Partial Deposit Insurance, Bank Runs, and Private Deposit Insurance

George G. Pennachi

Working Paper #17-84

THE WHARTON SCHOOL

University of Pennsylvania

Philadelphia, PA 19104

Partial Deposit Insurance, Bank Runs, and Private Deposit Insurance

by

George G. Pennacchi

Wharton School
University of Pennsylvania

Comments and suggestions by Stanley Fischer, Jim Poterba, and Julio Rotemberg are much appreciated. Any remaining errors are my own.

I. Introduction

Deposit insurance became widespread in the United States only after the Federal government passed the Banking Act of 1933 which, among other things, provided for the creation of the Federal Deposit Insurance Corporation (FDIC). Since this time, the provision of deposit insurance has remained almost exclusively the domain of the federal government. A significant proportion of bank liabilities, however, are not officially insured. Among these liabilities are included deposits in accounts over the $100,000 coverage limit, foreign deposits, subordinated debt, and federal funds. It is estimated that 27 percent ($419 billion) of total U.S. domestic deposits are not protected by federal deposit insurance.[1] This proportion of uninsured deposits is generally larger, the larger is individual bank size.

Historically the deposit insuring agencies have, however, handled most bank failures in such a manner as to de-facto insure all bank deposits. Instead of making direct payments to insured depositors at the time a bank is closed, a "purchase and assumption" transaction is usually made by the insuring agency, especially if the bank is a relatively large bank. This transaction entails the insuring agency taking ownership of the failed bank's assets which are presumed to bear significant default probabilities and then arranging for another acquiring bank (usually selected through a competitive bidding procedure) to assume all other assets and liabilities of the failed bank. Because in an arrangement of this sort even uninsured liabilities are assumed by the acquiring bank, the officially uninsured liabilities of the failed bank are in actuality insured.

Since the creation of the FDIC and until the 1982 Penn Square Bank failure, all depositors of failed banks with over $60 million in total deposits have not incurred any losses as a result of the bank's closing, since in each case failures have not been handled by direct payments to only insured depositors. Thus it is very likely that uninsured depositors, particularly those in the largest banks, have come to view their deposits to be risk-free in terms of default, and thus have probably not demanded much, if any, risk premium in lending their funds to these banks.

Studies by the FDIC and Federal Home Loan Bank Board (FHLBB), which were mandated by the Garn - St. Germain Depository Institutions Act of 1982, have proposed that greater degrees of "market discipline" rather than direct regulatory controls be used to restrain a bank's possible incentive toward excessive risk taking that would increase the deposit insuring agency's liability.[2] The FDIC states that reforms could be made by discontinuing the practice of purchase and assumption transactions that de-facto insure officially uninsured liabilities of banks. While failures of large banks could still be handled by arranging mergers with a stronger acquiring bank, it is recommended that uninsured depositors would only recover a portion of their claims on bank assets, losing a portion of their deposits equal to the FDIC's estimate of the negative net worth of the failed institution.

Fairly recently, the FDIC had stated, and confirmed by its actions, that it would institute the new policy described above on an experimental basis.[3] Starting with the Penn Square Bank failure and later with the merger of the failed Seminole State National Bank with West Texas Bancshares, Inc., uninsured depositors of the failed banks lost a portion of their promised claims. Apparently, this experiment was short lived. Soon after Continental Illinois Corporation's financial difficulties became apparent and a run on the bank by uninsured depositors commenced, the FDIC announced that it would de-facto insure the bank's officially uninsured deposits.

It is evident that the choice of policy concerning uninsured depositors made by the FDIC can have far-reaching consequences. Under a policy of only partial deposit insurance, large depositors might react by splitting their deposit accounts among many banks such that no account will have a deposit total greater than the $100,000 statuatory limit on insured accounts. A similar alternative, which had until recently gained popularity, was for large depositors to place their funds with Certificate of Deposit brokers who then split up these funds between banks instead of the depositors doing it directly. However, the FDIC and FHLBB have recently ruled that they will revoke insurance coverage for any brokered deposits beyond the first $100,000 placed in a given bank by an individual broker.[4]

Assuming that large depositors might prefer the convenience of maintaining their deposits in a single account, they will now require banks to pay a risk premium that will be an increasing function of the bank's probability of failure. This reaction is precisely what the goal of the FDIC's experimental policy change was to attain. Most of the burden of monitoring banks would be shifted from the FDIC to uninsured depositors. Banks which wished to pursue riskier investment strategies or lever themselves to a greater extent would have to "pay" for the greater risk that would be imposed on uninsured bank deposits by paying a higher risk premium. Banks' possible incentive for excessive risk taking would be mitigated. In order to aid large depositors in assessing individual bank risk, an expanded degree of bank financial information disclosure has been advocated by the insuring agencies.[5]

While a banking system which more efficiently prices the risk that bank managers choose to undertake may be a beneficial reaction to the FDIC's policy change, other reactions by uninsured depositors may not be quite as welcome.

The possibility of a run on the bank by uninsured depositors will now also exist. While one of the primary reasons for the establishment of deposit insurance may well have been to prevent the financial and monetary instability that resulted from widespread depositor withdrawals, this initial objective will be compromised in the FDIC's effort to reduce its liability.[6]

Bank runs and deposit insurance have been studied by other authors, including Kareken and Wallace (1978), Cone (1981), and Diamond and Dybvig (1983). Given that bank liabilities take the form of deposit contracts and bank assets are illiquid to some degree, deposit insurance is shown to provide a benefit by eliminating the incentive for bank runs which would cause real resource losses owing to the forced liquidation of bank assets in paying off depositors. Thus, avoiding these bankruptcy costs would provide an incentive for banks and depositors to always prefer a system with deposit insurance.

For a bank which issues both government insured and uninsured deposits, the cost of liquidating assets to pay uninsured depositors during a run may not be substantial. Banks could choose to sell their most liquid assets first, and borrowing via the Federal Reserve's discount window, which has occurred during some recent failures, would also tend to minimize real resource costs of bank runs.[7] Therefore it is not clear, a priori, that banks could have much preference for a system of complete insurance for all deposits over a system of partial deposit insurance with the possibility of bank runs by uninsured depositros. This may be especially true if the deposit insurance premium charged banks by the FDIC does not coincide with the individual bank's fair insurance rate.

If banks do prefer to have all deposits insured but the government insuring agency's policy is to insure only the liabilities of "small" depositors, then banks and uninsured depositors may prefer a situation in which non-government insured liabilities are insured by competitive private insurance companies that charge premia that vary with a bank's level of risk. This scheme could provide "market discipline" in terms of efficiently pricing banks' cost of funds (non-government insured funds, at least) while it would remove any adverse effects of bank panics by removing the incentive of deposit withdrawals by non-government insured depositors. This form of a government-private co-insurance plan is essentially equivalent to private firms providing insurance with the government taking on a portion of the liability for only the case of catastrophic losses, i.e. a negative net worth of the bank having larger magnitude than the total of non-government insured liabilities. The private insurance company's claim on bank assets is junior to that of the government insuring agency's claim. Thus for minor bankruptcies, the private insurance company would cover most if not all of the bank's negative net worth, with the government insuring agency only making payments with the occurence of major bankruptcies where the negative net worth of the bank exceeds the total of privately insured deposits. Some regulation of private insurance companies capital levels may be warranted so as to make certain that large depositors of privately insured banks won't question these insurance companies' ability to make payments in the event of widespread bank failures. However, as will be discussed below, it is theoretically possible for insurers to require only small amounts of equity capital if they hold the proper assets to hedge their liabilities. Essentially, they could reduce their net risk position to be that of only non-systematic risk. While the previous policies of the FDIC and FSLIC of de-facto insuring officially uninsured depositors have probably created little or no demand for private insurance, it is hoped that the increase in demand for this type of deposit insurance resulting from an FDIC decision to provide only partial insurance will induce entry by private firms.[9]

From the perspective of minimizing liquidation or bankruptcy costs, it may not matter much whether banks' deposits are fully or partially insured. However, from the point of view of avoiding monetary shocks to the economy caused by bank runs, it may matter a great deal whether the possibility of a panic by uninsured depositors exists.

This paper has two main goals. First is to formulate a model of the banking firm in which the value of bank equity, uninsured depositors' risk premia, and fair value deposit insurance rates can be derived. Second is to study what incentives banks may have to prefer a given structure of uninsured to government insured or possibly privately insured deposits.

In section II the size of uninsured depositors' risk premium is derived along with the fair pricing of government deposit insurance and the equilibrium value of bank equity. It is demonstrated that an increase in the tendency of bank panics to occur and the amount of deposits that can be withdrawn during a panic before the bank is closed will likely result in uninsured depositors requiring a lower risk premium on their deposits. If the government insuring agency fixes the premium it charges each bank for insured deposits, then this lowering of uninsured depositors' risk premium is shown to result in an increase in the equilibrium value of bank equity. An analysis of banks' incentive to issue government insured versus uninsured deposits is also made. Again if the government insuring agency charges the same premium to all banks, and deposit markets are competitive, it is found that relatively safe banks will prefer to issue a large proportion of uninsured deposits, while relatively risky banks will have an incentive to only issue government insured deposits.

