)^2(1 + a^2\sigma_u^2\sigma_x^2)$$

$$+ \frac{\sigma_\theta^2}{\sigma_x^2}(x - \overline{X}_i)(\overline{X}_i - Ex)(1 + a^2\sigma_u^2\sigma_x^2) + a^2\sigma_u^2\sigma_\theta^2 Ex(x - \overline{X}_i) \quad (35)$$

$$+ a\sigma_u^2(\theta - E\theta)(x - \overline{X}_i) + (\theta - E\theta + a\sigma_u^2 x)(\theta - E\theta + a\sigma_\theta^2 x)\Big] .$$

The cases where (33) is positive for everybody involve a, σ_u^2, σ_x^2 and $(\overline{X}_i - Ex)$ being sufficiently small. In the extreme case where these are negligible, it can be seen

$$\frac{dE[(uX_U + RM_i)|\theta, x]}{d\lambda}\Bigg|_{\lambda=0} \simeq - \frac{1}{a\sigma_\varepsilon^2}\left[\frac{\sigma_\theta^2}{\sigma_x^2}(x - \overline{X}_i)^2 + (\theta - E\theta)^2\right] . \qquad (36)$$

Hence for uninformed people the larger is $(x - \overline{X}_i)^2$ the greater is the reduction in the expected return of their portfolios when informed people are introduced into the market. The profits the informed earn are therefore predominantly at the expense of those uninformed with large $(x - \overline{X}_i)^2$. These profits mean that the informed are willing to pay a tax on information which is used to finance a lump sum grant of $\log(1 + \sigma_\theta^2/\sigma_\varepsilon^2)/2a - c$ to the uninformed. Thus the overall effect of introducing informed traders into the market can be that there is a redistribution from those uninformed people with high $(x - \overline{X}_i)^2$ to those with low $(x - \overline{X}_i)^2$. For example if $\theta = E\theta$ those uninformed people with $\overline{X}_i = x$ receive the lump sum grant and the return on their portfolios is unchanged so that overall they are better off: in contrast those with endowments such that $(x - \overline{X}_i)^2$ is large are worse off by an amount $\sigma_\theta^2(x - \overline{X}_i)^2/a\sigma_\varepsilon^2\sigma_x^2$ less the lump sum grant. This is precisely the type of transfer that insures people against variations in x.

If the distribution of wealth is unbounded then clearly (34) cannot be positive for everybody. However in this case it is still possible for average expected utility, or equivalently utilitarian social welfare, to be improved. For example, if $\overline{X}_i$ is normally distributed with mean Ex and variance $\sigma^2_{\overline{X}_i}$, then it can be shown that average expected utility will increase provided that in addition to a, c, σ_u^2 and σ_x^2, $\sigma^2_{\overline{X}_i}$ is also sufficiently small.

The reason improved public information about θ can make everybody worse off is the absence of possibilities to insure against variations in θ. In contrast, the reason asymmetric information about θ can make everybody better off is the absence of possibilities to insure against variations in x. If markets existed which allowed this, then the asymmetric information about θ would have no value.

Even though in the model used, Hakansson, Kunkel and Ohlsons' conditions for improved public information to have no social value are satisfied, it is in fact possible for everybody to be better off with allocations associated with asymmetric information. In considering the social value of information, it is therefore necessary to consider the allocations associated with asymmetric information as well as those with differing degrees of public information.

It is possible to combine (31) with a social welfare function, take the derivative with respect to λ, set it equal to zero and obtain a first order condition for the optimal level of λ. However, this yields a complex expression which cannot be solved explicitly and which gives little insight. A more useful approach is to use (32) to derive an upper bound on the optimal value of λ and a corresponding lower bound on the optimal tax.

Proposition 4

For any individualistic social welfare function, the optimal proportion of informed people, λ_o, must be such that

$$\lambda_o < \frac{a\sigma_u\sigma_x}{\sigma_\theta/\sigma_\varepsilon} \tag{37}$$

and the corresponding optimal tax, t_o, must be such that

$$t_o > \frac{1}{2a}\log\left(1 + \frac{\sigma_\theta^2/\sigma_\varepsilon^2}{1 + \sigma_u^2/\sigma_\varepsilon^2}\right) - c\ . \tag{38}$$

