THE LAW OF ONE PRICE IN INTERNATIONAL COMMODITY MARKETS: A REFORMULATION AND SOME FORMAL TESTS

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INTRODUCTION

The law of One Price (LOP) is an important ingredient in models of exchange rate determination, and it has been the subject of empirical analyses at least since Cassel (1918). In much of the empirical literature, tests are conducted for a generalized version of the LOP, the Purchasing Power Parity hypothesis (PPP). Tests of PPP, based on price indices, carried out by Gailliot (1970), Dornbusch (1976), Frenkel and Johnson (1976, 1978), Isard (1977), Genberg (1978), Kravis and Lipsey (1978), Richardson (1978), Frenkel (1981), and others, indicate that while PPP tends to hold in the "long-run," short-run deviations from it exist. Furthermore, these deviations tend to be persistent and predictable. However, this result is questioned by Rogalski and Vinso (1977), Roll (1979), and Adler and Lehrman (1983). Since it is likely that some of the deviations from PPP measured from constructed price indices are due to a variety of data and indexing problems, some authors have studied the behavior of individual commodity prices. Tests conducted by these authors are direct tests of the LOP for individual commodities. The results in Jain (1980), Crouhy-Veyrac, Crouhy and Melitz (1982), and Protopapadakis and Stoll (1983) also indicate that the LOP holds in the long-run but that significant short-run deviations are observed. These deviations are ascribed to transport costs and other impediments to commodity arbitrage. General reviews of the PPP and the LOP literature may be found in Officer (1976) and Shapiro (1983).

An important difficulty with the LOP tests for individual commodities is that the theory on which these tests are based explicitly predicts a discontinuity in the behavior of prices. The discontinuity arises because the LOP is based on the assumption that the supply of arbitrage funds is infinitely elastic outside a fixed transactions costs band, and it is zero inside the band. Tests reported to date do not provide an explanation of the price movements within the bands, and therefore they have no way to make a formal distinction between the short-run and the long-run.

In this paper we propose a reformulation of the LOP which is testable, and which distinguishes formally the long-run from the short-run. The proposed reformulation involves specifying both a continuous adjustment and an expectations formation mechanism. We execute formal tests of the long-run LOP and the short-run LOP using weekly data for twelve commodities traded internationally in organized commodities markets. The main conclusion that receives overwhelming support is that the LOP almost never holds in the short-run. The long-run LOP and our reformulation receive strong support when the commodity prices are for futures and forward contracts, but only modest support when the prices are spot.

The plan of this paper is as follows: After describing the standard formulation, we present our reformulation of the LOP. Formal tests of our model are described next. After a brief description of the data used in the study, the results are presented and conclusions drawn.

STANDARD FORMULATION OF THE LOP

The LOP requires that deviations from perfect equality of prices (adjusted for exchange rates) will not exceed transactions costs:

(1) $$-m \leq p - p^* - \pi \leq m,$$

where p, p^* are the logs of the domestic and foreign futures prices of a commodity, respectively, π is the log of the exchange rate for the corresponding maturity, and m is the corresponding proportional transactions cost. The transactions costs band $\pm$ m represents the points outside of which the supply elasticity of arbitrage funds is infinite and inside of which it is zero. Unfortunately, independent information on transactions costs is rarely available, even for financial markets, making impossible a direct test of this formulation of the LOP.

The standard formulation of the LOP fails to account properly for transactions costs, and empirical tests usually are carried out in the following regression framework:[1]

$$(2) \qquad p = \alpha_0 + \alpha_1 p^* + \alpha_2 \pi + \tilde{u}, \qquad -m < \tilde{u} < m.$$

Tests are conducted to determine if $\alpha_1 = \alpha_2 = 1$. However, once the existence of barriers to arbitrage is admitted, so that $m > 0$, difficulties arise. First, $\tilde{u}$ is likely to exhibit serial correlation as observations tend to cluster near one arbitrage band or the other. Second, it need not be the case that $\alpha_1 = \alpha_2 = 1$ within the arbitrage band, because this formulation of the LOP offers no explanation for the movements in international relative commodity prices within the transactions costs band. Finally, the single

[1] See Protopapadakis and Stoll (1983) for a more complete discussion of such regression tests of the LOP.

equation formulation, (2), is subject to simultaneous equations bias since p and p* are simultaneously determined.[2]

The difficulties with the standard formulation preclude a formal test of the LOP in the absence of independent information about transactions costs. If transactions costs are known, then the test would consist of comparing the deviations from LOP to the transactions costs.[3] But if transactions costs are not known, the researcher is confronted with data that are generated from two different mechanisms. If the LOP holds, observations that are on the border of either transactions costs band will obey the LOP. Least squares lines drawn through such observations will have a slope of unity but they will have different intercepts, because some observations will be on the lower band and some will be on the upper band. The remaining observations will be inside the band, where they do not obey the LOP. In order to test the LOP, the observations inside the band should be excluded from the tests. The LOP would be rejected if any observations are found outside the band. Since the researcher cannot know which observations are inside, on, or outside the band, a formally correct test of the standard formulation of the LOP is not possible.[4]

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[2]Since our tests deal with individual commodities, it is reasonable to assume exogeneity of the exchange rate, π. When LOP is tested using price indices, π also is simultaneously determined with the price indices.

[3]Frenkel and Levich (1975) use such an approach to test the interest rate parity proposition.

[4]For a more extensive discussion of this difficulty, see Protopapadakis and Stoll (1983).

We examine the adjustment of international commodity prices by using an alternative formulation of the LOP with transactions costs. This formulation assumes that any deviation from perfect equality of prices will give rise to some economic forces to restore the equilibrium. Though the precise specification of the impact of these economic forces is ad hoc, this alternative specification avoids the discontinuity in behavior implied by the standard framework. Furthermore, this specification models all the observations. In particular, we assume that an n'th-order lagged adjustment mechanism adequately summarizes the adjustment process.

Some informal evidence in favor of the lagged adjustment formulation comes from examining the plots of deviations from the LOP. None of the commodities examined shows any visual indication that "absorbing barriers" (a transactions costs band) play a role in the price formation mechanism. All the plots show the typical elliptical pattern around regression lines, without any flattening-out that would be expected if absorbing barriers constrain price movements.

A MORE GENERAL MODEL OF THE LOP

In this section, we formulate a general, multi-equation model of the LOP. The model has three building blocks. First, the LOP is formulated with respect to long-run prices rather than actual prices. Second, we introduce a lagged adjustment mechanism for the commodity price in each country. Third, we specify explicitly the process by which expectations of the commodity price in the other country are formed.

Long-Run Prices and the LOP:

It is likely that the LOP is not satisfied for all actual prices, since actual prices may deviate from the LOP as a result of transport costs and other barriers to arbitrage and trade. To allow for this possibility we formulate the LOP with respect to the long-run prices of the commodities. This is the first building block of the reformulation.

Furthermore, we allow the domestic long-run price, $\hat{p}$, to differ from foreigners' expectations of the domestic long-run, $\hat{p}_e$, because of information costs and other market imperfections. Similarly, the foreign long-run price, $\hat{p}^*$ may differ from the domestic arbitragers' expectations of the foreign long-run price, $\hat{p}^*_e$. The long-run LOP then may be written as follows:

$$(3i) \qquad \hat{p} = \alpha_0 + \alpha_1 \hat{p}^*_e + \alpha_2 \pi + \tilde{u},$$

$$(3ii) \qquad \hat{p}^* = -\alpha^*_0 + \alpha^*_1 \hat{p}_e - \alpha^*_2 \pi + \tilde{u}^*,$$

where a " ~" over a variable indicates a long-run price, and where (3i) is to be viewed as the LOP from the perspective of domestic commodity arbitragers; (3ii) is the LOP from the perspective of foreign commodity arbitragers. The long-run price is the price that would exist if adjustment in both economies were immediate. Arbitragers in each country form their expectations of the other country's long-run price $\hat{p}_e$, $\hat{p}^*_e$, without knowing the price expectations of their counterparts in the other country.[5] The parameters in (3) obviously are not independent of each

[5]The model is symmetric in the sense that (3) could be written with $\hat{p}_e$ and $\hat{p}_e$ on the LHS. In other words, $\hat{p}$, $\hat{p}^*$, $\hat{p}_e$ and $\hat{p}^*_e$ are simultaneously determined.

other because, for equilibrium to exist, the two equations must be consistent. This implies,

$$(4) \qquad \alpha_0^* = \frac{\alpha_0}{\alpha_1}, \quad \alpha_1^* = \frac{1}{\alpha_1}, \quad \alpha_2^* = \frac{\alpha_2}{\alpha_1} .$$

Furthermore, the LOP requires that $\alpha_1 = \alpha_2 = 1.00$. The value of α_0 depends on the characteristics of the stochastic process.

Price Adjustment:

The second building block is the specification of the price adjustment process. Deviations from LOP imply changes in the relative prices of commodities, which, in turn, induce changes in consumption and production that move prices to their desired levels, even in the absence of international commodity arbitrage. Depending on the anticipated duration of deviations, and depending on the nature of the commodity, producers, consumers, and storers of the commodity will change their production and consumption plans in response to such relative price changes. In addition, traders that face particularly low transport costs, either because of their physical location or because of joint-product aspects of their production, will engage in export or import activities that will change the national supply of the commodity, in response to deviations from prices that satisfy the LOP. These adjustments in the behavior of economic agents will result in a slow and partial elimination of deviations from the LOP, even in the absence of potential massive shipping when LOP deviations exceed some critical value (as implied by the standard formulation).

This slow adjustment process manifests itself both in the spot and in the futures prices, because the spot and futures prices are connected through the interest rate, the storage costs, and the "convenience yield" of the commodity. These three factors that connect the spot and futures prices tend to change slowly relative to the commodity prices, on a weekly basis. The slow change of these factors is responsible for the empirical regularity, namely that changes in spot and futures commodity prices tend to be highly correlated. However, the gap between current spot and futures prices is not fixed, and there is no reason to expect that the adjustment process in spot prices will be mirrored exactly in the futures prices.

The considerations just discussed do not have precise implications about how each commodity price might adjust within a country. They do suggest the possibility, however, of specifying a lagged adjustment mechanism to describe these deviations. Specifying an ad hoc adjustment mechanism that is not explicitly derived from maximizing principles is somewhat unsatisfactory. However, the alternative is to build a general equilibrium structural model with parameters that cannot be estimated in the absence of detailed transactions costs data. Furthermore, it has been shown that reduced forms of such structural models often can be approximated well by a lagged adjustment mechanism (see Muth (1960), Lucas (1975), Mussa (1975)).