Section III provides a framework for pricing the fair value of both private and government deposit insurance for the type of co-insurance system described above. The type of system envisioned also has the characteristic of being a "variable rate" insurance scheme where the private insurance company and possibly also the government insurance agency adjust their premium charged to the bank in response to changes in bank risk that are discovered at the time of a bank audit. The characteristics of this insurance system is shown to be quite similar to the system with uninsured deposits in section II, but with bank runs being ruled out. The incentive for banks and non-government insured depositors to prefer private insurance and the government insuring agency's insurance pricing scheme's effect on this incentive is studied. It is shown that under certain circumstances, banks may prefer a system with uninsured deposits and the possibility of bank runs, relative to a system with both private and government deposit insurance.

II. A Contingent Claims Model with Uninsured Depositors and Bank Runs

A. Assumptions

Much of the notation and assumptions used in this paper are similar to those in Merton (1978) and Pennacchi (1984).

A1. Banks and the government insuring agency are assumed to be able to bear no transactions costs in trading assets. Banks provide intermediary services to those savers (depositors) who would face costs of directly holding assets. Banks are assumed to hold financial assets and issue equity and both government insured and uninsured deposits. Banks may have some market power in their deposit markets, but are price takers in their asset market.

A2. It is assumed that the value of bank assets, V, follows the continuous time diffusion process:

(1) $$dV = (u_v V - C)dt + s_v V dz$$

where u_v is the instantaneous expected return on bank assets, C is the total net payouts per unit time from the bank, and s_v^2 is the instantaneous variance of the return on bank assets and is assumed to be constant. dz is a standard Weiner process.

A3. The current, time t, value of a riskless-in-terms-of-default discount bond that pays $1 in T periods, P(T, t) follows the process:

(2) $$dP(T) = u_p(T, t)Pdt + s_p(T)Pdq \ , \qquad P(0, t) = 1$$

where dq is also a Weiner process, $dqdz = \rho dt$, and $s_p(T)$ depends on the bond's time to maturity but is constant with respect to calendar time, t, given T.

A4. Banks issue both government insured and uninsured deposits. w is the proportion of insured deposits to total deposits, and is assumed constant over time, except possibly during bank runs. Thus (1 - w) times total deposits are uninsured. While the growth rate of total new deposits coming to the bank, n, is assumed constant, the growth rates of insured and uninsured deposits differ so as to maintain a constant proportion, w. These individual growth rates reflect differences in duration (i.e. "average" maturity), monopoly power, and risk premia between the two deposit classes. The stochastic process that total deposits follow is given by;

(3) $$dD/D = (u_d + n)dt + s_d dq$$

where if the duration of total deposits is T, $s_d = s_p(T)$ and $u_d = u_p(T) - m$

where m is a positive constant whose magnitude indicates the degree of monopoly power banks have in their deposit market.

A5. The government insuring agency audits banks at random intervals, audits having a Poisson probability distribution with mean = λ_g. Thus the probability of an audit over time interval dt is $\lambda_g dt$. The probability of an audit is assumed uncorrelated with any non-diversifiable risk. The insuring agency collects a continuous insurance premium of $\$h_g$ per unit time per dollar insured deposit. It incurs an auditing cost of $\$a_g$ per dollar total bank deposits at the time of each audit. If during an audit, the value of total deposits, discounted by the magnitude of monopoly rents attributable to the bank's ability to pay below market rates, ϕD, is greater than the market value of bank assets, V, the bank is closed, shareholders lose all claims on bank assets, the government insuring agency becomes the senior claimant, and the uninsured depositors become the junior claimant to bank assets.

A6. At the time of an audit, if the bank is solvent, the government insuring agency discloses all information on the bank's condition. Uninsured depositors then adjust their risk premium, k, accordingly. Uninsured depositors also are assumed to incur a monitoring cost of $\$a_g$ per dollar total deposits at this time. In addition, if inbetween audits the bank becomes insolvent, i.e., $V < \phi D$, uninsured depositors may privately learn of this information. The probability of this information being leaked to uninsured depositors is approximately $\lambda_j dt$. If the bank is revealed to be insolvent, a run on the bank by uninsured depositors commences. Uninsured depositors are able to withdraw a proportion, p, of their deposits before the government insuring agency steps in to close the bank, at which time an audit is made and the government insuring agency becomes the senior claimant.

Assumption A1 states that some investors face transactions costs such that they find it less costly to use banks' intermediary services, holding either insured or uninsured deposits rather than directly holding other assets. These services may be priced above marginal cost because of some sort of market imperfection or entry barrier within banking. Assumption A2 simply states that asset returns are instantaneously normally distributed, variance being constant over time, and assets following a continuous sample path. Assumption A3 gives bond price dynamics where the assumption that bond return variances are constant functions of maturity is utilized for analytical simplicity.

Assumption A4 gives the bank's behavior with respect to its holdings of insured relative to uninsured deposits and gives the correspondence between the return on total deposits and equivalent duration government bonds. The assumption of a constant proportion of insured to uninsured deposits over time is made for analytical simplicity, but in this context one can still allow for differences in durations and degrees of a bank's monopoly power between insured and uninsured deposit markets.[9] Appendix A of this chapter derives the stochastic processes for the growth rates of insured and uninsured deposits so as to maintain their relative proportions constant and such that the stochastic process for total deposits, (3), is satisfied.

Assumption A5 describes the auditing behavior and actions during bankruptcy of the deposit insuring agency. It is assumed that the insuring agency considers deposit market monopoly rents in its decision to close a bank, and a merger of the failed bank with an acquiring bank can be arranged so that the insuring agency is able to "sell" the failed bank's deposit market monopoly rents to the acquiring bank. In the current situation, $\phi = \left(n - \lambda_g(a_g + a_j)\right)/(n - m)$.

Assumption A6 models the type and timing of information received by uninsured depositors. For simplicity, bank runs are modelled to happen nearly instantaneously, being immediately followed by a closing of the bank. One might think of the private information received by one or more uninsured depositors as being akin to a "leak" of insider information that the bank is insolvent. The incentive for one or more uninsured depositors to withdraw deposits is assumed to lead to an overall panic by other uninsured depositors. While this analysis assumes that banks can costlessly liquidate assets in order to pay off those uninsured deposits which are withdrawn before the bank is closed, it would be a straightforward extension of the model to allow for a bankruptcy cost from "forced liquidation" during a bank run, so that the value of remaining bank assets after the bank is closed would be reduced.[10] This paper ignores liquidation costs not because it is thought they are always negligible, but because attention can now be focused on other factors influencing banks' liability decisions.

The occurence of bank runs when the bank has positive net worth has been ruled out. The model could be extended fairly simply to include this possibility, e.g., assuming the probability of a run when $V > \phi D$ is $\lambda_j' dt$ where this probability might reasonably be thought to be less than the probability of a run when $V < \phi D$. Assuming that solvent banks can always costlessly liquidate assets to pay off depositors or they have access to short term borrowing from a government lender of last resort, e.g., the Federal Reserve discount window, the aftermath of a run when the bank is solvent would be similar to a regular audit by the insuring agency. The bank wouldn't be closed, but the bank's insurance premium and the risk premium of the uninsured depositors who return to the bank might now be adjusted after the true condition of the bank's degree of positive net worth is revealed. The result of this extension would be equivalent to the model analyzed below, with a larger value of λ_g in the region $V > \phi D$. More generally, one might think that the probability of a bank run should be an increasing function of the bank's degree of insolvency, i.e., λ_j should rise with a fall in the level of bank capital. Modelling this assumption would be a straightforward extension of the analysis in this paper, though its effect on the qualitative results are likely to be minor.

Note that it is also assumed that in the event of a bank run, each uninsured depositor can only withdraw the same proportion, p, of his deposits. It may have been more realistic to assume that during the panic some depositors are able to withdraw all of their deposits while other depositors (those at the end of the line) can withdraw none. The analysis presented below will be valid under this alternative assumption for the case of risk neutral depositors or depositors who can somehow insure against receiving a lower than average payment following a panic. However, if this risk cannot be eliminated through some form of private insurance (e.g., holding a large fraction of all uninsured deposits in the bank) and depositors are risk averse, then the risk premium, k, that is derived below will be a lower bound for the risk premium needed for uninsured deposits to be issued by the bank. Note also that while the proportion, p, of deposits withdrawn is treated as exogenous, one might think of p as being a decreasing function of the maturity of uninsured deposits issued by the bank, i.e., the shorter the maturity of uninsured deposits, the greater the chance of more of them being withdrawn during a panic. Another parameter treated exogenous is the uninsured depositors monitoring costs, a_j. However, it might be reasonable to believe that the magnitude of a_j is inversely related to the amount of information disclosure by the insuring agency.