Proof

The result is derived by considering the sign of the first term in (32) which is the same for everybody. It can be shown using (29), (30) and the fact that $\log(1 + x) < x$ for $x > 0$ that

$$t + \lambda \frac{dt}{d\lambda} < \tfrac{1}{2} \frac{a\sigma_\varepsilon^2\sigma_x^2\sigma_\theta^2\eta}{\lambda^2(\lambda^2\sigma_\theta^2 + a^2\sigma_\varepsilon^2\sigma_x^2\sigma_u^2)} \left(\frac{a^2\sigma_x^2\sigma_u^2}{\sigma_\theta^2/\sigma_\varepsilon^2} - \lambda^2\right) . \tag{39}$$

Since for all individuals the sign of the second term in (32) is negative, the λ which is globally optimal for each individual must be such that the right-hand side of (39) is positive and this corresponds to (37) being satisfied. Hence the globally optimal λ for any individualistic social welfare function must be such that (37) is satisfied. It can then be shown that (29) and (37) imply (38). Thus the proposition is demonstrated.

One implication of (32) is that in order for an improvement in welfare to be possible a, σ_u^2 and σ_x^2 should be small whereas $\sigma_\theta^2/\sigma_\varepsilon^2$ should be large. Taking this together with (37) it can be seen that if a positive value of λ is optimal then this value will often be close to zero.

6. Release of information by firms

Diamond (1983) has used a very similar model to investigate the optimal release of information by firms, by interpreting the risky asset as being shares in a firm. He develops a noisy rational expectations equilibrium of the type considered by Hellwig (1980) and Diamond and Verrechia (1981) where information acquisition is similar to Grossman and Stiglitz (1980) and Verrechia (1982). The main difference between his model and the one above is that the informed observe different signals whereas here they all observe the same signal θ. Diamond shows that in situations where without any information release by the firm, the proportion of informed would be positive, the optimal policy is to release information with the maximum precision (i.e., the inverse of variance) such that nobody finds it worthwhile to privately gather information. The savings in resources because people no longer collect information and also an improvement in risk sharing due to the greater homogeneity of information make everybody better off. The firm should not release any information with lower precision than this level because, as in Proposition 1 above, the improved public information would make everybody worse off. By the same reasoning if $\lambda = 0$ when no announcement is made, then the optimal policy is not to release any information. Diamond points out that similar conclusions hold in the Grossman Stiglitz model.

The results obtained in Section 5 concerning the taxation of information gathering have counterparts in the context of the release of information by firms. Whereas Diamond considered the public release of information, the results above can be interpreted in terms of the private release of information to "insiders" who can then trade on the basis of this information.

A large literature on the desirability, or otherwise, of insider trading exists. In the U.S. trading on the basis of inside information is currently illegal. Manne (1966) has suggested that this prohibition is undesirable. He argued informally that insider trading may improve the incentives of both prospective and practicing managers. Dye (1984) has demonstrated formally in the context of a principal-agent model, that insider trading can indeed sometimes allow a Pareto improvement.

It is shown below by extending Proposition 3, that allowing insider trading can lead to a Pareto improvement even if there are no incentive problems. Dietrich-Campbell (1983) has used a result related to Proposition 2 to argue that insider trading is undesirable. However this does not take into account the point made by Manne that giving information to insiders and allowing them to trade on the basis of it may be a good way of compensating them. In Section 5 the informed make a payment of t to the government to finance a lump sum grant of λt to everybody. Alternatively the informed can be thought of as making a net payment of $(t - \lambda t)$ and the uninformed as receiving a net payment of λt. In the context of the optimal release of information by firms, this is equivalent to the insiders, who are the informed, having compensation which is $(t - \lambda t)$ lower than it would be if they were not able to trade on the inside information. This reduction in compensation allows the uninformed shareholders to receive dividend payments which are λt higher than they otherwise would be. The results of Section 5 suggest that, if, in addition to the public release strategy suggested by Diamond, a firm releases better information to insiders as a form of compensation, this may further improve the welfare of shareholders. This turns out to be the case.

Similarly to Diamond it will be assumed that the number of shareholders is sufficiently large that θ can be observed by the firm at a cost per shareholder of zero. After observing θ the firm announces publicly

$$h = \theta + \gamma \tag{40}$$

where γ is a normally distributed random variable with mean and variance $(0, \sigma_\gamma^2)$, which is independent of all the other random variables in the model. Hence h has mean $E\theta$ and variance

$$\sigma_h^2 = \sigma_\theta^2 + \sigma_\gamma^2 . \tag{41}$$

The public information release policy of the firm is its choice of σ_γ^2.

