Optimal Financial Structure in Exchange Economies

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Working Paper #18-84

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June, 1984

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I wish to thank Robert King and also Paul Romer and David Gordon for valuable help. Any errors remain my responsibility.

1. INTRODUCTION

Economists are beginning to examine financial institutions and structures in a new light. This approach starts from the premise that such financial structures arise to solve informational difficulties inherent in the underlying economy. In general, though this recent work has often shown how particular contracts or institutions can mitigate an information problem, it has rarely investigated the optimal arrangement for that particular economy. Such a distinction is particularly important if we consider free-entry and competition among the entrepreneurs who design and set-up institutions. Even work that has examined optimal arrangements has artifically restricted the options and opportunities of investors within the model. These restictions are felt particularly acutely because of the sophistication of modern financial markets. In part such restrictions were exogenously imposed by the state of the art of optimal contract and allocation theory; similarly, recent advances in that theory provide a richer framework for research.

This paper attempts to take a small step towards a positive theory of financial structure. It carefully specifies the basic structure of the economy and in a simple general equilibrium setting derives the optimal financial arrangements. In contrast to earlier work it explicitly considers coalitions and syndicates and permits the introduction of contrived uncertainty into contracts. These general arbitrage arguments and contractual possibilities hold promise of more fruitfully representing an increasingly deregulated financial world, where bankers and investment groups put together innovative financing and create new instruments and accounts.

The results from this general setting should also serve as a warning. When intuitively plausible economic environments are carefully specified it turns out that many types of financial institutions cannot arise. This is not to argue that banks do not exist but rather that a particular, popular, form of economic analysis is fraught with pitfalls. One goal of this paper is to clarify the assumptions that lead to different institutions. Seemingly innocuous specifications--such as the observability of consumption --can have major consequences for the resulting equilibrium.

Analyzing financial arrangements also provides a simple context for the exposition and clarification of several new concepts in mechanism theory, along with a chance to examine their usefulness in a particular setting.

This paper employs a one good, two period endowment model with individual uncertainty. This simplification (in the tradition of Lucas's "Asset Prices in an Exchange Economy" [1978] and Townsend's "Financial Structures as Communication Systems" [1984]) permits calculation of optimal contracts yet still provides time and uncertainty, the two essential elements of a financial economy. The work starts by formulating a maximization problem, solving it, and finding the market or institutional support for the optimal allocation determined by the problem.

The remainder of the paper is as follows. Section 2 describes the economy and sets up the mathematical program to be solved. Section 3 examines the solution to the basic problem with deterministic allocations and individual incentive compatibility constraints, and discusses the financial system that supports the solution. The next two sections address more elaborate versions of the problem. Section 4 allows random allocations while section 5 brings in multilateral incentive compatibility constraints and discusses support for the new allocations. Section 6 presents a summary and conclusions.

2. Description of the Economy

This exchange economy has one good x consumed in two periods $T = 1, 2$ with no storage between periods. At $T = 2$, all agents receive the identical endowment y_2. At $T = 1$, endowments differ. A lucky fraction t of the continuum of agents will receive a first period endowment of $y_1 + \theta_1 > y_1$, while an unlucky fraction $(1 - t)$ receives $y_1 + \theta_2 < y_1$, where for consistency $t\theta_1 + (1 - t)\theta_2 = 0$. The shock (θ_1, θ_2) is private information: it is impossible to verify if an agent has become "rich" or "poor". Notice that this endowment uncertainty is perfectly diversifiable, but that perfect diversification may not occur because of information (adverse selection) problems.

All agents have identical[1] preferences represented by a utility function $U(c_1, c_2)$ defined over consumption in both periods. The utility function is strictly increasing in c_i, continuous, twice differentiable and strictly concave. In addition it exhibits normality in consumption in both periods.

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[1] One may also represent the uncertainty by an additive shift in preferences as well, in which case endowments would be identical and preferences random. The two cases are more than just formally equivalent. Both model uncertainty arising from a household production function.

Since this model incorporates both time and uncertainty, an optimal contract specifies both intertemporal trade (borrowing and lending) and risk-pooling (insurance). Finding the optimal contract/institution involves first solving a maximization problem to determine the optimal allocation of the good across states and time, and then finding a contract that supports such allocations.

Before discussing the nature of the optimization problem, though, it may be useful to consider the ex post competitive equilibrium of this economy. This provides a reference point for comparison with the allocations and institutions discussed later. It may also serve to clarify the nature of the economy. Define an (ex post) competitive equilibrium to be a set of prices P_i and quantities $c_i(\theta_j)$ such that (i) each agent is maximizing utility given prices and his or her endowment, and (ii) supply equals demand for each good. That is, $c_i(\theta_j)$ solves

$$\max U(c_1, c_2)$$

subject to

$$p_1c_1 + p_2c_2 = p_1(y_1 + \theta_j) + p_2y_2$$

and in turn satisfies the aggregate consistency condition $\sum_j c_i(\theta_j)f(\theta_j) = \sum_j y_i(\theta_j)f(\theta_j)$ for all i. In other words, after the resolution of the uncertainty, this is the standard two-period exchange economy. Individuals engage in borrowing and lending to smooth consumption over the two periods. Since no storage is possible, economy-wide net borrowing will be zero in each period.