The following table partially summarizes the contingent actions and payments to uninsured depositors and the government insuring agency. It is assumed that $\phi > (1 - w)$, i.e., the analysis rules out the uninteresting case in which the value of monopoly rents, or bank charter value, is greater than the total of insured deposits. In this case, the insuring agency would never sustain any losses.

Bank Condition Region		Audit Occurs: Insuring Agency	Audit Occurs: Uninsured Depositors	Bank Run Occurs: Insuring Agency	Bank Run Occurs: Uninsured Depositors
I.	$V-\phi D > 0$	h_g adjusted	k adjusted	Run won't occur	
II.	$0>V-\phi D>-(1-w)(1-p)D$	0	$V-\phi D$	0	$V-\phi D$
III.	$-(1-w)(1-p)D> V-\phi D>-(1-w)D$	0	$V-\phi D$	$V-\phi D+ (1-w)(1-p)D$	$-(1-w)(1-p)D$
IV.	$-(1-w)D>V-\phi D> [(1-w)p-\phi]D$	$V-\phi D+(1-w)D$	$-(1-w)D$	$V-\phi D+ (1-w)(1-p)D$	$-(1-w)(1-p)D$
V.	$[(1-w)p-\phi]D>V-\phi D$	$V-\phi D+(1-w)D$	$-(1-w)D$	$-\phi D+(1-w)D$	$V-(1-w)D$

Bank Condition I denotes a positive level of bank capital so that if an audit occurs, the bank remains in operation, but insurance and risk premia will be adjusted. Note that for $0 > V - \phi D > -(1 - w)(1 - p)D$, i.e. Bank Condition II, uninsured depositors' loss is always equal to the total of the bank's negative net worth. Whether the bank is closed following an audit by the insuring agency or a bank run, the insuring agency will not incur any losses, since the magnitude of negative net worth of the bank is less than the amount of uninsured deposits that could not escape the bank after a run has started. One might wonder if it is in the best interests of the uninsured depositors to start a run in this situation. As a group, perhaps they would not wish to do so. However, if individual uninsured depositors believe they might be able to withdraw all their deposits before the bank is closed, the incentive for individuals to begin withdrawing deposits would be present.[11]

For Bank Condition III, IV, and V, uninsured depositors have less total losses if the bank is closed following a run than if the bank is closed after an audit. In Region III, uninsured depositors still lose only a portion of this deficit if a bank run occurs. Region IV denotes the bank capital condition in which uninsured depositors lose the total of their deposits remaining in the bank either after a run or an audit. Bank Condition V denotes the situation in which if a run occurs, the total of the bank's remaining tangible assets are paid to uninsured depositors, and the insuring agency can only retain the bank's charter value to offset its liability of the total of insured deposits.

B. Derivation of Uninsured Depositors' Risk Premium

Let $J(V, D, k, w)$ be the different in value between uninsured depositors' claims with the current risk premium, k, and the value of uninsured depositors claims if fair risk premium k^*, was charged. Then by definition, $J(V, D, k^*) = 0$. Therefore if $(1 - w)D$ is the value to uninsured depositors of their total deposit holdings given a fair premium is charged, $(1 - w)D + J$ is the value of uninsured deposits given the current condition of the bank and the actual risk premium, k. (Recall that risk premia are only assumed to be adjusted at insuring agency audits at which time the bank's condition is disclosed.)

Given the assumed stochastic process for bank assets, V, and total deposits, D, the stochastic process for J will be of the form;

$$
\begin{aligned}
(4) \quad dJ(V, D) = {} & \left[J_1(u_v V - C) + J_2(u_d + n)D + (1/2)J_{11}s_v^2V^2 \right. \\
& \left. + J_{12}\rho s_v s_d VD + (1/2)J_{22}s_d^2D^2\right]dt + J_1 s_v Vdz \\
& + J_2 s_d Ddq - I_{AUR}(a_j D + J) + I_{II\cap(AUR)}(V - \phi D) \\
& + I_{III\cap A}(V - \phi D) - I_{III\cap R}(1 - w)(1 - p)D - I_{IV\cap A}(1 - w)D \\
& - I_{IV\cap R}(1 - w)(1 - p)D - I_{V\cap A}(1-w)D - I_{V\cap R}\left((1-w)D-V\right)
\end{aligned}
$$

where subscripts of J denote partial derivatives and where "I" is the index operator, being 1 when its subscript is true, 0 otherwise. $\cup$ and $\cap$ are mathematical union and intersection symbols, A stands for the audit event, R for the bank run event, and II, III, IV, and V are the various Bank Condition Regions defined in the previous table. It is assumed that the form of the bank's net payouts per unit time, C, will be $C = -nD + \delta V + h_g wD + k(1 - w)D$ where δ is the bank's dividend - asset payout ratio.

Using arguments essentially similar to the derivation of the deposit insuring agency liability formulas found in Merton (1978) and extended in Pennacchi (1984), one can show that $j = J/D$ must satisfy the system of differential equations;

$$
\begin{aligned}
(5a) \quad (Q/2)x^2 j_1'' + \left[(m-n-\delta)x + n - wh_g - (1 - w)k\right]j_1' + (n-m-\lambda_g)j_1 + k(1-w) \\
- \lambda_g a_j = 0 \, , \qquad x > \phi
\end{aligned}
$$

(5b) $$(Q/2)x^2 j_2'' + \left[(m-n-\delta)x+n-wh_g-(1-w)k\right]j_2' + \left(n-m-(\lambda_g+\lambda_j)\right)j_2 + k(1-w)$$

$$+ (\lambda_g + \lambda_j)(x - \phi - a_j) = 0 \quad , \qquad \phi \geqslant x > \phi - (1-w)(1-p)$$

(5c) $$(Q/2)x^2 j_3'' + \left[(m-n-\delta)x + n-wh_g-(1-w)k\right]j_3' + \left(n-m-(\lambda_g+\lambda_j)\right)j_3 + k(1-w)$$

$$- (\lambda_g + \lambda_j)a_j - \lambda_j(1-w)(1-p)+\lambda_g(x-\phi) = 0 \quad , \ \phi-(1-w)(1-p) \geqslant x > -(1-w)$$

(5d) $$(Q/2)x^2 j_4'' + \left[(m-n-\delta)x+n-wh_g-(1-w)k\right]j_4' + \left(n-m-(\lambda_g+\lambda_j)\right)j_4 + k(1-w)$$

$$- (\lambda_g+\lambda_j)a_j - \lambda_j(1-w)(1-p) - \lambda_g(1-w) = 0 \ , \ \phi - (1-w) \geqslant x > (1-w)p$$

(5e) $$(Q/2)x^2 j_5'' + \left[(m-n-\delta)x+n-wh_g - (1-w)k\right]j_5' + \left(n-m-(\lambda_g+\lambda_j)\right)j_5 + k(1-w)$$

$$- (\lambda_g + \lambda_j)a_j - \lambda_j\left((1-w)-x\right) - \lambda_g(1-w) = 0 \quad , \ (1-w)p \geqslant x$$

where $x = V/D$ is the bank's asset to deposit ratio, $Q = s_v^2 + s_d^2(T) - 2\rho s_v s_d(T)$, is the variance of x, $j' = \partial j/\partial x$, and $j'' = \partial^2 j/\partial x^2$. j must also now satisfy the ten boundary conditions;

(A) $\lim_{x\to\infty} |j_1(x)| < \infty$

(B) $j_1(\phi) = j_2(\phi)$

(C) $j_1'(\phi) = j_2'(\phi)$

(D) $j_2\left(\phi-(1-w)(1-p)\right) = j_3\left(\phi-(1-w)(1-p)\right)$

(E) $j_2'\left(\phi-(1-w)(1-p)\right) = j_3'\left(\phi-(1-w)(1-p)\right)$

(F) $j_3\left(\phi-(1-w)\right) = j_4\left(\phi-(1-w)\right)$

(G) $j_3'\left(\phi-(1-w)\right) = j_4'\left(\phi-(1-w)\right)$

(H) $j_4\left((1-w)p\right) = j_5\left((1-w)p\right)$

(I) $j_4'\left((1-w)p\right) = j_5'\left((1-w)p\right)$

(J) $j_5(0) = -\left[\lambda_j(1-w)(1-p) + \lambda_g(1-w)\right]/(\lambda_j + \lambda_g)$

The solution of (5a) - (5e) subject to the boundary conditions (A) to (J) for $x \geqslant \phi$, i.e., for the case where the bank is currently solvent, is given by;

$$(6) \quad j_1(x)=c_j x^{r_{22}} F\left(-r_{22}, 1+u_2, -2\left(wh_g+(1-w)k-n\right)/Qx\right)+\left((1-w)k-\lambda_g a_j\right)/(m+\lambda_g-n)$$

where $u_2 = \left[\left(1 - (2/Q)(m-n-\delta)\right)^2 + 8(m+\lambda_g-n)/Q\right]^{1/2}$

$$r_{22} = (1/2)\left[1 - (2/Q)(m-n-\delta) - u_2\right]$$

and F(a, b, z) is the confluent hypergeometric function of "Kummer's function." The expression for c_j is given in Appendix B of the paper.