Our interpretation of the lagged adjustment model is that it adequately summarizes the behavior of the commodity price data within the sample, and that it provides a means to distinguish formally the short-run from the long-run behavior of commodity prices. To the extent that there have been important changes in the costs of international transactions or in the technology of such transactions--including the technology of information-gathering and processing--our lagged adjustment model is misspecified. Furthermore, since the adjustment parameters we estimate depend critically on the trading environment, the model cannot be used for forecasting purposes if the environment changes.

The general structure of a lagged adjustment model is that the variable of interest adjusts slowly to its <u>long-run</u> value. We apply Occam's razor and begin by assuming a first-order lagged adjustment process.[6] Each national price adjusts slowly to deviations from its long-run level $\hat{p}$, $\hat{p}^*$,

(5i) $$\tilde{p} = \beta\hat{p} + (1-\beta)p_{-1} + \tilde{v},$$

(5ii) $$\tilde{p}^* = \beta^*\hat{p}^* + (1-\beta^*)p^*_{-1} + \tilde{v}^*,$$

where $\tilde{p}$, $\tilde{p}^*$ are the observed commodity prices, $\tilde{v}$, $\tilde{v}^*$ are random disturbances, and where $0<\beta<1$ and $0<\beta^*<1$ are the adjustment parameters. As β, β^* approach unity the adjustment becomes faster.

Expectations Formation:

The third building block of the model is the specification of expectations. In our formulation it is not possible to specify "rational" expectations, because the model is based only on arbitrage relations, and it does not incorporate any exogenous economic variables that determine the

[6] In principle an n'th order adjustment could be specified. However, since such a polynomial will "fit" a small sample arbitrarily well, any tests based on such a specification have no content. Therefore we start by specifying a first-order adjustment. If the regression residuals are highly autocorrelated, the adjustment process can be readily augmented to second-order.

general equilibrium prices of the commodities.[7] We specify two alternative expectations processes that are sufficiently different so that the empirical results may differ as well. The expectations processes we specify should be thought of as alternative formulations that approximate expectations formation when gathering and processing information is costly.[8]

Myopic expectations: Under this specification, the arbitragers' expectation of the other country's long-run price equals the current actual price:

(6i) $\hat{p}^*_e = \tilde{p}^*$

(6ii) $\hat{p}_e = \tilde{p}$

We label this specification "myopic" because economic agents in each country do not take into account the slow adjustment process taking place in the other country. Thus they ignore the predictable movement of the long-run price in their country that comes from the lagged adjustment

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[7]Aside from the difficulties associated with specifying the underlying determinants of commodity prices, most of the data that would be needed are available only monthly or quarterly. In contrast, the commodity prices we examine are weekly.

[8]There is an alternative formulation that results in identical estimating equations (equations (7) and (10) below). The alternative formulation consists of asserting that the LOP holds exactly for the long-run prices $\hat{p}$, $\hat{p}^*$ (i.e. $\tilde{u} = \tilde{u}^* \equiv 0.00$), and that there is no information assymmetry (i.e. $\hat{p}_e = \hat{p}$, $\hat{p}^*_e = \hat{p}^*$). In this alternative formulation, the expectations equations (equations (6) and (9)) are interpreted as the determinants of the other country's long-run price to which each country adjusts.

process of the foreign price. We use this specification because it is the natural extension of the standard lagged adjustment models.

Using (5) to substitute for the unknown $\hat{p}$ and p^{*} on the LHS of (3), and using (6) to substitute for the unknown $\hat{p}$ and p^{*} on the RHS of (3) yields

(7i) $$\tilde{p} = \alpha_0 \beta + \alpha_1 \beta \tilde{p}^{*} + \alpha_2 \beta \pi + (1-\beta) p_{-1} + \tilde{w} \quad ,$$

(7ii) $$\tilde{p}^{*} = -\frac{\alpha_0 \beta^{*}}{\alpha_1} + \frac{\beta^{*}}{\alpha_1}\tilde{p} - \frac{\alpha_2 \beta^{*}}{\alpha_1}\pi + (1-\beta^{*})p^{*}_{-1} + \tilde{w}^{*}.$$

The unconstrained counterpart of (7) is (8) below:

(8i) $$\tilde{p} = \eta_0 + \eta_1 \tilde{p}^{*} + \eta_2 \pi + \eta_3 p_{-1} + \tilde{w} \quad ,$$

(8ii) $$\tilde{p}^{*} = \eta^{*}_0 + \eta^{*}_1 \tilde{p} + \eta^{*}_2 \pi + \eta^{*}_3 p^{*}_{-1} + \tilde{w}^{*} \quad .$$

Informed expectations: In contrast to myopic expectations, "informed" expectations assume that economic agents in each country are well-informed, and they are forward-looking. The informed expectations version of the model assumes that agents take account of the slow adjustment of prices in the other country, but that they do not know precisely the long-run price in the other country. Such a specification has been used rarely in the lagged adjustment literature. It is, however, more nearly consistent with rational expectations.

We adopt a very general framework by specifying the expectations of the other country's long-run price as a distributed lag function of all the variables in the system. Such a specification is very similar to the

Vector-Autoregressive (VAR) specification, discussed by Sims (1980).[9] The important difference between our specification and the standard VAR model is that we allow the current value of the exchange rate to appear in both equations, consistent with our assumption that the exchange rate is exogenous to both commodity prices.

$$\text{(9i)} \qquad \hat{p}_e^* = \theta_1^*(L)p_{-1}^* + \pi^*(L)\pi + \lambda_1^*(L)p_{-1} + \gamma_0^*,$$

$$\text{(9ii)} \qquad \hat{p}_e = \theta_1(L)p_{-1}^* + \pi(L)\pi + \lambda_1(L)p_{-1} + \gamma_0,$$

where we have adopted the convention $\chi(L)y \equiv \chi_0 y + \chi_1(L)y_{-1} \equiv \chi_0 y + \chi_1 y_{-1} + \chi_2(L)y_{-2}$; $\chi(L)$ is a distributed lag polynomial and y_{-i} is the i'th lag of y.

The assumption that the LOP holds for long run prices, equation (3), places restrictions on the distributed lag parameters of equation (9) because the processes generating $\hat{p}_e^*$ and $\hat{p}_e$ cannot be independent if any form of the LOP holds. Thus, tests of the LOP hypothesis are tests of the overidentifying restrictions on (9) that arise from (3).

Combining the first-order lagged adjustment specification (5), the distributed lag specification for expected prices (9), and the LOP equation (3), we get (details in Appendix B.1):

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[9]The VAR representation Sims proposes consists of a system of equations in which each variable is a linear function of the lagged values of all the variables in the system. The unconditional expectation of each variable is a function only of all the lagged variables in the system.

$$(10\text{i}) \qquad \tilde{p} = \alpha_1 \beta \theta_1^*(L) p_{-1}^* + (\alpha_1 \eta_0^* + \alpha_2)\beta\pi + \alpha_1 \beta \eta_1^*(L)\pi_{-1}$$

$$+ (1-\beta + \alpha_1 \beta \lambda_1^*) p_{-1} + \alpha_1 \beta \lambda_2^*(L) p_{-2} + \tilde{w} ,$$

$$(10\text{ii}) \qquad \tilde{p}^* = (1-\beta^* + \beta^* \theta_1^*) p_{-1}^* + \beta^* \theta_2^*(L) p_{-2}^* + \beta^* \eta^*(L)\pi$$

$$+ \beta^* \lambda_1^*(L) p_{-1} + \tilde{w}^* .$$

The unconstrained version of (10) is:

$$(11\text{i}) \qquad \tilde{p} = \delta_1(L) \tilde{p}_{-1}^* + \zeta(L)\pi + \xi_1(L) p_{-1} + \tilde{\sigma} ,$$

$$(11\text{ii}) \qquad \tilde{p} = \delta_1^*(L) p_{-1}^* + \zeta^*(L)\pi + \xi_1^*(L) p_{-1} + \tilde{\sigma}^* .$$

In Appendix B.2 we show that the system of equations (10) is overidentified, and that a meaningful test of the LOP restrictions can be performed.

FORMAL TESTS OF THE LOP

The LOP tests we propose for the "myopic" model and for the "informed" model are very similar in structure. In both cases we compare the log-likelihood value of the unconstrained equation (equation (8) for myopic, equation (11) for informed) to its constrained counterpart. Since the system of unconstrained equations (8 for myopic and 11 for informed) is just-identified, it is possible to test the partial adjustment model for both expectations specifications by testing the restrictions in (7) against (8), and in (10) against (11). The tests we discuss are nested, so that, asymptotically, failure of one hypothesis implies failure of all subsequent hypotheses. The test results are summarized in Table 1, and they are discussed below.

We test first the empirical relevance of the two models. If the log-likelihood value is significantly smaller for the constrained than for the unconstrained version, the model is rejected. This test of the two models is the first null hypothesis, and it is labeled H_0^1. The "weak long-run LOP" hypothesis, H_0^2, is whether the commodity price in a country responds differently to a change in the exchange rate than it does to a change in the commodity price of the other country. The H_0^2 hypothesis suggests that the response should be the same, and that $\alpha_1=\alpha_2$. The H_0^2 restriction is tested by imposing it on equations (7) and (10). The long-run LOP, H_0^3, requires that $\alpha_1=\alpha_2=1.00$; H_0^3 is tested by imposing the restriction on equations (7) and (10). The "weak short-run LOP," H_0^4, on the other hand, requires that in addition to $\alpha_1=\alpha_2=1.00$, it must be that $\beta=\beta^*$. Finally, the short-run LOP, H_0^5, requires that $\alpha_1=\alpha_2=1.00$, <u>and</u> that $\beta=\beta^*=1.00$. At $\beta=1.00$ the myopic model is no longer identified because the coefficient is zero on the only variable that is excluded from the "other" equation (the identifying variable). Thus, it is not possible to use a log-likelihood test in this case. Instead the test of H_0^5 is a t-test on whether the adjustment coefficient is different from unity. In the case of informed expectations, the model is identified for $\beta=1.00$, and the log-likelihood test can be used.

DATA

Spot, futures, and, in some cases, forward price data were collected for 12 standardized commodities traded in more than one country, and for which prices are quotes in more than one currency. A forward contract has a constant time to maturity--e.g., 3 months--and a changing maturity date; whereas a futures contract has a fixed maturity date--e.g., March--and a changing time to maturity.[10] Observations are taken on Wednesday of each week in the post-1972 period of floating exchange rates.[11] The data used in the study are summarized in Appendix A.