Going beyond the ex post competitive equilibrium to the social optimum involves solving a mathematical programming problem. Since all individuals are alike ex ante, the program must maximize the expected utility of a representative agent who has yet to learn his or her type. This chooses the best allocation (which may be random) subject to the resource constraints, the incentive compatibility constraints, and the multilateral incentive compatibility constraints. These constraints have straightforward interpretations. First, a distribution of the good among agents must be feasible. Second, since types are unverifiable, output must be distributed so that no agent gains by mis-representing his or her type. Finally, no group or coalition can improve upon an allocation by selective misrepresentation combined with trading among itself. With this in mind, we now proceed to the formal statement of the problem and its solution.

3. THE PROBLEM AND ITS SOLUTION

This first stage of the analysis will characterize optimal allocations, where optimizing involves maximizing expected utility subject to various constraints. Since a major task of this paper involves determining the effects of allowing random allocations and imposing group incentive compatibility constraints, we solve the problem in steps. The first step chooses non-random allocations subject only to the individual incentive compatibility constraints. The next step allows random allocations (lotteries), and the final step involves imposing the multilateral incentive compatibility constraints.

These constraints imposed upon the problem correspond to differing levels of the social planner's (and thus ultimately the contracting parties') ability to monitor the actions of the private agents. Fully observable income permits allocations contingent on agent type. Only the resource constraints would need to be considered; the incentive compatibility constraints would be superfluous. However, when the social planner observes only consumption, incentive compatibility constraints are required, since the planner cannot identify agents by type beforehand during the distribution of allocations. Still, there is no need to impose group or multilateral constraints since agents cannot trade or make side payments among themselves: the social planner can make sure they consume what they are given. Without observable consumption, however, group incentive constraints come into play. The allocated goods can now be traded and bargained with. The sequence of constraints we consider forms a natural progression following the technology of communication and monitoring present in the economy.

To solve the problem, then, one looks for allocations x_{ij} where $i = 1, 2$ indexes period and $j = 1, 2$ indexes agent type. These are the solutions to

(I) $\text{MAX } tU(x_{11} + \theta_1, x_{21}) + (1 - t)U(x_{12} + \theta_2, x_{22})$

subject to

$$tx_{11} + (1 - t)x_{12} = y_1 \quad \text{(RESOURCE)}$$

$$tx_{21} + (1 - t)x_{22} = y_2 \text{ and}$$

$$U(x_{11} + \theta_1, x_{21}) > U(x_{12} + \theta_1, x_{22}) \quad \text{(IC)}$$

$$U(x_{12} + \theta_2, x_{22}) > U(x_{11} + \theta_2, x_{21})$$

which incorporates individual incentive compatibility conditions, or (II), which is (I) plus coalition incentive compatibility conditions

$$U\left(\gamma x_{11} + (1 - \gamma)x_{12} + \theta_1, \gamma x_{21} + (1 - \gamma)x_{22}\right) \leq U(x_{11} + \theta_1, x_{21}) \quad \text{(MIC)}$$

$$U\left(\beta x_{11} + (1 - \beta)x_{12} + \theta_2, \beta x_{21} + (1 - \beta)x_{22}\right) \leq U(x_{12} + \theta_2, x_{22})$$

where $0 < \gamma, \beta < 1$ and denote the fraction of a coalition's members representing themselves as type 1's. The form of this constraint appears to restrict the bargaining process since coalition members receive payment exactly in proportion to the number of people claiming to be each type. In fact, this is no constraint because each individual is free to choose which coalition to enter (or form), and by varying the proportions of agents, can achieve any proportion possible in a bargaining situation.

The lagrangian, and hence the first order conditions for problem (I) will involve four multipliers: two from the resource constraint, λ_1, λ_2, and two for the incentive compatibility constraints, μ_1, μ_2. A constraint is said to bind if the solution satisfies it with equality, i.e., the associated lagrange multiplier is non-zero.

We do not show the first order conditions for problems (I) and (II) for two reasons. First, the more interesting facts about the solution do not easily follow from the first order conditions, and secondly the constraints need not be convex. Other techniques, along the lines of Holmstrom (1982) and Holmstrom and Weiss (1983) will prove more helpful.

The following three propositions exhibit the basic properties of the solution. First, however, we establish a useful benchmark:

the full information or unconstrained optimum, the allocation attainable if θ_j were public information. With agent type observable, allocations can be made contingent on that type. Each agent then receives $(y_1 - \theta_j, y_2)$ and thus consumes (y_1, y_2). The public information allows perfect insurance of the ex ante identical agents since the risk is diversifiable: agents consume the expected value of income.

PROPOSITION 1: The full information social optimum does not solve (I).

Proof: The full information optimum removes all uncertainty. This implies equal consumption for all, or

$$x_{11} + \theta_1 = y_1 = x_{12} + \theta_2 \text{ and } x_{21} = y_2 = x_{22}$$

but then $x_{12} + \theta_1 > x_{11} + \theta_1$ since $\theta_2 < 0$, showing

$$U(x_{12} + \theta_1, x_{22}) > U(x_{11} + \theta_1, x_{21})$$

so this allocation is not incentive compatible.

Q.E.D.