Note that in (6) the last term is equal to the present value of the expected premium received until the next audit minus the present value of the expected monitoring cost by uninsured depositors at the next audit. Clearly if there was no probability of bank default, in which case the first term would be zero, the fair value of k would just be the uninsured depositors' average total monitoring costs per dollar insured deposits. Thus the first term of (6) reflects the option component of the fair risk premium valuation, attributable to the possibility of the bank's default.

C. Derivations of the Equilibrium Value of Bank Equity and the Deposit Insuring Agency's Liability for a Variable Rate Insurance Scheme

Deriving the equilibrium value of bank equity is somewhat easier then for uninsured depositors' claims or the insuring agency's liability as the complexity of equity holders' payoff structure is less. While the uninsured depositors' claims or the insuring agency's liability as the complexity of equity holders' payoff structure is less. While the bank is in operation, equity holders receive dividends, while if the bank has negative net worth, either a bank run or an audit by the insuring agency will cause any claims on the bank's assets by shareholders to be terminated. Thus the stochastic process for bank equity will be;

(7) $$dB(V,\ D) = \left[B_1(u_v V - C) + B_2(u_d + n)D + (1/2)B_{11}s_v^2V^2 + B_{12}\rho s_v s_d VD + (1/2)B_{22}s_d^2D^2\right]dt + B_1 s_v Vdz + B_2 s_d Ddq + I_{A\cap I}(V - \phi D) - I_{AUR}B$$

Note that at each audit where the government re-adjusts the deposit insurance premium it charges the bank to a new fair value rate, and discloses the bank's financial condition to uninsured depositors who are assumed to adjust the risk premium charged the bank on their deposits, both the government's liability, -G, and the value of uninsured depositors' claims over that value in which a fair premium is charged, J, both return to zero. Since our formulation assumes the deposit insuring agency arranges mergers following bankruptcy, there are no bankruptcy costs. Therefore, we have the Modigliani-Miller theorem applying and $V = B + J + G + \phi D$. Since at an audit when the bank is solvent $J = G = 0$, it then follows that $B = V - \phi D$. The last two terms in (7) reflect this.

In a similar manner to the derivation in part B, one can show that the equilibrium value of equity per dollar deposit, $b = B/D$, must satisfy the set of differential equations;

(8a) $$(Q/2)x^2b_1'' + \left[(m-n-\delta)x+n-k(1-w)-h_g w\right]b_1' + (n-m-\lambda_g)b_1 + (\delta + \lambda_g)x - \lambda_g = 0 \quad , \quad x \geq \phi$$

(8b) $(Q/2)x^2 b_2'' + \left[(m - n - \delta)x + n - k(1 - w) - h_g w\right]b_2' + \left(n-m-(\lambda_g+\lambda_j)\right)b_2$

$$+ \delta x = 0, \quad x < \phi$$

subject to the boundary conditions;

(A) $\lim_{x\to\infty}(b_1/x) = 1$

(B) $b_2(0) = 0$

(C) $b_1(\phi) = b_2(\phi)$

(D) $b_1'(\phi) = b_2'(\phi)$

The solution to the above is for $x \geq \phi$;

(9) $$b = x + c_b x^{r_{22}} F\left(-r_{22},\ 1 + u_2, -2\left(k(1-w)+h_g w-n\right)/Qx\right)$$

$$+ \left(h_g w + k(1 - w) + \lambda_g \phi - n\right)/(n - m - \lambda_g)$$

where the expression for c_b is given in Appendix B.

One could derive the formula for the government insuring agency's liability in the same direct manner as was done for j and b. However, for the case $x \geq \phi$, one can simply use (6), (9), and the Modigliani-Miller relation

$$x = b + j + g + \phi\ , \text{ or}$$

$$x = x + (c_b + c_j)x^{r_{22}} F\left(-r_{22},\ 1 + u_2,\ -2\left(wh_g+(1-w)k-n\right)/Qx\right)$$

$$+ (-h_g w - \lambda_g a_j - \lambda_g \phi + n)/(m + \lambda_g - n) + g + \phi$$

which implies;

(10) $$g=-(c_b+c_j)x^{r_{22}} F\left(-r_{22},1+u_2,\ -2\left(wh_g+(1-w)k-n\right)/Qx\right)+(h_g w-\lambda_g a_g)/(m+\lambda_g-n)$$

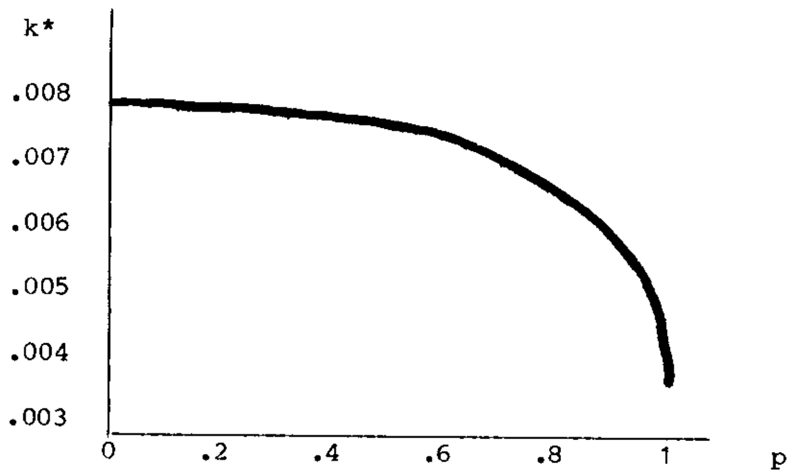
In order to gain some feel for the size of the risk premium uninsured depositors would require and the corresponding equilibrium values of bank equity and the government insuring agency's liability, k* is calculated below for some typical bank risk parameter values. This will allow us to investigate how increasing the amount of deposits that can be withdrawn if a run occurs, p, and a greater tendency of bank runs to start, λ_j, will affect uninsured depositors' risk premium and the value of bank equity if the government insuring agency maintains a fixed premium. Equation (6) is equated to zero and numerical methods are used to find that k which satisfies the relation. The equilibrium values of bank equity and the insuring agency's claim are calculated from (9) and (10) respectively.

Most bank parameter values were selected to be typical of those estimated from the sample of 23 commercial banks studied in Pennacchi (1984). These are;

$Q = .0002$	$\delta = .002$	$\lambda_g = 1$
$x = 1.03$	$n = 0$	$a_g = w(.00013)$
$h_g = .0005$		$a_j = (1 - w)(.00013)$

Thus this "typical" bank has three percent capital and is audited by the government on average once per year. It is assumed that the insuring agency and uninsured depositors incur auditing (or monitoring) costs in proportion to their share of total deposits.

The effect of changes in the proportion of uninsured deposits that are withdrawn from the bank given a bank run occurs is first analyzed. $\lambda_j dt$, the probability of a run occuring given the bank is insolvent, is assumed to be once per year, so that there is an equal probability of the bank being closed by a run or by a normal audit. Free entry into banking is assumed so that



$m = \lambda_g(a_g + a_j)$. The proportion of government to total deposits is assumed to be $w = .95$.

Because a fixed government deposit premium of $h_g = .0005$ on insured deposits was assumed,[12] one can see from Table 1 that since $b < x - 1$ and $g > 0$ in this example, it follows that this premium is greater than the insuring agency's fair value rate, though it would have been less than the government's fair value rate of $h_g = .0006$ if there was de-facto insurance of uninsured depositors, as will be shown subsequently in section III. Note, however, as p increases, i.e., the proportion of deposits that uninsured depositors can withdraw during a run increases, that the value of equity, b, and the insuring agency's liability, -g, increase. That the value of bank equity should increase might be counter-intuitive, but as p increases the insuring agency will be "insuring" a greater proportion of uninsured deposits during a run, i.e., those deposits that escape, at the same cost to the bank as before, so that the bank receives, in a sense, more "insurance" for its money. Thus we could expect that the bank and uninsured depositors would prefer a larger value of p and might institute behavioral changes to bring that about. One change that might unable uninsured depositors to withdraw a greater percentage of deposits after a bank run has started but before the bank can be closed would be a shortening of the maturity of uninsured deposits, i.e., more "demand deposits", relative to the maturity that would exist if deposits were all de-facto insured. Other agency cost effects might also lead uninsured deposits to have short maturities, as described in Myers (1977) and Pennacchi (1983).

Observe that uninsured depositors' risk premium is, at least for this range of parameter values, a concave function of p. The intuition behind this result is that small values of p imply a larger Bank Condition Region II, that region where if a run occurs, uninsured depositors end up bearing the loss of the bank's total negative net worth. As p increases, this region decreases, thus decreasing the expected loss by uninsured depositors given a run. This expected loss decreases at an increasing rate with p, since most the probability mass of the distribution of the value of a bank's net worth given a run, will be closer to the boundary of Regions I and II than to the boundary of Regions II and III, which is farther out in the tail of the distribution. Thus the expected loss by uninsured depositors is a concave function of p. Since k can be thought of as approximately linearly related to expected loss, it will also be a concave function of p.