For 4 metals in our sample--copper, tin, lead, and zinc--only spot data are available. For silver, spot data, 3-month forward data, and 6-month forward data are used in the analysis. In the case of 5 of the agricultural commodities--coffee, cocoa, soybean meal, sugar, and wheat--data for two or more futures contracts were available. For each commodity, these data are divided into "near" data that constitute a time series of weekly price observations with less than 6 months to maturity, and "far" data that constitute a time series of weekly price observations with 6 to 12 months to maturity. For the other agricultural commodities--wool and rubber--only spot data are available. In total there are 19 data series which are analyzed for the twelve commodities. These 19 separate sets of data clearly are not independent.

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[10]For Copper and Silver only forward prices are available for the U.K. For these two commodities we match the forward U.K. quotes to the futures U.S. quotes, though there are theoretical reasons why future and forward prices need not be equal (see Cox, Ingersoll and Ross (1981).

[11]Where possible, U.S. opening and foreign closing prices are used in order to match transactions times as closely as possible.

TEST RESULTS FOR MYOPIC EXPECTATIONS[12]

Myopic expectations results are discussed first. The results obtained from estimating equations (7) and (8) and from testing the hypotheses detailed in Table 1 are not very satisfactory. Of the 19 separate sets of simultaneous equations, the model itself is rejected in 6 cases, with 3 indeterminate cases, at the 99-percent level.[13] The long-run LOP is rejected in 9 of 19 cases, with 1 case indeterminate, at the 99-percent level. The short-run LOP is rejected in all cases for any conventional confidence level.

There are two important difficulties with the results of the myopic expectations model. The first is that the frequency with which the model is rejected is troublesome, and it casts doubt on the model's ability to describe the sample data adequately. The second is that the estimated residuals are autocorrelated in many instances. This autocorrelation not only can cause the standard errors to be understated, but it can result in inconsistent estimates of the model parameters because of the presence of the lagged dependent variables. While it is possible to adjust the

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[12]The results reported in this paper are calculated using TSP.

[13]We say that the test of the j'th null hypothesis is "indeterminate" if the following conditions occur: (i) the estimating algorithm does not reach a global maximum. We infer that the algorithm is not at a global maximum if the likelihood value of a subsequent, more restrictive hypothesis, is higher (j+i'th hypothesis, $i \geq 1$), (ii) the j+i'th hypothesis is rejected. If the j+i'th hypothesis is rejected, the j'th hypothesis may or may not be rejected. Rejection of the j+i'th hypothesis provides no information on the validity of the j'th hypothesis.

If, instead, the j+i'th hypothesis is not rejected, then the j'th hypothesis also is not rejected, in large samples. In principle, a grid search should reveal the global maximum in every case, but because such a procedure is very expensive, we did not attempt it.

standard errors to reflect the autocorrelation of the residuals, no adjustment is available to correct for the potential inconsistency of the estimates.[14]

We tried to remedy the residual autocorrelation by allowing for a second-order adjustment process.[15] The rejection rate for the model with the second-order adjustment is higher. With the second-order adjustment process the model is rejected in 11 of 19 cases, with 3 cases indeterminate. These findings suggests that the first-order adjustment restriction on the model is not likely to be responsible for the poor performance of the model. The residual autocorrelation is less severe, but it is not eliminated. A difficulty with the estimates of β_2, β_2^* is that they are generally negative. Though negative β_2's need not result in instability, it is not clear how to interpret them as adjustment parameters.

Since the myopic model performs poorly, we do not report in detail tests of all the hypotheses specified in Table 1, nor do we report any regression results. The results for the myopic model are available from the authors as a separate Appendix (Appendix C).

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[14]The approximate correction to standard errors for autocorrelation is $\frac{1+\rho^2}{1-\rho^2}$. Assuming $\rho = 0.50$ the correction factor is 1.67.

[15]We do not use the standard procedure of allowing for AR1 errors in this case, because the AR1 error process can be interpreted as an alternative lagged adjustment process. Introducing both lagged price adjustment and AR1 errors does not allow for a simple interpretation of the meaning of short-run LOP.

TEST RESULTS FOR INFORMED EXPECTATIONS

Table 2 summarizes the test results for the five hypotheses associated with the LOP and Table 3 presents some of the detailed regression results. A very encouraging result is that the informed expectations specification is rejected much less frequently than the myopic expectations. The model is rejected in at most 4 of the 19 cases, at the 99-percent level.[16] However, during the estimation of the informed expectations specification, the algorithm fails to reach a global maximum much more frequently than for myopic expectations for the least restrictive null hypotheses (9 of 19 for informed, compared to 3 of 19 for myopic in tests of H_0^1). The reason for the increased failure rate seems to be that near under-identification occurs much more frequently for informed expectations in the least restrictive null hypotheses, because there are more free parameters to be estimated relative to the myopic expectations. Examination of the computation sequences reveals that in most cases the underidentification is confined to a subset of the parameters. Another feature of the informed expectations specification is that the residuals are generally well-behaved, and there is little need to estimate a second-order adjustment process.

The strongest conclusion that emerges from Table 3 is that the short-run LOP, H_0^5, is rejected at any level of significance for all commodities except for Silver 3-Mo. and Silver 6-Mo. And, except for the three Silver

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[16]There are 2 outright rejections and 2 indeterminate cases. If both of the indeterminate cases are assumed to be rejections then the model is rejected in 4 of the 19 cases.

maturities, the estimates of β and β^* are numerically very different from unity.

The long-run LOP fares reasonably well with informed expectations. Of the seven agricultural commodities, the long-run LOP is rejected only for Cocoa Far and Wool at the 99-percent level. For the metals, however, the long run LOP is rejected for Copper, Silver Spot, and Tin. Overall, the long-run LOP is rejected in 5 of 19 cases.

Table 3 reports the regressions that incorporate the H_0^2 restriction ($\alpha_1=\alpha_2$) and the regressions that incorporates the H_0^4 restrictions ($\alpha_1=\alpha_2=1.00$, $\beta=\beta^*$). An examination of the estimated values of α (under the $\alpha_1=\alpha_2$ restriction) shows that the estimates range from .877 to 1.208, with an average value of 1.040. Of these estimates only those for Silver Spot and Silver 6-Mo. are statistically different from 1.00 at the 99-percent level (values are .992 and 1.019, respectively). For the cases in which H_0^3 is rejected, the estimates of α (available for Cocoa Far, Silver Spot, Tin, and Wool) are 1.000, .992, .993, and 1.006, respectively. The deviations of these values from 1.00 seem to have little economic significance. The rejections of the long-run LOP are concentrated on cases for which the data are spot prices. The long-run LOP is rejected in 4 of the 7 cases for which the data are spot, but it is rejected only in 1 of 12 cases for which data are futures or forward. Similarly, the model itself is rejected only in cases for which data are spot. The case of Silver offers a good illustration. The long-run LOP, and possibly the model, is rejected for Silver Spot. Furthermore, the point estimate of α is statistically different from unity. In contrast, neither the model nor the long-run LOP is rejected by either the 3-month or the 6-month Silver. The point estimate of α for 3-month Silver is not statistically different from unity, but the one for 6-month Silver is just significantly different from unity (values are 1.030 and 1.019, respectively).

The results lead to the conclusion that the specification with first-order lagged adjustment and informed expectations describes the behavior of futures and forward commodity prices quite well. However, this specification is not as successful in describing the behavior of spot prices. This conclusion is not too surprising, since spot prices probably respond to market influences, not accounted for here, that are sufficiently short-lived to have an appreciably smaller impact on futures prices. A similar conclusion, that future prices adhere to the LOP better than spot prices, was reached in Protopapadakis and Stoll (1983) through some informal tests.

The estimates of β vary considerably across commodities but none are negative, and they are generally higher than the estimates from the myopic specification. The average value of β for informed expectations (when $\beta = \beta^*$) is .204 with a range .019 to .700, excluding the three Silver maturities. For the three Silver maturities the values of β are .788, 1.001 and .969, for spot, 3-Mo., and 6-Mo., respectively. Only the value of β for Silver Spot is statistically different from unity. The low value of β (.204) implies that 68 percent of the deviation from the long-run price will be eliminated by 5 weeks, and 90 percent of the deviation will be eliminated by 10 weeks. The high value of β (.700), on the other hand, implies that 90 percent of the deviation will be eliminated within 2 weeks.

CONCLUSION

In this paper we propose and execute formal tests of whether the Law of One Price holds across countries in the long-run and in the short-run for commodities that are traded in organized commodity markets.

In order to construct a formal test of the LOP, we develop a model in which prices in each country adjust with a lag. In addition we specify explicitly two distinct expectations structures. The "myopic" expectations specification assumes that traders ignore the price adjustment process in the other country, while the "informed" expectations specification allows traders to take account of the foreign price adjustment.

The empirical results summarized in this paper support our lagged adjustment model coupled with the "informed" expectations specification. Informed expectations appear to explain the data much better than myopic expectations.

The hypothesis that the Law of One Price holds in the long-run is supported by the data. The support is largely due to the cases for which data are futures or forward prices. In the cases for which data are spot prices, the support is much more limited. However, even in the instances in which the long-run LOP hypothesis, or the hypothesis that the model is adequate, fails, the point estimates of the slope coefficient, α_1, are reasonably close to unity. It is likely that our model ignores variables that may be important to the behavior of spot prices but not so important to the behavior of futures prices.

The hypothesis that the Law of One Price holds in the short-run is uniformly rejected except for the forward Silver prices. The values of the lagged adjustment parameters, excluding Silver, imply that 90 percent of a deviation from the LOP would be expected to be eliminated from within 10 weeks (high) to within 2 weeks (low).

The overall conclusion is that long-run LOP is a usable approximation of the behavior of commodity prices for macroeconomic purposes.

APPENDIX A
Data Sources:

The data used in this study are summarized in Table A-1 which is largely self-explanatory.

In the U.K., all the metals are traded on the London Metal Exchange (LME), and spot and forward prices are available. In the U.S., Silver and Copper are traded on a futures market, but no futures prices exist for Tin, Lead and Zinc. Even where futures prices exist, as in Copper, maturities do not always match forward maturities, which results in a reduction in the sample size.