The uninformed now condition their demands on h as well as w_λ. Thus β is changed to

$$\beta_h(\lambda) = \frac{\sigma_\varepsilon^2}{Var(u|w_\lambda, h)} . \tag{42}$$

Similarly to the Grossman Stiglitz model, it can be shown there are three types of equilibrium as in Section 3 with β_h replacing β.

In the context of this model, Diamond's main result is to show that if $\lambda > 0$ for the equilibrium when no information is released, the firm's optimal choice of σ_γ^2 is such that

$$e^{ac}\beta_h(0)^{1/2} = 1 \tag{43}$$

so that all information gathering by individuals is eliminated. If $\lambda = 0$ with no information release the firm should not give out any information, or equivalently it should set $\sigma_\gamma^2 = \infty$. Hence Diamond's result can be stated as:

<u>Proposition 5</u>

The firm's optimal choice of σ_γ^2 is the maximum value such that its public announcement of h leads to no information gathering by individuals.

If insider trading is legal then Proposition 5 does not always give the optimal information release policy for the firm.

<u>Proposition 6</u>

The firm may be able to improve the welfare of all its shareholders if in addition to publicly releasing h as in Proposition 5 it announces θ to a group of insiders as a form of compensation.

<u>Proof</u>

There are two cases to be considered. If $\lambda = 0$ in the equilibrium with no information release, then it follows directly from (33) that the firm may be able to improve the welfare of all its shareholders by releasing θ to a group of insiders, reducing direct compensation and increasing dividends appropriately.

If $\lambda > 0$ in the equilibrium with no information release, then the firm chooses σ_γ^2 so that (43) is satisfied. All the uninformed then condition their demands on the firm's announcements of h and on w_λ. It can be shown (see Appendix) that

$$\left.\frac{dEV_{Ui}^{th}}{d\lambda}\right|_{\lambda=0} = -\left.EV_{Ui}^{h}\right|_{\lambda=0}\Big[\tfrac{1}{2}\log\Big(1 + \frac{\sigma_\theta^2}{\sigma_\varepsilon^2}\frac{\sigma_\gamma^2}{\sigma_h^2}\Big) \tag{44}$$

$$- a^2 \mathrm{Var}(u|h)\frac{\sigma_\theta^2}{\sigma_\varepsilon^2}\frac{\sigma_\gamma^2}{\sigma_h^2}\Big(\frac{\sigma_x^2}{1 + a^2\mathrm{Var}(u|h)\sigma_x^2} + \frac{(\overline{X}_i - Ex)^2}{(1 + a^2\mathrm{Var}(u|h)\sigma_x^2)^2}\Big)\Big]$$

where

$$\left.EV_{Ui}^{h}\right|_{\lambda=0} = -\left(1 + a^2\mathrm{Var}(u|h)\sigma_x^2\right)^{-1/2} \tag{45}$$

$$\exp\Big[-a\Big(\overline{RM}_i + \overline{X}_i E\theta + \frac{a}{2}\Big(\frac{\mathrm{Var}(u|h)(\overline{X}_i - Ex)^2}{1 + a^2\mathrm{Var}(u|h)\sigma_x^2} - \sigma_u^2\overline{X}_i^2\Big)\Big)\Big] ,$$

$$\mathrm{Var}(u|h) = \sigma_u^2 - \frac{\sigma_\theta^4}{\sigma_h^2} . \tag{46}$$

(In contrast to (33) c = 0 in (44) since the cost of releasing θ to insiders is zero.) It can be seen that as with (33), (44) can be positive for everybody so that releasing information to insiders can again lead to a Pareto improvement. Hence the proposition is demonstrated.

In Section 5 an upper bound on the optimal tax was found. However, it is not possible to characterize the optimal information release policy in a similar way. Proposition 6 is concerned with the case where the firm announces h publicly and θ to insiders. In general the firm could adopt much more sophisticated policies by announcing $h_i = \theta + \gamma_i$ to group i. Even this is not the most general information release policy, as in addition $\sigma^2_{\gamma_i}$ could, for example, be made to depend on θ. All these changes may in certain circumstances improve risk sharing and allow all shareholders to be better off. The optimal information release policy of the firm could thus be exceedingly complex.