Proposition 1 should not be surprising. Imperfect information creates the need for the IC constraints, which preclude reaching the unconstrained social optimum. In relation to the other benchmark, proposition 8 (below) shows that the optimum allocation improves upon the ex post competitive equilibrium of this exchange economy.

The IC constraints, coupled with the desire to smooth consumption over time, and the assumption that all goods are normal, imply a time pattern in the allocations: $x_{11} < x_{12}$ and $x_{21} > x_{22}$.

People with a positive draw in T = 1 desire to save and consume some of the windfall tomorrow. Those with a negative draw wish to borrow and spread the loss over both periods. This enables the planner to separate the two groups, but care must be taken not to give either group more in both periods. This background leads to

PROPOSITION 2: Only one IC constraint binds at the optimum of (I).

Proof: Without loss of generality, consider the case where the poor IC constraint binds. That implies $U(x_{12} + \theta_2, x_{22}) = U(x_{11} + \theta_2, x_{21})$ (i.e., on the same indifference curve). If the rich constraint also binds, then $U(x_{11} + \theta_1, x_{21}) = U(x_{12} + \theta_1, x_{22})$. The indifference curve through $(x_{11} + \theta_1, x_{21})$ must be flatter than the curve through $(x_{11} + \theta_2, x_{21})$ since consumption of both goods is normal. Similarly, the slope at $(x_{12} + \theta_1, x_{22})$ is flatter than at $(x_{12} + \theta_2, x_{22})$ (see Figure 1). Normality and convexity preclude mutual satisfaction of these two conditions. The rich IC constraint either does not bind or is not satisfied.

Q.E.D.

One and only one lagrangian multiplier associated with the Incentive Compatibility constraints can have a non-zero value. Intuitively, it seems unlikely that both groups will envy each other at an optimum allocation.

Since reducing risk involves redistributing income from the lucky (θ_1) to the unlucky (θ_2), one expects that it is the IC constraint of the lucky rich that precludes the full information optimum.

PROPOSITION 3: The IC constraint of the rich binds at the optimum of (I).

Proof: From proposition 2, one IC constraint must bind since proposition 1 rules out the unconstrained solution. Now assume the constraint binds the poor, not the rich. Then

(i) $U(x_{11} + \theta_1, x_{21}) > U(x_{12} + \theta_1, x_{22})$ and

(ii) $U(x_{12} + \theta_2, x_{22}) = U(x_{11} + \theta_2, x_{21})$

but recall $x_{11} < x_{12}$ and $x_{21} > x_{22}$. This implies we can take dx_{21} from the rich and give dx_{22} to the poor, increasing x_{22} and decreasing x_{21}, provided $tdx_{21} + (1 - t)dx_{22} = 0$. Notice (i) still holds for small transfers while (ii) becomes $U(x_{12} + \theta_2, x_{22}) > U(x_{11} + \theta_2, x_{21})$. The IC and resource constraints remain satisfied, but the redistribution reduces uncertainty, increasing expected utility. Put another way, relaxing a constraint increases the maximum.

Q.E.D.

The optimal allocation characterized above dominates that supported by a simple ex post market for securities which are claims (not state contingent) on consumption in a given period. The allocation requires some form of intermediary which can transform those 'primitive' assets of agents' endowments into more sophisticated securities. Following recent work stressing the insurance and liquidity role of the banking sector (Diamond and Dybvig [1983], Jacklin [1984], Haubrich and King [1984], Smith [1984]) we may consider the institution a bank. Imagine the following scenario. Agents agree to deposit $y_1 + \theta_2$ (the maximum verifiable amount) in the bank in period 1 and also agree to deposit y_2 in period 2. They are then given a choice between two withdrawal streams: (d_{11}, d_{21}) and (d_{12}, d_{22}) where $d_{11} - \theta_2 = x_{11}$, $d_{21} = x_{21}$, $d_{12} = x_{12}$ and $d_{22} = x_{22}$. Like other theoretical banks posited in the literature, this one sets an interest rate below the no-bank level. This serves to redistribute income from the rich, who tend to lend, to the poor, who wish to borrow. In this simple set up, of course, the interest rate is only implicit. The agents' opportunity set is not a line, but rather two points. Later, in section 5, we shall see how this bank exhibits yet another quality of recent banking models, namely, the need for exclusivity. The bank must be able to guarantee that agents consume their allocation, either by directly observing consumption or by excluding markets for assets that are claims to income. As the reason for such exclusivity involves coalitions, we postpone further discussion of this until section 5.

4. LOTTERIES AS ALLOCATIONS

In an important recent contribution, Prescott and Townsend [1984] have argued for the consideration of random allocations, which they term lotteries. One possible benefit of such randomness, as Prescott and Townsend have pointed out (in a slightly different context) is to convexify Problem (I). Figure 3 shows that the incentive compatibility constraints of (I) are not convex. Let point A denote the consumption level of the rich agent when the rich agent tells the truth. The IC constraints imply that the utility of this "truth-telling" bundle must be at least as high as the bundle received when the rich agent claims to be poor. Thus a rich person's "cheat point," for example B, must lie on or below the indifference curve through a. Point B satisfies the constraint, but convex combinations of A and B do not.