Care was taken to control for units of measurement. In general, U.K. commodity prices are quoted in pounds per metric ton, while U.S. prices are generally quoted in cents per pound. Only Wheat created some difficulties, since the U.K. measure, long tons, is a weight measure, whereas the U.S. measure, bushels, is a volume measure. We converted based on the volume standards required of the U.K. contract. In any event an improper conversion would be reflected in the intercept of our regression tests and would not affect the slope coefficient.

There are a variety of difficulties with this data set. Frequently we were forced to use bid prices, not transactions prices; and transactions prices are not exactly comparable in time because commodity exchanges do not open and close simultaneously. Futures contracts in different countries do not always refer to identical commodities, and spot prices may refer to a range of commodity qualities. Despite these difficulties, we argue that these are the most precise data presently available, not only in terms of comparability of commodities, but also in comparability of the observation time. The most detailed data examined previously are in Crouhy-Veyrac, Crouhy and Melitz (1982), and they rely on monthly data for which neither the timing of the price collection nor the exact commodity definitions are as clear.

A useful source for information on the world's commodity exchanges is De Keyser (1979).

TABLE 1

Nested Null Hypotheses Associated With The LOP

	Null Hypotheses	Restrictions	Test Statistic, Myopic	Tes Statistic, Informed[a]
H_0^1:	Model Test: Lagged adjustment model is adequate.	$\eta_0 = \alpha_0 \beta$, $\eta_0^* = \frac{-\alpha_0 \beta^*}{\alpha_1}$, $\eta_1 = \alpha_1 \beta$, $\eta_1^* = \frac{\beta^*}{\alpha_1}$, $\eta_2 = \alpha_2 \beta$ $\eta_2^* = \frac{\alpha_2 \beta^*}{\alpha_1}$, $\eta_3 = 1-\beta$, $\eta_3^* = 1-\beta^*$.	H_0^1 restrictions imposed on equation (8), log-likelihood ratio (LLR) test with 3 degrees of freedom (DF).	H_0^1 restrictions imposed on equation (11), log-likelihood ratio (LLR) test with 9 degrees of freedom (DF). The restrictions are listed in Appendix B.2.
H_0^2:	Weak Long-Run LOP: The response of $p(p^*)$ to $p^*(p)$ and π is identical	H_0^1 and $\alpha_1 = \alpha_2$.	H_0^1 and H_0^2 restrictions imposed on equation (8), LLR test with 4DF.	H_0^1 and H_0^2 restrictions imposed on equation (11) LLR test with 10DF.
H_0^3:	Long-Run LOP: The LOP holds in the long run only.	H_0^1 and $\alpha_1 = \alpha_2 = 1.00$.	H_0^1, H_0^2 and H_0^3 restrictions imposed on equation (8), LLR test with 5DF.	H_0^1, H_0^2 and H_0^3 restrictions imposed on equation (11), LLR test with 11DF.
H_0^4:	Weak Short-Run LOP: The lagged adjustment is the same in the two countries.	H_0^3 and $\beta = \beta^*$.	H_0^1 - H_0^4 restrictions imposed on equation (8), LLR test with 6DF.	H_0^1 - H_0^4 restrictions imposed on on equation (11), LLR test with 12DF.
H_0^5:	Short-Run LOP: The LOP holds in the long run and in the short run.	H_0^3 and $\beta = \beta^* = 1.00$.	H_0^1 - H_0^4 restrictions imposed on equation (8). Test whether the estimated value of β is less than 1.00. The estimate of β is normally distributed.	H_0^1 - H_0^5 restrictions imposed on equation (11), LLR test with 13DF.

[a]The degrees of freedom for the LLR test are based on including 4 lags of each variable in equations (10).

TABLE 2

Formal Tests of the Law of One Price For Informed Expectations
(1st Order Adjustment)

Commodity		H_0^1 (9DF)	H_0^2 (10DF)	H_0^3 (11DF)	H_0^4 (12DF)	H_0^5 (13DF)
Cocoa	NR	*	*	13.17	39.37 ‡	226.11 ‡
	FR	19.18†	31.28 ‡	31.28 ‡	62.80 ‡	181.86 ‡
Coffee	NR	12.36	14.17	14.17	22.82†	378.30 ‡
	FR	*	*	*	24.11†	419.72 ‡
Soybeans	NR	*	8.89	9.89	17.11	109.14 ‡
	FR	3.67	5.10	7.83	24.34†	115.98 ‡
Sugar	NR	3.78	4.56	19.88†	32.29	49.77 ‡
	FR	7.54	12.22	16.52	24.70†	122.03 ‡
Wheat	NR	*	*	*	19.11	463.58 ‡
	FR	*	*	*	23.99†	607.10 ‡
Rubber	SP	*	9.61	14.12	37.66 ‡	541.34 ‡
Wool	SP	24.68 ‡	33.82 ‡	37.58 ‡	39.84 ‡	221.15 ‡
Copper	SP	*	*	32.78 ‡	247.34 ‡	1,108.86 ‡
Silver	SP	*	*	53.12 ‡	111.78 ‡	135.32 ‡
	3-M	8.32	8.44	13.11	20.67	20.68
	6-M	11.08	11.65	21.16†	21.19†	21.57
Lead	SP	7.82	7.82	8.74	265.98 ‡	1,031.64 ‡
Tin	SP	49.28 ‡	49.70 ‡	50.06 ‡	227.34 ‡	331.82 ‡
Zinc	SP	*	*	18.06	255.68 ‡	1,627.36 ‡

Notes for Table 2

The test statistics for the null hypotheses H_0^1, H_0^2, H_0^3, H_0^4, and H_0^5 are twice the difference of the log-likelihood between the unconstrained regression and the regression constrained according to the hypothesis that is tested. These statistics are asymptotically χ^2-distributed at the indicated degrees of freedom. The null hypothesis is rejected if the test statistic is larger than a critical value.

(†) significant at the 95-percent level,
(‡) significant at the 99-percent level,
(*) did not attain the global maximum.

χ^2-Distribution

Significance Level	**9**	**10**	**11**	**12**	**13**	**14**
		Degrees of Freedom				
95	16.92	18.31	19.68	21.03	22.36	23.68
99	21.67	23.21	24.72	26.22	27.69	29.14

TABLE 3

Results from Selected LOP Test Regressions for Informed Expectations[a,b]
(1st Order Adjustment)

Commodity	α_1	β	β^*	No. of Observ.	H-Statistic and D.W. Coefficient[c] Dom.	For.
Cocoa Near	Convergence Difficulties					
	--	.244 ‡ (.034)	--	215	.10 (1.76)	1.42 (1.76)
Cocoa Far	1.000 (.016)	1.136 (.722)	.295 ‡ (.078)	283	-.42 (2.02)	.14 (2.12)
	--	.440 ‡ (.049)	--		1.19 (1.91)	1.55 (1.92)
Coffee Near	.877 (3.429)	.004 ‡ (.117)	.002 ‡ (.045)	141	-.85 (1.80)	-1.22 (1.80)
	--	.045 ‡ (.028)	--		-.49 (1.79)	-.76 (1.88)
Coffee Far	Convergence Difficulties					
	--	.037 ‡ (.024)	--	176	2.07 (1.63)	2.12 (1.50)
Soybeans Near	1.391 (2.146)	.553 (3.347)	.036 ‡ (.105)	91	-1.47 (1.94)	-.88 (2.09)
	--	.186 ‡ (.076)	--		-1.83 (1.88)	-.53 (1.84)
Soybeans Far	.898 (.063)	.368 ‡ (.143)	.980 (.828)	75	1.08 (1.74)	-.08 (1.85)
	--	.246 ‡ (.076)	--		1.20 (1.88)	.92 (1.84)

TABLE 3 (Cont'd Page 2)

Commodity	α_1	β	β^*	No. of Observ.	H-Statistic and D.W. Coefficient[c] Dom.	H-Statistic and D.W. Coefficient[c] For.
Sugar Near	1.198 (.154)	.214 ‡ (.126)	.578 (.268)	82	-.04 (1.80)	1.06 (1.52)
	--	.391 ‡ (.248)	--		.38 (1.73)	.36 (1.61)
Sugar Far	1.039 (.014)	.347 ‡ (.078)	.422 ‡ (.084)	134	1.09 (1.59)	-.17 (1.74)
	--	.345 ‡ (.066)	--		1.12 (1.66)	-.05 (1.84)
Wheat Near	Convergence Difficulties			129		
	--	.026 ‡ (.016)	--		-1.06 (1.61)	-.55 (1.92)
Wheat Far	Convergence Difficulties			159		
	--	.019 ‡ (.016)	--		.54 (1.83)	2.63 ‡ (1.78)
Rubber Spot	1.216 (.139)	.072 ‡ (.019)	.673 ‡ (.878)	217	-.18 (1.76)	-1.51 (1.86)
	--	.057 ‡ (.022)	--		-.22 (1.93)	-.54 (2.00)
Wool Spot	1.006 (.110)	.143 ‡ (.057)	36.557 (2362.03)	231	.66 (2.38)	-.44 (2.02)
	--	.304 ‡ (.033)	--		.37 (2.14)	.14 (1.93)

TABLE 3 (Cont'd Page 3)

Commodity	α_1	β	β^*	No. of Observ.	H-Statistic and D.W. Coefficient[c] Dom.	For.
Silver Spot	.992 ‡ (.002)	.903 (.054)	.841 ‡ (.039)	390	1.58 (1.78)	1.10 (2.01)
	—	.788 ‡ (.037)	—		1.66 (1.67)	1.11 (1.88)
Silver 3-Mo.	1.030 (.045)	1.065 (.281)	.897 (.285)	42	-1.88 (2.13)	-1.96 (2.12)
	--	1.001 (.064)	--		-1.48 (2.24)	-1.55 2.27)
Silver 6-Mo.	1.019 ‡ (.009)	.940 (.054)	1.007 (.057)	31	-.68 (1.76)	-.56 (1.69)
	--	.969 (.064)	--		-.64 (1.71)	-.56 (1.58)
Copper Spot	Convergence Difficulties			408		
	--	.093 ‡ (.016)	--		-1.16 (1.75)	-1.94 (1.91)
Zinc Spot	Convergence Difficulties			408		
	—	.030 ‡ (.009)	—		2.37 (1.80)	1.13 (1.55)
Tin Spot	.993 (.017)	.350 ‡ (.030)	.257 ‡ (.030)	401	3.43 ‡ (1.42)	2.08 (2.27)
	--	.700 ‡ (.026)	--		4.13 ‡ (1.67)	2.16 (1.88)

TABLE 3 (Cont'd Page 4)

Commodity	α_1	β	β^*	No. of Observ.	H-Statistic and D.W. Coefficient[c] Dom.	For.
Lead Spot	1.208 (.264)	.046 ‡ (.018)	.033 ‡ (.012)	408	2.74 ‡ (1.95)	1.13 (1.94)
	--	.114 ‡ (.020)	--		2.62 ‡ (1.70)	.54 (1.68)

Notes for Table 3

[a]In the first regression reported for each commodity, $\alpha_2=\alpha_1$. In the second regression $\alpha_1=\alpha_2=1.00$, $\beta=\beta^*$. The vector-autoregression parameters that are estimated along with the parameters of interest are suppressed to save space. n-statistics are reported in parentheses below the coefficients. These statistics are asymptotically normally distributed. The significance notation for α_1 is with respect to the null hypothesis $\alpha_1=1.00$ (Long run LOP). The significance notation for β, β^* is with respect to the null hypothesis $\beta, \beta^*=1.00$.