Using our model, one can also analyze how changing the probability of a run occuring, given the bank has negative net worth, affects uninsured depositors' risk premium. Larger values of λ_j might be a rough proxy for the amount of private information on the bank's financial condition to which uninsured depositors have access.

From Table 2, we see that in each instance, higher frequencies of bank runs lower uninsured depositors' risk premium. For this range of parameter values, because of the insuring agency's fixed insurance premium, an increased frequency of bank runs implies a greater liability to the insuring agency, and will therefore increase the value of bank equity slightly. A greater possibility of bank runs occuring appears to have one quality similar to that of a higher frequency of bank audits in that there is less chance that capital will become significantly negative before the bank is closed. Thus uninsured depositors' expected loss in the event of capital becoming negative is less. This implies that uninsured depositors would require a smaller risk premium. It also would have the effect of tending to lower the insuring agency's liability (raising g). A second effect from a greater chance of bank runs would tend to increase the government insuring agency's liability, and increase shareholders' equity, b. While for Bank Region II, an increase in λ_j will lessen the government's liability in that the bank will be closed sooner, for Regions III, IV, and V, an increased probability of bank runs will produce the second effect of increasing the government's expected losses, since its payouts will be greater if a run occurs rather than an audit. In the above example, it appears that this second effect outweighs the first. This result implies that one should be cautious about advocating more liberal private access to bank financial information (which might tend to increase λ_j) as a method of minimizing the liability of the government insuring agency.

If the deposit insuring agency charged a fair risk premium, according to equation (10), the bank would be indifferent as to whether it issued insured versus uninsured deposits, assuming as we have been that there are no liquidity costs of bank runs, auditing information costs are shared proportionately, and the bank has equivalent interest rate margins (i.e., degrees of monopoly power) in both insured and uninsured markets. However, if the insuring agency charges the same insurance premium per dollar insured deposits, without regard to the bank's proportion of insured to total deposits, w, the bank's selection of insured versus uninsured deposits will affect the market value of equity. This is illustrated in Table 3, below. Again, the same parameter values are used as before, including $p = .5$, $\lambda_j = 1$, $h_g = .0005$, and deposit markets are assumed competitive.

In Table 3, it is apparent that as the bank starts from a situation of being financed issuing only insured deposits and then begins to issue uninsured deposits, there is a fall in equity value until approximately $w = .95$ after which equity begins to rise again. Because uninsured depositors are junior claimants after a bank failure, a fairly high risk premium need initially be paid for the first few uninsured depositors. Observe that the insuring agency's fair value insurance premium quickly falls, from, in this example, being $h_g = .0006$ at $w = 1$, to under .0005 at $w = .99$. This follows from the fact that while the actual premium charged, $h_g = .0005$, remains constant, at $w = 1$, $g < 0$ $(b > x - 1)$, while at $w = .99$, $g > 0$ $(b < x - 1)$. As w decreases, b continues to decrease over some range, as the fixed insurance rate it pays on insured deposits becomes more and more "over-priced." However, after w falls far enough, the marginal benefit of replacing over-priced insured deposits with fair priced uninsured deposits will exceed the marginal cost of increasingly over-priced insured deposits, such that the value of bank equity will rise again. However, in this example, we know that the bank's global optimum with respect to its choice of w will be at $w = 1$, all insured deposit finance, since while equity monotonically increases after approximately $w = .95$, at $w = 0$, $b = .03$ if uninsured depositors continue to require a fair risk premium.

Conversely, if at $w = 0$, $b < x - 1$, i.e., insured deposits are over-priced even when all liabilities are insured deposits, examples can be constructed in which b will still be a "U"-Shaped" function of w (this will always be the case unless insured deposits are exceedingly over-priced in which case b will be a monotonically decreasing function of w). However, now the bank's global optimum with respect to w will be at w = 0, i.e., all liabilities will be uninsured liabilities. Therefore for competitive deposit markets and a fixed rate government insurance scheme, equilibrium would imply that higher risk banks will choose to issue all government insured deposits while lower risk banks will choose to issue mainly uninsured deposits.

III. A Contingent Claims Model of a Joint Public-Private Insurance System

This section studies the fair pricing of fair value deposit insurance for a joint public-private co-insurance system, where private insurance companies insure those bank liabilities not covered by government deposit insurance. As in the previous section with uninsured depositors, private insurance companies are assumed to be legally a junior claimant with regard to the government insuring agency. There is a very close relationship between pricing uninsured depositors' risk premium and the pricing of a private insurance firm's insurance premium, as will be indicated below.

It is assumed that the government insuring agency acts identically to the behavior assumed in section II, auditing with frequency λ_g, collecting premium h_g, incurring auditing costs a_g. In addition, the government insuring agency shares (or sells) the information from its audits to the private insurance firm, which incurs costs a_j per dollar total deposits in monitoring costs at this time.[13] Because of the existence of private insurance, there is no incentive for non-government insured depositors to commence a run on the bank, so that in the analysis of section II, one now sets $\lambda_j = 0$, and $p = 0$.[14] Since non-government insured depositors will now not require banks to pay a risk premium on their deposits, k will be equal to zero. However, banks must now instead pay an insurance premium of h_j per dollar of non-government insured deposits. A variable rate insurance scheme is assumed to be administered by the private insurance firm so that their premium, h_j, is assumed to be re-adjusted at each bank audit. Letting $J(V, D, h_j)$ now denote the value of the private insuring company's claim on bank assets, i.e., minus the private insurance firm's liability, one can show that the contingent payments to the private insuring firm is quite similar to the payments to uninsured depositors, in section II. By replacing k with h_j, setting $\lambda_j = p = 0$, and noting that Bank Condition Region III is redundant and Region V is irrelevant, it is straightforward to see that the private firm's claim must follow the stochastic process;

(11)
$$dJ(V, D) = \left[j_1(u_v V - C) + J_2(u_d + n)D + (1/2)J_{11}s_v^2 V + J_{12}\rho s_v s_d VD + (1/2)J_{22}s_d^2 D^2\right]dt + J_1 s_v Vdz + J_2 s_d Ddq - I_A(a_j D + J) + I_{A\cap II}(V - D) - I_{IV\cap A}(1 - w)D$$

which will lead to the system of differential equations;

(12a)
$$(Q/2)x^2 j_1'' + \left[(m - n - \delta)x + n - wh_g - (1 - w)h_j\right]j_1' + (n - m - \lambda_g)j_1 + h_j(1 - w) - \lambda_g a_j = 0, \quad x \geqslant \phi$$

(12b)
$$(Q/2)x^2 j_2'' + \left[(m - n - \delta)x + n - wh_g - (1 - w)h_j\right]j_2' + (n - m - \lambda_g)j_2 + h_j(1 - w) - \lambda_g a_j + \lambda_g(x - \phi) = 0, \quad \phi > x \geqslant \phi - (1 - w)$$

(12c)
$$(Q/2)x^2 j_3'' + \left[(m - n - \delta)x + n - wh_g - (1 - w)h_j\right]j_3' + (n - m - \lambda_g)j_3 + h_j(1 - w) - \lambda_g a_j - \lambda_g(1 - w)D = 0 \quad , \phi - (1 - w) > x$$

In deriving (12) from (11), one can make use of a variation of the classical hedge arguement to derive equilibrium conditions between the means and variances of the different securities in this economy. By forming a portfolio of three principle securities, the types of assets held by the bank, V, the private insuring agency's claim, J, and default free bonds of maturity T, P(t, T), that requires zero net investment and produces a return with zero systematic (non-diversifiable) risk, it must be true that, in the absence of arbitrage opportunities, this return has an expected value of zero. The quantities of the bank's assets and default free bonds in this portfolio can be likened to the securities that the private insurance company must hold in order to reduce its liability from a given bank to purely non-systematic risk. By insuring many different banks, i.e., holding a number of these different hedge portfolios, the total non-systematic risk of the insurance company can be made arbitrarily small. If the insurance company does hedge its risk from insuring deposits correctly, it is theoretically able to reduce its own probability of default to any arbitrarily small amount for any degree of its own capitalization, provided it is able to insure as many banks as its wishes. Note that this result does not depend on the timing of different bank failures being uncorrelated. Therefore the feasibility of private deposit insurance relative to government deposit insurance does not depend per se on the intertemporal correlation of the risk being insured, as might first have been thought. However, as a practical matter, the feasibility of a private insuror of a large bank might depend on its ability to insure enough other large banks in order to adequately reduce the non-systematic risk to which it is exposed.