Define a lottery L_j as a probability distribution over all possible allocations x_{ij} in the commodity space. The utility of a lottery L_j will be $E\big(U(L_j)\big) = \int U(x_{1j} + \theta_j, x_{2j}) z(x_{1j}, x_{2j}) dx_{1j} dx_{2j}$ where $z(x_{1j}, x_{2j})$ is the probability of a bundle (x_{1j}, x_{2j}). The incentive compatibility constraints become

(LIC) $$\int U(x_{1j} + \theta_j, x_{2j}) z(x_{1j}, x_{2j}) da \geq$$

$$\int U(x_{1k} + \theta_j, x_{2k}) z(x_{1k}, x_{2k}) dx, \quad k \neq j .$$

Any allocation can be specified by the probability weights z, since z is from a distribution over the entire commodity space. Thus, by the expected utility theorem, utility is linear in z, and hence the LIC are linear in z, and constitute convex constraints. A deposit contract now gives you a choice of lotteries--you withdraw a randomized amount. For example, your place in line during a bank run (the Diamond-Dybvig sequential service constraint) provides a lottery. You might get your money out, or you might not.

By curing non-convexities lotteries allow many problems involving uncertainty to be studied with the tools of modern general equilibrium theory. The next two propositions however show that lotteries change the qualitative nature of the optimal allocation less than might initially be expected. Notice the tension inherent in contrived randomness: the purpose of a bank is to reduce uncertainty, but a lottery adds uncertainty.

PROPOSITION 4: If agents are risk averse, lotteries cannot achieve the full information social optimum.

Proof: Assume using lotteries increases expected utility, that is, assume the optimal lottery is not degenerate. Hence there must be randomness in the allocation. The full information solution to (I), however, is a non-random amount, so with risk aversion the optimal lottery allocation is strictly inferior to the unconstrained optimum.

Q.E.D.

Prescott and Townsend in their section 6 present an example where the use of lotteries leads to the full information solution. A necessary part of that example, however, is that one party be risk neutral and thus able to bear a lottery with no loss of utility. Proposition 4 may serve as a counterweight to that example by showing that in general such extreme results are not possible. It seems intuitive that lotteries cannot totally remove the effect of imperfect, incomplete information. Often, though, they do considerably less.

PROPOSITION 5: With decreasing absolute risk aversion and $t \leq (1 - t)$ lotteries cannot improve upon the optimal non-stochastic allocation.

Proof: First consider the deterministic allocation. From proposition 3, only the rich IC constraint binds at the solution. Hence, $U(x_{11} + \theta_1, x_{21}) = U(x_{12} + \theta_1, x_{22})$. To dominate this a solution with lotteries must increase ex ante expected utility, that is, decrease the uncertainty of utility by redistributing income from the rich to the poor. Since the rich are risk averse, we can find a lottery L_2 with $E(L_2) = (x_{12}, x_{22})$ such that $U(x_{11} + \theta_1, x_{21}) > U(L_2 + \theta_1)$. Now define the risk premium δ_j (following Pratt) as the amount that must be paid to agent j to make the agent indifferent between L_2 and $E(L_2) + \delta_j$. Thus, $U(x_{11} + \theta_1, x_{21}) = U(L_2 + \theta_1 + \delta_j) > U(L_2 + \theta_1)$. The theoretical maximum that we could redistribute from the rich to the poor is $(t/1 - t)(\delta_1)$. However, with decreasing absolute risk aversion, and $t < 1/2$, $\delta_1(t/1 - t)$ will not compensate the poor for the risk they bear.

Q.E.D.

Intuitively, the binding constraint is that the rich cannot act like the poor. Already, the poor do not want to act like the rich, who get taxed. Hence, any uncertainty in the rich allocation is useless. Adding uncertainty to the poor allocation is also useless, since the rich are more willing than the poor to bear that risk. Under common preference restrictions and for plausible shocks contrived randomness does not help.

5. Multilateral Incentive Compatibility

If the social planner cannot explicitly specify consumption nor exclude the possibility of exchanges between individuals, individual incentive compatibility constraints alone are not enough to guarantee that people actually consume their assigned allocations. A simple example is when the IC constraints put an allocation off the contract curve. Agents then trade from this allocation to reach the Contract Curve (CC). The resulting point is not incentive compatible: if it were, the allocation could have been there, since it is Pareto superior to the original point. The trade makes both individuals better off, so E(U) rises. This makes the original allocation not incentive compatible, given the possibility of trade. More formally, an allocation is multilateral incentive compatible (MIC) if no coalition can improve upon the allocation. The strategies open to the coalition involve the types claimed by members and the exchanges between members. Of course, the coalition cannot verify agents' income shocks any more than the planner can. Still, as the above example shows, that is not necessary for coalition formation to restrict feasible allocations. After the allocation, and after the income shock, there can be incentives (arbitrage possibilities) for groups. One noticeable restriction concerns lotteries.[2]

PROPOSITION 6: If lottery results are public information then lotteries cannot improve upon the deterministic solution to (II).

Proof: Without loss of generality, consider a lottery given to the poor, (L_2). Then consider the coalition consisting of all agents claiming to be poor (and thus receiving the poor allocation). The public nature of the lottery enables the coalition to collect each agent's realization and dispense $E(L_2)$ to each. This removes the uncertainty and makes the allocation deterministic. Since agents are risk averse, they will join the coalition.