[b](†) 95 percent significance level.

(‡) 99 percent significance level.

[c]The H-statistics reported are approximate, and they are calculated by regressing the lagged values of the residuals and all the other regressors on the current value of the residuals. The value reported is the t-statistic of the coefficient of the lagged residual. The number in parentheses below the H-statistics is the Durbin-Watson coefficient of the report regression.

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APPENDIX TABLE A-1
Description of Commodity Price Data

Commodity	Contract U.S. or other country	Contract U.K.	Period Covered	Data[a] Source
Silver	U.S. spot	spot	1/5/72-5/28/80	Samuel Montagu
Silver	U.S. futures: various Comex cents/lb	forward: 3,6 mos LME £/metric ton	1/5/72-5/28/80	JC
Copper	U.S. spot	spot	1/5/72-7/30/80	JC
Copper	U.S. futures: various Comex cents/lb	forward: 3 mos LME £/metric ton	12/6/72-6/27/79	JC
Tin	U.S. spot	spot	1/5/72-7/30/80	JC
Lead	U.S. spot	spot	1/5/72-7/30/80	JC
Zinc	U.S. spot	spot	1/5/72-7/30/80	JC
Coffee	U.S. futures: March, Sept. NY Coffee & Sug. Exch. Coffee "C", arabica Cents/lb	futures: March, Sept. Coffee Terminal Mkt. Robusta £/long ton prior to 2/20/74 £/metric ton as of 2/20/74	10/4/72-6/27/79	CSFM
Cocoa	U.S. futures: March, Sept., Dec. NY Cocoa Exch. cocoa beans cents/lb	futures: March, Sept., Dec. London Cocoa Terminal Mkt. Assoc. cocoa beans £/metric ton	5/17/72-6/27/79	CSFM

APPENDIX TABLE A-1 (Cont'd)

Sugar	U.S. futures: March, Sept. NY Coffee & Sug. Exch. #11 sugar cents/lb	futures: March United Terminal Sugar Mkt. Assoc. raw sugar £/metric ton	4/5/72-2/28/79	JC
Soybean meal	U.S. futures: Dec. Chicago Board of Trade soybean meal dollars/short ton	futures: Dec., June London Soybean Meal futures Mkt. toasted extracted soybean meal £/metric ton	1/8/75-12/26/79	JC
Wheat	U.S. futures: March, Sept. Chicago Board of Trade #2 Soft red dollars/bushel	futures: March, Sept. London Grain Futures Market EEC Wheat, weight to exceed 72.5 kg per hectolitre £/long ton	4/2/75-9/19/79	JC
Rubber	Malaya spot	spot	1/2/74-12/26/79	LFT
Greasy Wool	Australia spot	spot	1/2/74-12/26/79	LFT
British pound	U.S. spot futures: March, Sept., Dec.		1/5/72-7/30/80 1/5/72-7/30/80	JC CSFM
Australian$		spot	1/2/74-12/26/79	LFT
Malaysian Ringat		spot	1/2/74-12/26/79	LFT

a. Abbreviations: JC = Journal of Commerce. CSFM = Columbia Center for the Study of Futures Markets. LFT = London Financial Times.

APPENDIX B

B.1. Equations (10i) and (10ii) are derived as follows: Define any distributed lag polynomial $\phi(L)y = \chi_0 y + \chi_1 y_{-1} + \chi_2 y_{-2} + \cdots \chi_i(L) y_{-i}$.

Solve equations (5i) and (5ii) for $\hat{p}$ and $\hat{p}^*$,

(B.1-1)
$$\hat{p} = \frac{1}{\beta}\tilde{p} - \frac{(1-\beta)}{\beta} p_{-1} - \frac{1}{\beta}\tilde{v},$$
$$\hat{p}^* = \frac{1}{\beta}\tilde{p}^* - \frac{(1-\beta^*)}{\beta^*} p_{-1}^* - \frac{1}{\beta^*}\tilde{v}^*,$$

and substitute $\hat{p}$, $\hat{p}^*$ in equations (3i) and (3ii),

(B.1-2)
$$\tilde{p} - (1-\beta)\, p_{-1} - \tilde{v} = \alpha_0 \beta + \alpha_1 \beta \hat{p}_e^* + \alpha_2 \beta\pi + \tilde{u},$$
$$\hat{p}^* - (1-\beta^*)\, p_{-1}^* - \tilde{v}^* = -\frac{\alpha_0\, \beta^*}{\alpha_1} + \frac{\beta^*}{\alpha_1}\hat{p}_e - \frac{\alpha_2\, \beta^*}{\alpha_1}\pi + \tilde{u}^*.$$

Now substitute the expectations specification (9i) and (9ii) in (B.1-2) to get,

(B.1-3)
$$\tilde{p} = \alpha_1 \beta\theta_1^*(L)\, p_{-1}^* + \alpha_1 \beta\eta^*(L)\,\pi + \alpha_1 \beta\lambda_1^*(L)\, p_{-1}$$
$$+ \alpha_2\, \beta\pi + (1-\beta) p_{-1} + \tilde{v} + \tilde{u},$$

(B.1-4)
$$\tilde{p}^* = \frac{\beta^*}{\alpha_1}\theta_1(L)\, p_{-1}^* + \frac{\beta^*}{\alpha_1}\eta(L)\,\pi + \frac{\beta^*}{\alpha_1}\lambda_1(L)\, p_{-1}$$
$$- \frac{\alpha_2\, \beta^*}{\alpha_1}\pi + (1-\beta^*)\, p_{-1}^* + \tilde{v}^* + \tilde{u}^*,$$

where the constant term has been suppressed for expositional economy (the constant term is unconstrained). Combining the terms in (B.1-3) we get (10i).

$$(10i) \qquad \tilde{p} = \alpha_1 \beta\theta_1^*(L)\ p_{-1}^* + (\alpha_1\ \eta_0^* + \alpha_2)\beta\pi + \alpha_1 \beta\eta_1^*(L)\ \pi_{-1}$$

$$+ (1-\beta + \alpha_1 \beta\lambda_1^*)\ p_{-1} + \alpha_1 \beta\lambda_2^*(L)p_{-2} + \tilde{w}.$$

An equation similar to (10i) can be derived from (B.1-4) for $\hat{p}^*$. However, the imposition of long-run PPP implies that the parameters of $\theta^*(L)$, $\eta^*(L)$ and $\lambda^*(L)$ are related to those of $\theta(L)$, $\eta(L)$and $\lambda(L)$, respectively.

More specifically, for an equilibrium consistent with long-run LOP to exist it must be $\hat{p}_e$ from (9ii), and $\hat{p}_e$ derived from taking expectations of (3i) must result in the same expression. This relation holds also for $\hat{p}_e^*$ from (9i) and $\hat{p}_e^*$ from (3ii) and it results in the same restrictions.

To get these restrictions, substitute $\hat{p}_e^*$ from (9i) into (3i)

$$(B.1\text{-}5) \qquad \hat{p}_e = \alpha_1\ \theta_1^*(L)p_{-1}^* + \alpha_1\ \eta^*(L)\ \pi + \alpha_1\ \lambda_1^*(L)p_{-1} + \alpha_2\ \pi.$$

(B.1-5) and (9ii), reproduced below, must give the same results.

$$(9ii) \qquad \hat{p}_e = \theta_1(L)p_{-1}^* + \eta(L)\ \pi + \lambda_1(L)p_{-1}.$$

It follows immediately that:

$$(B.1\text{-}6) \qquad \theta_1(L) = \alpha_1\ \theta_1^*(L) \quad \text{for all L,} \qquad \lambda_1(L) = \alpha_1\ \lambda_1^*(L) \quad \text{for all L,}$$

$$\eta_0 = \alpha_1\ \eta_0^* + \alpha_2, \quad \eta_1(L) = \alpha_1\ \eta_1^*(L) \quad \text{for all L.}$$

Substituting these restrictions into (B.1-4) yields equation (10ii) in the text, (10ii) $$\tilde{p}^* = (1-\beta^* + \beta^* \theta_1^*)\ p_{-1}^* + \beta^* \theta_2^*(L)\ p_{-2}^* + \beta^* \eta^*(L)\,\pi + \beta^* \lambda_1^*(L)\ p_{-1} + \tilde{w}^*.$$

To gain some additional intuition about why the restrictions in (B.1-6) are valid, consider the extreme case of $\alpha_1 = \alpha_2 = \beta = \beta^* = 1.00$, i.e., LOP holds instantaneously. Evaluate (10i) and (10ii) at these parameter values and subtract to get:

(B.1-7) $$\tilde{p} - \tilde{p}^* = \pi,$$ the LOP holds instantaneously.

B.2.