7. Concluding remarks

In contrast to the question of the existence of equilibrium in rational expectations models, there has been relatively little work on the welfare properties of rational expectations equilibria (for exceptions see Grossman (1981) and Laffont (1983)). Although the analysis above is concerned with a special example, it has implications for more general models. In rational expectations models, prices reveal information and similarly to Hirshleifer (1971) the more information that is publicly revealed the worse off everybody can be. However, the results of this paper indicate it is not the case that equilibria where no information is publicly revealed are necessarily Pareto optimal. In noisy rational expectations models it may be better to have prices which reveal some information about payoff relevant random variables (here θ), if this is associated with some mechanism such as the taxation of information gathering or insider trading which permits risk sharing across non-payoff relevant random variables (here x).

Finally it should be pointed out that the welfare comparisons above are very sensitive to the type of ex-ante insurance markets which are available. In particular, information makes prices more variable ex-post. Hence ex-ante markets which allow people to trade claims contingent on the realization of ex-post prices could serve a valuable insurance role.

Appendix

Proof of Proposition 1

The result is demonstrated by evaluating the two levels of utility and comparing them.

(i) $EV_{Ii}\big|_{\lambda=1,c=0}$

Putting c = 0 and substituting (8) into (7) it follows that the expected utility of the informed in this case is given by

$$E\left(V_I(W_{1i})\,|\,\theta,\ x\right)\Big|_{c=0} = -\exp\left[-a\left(R\bar{M}_i + RP\bar{X}_i + \frac{(\theta - RP)^2}{2a\sigma_\varepsilon^2}\right)\right] . \tag{A1}$$

When $\lambda = 1$ equilibrium requires $X_I = x$ so that using (8)

$$RP = \theta - a\sigma_\varepsilon^2 x . \tag{A2}$$

Hence

$$E\left(V_I(W_{1i})\,|\,\theta,\ x\right)\Big|_{\lambda=1,c=0} = -\exp\left[-a\left(R\bar{M}_i + (\theta - a\sigma_\varepsilon^2 x)\bar{X}_i + \frac{a}{2}\sigma_\varepsilon^2 x^2\right)\right] . \tag{A3}$$

Taking expectations over θ gives

$$E\left(V_I(W_{1i})\,|\,x\right)\Big|_{\lambda=1,c=0} = -\exp\left[-a\left(R\bar{M}_i + \bar{X}_i E\theta - \frac{a}{2}\sigma_\theta^2\bar{X}_i^2 - a\sigma_\varepsilon^2 x\bar{X}_i + \frac{a}{2}\sigma_\varepsilon^2 x^2\right)\right] . \tag{A4}$$

(ii) $EV_{Ui}\big|_{\lambda=0}$

Similarly to (7) it can be shown that when $\lambda = 0$ and the uninformed people's prior on u is $(E\theta,\ \sigma_u^2)$ then

$$E\left(V_U(W_{1i})\,|\,\theta,\ x\right)\Big|_{\lambda=0} = -\exp\left[-a\left(R\bar{M}_i + RP\bar{X}_i + (E\theta - RP)X_U - \frac{a}{2}\sigma_u^2 X_U^2\right)\right] . \tag{A5}$$

Choosing X_U to maximize this gives

$$X_U = \frac{(E\theta - RP)}{a\sigma_u^2} . \tag{A6}$$

Substituting back into (A5) gives

$$E\left(V_U(W_{1i})\middle|\theta,\ x\right)\Big|_{\lambda=0} = -\exp\left[-a\left(R\overline{M}_i + RP\overline{X}_i + \frac{(E\theta - RP)^2}{2a\sigma_u^2}\right)\right] \ . \tag{A7}$$

In the equilibrium with $\lambda = 0$, $X_U = x$ so that

$$RP = E\theta - a\sigma_u^2 x \ . \tag{A8}$$

Hence

$$E\left(V_U(W_{1i})\middle|x\right)\Big|_{\lambda=0} = -\exp\left[-a\left(R\overline{M}_i + (E\theta - a\sigma_u^2 x)\overline{X}_i + \frac{a}{2}\sigma_u^2 x^2\right)\right] \ . \tag{A9}$$

Using the fact that $\sigma_u^2 = \sigma_\theta^2 + \sigma_\varepsilon^2$ it can be shown

$$E\left(V_U(W_{1i})\middle|x\right)\Big|_{\lambda=0} = E\left(V_I(W_{1i})\middle|x\right)\Big|_{\lambda=1,c=0}\exp\left[-\frac{a^2\sigma_\theta^2}{2}(x - \overline{X}_i)^2\right] \ . \tag{A10}$$

Now since $\exp[-a^2\sigma_\theta^2(x - \overline{X}_i)^2/2] \leq 1$, x is normally distributed and utility is negative,

$$EV_{Ui}\Big|_{\lambda=0} > EV_{Ii}\Big|_{\lambda=1,c=0} \tag{A11}$$

and the proposition follows immediately.