Q.E.D.

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[2]I wish to thank Robert King and Robert Townsend for pointing this out to me.

Notice the important place of observability in the proof of Proposition 6. If lottery results were private information, coalitions could not diversify away the contrived risk.

The following proposition proves that the bank allocation is not MIC, showing that additional asset markets enable agents to redistribute the coalition's proceeds. With such coalitions possible, some groups can do better than the bank allocation with a strategy of misrepresentation and redistribution. This is the reason that we have stressed the exclusive nature of the bank, and the need for the absence of other asset markets.

PROPOSITION 7: The solution to problem (I) does not solve problem (II).

Proof: Since the fractions t and (1 - t) will cancel out, consider the case where $t = \frac{1}{2}$. We showed above how points off the contract curve do not solve (II) so we concentrate on solutions to (I) which lie on the contract curve. For any such point A (see figure 3) corresponding to $(x_{11} + \theta_{21}, x_{21})$, find the "cheat point" B of $(x_{12} + \theta_1, x_{22})$. These points lie on the same indifference curve by Proposition 3. By the MIC constraints in (II) agents can consume anywhere along the chord AB by having different proportions of the coalition membership misrepresent their type. Convexity of preferences implies this is preferred to A.

(Q.E.D.)

In fact, the multilateral incentive constraints put rather stringent limits on the insurance possibilities of problem (II).

PROPOSITION 8: An allocation solves problem (II) if and only if it is an ex post competitive equilibrium for the economy.

Proof: As in the previous proposition, it suffices to examine $t = 1/2$. The proof shows that any solution to (I) and the corresponding "cheat point" lie along a price line through the original endowment point, $(y_1 + \theta_1, y_2; y_1 + \theta_2, y_2)$. This implies (i) any allocation A that is a competitive equilibrium is MIC, and (ii) any other allocation is not MIC (by a variant of the proof of proposition 7). Let E be the initial endowment, $E = (y_1 + \theta, y_2)$. The allocation point is $A = (x_{11} + \theta, x_{12})$, while the "cheat point" $B = (x_{12} + \theta, x_{22}) = (2y_1 - x_{11} + \theta, 2y_2 - x_{21})$. (See figure 4.a,b.) The slope from E to A is

$$(x_{21} - y_2)/\left(x_{11} + \theta - (y_1 + \theta)\right)$$

while the slope from B to E is

$$y_2 - (2y_2 - x_{21})/\left[y_1 + \theta - (2y_1 - x_{11} + \theta)\right]$$

both of which equal

$$[x_{21} - y_2]/[x_{11} - y_1] .$$

Now if A is a competitive equilibrium for E, it must also be for B. Hence A is incentive compatible (also MIC). Should A not be an equilibrium, some section of the line segment AB will be above an indifference curve through A. Convexity implies one coalition can improve upon A (though it may be a coalition of the poor).

Q.E.D.

This result, that the only multilaterally incentive compatible financial structure is the competitive equilibrium, bears a close relation to other work. In particular Hammond [1983] has shown under quite general conditions that MIC precludes non-linear pricing of exchangeable goods. In the economy considered here, exchangeability depends upon observability. If the central planner or Bank cannot observe consumption directly, agents can trade income. In effect, agents have access to a loan market. Income becomes an exchangeable commodity. Furthermore, the bank allocation (solution to (I)), based on discrete deposits and differential payments, is decidedly non-linear. In that case, Hammond's results suggest that the non-linear solution to (I) (without MIC) will not survive coalition formation. From an ex ante perspective, everyone is better off with a bank (I) than with only a loan market (II). However, the extra benefit of insurance comes from non-linearities that get arbitraged away by groups.

Economically, there is some reason to fear results that depend too heavily on the formation of coalitions. We intuitively expect that organization, agency, and transaction costs severely restrict coalition formation. However, a non-cooperative approach to the problem reveals that much weaker conditions yield the same results. The proof of proposition 8 does not require complicated coalitions. All that is really needed is for one individual to claim an incorrect type, move to the "cheat point" B and offer to trade with other agents. This results in an allocation along AB. In the language of game theory, the outcome is not a Nash Equilibrium. If everyone else has told the truth and received the non-MIC allocation, it remains in the interest of the remaining player[3] to misrepresent his type and trade with others. The real force behind multilateral incentive compatibility is the ability of agents to exchange goods, not the complex machinations of coalitions. As Hammond puts it, "consumers need rely on each other only to make mutually advantageous exchanges."

Since the solution to problem (II) must be a competitive equilibrium one type of support is simply to let agents trade claims to income. Of course, other equivalent mechanisms are possible. For instance, a bank taking in deposits and allowing withdrawals can exist side by side with a loan market. The rate of return on bank deposits must equal the competitive return on other assets.

The lower expected utility when income is exchangeable creates an incentive to somehow make income non-tradeable, and thus allow a bank to redistribute income.[4] Indeed, some government regulation of the banking system may rest on this incentive. For example, some assets, such as IRA's, not only cannot be redeemed early, but, according to regulations cannot be used as collateral for loans. This seems an attempt to restrict the exchangeability of income.