Solving (12) subject to the boundary conditions;

(A) $\lim_{x\to\infty} |j_1(x)| < \infty$

(B) $j_1(\phi) = j_2(\phi)$

(C) $j_1'(\phi) = j_2'(\phi)$

(D) $$j_2\big(\phi - (1 - w)\big) = j_3\big(\phi - (1 - w)\big)$$

(E) $$j_2'\big(\phi - (1 - w)\big) = j_3'\big(\phi - (1 - w)\big)$$

(F) $$j_3(0) = -(1 - w)$$

gives the solution for $x \geqslant \phi$ as;

(13) $$j_1(x) = c_p x^{r_{22}} F\big(-r_{22}, 1 + u_2, -2\big(wh_g + (1 - w)h_j - n\big)/Qx\big) + \big((1 - w)h_j - \lambda_g a_j\big)/(m + \lambda_g - n)$$

and the value of c_p is given at the end of Appendix B.

In the context of this joint private-public insurance system, expressions for the government insuring agency's liability per dollar total deposits, $-g(x)$, and the equilibrium value of bank equity per dollar deposit, $b(x)$, are nearly identical to (10) and (9), respectively, but with k replaced by h_j and $\lambda_j = p = 0$.

In a similar manner to the preceding section, the fair value private and government deposit insurance premia can be calculated. This is done below, for the most part using the same parameter values as before, essentially a bank with three percent capital with competitive markets that is audited on average once per year. The private and government insurors are assumed to share total auditing and monitoring costs of .00013 per dollar deposit according to the relative proportions of deposits they insure. Since both insurers charge fair value rates, $b = x - 1 = .03$.

In Table 4, note the very high private insurance premium that would need be charged the bank for the first few non-government insured deposits it issues, if it has previously issued all government insured deposits. This reflects the junior claimant status of the private insurance firm, the fact that if a bank failure does occur, the private insurance company will likely have a liability of most if not all of the deposits it insures. Observe that the fair value government insurance premium is approximately equal to average auditing costs, .00013, for $w < .95$. Thus for banks with any substantial level of privately insured deposits, the government's pricing problem is extremely simple, just charge their average auditing costs. The government insuring agency's role becomes that of a catastrophic insurer. It will be very rare that they would incur any losses from a bank failure.

If the government insuring agency maintained a fixed insurance rate, this might have a profound effect on the proportion of privately insured versus government insured deposits banks would select. This is illustrated below. The government insuring agency is assumed to charge the premium $h_g = .0005$, which remains constant with changes in w. Two different cases are shown in Table 5, one in which deposit markets are assumed competitive, and the other in which deposit market monopoly rents (for both government and privately insured markets) are equal to one percent of deposits.

Similar to Table 3 of section II, it is apparent that the market value of equity is a "U-shaped" function of the proportion of government insured to total deposits, w. Thus under our assumptions, the bank's global optimum with respect to its choice of w will be at either $w = 1$ or $w = 0$. If the government's fixed rate premium, h_g, is less than the fair premium, h_g^*, at $w = 1$, the bank will choose all government insured deposits, otherwise the bank will choose all privately insured deposits.

If the assumption that banks have equal interest rate margins in government insured and non-government insured markets is relaxed, the bank's average total interest margin, $m = wm_i(w) + (1 - w)m_u(w)$, (See Appendix A) will then be a function of w. In this more general case, it will be possible for bank's optimal choice of w to have an "interior" solution, i.e., $0 < w <$

1. It would be straightforward to use the above framework to numerically solve for the optimal w given selected parameter values and functional forms for $m_i(w)$ and $m_u(w)$.

The question of what incentive would exist for banks and uninsured depositors to prefer a joint public-private insurance system can be analyzed using the preceeding analysis. It will be seen that the preference for an uninsured versus a privately insured system will depend on a number of factors. Comparing equilibrium values of equity in Table 3 and Table 5 for the competitive deposit market case and for given values of w, one notes that for very high values of w (w = .99), the market value of equity is less for the uninsured depositor case than for private insurance, while for lower values of w (w = .97), the opposite result obtains. The existence of bank runs by uninsured depositors appears to be the critical factor in explaining the differences. Ex ante, the possibility of bank runs can be beneficial to the bank's equity holders because for p > 0, bank runs would cause the government insuring agency sharing on losses in situations in which they wouldn't if only normal government audit were permitted to close the bank, (i.e., Bank Condition III, IV, and V), lowering the value of the government's claim for a fixed insurance rate and therefore raising the market value of equity. However, bank runs have also a second detrimental effect on equity in that they will tend to close a bank with negative net worth sooner than if only government audits were permitted to do so. This tends to increase the value of the government's claim, for a fixed insurance rate, thus lowering the value of bank equity. Apparently, in our example, this second detrimental effect outweighs the beneficial effect for very high values of w, but the relative magnitudes are reversed at lower values. This makes sense since for w close to one, i.e., few non-government insured deposits, the additional liability to the government agency (benefits to equity holders) resulting from the small amount of uninsured deposits escaping the bank during a run is limited. For greater levels of non-government insured deposits, the government liability (benefit to equity holders) can be much larger.

The above comparison has assumed equivalent monitoring costs by uninsured depositors and private insurors and also that the occurence of a bank run did not cause any inefficiencies in banks' asset liquidation (i.e., there were no bankruptcy costs). It is clear, however, that "forced liquidation" costs from bank runs and higher monitoring costs by uninsured depositors relative to a private deposit insuring agency could also be significant factors in determining whether banks would favor a private-public insurance system over a system with just partial government insurance. It is straightforward to study the effect of these additional factors using the above framework of analysis.

IV. Conclusion

This paper presented a model of the banking firm in which the possibility of uninsured depositors initiating a run on the bank was considered. Formulas for the risk premium that banks would need to pay uninsured depositors, as well as the fair government deposit insurance rate and the equilibrium value of bank equity were derived. For plausible parameter values, the formulas gave evidence that if government insuring agencies ended de-facto insurance of legally uninsured bank liabilities, banks with small proportions of uninsured deposits would have to pay sizable risk premia on these deposits. Also, uninsured depositors' risk premium tended to decline with greater frequencies of bank runs, and this was shown to possibly be ex-ante beneficial to bank equity holders at the expense of the government insuring agency. Therefore, a policy of increased informtion disclosure by banks to the public may have some negative effects to the government insurance system if information disclosure makes bank runs more likely.

If government insuring agencies maintained fixed insurance premia that did not discriminate between differences in banks' proportion of insured to total deposits, as is currently the practice in the United States, higher risk banks would have an incentive to choose to issue only government insured deposits while lower risk banks would choose to issue primarily uninsured deposits. It may well be that the purported benefits of inducing increased "market discipline," i.e., the monotoring of banks by uninsured depositors, by removing the de-facto insurance of non-officially insured liabilities, are exaggerated. Higher risk banks might discontinue issuing non-government insured liabilities, thus escaping the control by private market forces. However, if substantial monopoly rents could be attained in non-government insured deposit markets, removing de-facto insurance may induce banks to pursue a safer capital structure and finance themselves mainly by uninsured liabilities. But, as shown in Pennacchi (1984), banks with substantial degrees of market power in their deposit markets would probably choose to pursue safe investment strategies in any case, even under government fixed rate de-facto insurance of all liabilities.

A private-public deposit insurance system was also considered in this analysis. It was shown how the derivation of formulas for the private insurance firm's fair deposit insurance premium was similar to that of the uninsured depositors' risk premium. As in the partial government deposit insurance model, if the government insuring agency charged a fixed premium, irrespective of individual bank risk, there would exist distorting incentives in which risky banks might choose to issue only government insured deposits. This distortion could be eliminated by the government insuring agency instituting a variable rate insurance scheme. Fortunately, for the great proportion of large banks which have significant non-government insured liabilities, the government insuring agency's fair pricing scheme is relatively simple. Evidence from the derived formulas indicates that if uninsured liabilities are not de-facto insured, the fair government deposit insurance premium for these banks will be only slightly greater than the premium required to cover the insuring agency's average auditing costs.

The question of whether a system with partial government insurance would have the incentive to evolve to a system of joint private-public insurance was posed. It was shown that the existence of bank runs in a partial fixed rate government insurance system might actually be ex ante beneficial to bank equity holders. Preferences for private insurance would have to depend on incentives to eliminate any "forced liquidation" costs stemming from bank runs, or private insurers having lower monitoring costs than uninsured depositors.

Footnotes

[1]Federal Deposit Insurance Corporation (1983) page F-4.

[2]See "Deposit Insurance in a Changing Environment," FDIC (1983) and "Agenda for Reform," FHLBB (1983).

[3]"Big Depositors at 2 Failed Banks May Lose Some Funds Due to New FDIC Approach," The Wall Street Journal, March 20, 1984.

[4]"Curb is Put on Broker Deposits: Bank Agencies Clear Modified Insurance Plan," The New York Times, March 27, 1984.

[5]There seems to be an apparent contradiction in the FDIC's reasoning concerning the feasibility of pricing deposit insurance premia according to individual bank risk. The FDIC (1983) advocates more information disclosure to uninsured depositors to aid these investors' assessment of the fair risk premium banks would have to pay on uninsured liabilities. However, the FDIC states that it is currently infeasible to estimate accurately enough a fair deposit insurance premium, while this is identical to what they expect uninsured depositors to do (see section III).