Consider the system of equations (10) written out explicitly for 3 lags (more lags increase overidentification):

(B.2-1)
$$\tilde{p} = \alpha_1 \beta\theta_1^* p_{-1}^* + \alpha_1 \beta\theta_2^* p_{-2}^* + \alpha_1 \beta\theta_3^* p_{-3}^* + (\alpha_1 \eta_0^* + \alpha_2)\beta\pi + \alpha_1 \beta\eta_1^* \pi_{-1} + \alpha_1 \beta\eta_2^* \pi_{-2} + \alpha_1 \beta\eta_3^* \pi_{-3} + (1-\beta+\alpha_1 \beta\lambda_1^*)p_{-1} + \alpha_1 \beta\lambda_2^* p_{-2} + \alpha_1 \beta\lambda_3^* p_{-3} + \tilde{w},$$

$$\tilde{p}^* = (1-\beta^* + \beta\theta_1^*)p_{-1}^* + \beta^* \theta_2^* p_{-2}^* + \beta^* \theta_3^* p_{-3}^* + \beta^* \eta_0^* \pi + \beta^* \eta_1^* \pi_{-1} + \beta^* \eta_2^* \pi_{-2} + \beta^* \eta_3^* \pi_{-3} + \beta^* \lambda_1^* p_{-1} + \beta^* \lambda_2^* p_{-2} + \beta^* \lambda_3^* p_{-3} + \tilde{w}^*.$$

The corresponding reduced form system can be written as:

(B.2-2)
$$\tilde{p} = c_1 p_{-1}^* + c_2 p_{-2}^* + c_3 p_{-3}^* + c_4 \pi + c_5 \pi_{-1} + c_6 \pi_{-2} + c_7 \pi_{-3} + c_8 p_{-1} + c_9 p_{-2} + c_{10} p_{-3} + \tilde{z},$$

$$\tilde{p}^* = c_1^* p_{-1}^* + c_2^* p_{-2}^* + c_3^* p_{-3}^* + c_4^* \pi + c_5^* \pi_{-1}$$

$$+ c_6^* \pi_{-2} + c_7^* \pi_{-3} + c_8^* p_{-1} + c_9^* p_{-2} + c_{10}^* p_{-3} + \tilde{z}^* .$$

Each equation contains 10 RHS variables. The reduced form equations have 20 coefficients while the constrained model has 14 parameters. To show that the system is at least just-identified it is necessary to show that the 14 parameters in (B.2-1) have at least one solution that is a function only of the 20 coefficients of (B.2-2). If it is possible to get more than one solution, then the system is overidentified. Equating the coefficients of (B.2-1) and (B.2-2) gives:

(i) $\alpha_1 \beta \theta_1^* = c_1$, (i*) $1 - \beta^* + \beta^* \theta_1^* = c_1^*$,

(ii) $\alpha_1 \beta \theta_2^* = c_2$, (ii*) $\beta^* \theta_2^* = c_2^*$,

(iii) $\alpha_1 \beta \theta_3^* = c_3$, (iii*) $\beta^* \theta_3^* = c_3^*$,

(iv) $\alpha_1 \beta \eta_0^* + \alpha_2 \beta = c_4$, (iv*) $\beta^* \eta_0^* = c_4^*$,

(v) $\alpha_1 \beta \eta_1^* = c_5$, (v*) $\beta^* \eta_1^* = c_5^*$,

(vi) $\alpha_1 \beta \eta_2^* = c_6$, (vi*) $\beta^* \eta_2^* = c_6^*$,

(vii) $\alpha_1 \beta \eta_3^* = c_7$, (vii*) $\beta^* \eta_3^* = c_7^*$,

(viii) $1 - \beta + \alpha_1 \beta \lambda_1^* = c_8$, (viii*) $\beta^* \lambda_1^* = c_8^*$,

(ix) $\alpha_1 \beta \lambda_2^* = c_9$, (ix*) $\beta^* \lambda_2^* = c_9^*$,

(x) $\alpha_1 \beta \lambda_3^* = c_{10}$. (x*) $\beta^* \lambda_3^* = c_{10}^*$.

Denote a solution for X by $\hat{X}$. From (i*):

$$\hat{\beta}^* = \frac{c_1^* - 1}{\theta_1^* - 1} \quad .$$

(ii*) through (x*) then give $\hat{\theta}_2^*$, $\hat{\theta}_3^*$, $\hat{\eta}_0^*$, $\hat{\eta}_1^*$, $\hat{\eta}_2^*$, $\hat{\eta}_3^*$, $\hat{\lambda}_1^*$, $\hat{\lambda}_2^*$, $\hat{\lambda}_3^*$ as functions of θ_1^*.

From (i) and (ii) we have

$$\theta_1^* = \frac{\hat{\theta}_2^*(\theta_1^*)\ c_1}{c_2} \ ,$$

which gives

$$\theta_1^* = \frac{c_2^* c_1}{c_2(c_1^* - 1)} \ (\theta_1^* - 1) \ .$$

Let $\frac{c_2^* c_1}{c_2(c_1^* - 1)} = Z$, then

(B.2-3) $$\hat{\theta}_1^* = \frac{-Z}{1-Z} \ , \quad \hat{\beta}^* = (1 - c_1^*)\ (1 - Z).$$

Thus, using (i), (ii) and (ii*) through (x*) gives $\hat{\theta}_1^*$, $\hat{\theta}_2^*$, $\hat{\theta}_3^*$, $\hat{\eta}_0^*$, $\hat{\eta}_1^*$, $\hat{\eta}_2^*$, $\hat{\eta}_3^*$, $\hat{\lambda}_1^*$, $\hat{\lambda}_2^*$, $\hat{\lambda}_3^*$, $\hat{\beta}^*$ in terms of c_i, c_i^*'s only. It remains to calculate α_1, α_2, β .

From (iii)

(B.2-4) $$(\alpha_1 \beta) = \frac{c_2}{\hat{\theta}_3^*}$$

Substitute B.2-4 into (viii) to get $1 - \beta + \frac{c_2 \hat{\lambda}_1^*}{\hat{\theta}_3^*} = c_8$, and

(B.2-5) $$\hat{\beta} = 1 + \frac{c_2 \hat{\lambda}_1^*}{\hat{\theta}_3} - c_8 \ .$$

From (iii) we have,

(B.2-6) $$\hat{\alpha}_1 = \frac{c_2}{\theta_3^* \beta} .$$

From (iv)

$$\hat{\alpha}_1 \hat{\beta} \hat{\eta}_0^* + \alpha_2 \hat{\beta} = c_4 , \text{ thus}$$

(B.2-7) $$\hat{\alpha}_2 = \frac{c_4}{\beta^*} - \hat{\alpha}_1 \hat{\eta}_0^* .$$

This completes the proof that (B.2-1), (10) in the text, is at least just-identified. In calculating this particular solution we did not use equations (ii), (v), (vi), (vii), (ix) and (x). Thus (10) is overidentified at least with respect to some parameters.

APPENDIX C FOR:
THE LAW OF ONE PRICE IN INTERNATIONAL COMMODITY MARKETS: A REFORMULATION AND SOME FORMAL TESTS

MYOPIC EXPECTATIONS SPECIFICATION

Aris Protopapadakis

and

Hans Stoll

APPENDIX C
REGRESSION RESULTS FOR MYOPIC EXPECTATIONS

C-1: APPENDIX TABLE C-1

Formal Tests of the Law of One Price For Myopic Expectations (1st Order Adjustment)

Commodity		Unconditionala					Conditional
		H_0^1 (3DF)	H_0^2 (4DF)	H_0^3 (5DF)	H_0^4 (6DF)	H_0^5 (n-stat)	H_0^3 (2DF)
Cocoa	NR	21.10‡	21.78‡	21.78‡	23.60‡	58.03{:	0.68
"	FR	16.08‡	17.50‡	21.88‡	22.16‡	45.12‡	5.80
Coffee	NR	15.16*	22.37*	25.30*	19.02‡	88.53‡	10.15b
"	FR	14.07*	17.47‡	19.93‡	22.22‡	107.92‡	5.86c
SoyB	NR	6.97	12.24†	12.33†	14.28†	35.63‡	5.36
"	FR	15.16‡	16.10‡	17.54‡	19.16‡	23.82‡	2.38
Sugar	NR	3.87	5.11	13.08†	13.72†	32.22‡	9.21†
"	FR	4.50	7.82	13.69†	15.16†	35.19‡	9.19†
Wheat	NR	1.76	3.03	7.34	8.31	176.16‡	5.58
"	FR	1.92	4.29	4.29	4.40	70.64‡	2.37
Rubber	SP	13.22‡	17.68‡	20.30‡	22.58‡	102.39‡	7.08†
Wool	SP	2.44	3.22	7.60	7.60	103.75‡	5.16
Copper	SP	6.48	8.08	8.50	13.90†	314.23‡	2.02
Silver	SP	17.96‡	19.92‡	51.30‡	52.24‡	78.03‡	33.34‡
"	3-M	12.61*	10.61†	17.36d	12.49	25.00‡	4.76
"	6-M	4.89	5.45	27.25‡	29.22‡	22.22‡	22.36‡
Lead	SP	9.60†	19.56‡	19.56‡	21.04‡	243.00‡	9.96‡
Tin	SP	12.28‡	17.54‡	22.64‡	24.68‡	77.89‡	10.36‡
Zinc	SP	7.00	8.58	9.70	16.44†	495.00‡	2.70
Simultaneous Estimation With Two Maturities:		(5DF)	(6DF)	(7DF)	(8DF)		
Cocoa		12.64†	13.08†	14.50†	16.28†	86.50‡	1.76
Coffee		4.94	5.94	6.12	7.78	136.71‡	1.18
Sugar		22.14‡	22.14‡	23.88‡	38.90‡	339.00‡	1.74
Wheat		13.86†	14.08†	14.10†	14.36	247.50‡	0.26

Notes for Table C-1

The test statistics for the null hypotheses H_0^1, H_0^2, H_0^3, and H_0^4 are asymptotically χ^2-distributed at the indicated degrees of freedom. The null hypothesis is rejected if the test statistic is larger than a critical value.

The test statistic for H_0^5 is asymptotically normally distributed (n-statistic), and it is the number of standard deviations by which the estimate of β differs from 1.00. The reported estimate of β is from the regression in which $\beta = \beta^*$.

(†) significant at the 95-percent level,

(‡) significant at the 99-percent level,

(*) did not attain the global maximum.

[a]Unconditional tests of the H_0^i's use equation (7) as a reference. These tests are based on the list of variables implied by the lagged adjustment model. The conditional test of H_0^3 is conditioned on the maintained hypothesis that the first-order lagged adjustment specification is the "correct" model.

[b]The hypothesis appears to be rejected. However, since both the regressions used to test H_0^1 and H_0^3 may not have attained a global maximum, it is not possible to perform the test. The value of the statistic is between 0.00 and 25.30.

[c]The hypothesis does not appear to be rejected. However since the regression used to test H_1^0 may not have attained a global maximum, it is not possible to perform the test adequately. The value of the statistic is between 5.86 and 19.93.