6. SUMMARY AND CONCLUSIONS

Rather than coming to any definitive conclusions on the nature of financial structure, this paper has been an analytical first step in building a model that captures the important features of

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[3]More precisely, a set of positive measure must follow this strategy. It remains true that each member of the set acts individually to take advantage of the arbitrage opportunity.

[4]Jacklin [1984] also makes this point.

financial systems. This paper has shown (I hope) that such an exercise is possible, and has raised several important issues along the way. Despite the extreme simplicity of the economy and the stark formalism of the method, recognizable institutions emerged as the outcome of an explicit optimization process. Much more work needs to be done to clarify many theoretical issues and to produce empirically testable structures that can provide guidance in matters of policy.

This study may also serve as a warning. Justifying a financial institution as the outcome of a mathematical programming problem requires careful consideration of the contracts and constraints. Traditional assumptions, such as unobserved income, are usually not enough to produce traditional solutions. This paper, however, attempts to go beyond that negative result by clarifying when the new possibilities and restrictions will be applicable.

For example, contrived uncertainty has a rather limited scope in models of this general class, especially when the primary good, in this case income, is exchangeable. In some fields, however, it will be most plausible to posit non-exchangeability; for example, in labor contracts. Prescott and Townsend [1984] have had success in this area. More work can help delineate the circumstances under which lotteries will form a significant part of the institutional make-up.

Perhaps more importantly, we have seen that multilateral incentive compatibility constraints deserve serious consideration. These constraints dramatically limit potential allocations and institutions. Equally important, their strength comes from the basic notion of arbitrage. It seems important to proceed with this concept on at least two fronts. A richer model may be needed to explain banks and other financial institutions, perhaps containing transactions costs, information production, or government regulation. Secondly, current and future models of contractual arrangements should be subject to the full discipline of arbitrage arguments.

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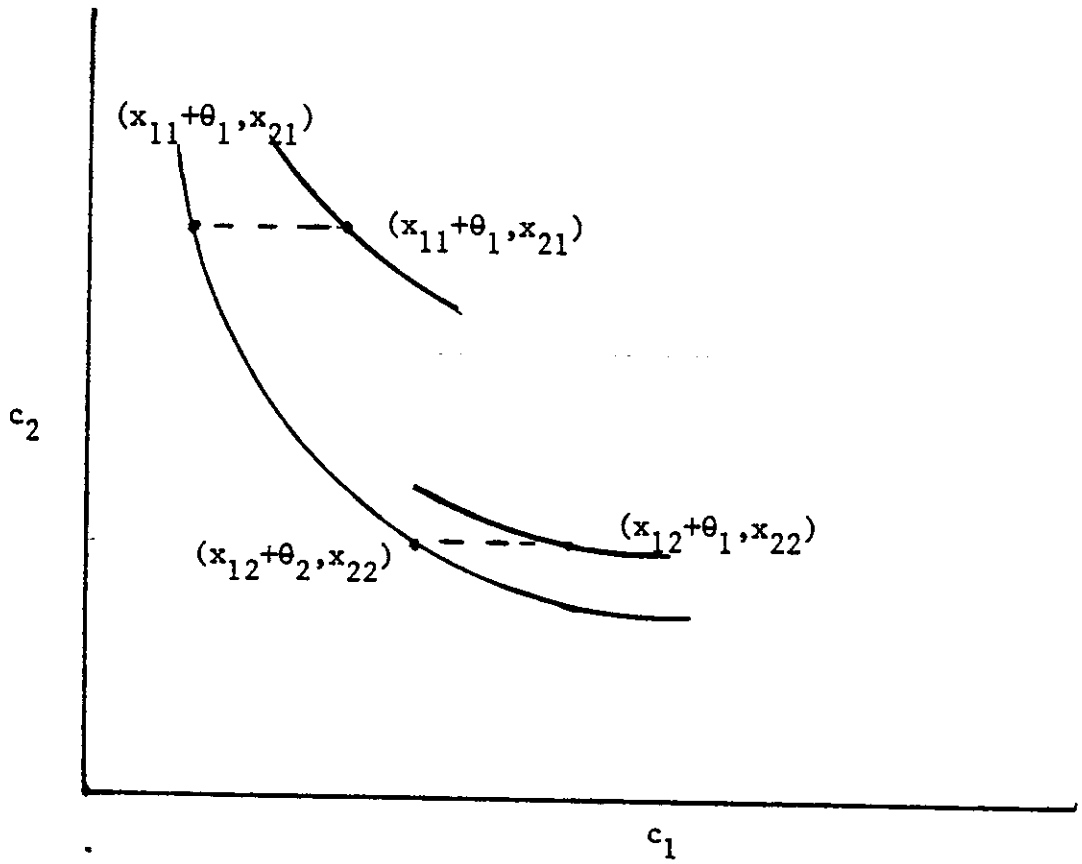