[6]Some might argue that the primary reason for deposit insurance is to protect the small unsophisticated depositor. See Karken (1983) for discussion.

[7]In recent years, more secondary markets have arisen that have undoubtably lowered the transactions cost of selling many types of bank assets, such as mortgages.

[8]Chapter VII in FDIC (1983) discusses the feasibility of private excess deposit insurance.

[9]One might think that duration differences between insured and uninsured deposits would be substantial. While the duration of insured deposits might well be arbitrary for a given risk premium, it is reasonable to expect

uninsured depositors to prefer very short maturity deposits as these may increase their ability to withdraw a greater proportion of deposits, p, after a bank run begins, and before the bank is closed. It is also reasonable to suppose that the bank would possess different degrees of market power in insured and uninsured deposit markets, as uninsured depositors tend to be large depositors who have access to competing borrowers, while government insured depositors might tend to face higher transactions costs in lending funds to other borrowers, thus giving banks greater monopoly power over this group.

[10]The modelling of liquidation costs could be done similarly to the bankruptcy cost model of Turnbull (1979).

[11]Compare with Diamond and Dybvig (1983).

[12]For computational convenience, it is assumed that while the government insuring agency currently charges $h_g = .0005$, this rate is adjusted to the bank's fair value rate at the next audit, i.e., a variable rate scheme will then commence, instead of a fixed rate being maintained forever. For our example where $h_g > h_g^*$, the same qualitative results as those for a maintained fixed rate scheme will be had, but our value for the equilibrium value of equity will be somewhat overstated while our value for the insuring agency's claim will be slightly understated, since the insuring agency would expect to be "overcharging" for more than one auditing period if a fixed rate scheme was maintained. However, our value for the risk premium, k*, as computed in the text, is completely correct, being the same in either case. The problem with assuming a fixed rate scheme being maintained forever is the difficulty in deriving the insuring agency's claim and the equilibrium value of equity in closed form when the uninsured depositors' risk premium is varying at each auditing period.

[13]Alternatively, one could have modelled the insuring agency and private insuring firm auditing at separate times with perhaps different frequencies, λ_g and λ_j, respectively. As long as information was shared or sold (i.e. sharing auditing costs), which seems reasonable, the derivation will be similar to the derivation in the text where now λ_g is replaced by $\lambda_g + \lambda_j$.

[14]It is implicitly assumed that non-government insured depositors place complete faith in the private insurance firm's ability to make payment in the event of a bank closing.

Appendix A

This appendix derives the stochastic processes that the growth rates of insured and uninsured deposits must follow given that these two classes of deposits have different durations and/or the bank possesses different degrees of monopoly power in these two markets.

Let the return on insured deposits be given by;

$$dD_i/D_i = u_{d_i} dt + s_d(T_i)dq$$

where = $u_{d_i}(T_i) = u_p(T_i) - m_i$ and T_i is the duration of insured deposits.

Let the return on uninsured deposits, inbetween audits, be given by;

$$dD_u/D_u = u_{d_u} dt + s_d(T_u)dq$$

where $u_{d_u}(T_u) = u_p(T_u) - m_u + k$ and T_u is the duration of uninsured deposits. Then the stochastic processes for the change in insured and uninsured deposits are given by;

$$dD_i = u_{d_i} D_i dt + D_i s_d(T_i)dq + D_i dn_i$$

$$dD_u = u_{d_u} D_u dt + D_u s_d(T_u)dq + D_u dn_u$$

where $$dn_i = a_i dt + b_i dq \quad \text{and}$$

$$dn_u = a_u dt + b_u dq$$

are the (stochastic) growth rates for new insured and uninsured deposits, respectively.

The change in total deposits given in equation (3) will be the sum of the changes of insured and uninsured deposits. In order to maintain the proportions of insured to uninsured deposits constant, equal to $w/(1 - w)$, and have;

$$s_d(T) = ws_d(T_i) + (1 - w)s_d(T_u) \quad \text{and}$$

$$m = wm_i + (1 - w)m_u$$

and have the growth rate of total new deposits equal n, it can be verified that the parameters of the growth rate processes for insured and uninsured deposits will equal;

$$a_i = (1 - w)(u_{d_u} - u_{d_i}) + n$$

$$a_u = w(u_{d_i} - u_{d_u}) - k + n$$

$$b_i = \left(1 - w)(s_d(T_u) - s_d(T_i)\right)$$

$$b_u = w\left(s_d(T_i) - s_d(T_u)\right)$$

Appendix B

This appendix gives the values of c_j, c_b and c_p which are used in the solutions for uninsured depositors' contingent claim, the equilibrium value of bank equity, and the private insurance company's claim which are equations (6), (9), and (13), respectively.

Maintaining the notation used in the text, define the following symbols;

$$\phi_1 = \phi - (1 - w)(1 - p)$$

$$\phi_2 = \phi - (1 - w)$$

$$\phi_3 = (1 - w)p$$

$$g = h_g w + k(1 - w)$$

$$M = 2(h - n)/Q$$

$$u_2' = \left[\left(1 - (2/Q)(m - n - \delta)\right)^2 + 8(m + \lambda_g + \lambda_j - n)/Q\right]^{1/2}$$

$$r_{21}' = (1/2)\left[1 - (2/Q)(m - n - \delta) + u_2'\right]$$

$$r_{22}' = r_{21}' - u_2'$$

$$\lambda' = \lambda_g + \lambda_j$$

$$y_1(x) = x^{r_{21}'} F(-r_{21}', 1 - u_2', - M/x)$$

$$y_2(x) = x^{r_{22}'} F(-r_{22}', 1 + u_2', - M/x)$$

$$y_3(x) = r_{21}' x^{r_{21}'-1} F(1 - r_{21}', 1 - u_2', M/x)$$

$$y_4(x) = r_{22}' x^{r_{22}'-1} F(1 - r_{22}', 1 + u_2', M/x)$$

$c_j = |A| / |B|$, where A and B are each 9 × 9 matrices. Let the i, j^{th} element of A be given by a_{ij}. Then

$$a_{11} = \lambda'(\phi-(h-n)/(m + \lambda' - n))/(\lambda' + \phi)+[k(1-w)-\lambda'(a_j + \phi)]/(m+\lambda'-n)$$
$$- [k(1 - w) - \lambda_g a_j]/(m + \lambda_g - n)$$

$$a_{21} = \lambda'(\lambda' + \delta)$$

$$a_{31} = -\lambda_j[\phi_1-(h-n)/(m+\lambda'-n)]/(\lambda'+\delta)+\lambda_j(\phi-(1-w)(1-p))/(m+\lambda'-n)$$

$$a_{41} = - \lambda_g/(\lambda' + \delta)$$

$$a_{51} = -\lambda_g[\phi_2 - (h-n)/(m+\lambda'-n)]/(\lambda' + \delta) + \lambda_g(\phi - (1 - w))/(m+\lambda'-n)$$

$$a_{61} = - \lambda_g(\lambda' + \delta)$$

$$a_{71} = \lambda_j[\phi_3 - (h - n)/(m+\lambda'-n)]/(\lambda' + \delta) - \lambda_j p/(m + \lambda - n)$$

$$a_{81} = \lambda_j/(\lambda' + \delta)$$

$$a_{91} = -(1 - w) + \lambda_j(p(1 - w)/\lambda' + (h - n)/[(\lambda' + \delta)(m+\lambda'-n)])$$
$$-[k(1 - w) + \lambda'(a_j + (1 - w)]/(m + \lambda' - n))$$

$$a_{98} = M^{-r_{21}}F(1 - u_2')/\Gamma(1 + r_{22}')$$

$$a_{99} = M^{-r_{22}}\Gamma(1 + u_2')/\Gamma(1 + r_{21}')$$

$a_{12} = -y_1(\phi)$ $\qquad$ $a_{13} = - y_2(\phi)$

$a_{22} = -y_3(\phi)$ $\qquad$ $a_{23} = - y_4(\phi)$

$a_{32} = y_1(\phi_1)$ $\qquad$ $a_{33} = y_2(\phi_1)$

$a_{42} = y_3(\phi_1)$ $\qquad$ $a_{43} = y_4(\phi_1)$

$a_{34} = -y_1(\phi_1)$ $\qquad$ $a_{35} = y_2(\phi_1)$

$a_{44} = -y_3(\phi_1)$ $\qquad$ $a_{45} = - y_4(\phi_1)$

$a_{54} = y_1(\phi_2)$ $\qquad$ $a_{55} = y_2(\phi_2)$

$a_{64} = y_3(\phi_2)$ $\qquad$ $a_{65} = y_4(\phi_2)$

$a_{56} = -y_1(\phi_2)$ $\qquad$ $a_{57} = - y_2(\phi_2)$

$a_{66} = -y_3(\phi_2)$ $\qquad$ $a_{67} = - y_4(\phi_2)$

$a_{76} = y_1(\phi_3)$ $\qquad$ $a_{77} = y_2(\phi_3)$

$a_{86} = y_3(\phi_3)$ $\quad a_{87} = y_4(\phi_3)$

$a_{78} = -y_1(\phi_3)$ $\quad a_{79} = -y_2(\phi_3)$

$a_{88} = -y_3(\phi_3)$ $\quad a_{89} = -y_4(\phi_3)$

The rest of the elements, a_{ij}, are equal to zero.