[d]H_0^1 and H_0^3 are apparently rejected, but the algorithm did not reach the global maximum. The minimum log likelihood value that could be assigned to the regression used to test H_0^1 is that of H_0^2, (10.61) and to H_0^3 that of H_0^4(12.49). These values lead to no rejection of H_0^1 and H_0^3 at the 99-percent level.

χ^2 - Distribution

Significance Level	Degrees of Freedom 1	2	3	4	5	6	7	8	9	10
95	3.84	5.99	7.81	9.49	11.07	12.59	14.07	15.51	16.92	18.31
99	6.63	9.21	11.34	13.28	15.09	16.81	18.48	20.09	21.67	23.21

APPENDIX TABLE C-2

Results From Selected LOP Test Regressions For Myopic Expectations[a,b]
(1st Order Adjustment)

Commodity	α_0	α_1	β	β^*	No. of Observ.	H-Statistic and D.W. Coefficient[c] Dom.	For.	Notes
Cocoa Near	-.042 (.027)	.999 (.013)	.056 ‡ (.028)	.212 ‡ (.018)	281	.33 (1.72)	.76 (1.69)	
	-.040 (.006)	--	.147 ‡ (.015)	--		.38 (1.69)	.78 (1.68)	
Cocoa Far	-.019 (.011)	.990† (.005)	.264 ‡ (.024)	.331 ‡ (.020)	324	-1.08 (2.08)	-2.51† (2.13)	
	-.041 (.003)	--	.278 ‡ (.016)	--		-1.05 (2.06)	-2.50† (2.14)	
Coffee Near	Convergence Difficulties				215			Convergence Difficulties
	.182 (.029)	--	.035 ‡ (.011)	--		1.26 (1.48)	1.35 (1.48)	
Coffee Far	.426 (.093)	.891† (.049)	.036 ‡ (.023)	.56 ‡ (.021)	248	1.69 (1.45)	.79 (1.59)	Convergence Difficulties
	.200 (.034)	--	.029 ‡ (.009)	--		1.68 (1.44)	.81 (1.60)	

Table C-2 (Cont'd. - Page 2)

Commodity	α_0	α_1	β	β^*	No. of Observ.	H-Statistic and D.W. Coefficient[c] Dom.	For.
Soybeans Near	-.090 (.197)	.964 (.086)	.227 ‡ (.045)	-.004 ‡ (.056)	110	.13 (1.89)	.26 (1.73)
	-.165 (.014)	--	.106 ‡ (.025)	--		-.02 (1.97)	.32 (1.66)
Soybeans Far	.004 (.076)	.936 (.033)	.358 ‡ (.041)	-.134 ‡ (.062)	95	.25 (1.75)	1.09 (1.46)
	-.152 (.010)	--	.142 ‡ (.036)	--		.51 (1.71)	1.14 (1.36)
Sugar Near	-.226 (.060)	1.052† (.023)	.252 ‡ (.032)	.329 ‡ (.041)	123	1.03 (1.46)	-.24 (1.46)
	-.090 (.012)	--	.191 ‡ (.025)	--		1.12 (1.41)	-.20 (1.51)
Sugar Far	-.158 (.026)	1.030 ‡ (.010)	.172 ‡ (.047)	.298 ‡ (.021)	161	1.55 (1.61)	1.27 (1.73)
	-.084 (.005)	--	.205 ‡ (.023)	--		1.55 (1.58)	1.38 (1.72)
Wheat Near	-10.827 (14.205)	4.863 (5.202)	.011 ‡ (.012)	.010 ‡ (.013)	160	1.77 (1.52)	-.76 (2.02)
	-.276 (.082)	--	.014 ‡ (.006)	--		1.78 (1.52)	-.73 (2.02)

Table C-2 (Cont'd. - Page 3)

Commodity	α_0	α_1	β	β^*	No. of Observ.	H-Statistic and D.W. Coefficient[c] Dom.	For.
Wheat Far	-.142 (5.258)	.911 (1.922)	.014 ‡ (.012)	.009 ‡ (.016)	182	1.09 (1.82)	-1.48 (2.20)
	-.392 (.156)	--	.010 ‡ (.006)	--		1.09 (1.83)	-1.48 (2.20)
Rubber Spot	-.636 (.218)	1.187† (.072)	.055 ‡ (.014)	.038 ‡ (.025)	263	3.65 ‡ (1.42)	.43 (1.78)
	-.082 (.024)	--	.038 ‡ (.009)	--		3.67 ‡ (1.43)	.42 (1.76)
Wool Spot	-.180 (.048)	1.082† (.040)	.158 ‡ (.011)	.195 ‡ (.020)	273	-4.08 ‡ (2.38)	-1.25 (2.14)
	-.081 (.006)	--	.170 ‡ (.008)	--		-4.05 ‡ (2.36)	-1.37 (2.15)
Copper Spot	.105 (.357)	.974 (.166)	.035 ‡ (.005)	.002 ‡ (.007)	432	-.62 (2.03)	-4.09 ‡ (2.35)
	.036 (.048)	--	.026 ‡ (.003)	--		-.64 (2.04)	-4.17 ‡ (2.28)
Silver Spot	.016 (.002)	.990 ‡ (.001)	.877 ‡ (.007)	.987 (.007)	414	4.55 ‡ (1.47)	-1.79 (1.58)
	-.0002 (.001)	--	.719 ‡ (.004)	--		3.69 ‡ (1.48)	-5.14 ‡ (2.48)

Table C-2 (Cont'd. - Page 4)

Commodity	α_0	α_1	β	β^*	No. of Observ.	H-Statistic and D.W. Coefficient[c] Dom.	For.
Silver 3-Mo.	.006 (.021)	.996† (.002)	.998† (.001)	1.000 (.001)	69	.70 (1.55)	.68 (1.55)
	-.005 (.002)	--	.999 ‡ (.0001)	--		.67 (1.56)	.66 (1.56)
Silver 6-Mo.	-.269 (.040)	1.019 ‡ (.003)	.998† (.001)	1.000 (.001)	54	.86 (1.44)	.88 (1.43)
	-.001 (.001)	--	.998 ‡ (.0001)	--		.28 (1.45)	.29 (1.46)
Lead Spot	.072 (.346)	.999 (.100)	.032 ‡ (.005)	.013 ‡ (.010)	432	-.44 (1.89)	-1.57 (2.00)
	.053 (.039)	--	.028 ‡ (.004)	--		-.46 (1.89)	-1.60 (1.98)
Tin Spot	.122 (.041)	.980 (.011)	.241 ‡ (.018)	.136 ‡ (.023)	428	1.80 (1.82)	-2.77 ‡ (2.21)
	.048 (.004)	--	.190 ‡ (.010)	--		1.70 (1.84)	-2.94 ‡ (2.16)
Zinc Spot	1.169 (1.120)	.687 (.325)	.016 ‡ (.004)	-.007 ‡ (.003)	432	-.80 (2.04)	1.38 (1.83)
	.087 (.137)	--	.010 ‡ (.002)	--		-.79 (2.05)	1.38 (1.78)

TABLE C-2 (Cont'd. Page 5)

Simultaneous Two-Maturity Estimation

Commodity	α_0	α_1	β	β^*	Observ.	H-Statistics[c] Dom.	For.	
Cocoa	-.018	.989	.092 ‡	.185 ‡	259	.28	.07	Near
	(.020)	(.010)	(.020)	(.013)		-.75	-1.99	Far
	-.040	--	.135 ‡	--		.30	.13	Near
	(.005)		(.010)			-.69	-1.90	Far
Coffee	.251	.973	.019 ‡	.066 ‡	186	.54	.23	Near
	(.102)	(.046)	(.014)	(.012)		1.60	.98	Far
	.200	--	.043 ‡	--		.50	.20	Near
	(.022)		(.007)			1.56	.98	Far
Sugar	.572	.692 ‡	-.043 ‡	.020 ‡	119	.46	-.22	Near
	(.289)	(.096)	(.006)	(.008)		.24	-.17	Far
	-.189	--	.017 ‡	--		.59	-.07	Near
	(.107)		(.003)			.44	.07	Far
Wheat	-.592	1.104	.005 ‡	.013 ‡	136	1.34	-.81	Near
	(2.937)	(1.091)	(.009)	(.011)		.67	-1.42	Far
	-.345	--	.010 ‡	--		.84	-.80	Near
	(.097)		(.004)			.66	-1.41	Far

Notes for Table C-2

[a]In the first regression reported for each commodity, $\alpha_2 = \alpha_1$. In the second regression, $\alpha_1 = \alpha_2 = 1.00$, $\beta = \beta^*$. n-statistics are reported in parentheses below the coefficients. These statistics are asymptotically normally distributed. The significance notation for α_1 is with respect to the null hypothesis $\alpha_1 = 1.00$(Long run LOP). The significance notation for β, β^* is with respect to the null hypothesis $\beta, \beta^* = 1.00$.

[b](†) 95 percent significance level.

(‡) 99 percent significance level.

[c]The H-statistics reported are approximate, and they are calculated by regressing the lagged values of the residuals and all the other regressors on the current value of the residuals. The value reported is the t-statistic of the coefficient of the lagged residual. The number in parentheses below the H-statistic is the Durbin-Watson coefficient of the reported regression.