Figure 1
Only one IC constraint binds

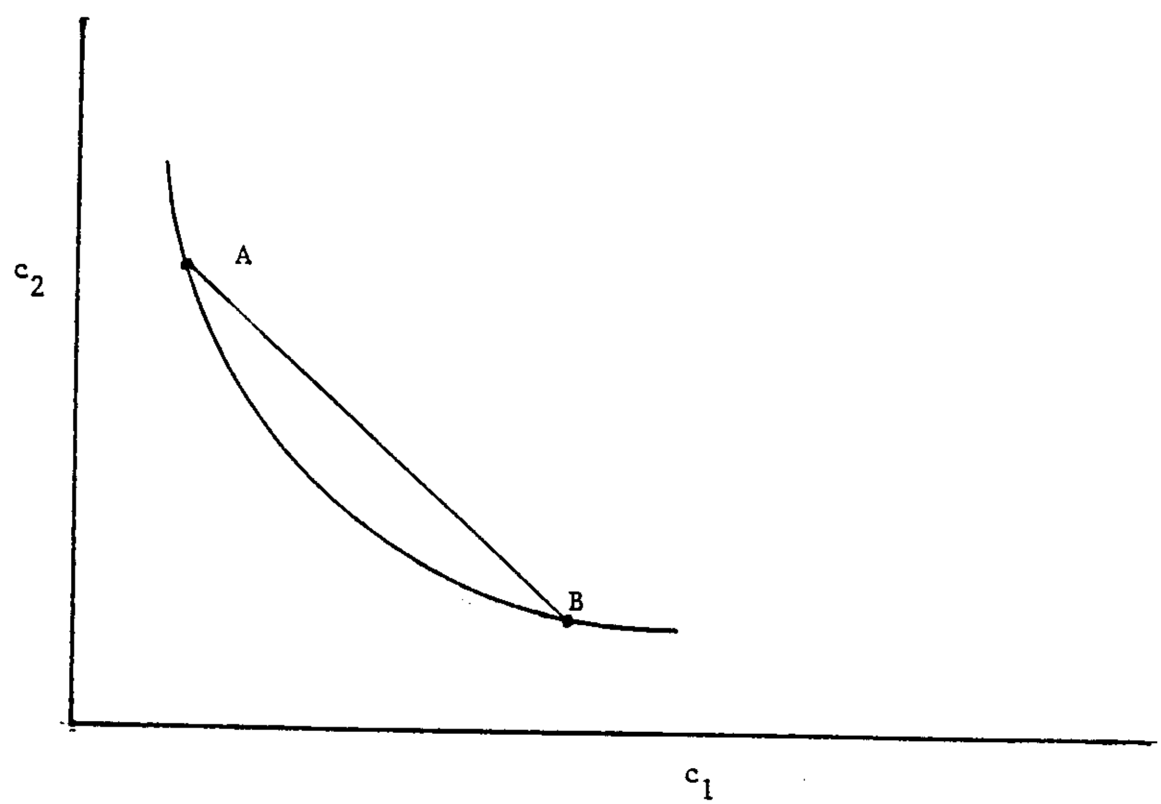

Figure 2
IC constraints not convex

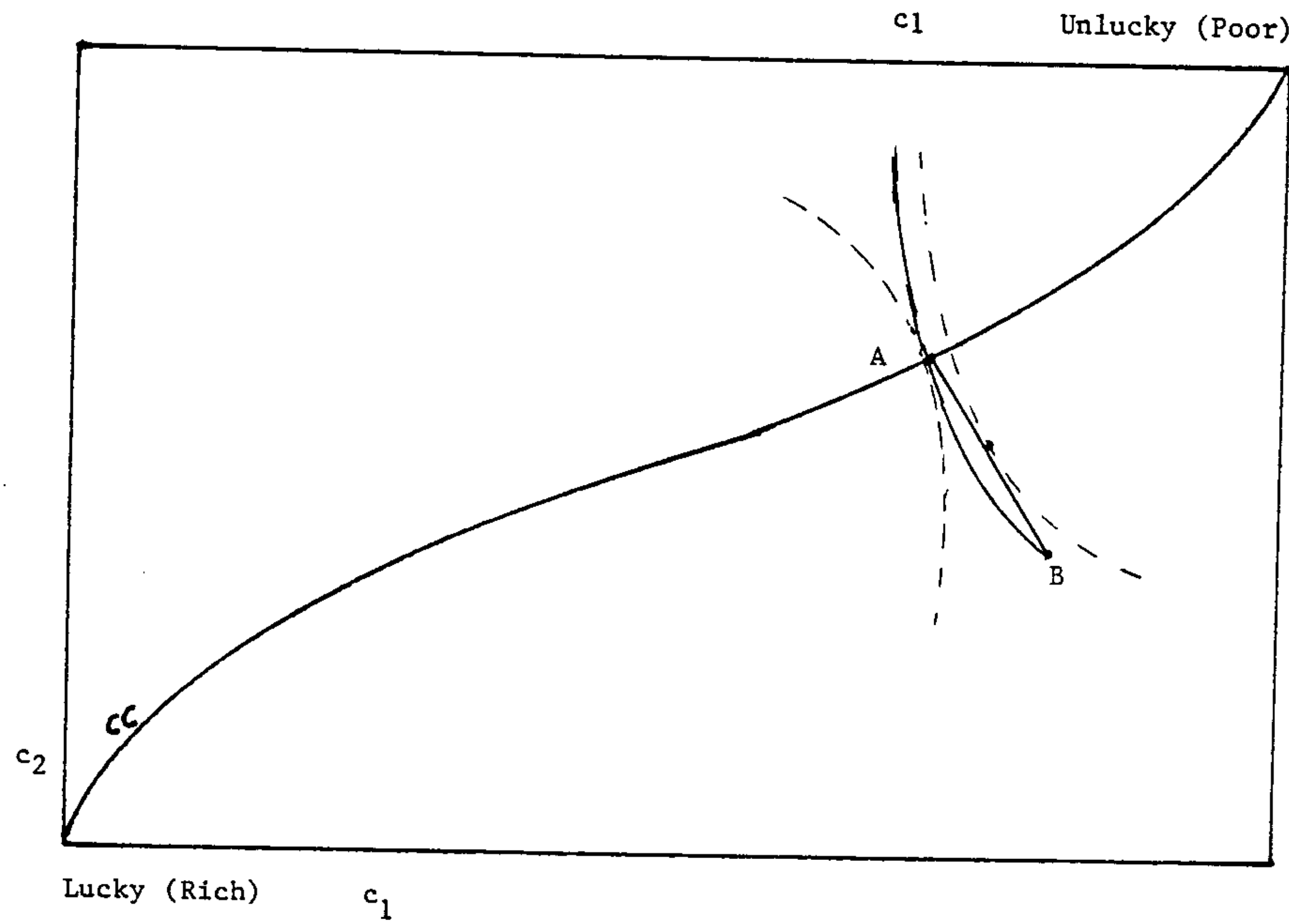