The elements of matrix B, b_{ij}, are equivalent to the corresponding elements of matrix A except for the following;

$$b_{11} = \phi^{r_{22}} F(-r_{22}, 1 + u_2, -M/\phi)$$

$$b_{21} = r_{22}\phi^{r_{22}-1} F(1 - r_{22}, 1 + u_2, -M/\phi)$$

$$b_{31} = b_{41} = b_{51} = b_{61} = b_{71} = b_{81} = b_{91} = 0 .$$

The value for c_b can also be written as a ratio of two determinants;

$c_b = |A| \,/\, |B|$ where A and B are now 3 × 3 matrices with the following elements;

$$a_{11} = -\delta(n-h)/[(\delta+\lambda')(n-m-\lambda')] - (h-n+\phi\lambda_g) - \lambda'\phi/(\delta + \lambda')$$

$$a_{21} = -\lambda'/(\delta + \lambda')$$

$$a_{31} = \delta(n - h)/[(\delta + \lambda')(n - m - \lambda')$$

$a_{12} = -y_1(\phi)$ $\quad a_{31} = -y_2(\phi)$

$a_{22} = -y_3(\phi)$ $\quad a_{32} = -y_4(\phi)$

$$a_{31} = M^{-r'_{21}}\Gamma(1 - u'_2)/\Gamma(1 + r'_{22})$$

$$a_{33} = M^{-r'_{22}}\Gamma(1 + u'_2)/\Gamma(1 + r'_{21})$$

The elements of matrix B, b_{ij}, are equivalent to the elements of matrix A except for;

$$b_{11} = \phi^{r_{22}} F(-r_{22},\ 1 + u_2,\ -M/\phi)$$

$$b_{21} = r_{22}\phi^{r_{22}-1} F(1 - r_{22},\ 1 + u_2,\ -M/\phi)$$

and $b_{31} = 0$.

The value for c_p is the ratio of two 5 × 5 determinants. Denoting c_p by; $c_p = |A|/|B|$. Redefine the following symbols;

$$h = wh_g + (1 - w)h_j$$

$$y_1(x) = x^{r_{21}} F(-r_{21},\ 1 - u_2,\ M/x)$$

$$y_2(x) = x^{r_{22}} F(-r_{22},\ 1 + u_2,\ -M/x)$$

$$y_3(x) = r_{21}x^{r_{21}-1} F(1 - r_{21},\ 1 - u_2,\ -M/x)$$

$$y_4(x) = r_{22}x^{r_{22}-1} F(1 - r_{22},\ 1 + u_2,\ -M/x)$$

Then we have for the elements of matrix A;

$$a_{11} = \lambda_g\left[\phi - (h-n)/(m+\lambda_g-n)\right]/(\lambda_g+\delta) - \lambda_g\phi/(m+\lambda_g-n)$$

$$a_{21} = \lambda_g/(\lambda_g + \delta)$$

$$a_{31} = -\lambda_g\left[\phi_2 - (h - n)/(m+\lambda_g-n)\right]/(\lambda_g+\delta) + \lambda_g\left(\phi - (1 - w)\right)/(m+\lambda_g - n)$$

$$a_{41} = \lambda_g/(\lambda_g + \delta)$$

$$a_{51} = -(1 - w) - \left(h_j - \lambda_g\left(a_j + (1 - w)\right)/(m+\lambda_g-n)\right.$$

$a_{12} = -\ y_1(\phi)$ $\quad$ $a_{13} = -\ y_2(\phi)$

$a_{22} = -\ y_3(\phi)$ $\quad$ $a_{23} = -\ y_4(\phi)$

$a_{32} = \ y_1(\phi_2)$ $\quad$ $a_{33} = \ y_2(\phi_2)$

$a_{42} = \ y_3(\phi_2)$ $\quad$ $a_{43} = \ y_4(\phi_2)$

$a_{34} = -\ y_1(\phi_2)$ $\quad$ $a_{35} = -\ y_2(\phi_2)$

$a_{44} = -\ y_3(\phi_2)$ $\quad$ $a_{45} = -\ y_4(\phi_2)$

$$a_{54} = M^{-r_{21}}\Gamma(1 - u_2)/\Gamma(1 + r_{22})$$

$$a_{55} = M^{-r_{22}}\Gamma(1 + u_2)/\Gamma(1 + r_{21})$$

and the remaining elements of matrix A are equal to zero. The elements of the matrix B, b_{ij}, are equal to the corresponding elements of matrix A except for;

$$b_{11} = \phi^{r_{22}}F(-r_{22},\ 1 + u_2,\ -M/\phi)$$

$$b_{21} = r_{22}\phi^{r_{22}-1}F(1 - r_{22},\ 1 + u_2,\ -M/\phi)$$

and $b_{31} = b_{41} = b_{51} = 0$.

References

Cone, K., 1981, "Instability in Financial Intermediation: Do Money Market Funds Need Deposit Insurance?" mimeo, Stanford University, September.

Diamond, D. and P. Dybvig, 1983, "Bank Runs, Deposit Insurance, and Liquidity," Journal of Political Economy.

Federal Deposit Insurance Corporation, 1983, "Deposit Insurance in a Changing Environment," Washington, D. C.

Federal Home Loan Bank Board, 1983, "Agenda for Reform," Washington, D. C.

Kareken, J., 1983, "The First Step in Bank Deregulation: What About the FDIC?" American Economic Review 73, (May) pp. 198-203.

Kareken, J. and N. Wallace, 1978, "Deposit Insurance and Bank Regulation: A Partial-Equilibrium Exposition," Journal of Business 51, pp. 413-438.

Merton, R., 1978, "On the Cost of Deposit Insurance When There Are Surveillance Costs," Journal of Business 51, pp. 439-452.

Myers, S., 1977, "Determinants of Corporate Borrowing," Journal of Financial Economics (May).

Pennacchi, G., 1984, "Alternative Systems of Deposit Insurance," unpublished Ph.D. dissertation, Massachusetts Institute of Technology.

__________, 1983, "Maturity Structure in a Model of Unregulated Banking," unpublished mimeo.

Turnbull, S., 1979, "Debt Capacity," Journal of Finance 34, pp. 931-940.

Table 1

	Fair Value Risk Premium	Equity	Agency Claim
	k*	b	g
p = 0	.007901	.029661	.000339
p = .1	.007895	.029661	.000339
p = .2	.007884	.029661	.000338
p = .3	.007863	.029662	.000338
p = .4	.007823	.029664	.000336
p = .5	.007744	.029668	.000332
p = .6	.007594	.029675	.000325
p = .7	.007306	.029688	.000313
p = .8	.006761	.029712	.000288
p = .9	.005736	.029758	.000242
p = 1	.003834	.029844	.000156

<u>Table 2</u>

	k^*	b	g
$p = .5$			
$\lambda_j = .5$	.008632	.029666	.000334
$\lambda_j = 1$	.007744	.029668	.000332
$\lambda_j = 2$	.006659	.029668	.000331
$p = .9$			
$\lambda_j = .5$	.007022	.029738	.000262
$\lambda_j = 1$	.005736	.029758	.000242
$\lambda_j = 2$	.004662	.029759	.000241

Table 3

	k*	b	g
w = 1	-	.030089	-.000089
w = .99	.02279	.029793	.000206
w = .97	.01198	.029683	.000316
w = .95	.00774	.029668	.000332
w = .93	.00565	.029669	.000331
w = .91	.00444	.029675	.000325

Table 4

Private and Public Fair Deposit Insurance Rates

	h_j	h_g
w = 1	-	.0006003
w = .9999	.04649	.0005957
w = .999	.04061	.0005603
w = .99	.02794	.0003242
w = .98	.01975	.0002096
w = .97	.01476	.0001623
w = .96	.01158	.0001430
w = .95	.00944	.0001352
w = .9	.00483	.0001301
w = .8	.00248	.0001300
w = .5	.00107	.0001300
w = 0	.00060	-

Table 5

	Competitive Deposit Market			1 Percent Monopoly Rent		
	h_j	b	g	h_j	b	g
w = .99	.02892	.029835	.000165	.011618	.039688	.000288
w = .97	.01589	.029677	.000323	.006247	.039631	.000346
w = .95	.01024	.029655	.000346	.004023	.039627	.000349
w = .93	.00742	.029657	.000343	.002930	.039633	.000344
W = .91	.00580	.029663	.000337	.002305	.039640	.000337
W = .90	.00523	.029666	.000333	.002088	.039644	.000333
W = .80	.00266	.029704	.000296	.001099	.039680	.000296
W = .50	.00111	.029815	.000185	.000506	.039791	.000185
W = .25	.00077	.029907	.000092	.000374	.039884	.000092