Derivation of equations

The derivations of some of the equations in the text are somewhat complex. Outlines of the most difficult of these are given below. Full details of all the derivations may be obtained on request from the author.

Equation (25)

Using (23) and (24) it can be shown

$$E\left(V_U(W_{1i})\,\middle|\,w_\lambda\right) = -\exp\Big[-a\Big(\overline{RM}_i+\Big(E\theta-\frac{Ex}{D}\Big)\overline{X}_i + \frac{1}{D}\Big[\frac{\lambda}{a\sigma_\varepsilon^2}+\frac{(1-\lambda)}{a\mathrm{Var}(u|w_\lambda)}\eta\Big]\overline{X}_i(w_\lambda - E\theta) \tag{A12}$$

$$+\frac{1}{2aD^2\mathrm{Var}(u|w_\lambda)}\Big(\frac{\lambda}{a\sigma_\varepsilon^2}(1-\eta)(w_\lambda - E\theta) - Ex\Big)^2\Big)\Big]$$

where

$$D = \frac{\lambda}{a\sigma_\varepsilon^2}+\frac{(1-\lambda)}{a\mathrm{Var}(u|w_\lambda)}\ . \tag{A13}$$

Taking expectations over w_λ and using the linear transformation

$$w^*_\lambda = w_\lambda - E\theta + \frac{\zeta}{\xi} \tag{A14}$$

where

$$\zeta = \frac{a}{D}\Big(\frac{\lambda}{a\sigma_\varepsilon^2}+\frac{(1-\lambda)}{a\mathrm{Var}(u|w_\lambda)}\eta\Big)\overline{X}_i - \frac{\lambda(1-\eta)Ex}{a\sigma_\varepsilon^2 D^2\mathrm{Var}(u|w_\lambda)}\ , \tag{A15}$$

$$\xi = \frac{\lambda^2(1-\eta)^2}{a^2\sigma_\varepsilon^4 D^2\mathrm{Var}(u|w_\lambda)}+\frac{1}{\mathrm{Var}\ w_\lambda} \tag{A16}$$

the integral can be eliminated and it can be shown

$$EV_U(W_{1i}) = -\frac{\xi^{-1/2}}{\sqrt{\mathrm{Var}\ w_\lambda}}\exp\Big[-a\Big(\overline{RM}_i + \Big(E\theta - \frac{Ex}{D}\Big)\overline{X}_i + \frac{1}{2a}\Big(\frac{(Ex)^2}{D^2\mathrm{Var}(u|w_\lambda)} - \frac{\zeta^2}{\xi}\Big)\Big)\Big]\ . \tag{A17}$$

Simplifying this expression using the fact that

$$a\Big[\frac{\lambda(1-\eta)}{a\sigma_\varepsilon^2}\ \frac{\mathrm{Var}\ w_\lambda}{D}\Big(\frac{\lambda}{a\sigma_\varepsilon^2}+\frac{(1-\lambda)}{a\mathrm{Var}(u|w_\lambda)}\eta\Big) - D\mathrm{Var}(u|w_\lambda)\mathrm{Var}\ w_\lambda\ \xi\Big] = -1 \tag{A18}$$

it can be shown that (A17) simplifies to (25).

Equation (44)

This is derived by first obtaining the equivalent of (A12). The difference is that $E(u|w_\lambda)$ and $Var(u|w_\lambda)$ are replaced by

$$E(u|w_\lambda, h) = E\theta + \gamma_1(w_\lambda - E\theta) + \gamma_2(h - E\theta) \qquad (A19)$$

where

$$\gamma_1 = \frac{\sigma_\theta^2\sigma_\gamma^2}{\sigma_h^2 Var\ w_\lambda - \sigma_\theta^2} \qquad (A20)$$

$$\gamma_2 = \frac{\sigma_\theta^2 Var\ w_\lambda - \sigma_\theta^4}{\sigma_h^2 Var\ w_\lambda - \sigma_\theta^4} \qquad (A21)$$

and

$$Var(u|w_\lambda, h) = \sigma_u^2 - \sigma_\theta^2[\gamma_1 + \gamma_2] \ . \qquad (A22)$$

To obtain the unconditional expected utility it is necessary to take expectations over and to linearly transform both w_λ and h. It is then possible to proceed similarly to the derivation of (33).

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