Figure 3
Point A is not multilateral incentive compatible

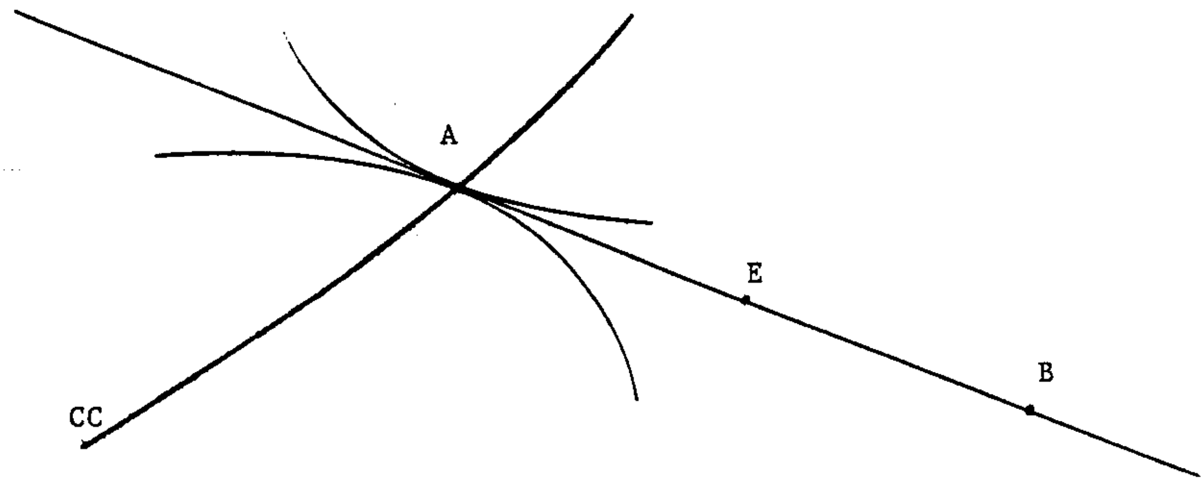

Figure 4.a

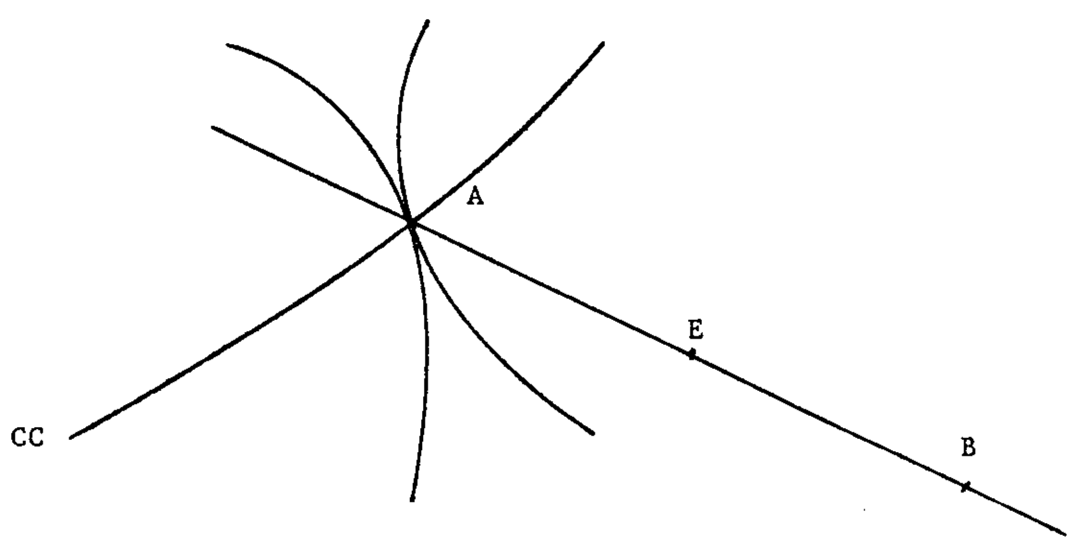

Figure 4.b