APPENDIX TABLE C-3

Formal Tests of the Law of One Price For Myopic Expectations
(2nd Order Adjustment)

Commodity		Unconditional[a]						Conditional[a]
		H_0^1 (3DF)	H_0^2 (4DF)	H_0^3 (5DF)	H_0^4 (7DF)	H_0^5 (n-stat) β_1	β_2	H_0^3 (2DF)
Cocoa	NR	34.40 ‡	47.94 ‡	47.98 ‡	53.14	43.59 ‡	3.12 ‡	13.58 ‡
"	FR	49.30 ‡	54.62 ‡	57.36 ‡	59.12 ‡	39.00 ‡	5.68 ‡	8.06†
Coffee	NR	15.51 ‡	26.07 ‡	26.09 ‡	30.23 ‡	34.28 ‡	0.96	10.58 ‡
"	FR	29.13*	37.38 ‡	39.40 ‡	42.96 ‡	46.27 ‡	1.64	10.27 ‡
Soybeans	NR	12.38 ‡	13.43 ‡	13.43 ‡	16.85†	24.98 ‡	3.04 ‡	1.04
"	FR	18.89 ‡	19.55 ‡	21.84 ‡	27.78 ‡	16.47 ‡	0.54	2.94
Sugar	NR	13.19 ‡	13.98 ‡	17.65 ‡	26.83 ‡	25.59 ‡	0.74	3.40
"	FR	10.65†	13.31 ‡	17.58 ‡	19.64†	28.25 ‡	0.39	6.93†
Wheat	NR	6.74	9.47	10.81	13.81	38.75 ‡	1.57	4.07
"	FR	10.69†	12.71†	12.81†	16.48†	99.80 ‡	2.57 ‡	5.80
Rubber**	SP	37.80*	38.12 ‡	44.88 ‡	49.26 ‡	37.20 ‡	0.36	7.08†[b]
Wool**	SP	5.90	6.58	9.78	10.32	96.00 ‡	10.88 ‡	3.88
Copper**	SP	25.20 ‡	28.30 ‡	28.76 ‡	42.32 ‡	80.93 ‡	6.17 ‡	3.56
Silver**	SP	33.20 ‡	33.28 ‡	71.50 ‡	120.92 ‡	76.78 ‡	6.92 ‡	38.30 ‡
"	3-M	16.94 ‡	93.31*	19.11 ‡	23.04 ‡	0.00	0.50	2.17
"	6-M	25.71*	24.13*	18.14 ‡	21.55 ‡	14.28 ‡	1.00	0.00c
Lead	SP	10.18†	21.02 ‡	21.24 ‡	23.20 ‡	45.86 ‡	0.45	11.06 ‡
Tin**	SP	55.20 ‡	58.38 ‡	61.80 ‡	93.10 ‡	56.14 ‡	1.47	6.60†
Zinc	SP	23.66 ‡	24.50 ‡	24.88 ‡	34.68 ‡	53.00 ‡	0.89	1.22

Notes for Table C-3

The test statistics for the null hypotheses H_0^1, H_0^2, H_0^3, and H_0^4 are asymptotically χ^2-distributed at the indicated degrees of freedom. The null hypothesis is rejected if the test statistic is larger than a critical value.

Each test statistic for H_0^5 is asymptotically normally distributed (n-statistic), and it is the number of standard deviations by which the estimate of β_1 differs from 1.00, and the estimate of β_2 differ from 0.00. The reported estimates of the β's are from the regression in which $\beta_i = \beta_i^*$.

(†) significant at the 95 percent level,

(‡) significant at the 99 percent level,

(*) did not attain the global maximum,

(**) the first-order adjustment model residuals of this commodity have significant autocorrelation.

[a] Unconditional tests of the H_0^i's use equation (7) as a reference. These tests are based on the list of variables implied by the lagged-adjustment model. The conditional test of H_0^3 is conditioned on the maintained hypothesis that the second-order lagged adjustment specification is the "correct" model.

[b] The hypothesis does not appear to be rejected at the 99-percent confidence level. However since the regression used to test H_0^1 may not have attained a global maximum, it is not possible to perform the test adequately. The value of the statistic is between 7.08 and 44.88.

[c] A mechanical calculation of the test statistic yields -7.57, an improper result. This is because the regression used to test H_0^1 did not attain a global maximum while the regression for H_0^3 did. The value of the statistic is between 0.00 and 18.14.

χ^2 - Distribution

Significance Level	Degrees of Freedom 1	2	3	4	5	6	7	8	9	10
95	3.84	5.99	7.81	9.49	11.07	12.59	14.07	15.51	16.92	18.31
99	6.63	9.21	11.34	13.28	15.09	16.81	18.48	20.09	21.67	23.21

APPENDIX TABLE C-4

Results from Selected LOP Test Regressions For Myopic Expectations[a,b]
(2nd order Adjustment)

Commodity	α_0	α_1	β_1	β_1^*	β_2	β_2^*	No.of Obs.	H-Statistic and D.W. Coefficient[c] Dom.	For.
Cocoa Near	-.060 (.038)	1.002 (.018)	.008 ‡ (.032)	.332 ‡ (.023)	-.119 ‡ (.043)	-.089 ‡ (.023)	258	.22 (1.57)	-1.17 (1.54)
	-.044 (.007)	-- --	.198 ‡ (.018)	-- --	-.073 ‡ (.023)	-- --		.06 (1.54)	-1.15 (1.56)
Cocoa Far	-.023 (.014)	.990 (.007)	.338 ‡ (.025)	.427 ‡ (.022)	-.142 ‡ (.031)	-.113 ‡ (.024)	310	-1.09 (2.08)	-2.51† (2.13)
	-.042 (.004)	-- --	.376 ‡ (.016)	-- --	-.125 ‡ (.022)	-- --		1.14 (1.74)	.26 (1.80)
Coffee Near	.249 (.122)	.998 (.060)	.140 ‡ (.041)	-.019 ‡ (.038)	-.060 (.038)	-.001 (.035)	188	-.13 (1.42)	-.74 (1.51)
	.203 (.027)	-- --	.064 ‡ (.027)	-- --	-.027 (.028)	-- --		.04 (1.47)	-.67 (1.43)
Coffee Far	.438 (.103)	.886† (.046)	.057 ‡ (.031)	.086 ‡ (.036)	-.033 (.027)	-.018 (.037)	222	-.46 (1.48)	-.67 (1.57)
	.205 (.042)	-- --	.061 ‡ (.020)	-- --	-.034 (.021)	-- --		-.40 (1.46)	-.75 (1.56)

TABLE C-4 (Cont'd. Page 2)

Results from Selected LOP Test Regressions
(2nd Order Adjustment and Myopic Expectations)

Commodity	α_0	α_1	β_1	β_1^*	β_2	β_2^*	Obs.	H-Statistic and D.W. Coefficient[c] Dom.	For.
Soybeans Near	-.217 (.474)	1.022 (.207)	.297 ‡ (.062)	.078 ‡ (.062)	-.179 ‡ (.066)	-.061 (.071)	103	-.10 (1.64)	-1.77 (1.69)
	-.165 (.024)	-- --	.178 ‡ (.033)	-- --	-.107 ‡ (.035)	-- --		.-- (1.85)	.-- (1.52)
Soybeans Far	.072 (.084)	.907† (.036)	.482 ‡ (.047)	-.421 ‡ (.088)	-.116† (.045)	.059 (.103)	88	-.65 (1.67)	.24 (1.58)
	-.154 (.011)	-- --	.155 ‡ (.051)	-- --	-.025 (.046)	-- --		-.20 (1.71)	.18 (1.30)
Sugar Near	-.218 (.063)	1.048 (.025)	.145 ‡ (.046)	.412 ‡ (.056)	.108 (.079)	.055 (.113)	108	-1.54 (1.77)	-1.96† (1.42)
	.093 (.012)	-- --	.163 ‡ (.033)	-- --	.058 (.079)	-- --		-1.25 (1.57)	-1.83 (1.61)
Sugar Far	-.155 (.026)	1.029 ‡ (.010)	.163 ‡ (.051)	.249 ‡ (.027)	.032 (.054)	.029 (.042)	152	.01 (1.63)	.31 (1.74)
	-.085 (.005)	-- --	.189 ‡ (.029)	-- --	.016 (.042)	-- --		-.09 (1.58)	.43 (1.70)

TABLE C-4 (Cont'd. Page 3)

Results from Selected LOP Test Regressions
(2nd Order Adjustment and Myopic Expectations)

Commodity	α_0	α_1	β_1	β_1^*	β_2	β_2^*	Obs.	H-Statistic and D.W. Coefficient[c] Dom.	For.
	-273.458	100.778	.002 ‡	.027 ‡	-.001	-.016		.61	-.23
	(6917.75)	(2526.32)	(.043)	(.038)	(.033)	(.044)		(1.57)	(1.48)
Wheat Near							149		
	-.28	--	-.027 ‡	--	.040	.--		.34	-.26
	(.087)	--	(.026)	--	(.026)	.--		(1.58)	(2.08)
	.372	.710	-.036 ‡	.137 ‡	.048	-.129 ‡		-2.24†	-.55
	(.677)	(.248)	(.061)	(.046)	(.062)	(.042)		(1.94)	(2.01)
Wheat Far							174		
	-.442	--	.065 ‡	--	-.056†	.--		-1.44	-.65
	(.209)	--	(.022)	--	(.022)	.--		(1.74)	(2.12)
	-.982	1.295 ‡	.021 ‡	.106 ‡	.022	-.056		-5.13 ‡	.47
	(.316)	(.103)	(.022)	(.044)	(.029)	(.036)		(1.58)	(1.63)
Rubber Spot							247		
	-.080	--	.044 ‡	--	-.010	--		-5.19 ‡	.37
	(.026)	--	(.026)	--	(.027)	--		(1.52)	(1.69)
	-.186	1.087	.332 ‡	.324 ‡	-.194 ‡	-.144		-.74	.23
	(.060)	(.049)	(.022)	(.029)	(.027)	(.365)		(2.10)	(1.86)
Wool Spot							258		
	-.080	--	.328 ‡	--	-.174 ‡	--		-.59	.37
	(.008)	--	(.007)	--	(.016)	--		(2.10)	(1.82)

TABLE C-4 (Cont'd. Page 4)

Results From Selected LOP Test Regressions
(2nd Order Adjustment and Myopic Expectations)

Commodity	α_0	α_1	β_1	β_1^*	β_2	β_2^*	No. of Obs.	H-Statistic and D.W. Coefficient[c] Dom.	For.
Copper Spot	.330 (.311)	.855 (.142)	.030 ‡ (.023)	.192 ‡ (.014)	.007 (.023)	-.194 ‡ (.017)	414	.09 (2.04)	1.29 (2.01)
	.037 (.055)	-- --	.102 ‡ (.011)	-- --	-.074 ‡ (.012)	-- --		.06 (1.94)	1.64 (2.13)
Silver Spot	.013 (.003)	.991 ‡ (.001)	.731 ‡ (.006)	1.061 ‡ (.006)	.081 ‡ (.007)	-.105 ‡ (.004)	406	-.96 (1.75)	-1.86 (1.78)
	-.0005 (.001)	-- --	.716 ‡ (.004)	-- --	-.042 ‡ (.006)	-- --		2.77 ‡ (1.33)	2.89 ‡ (2.27)
Silver 3 mos.	-.096 (.007)	.994 ‡ (.001)	1.000 (.001)	1.000 (.001)	-.001 (.001)	.001 (.001)	58	.41 (1.43)	.43 (1.43)
	-.004 (.002)	-- --	1.000 (.001)	-- --	-.001 (.001)	-- --		-.33 (1.58)	-.33 (1.58)
Silver 6 mos.	.244 (.066)	1.017 ‡ (.005)	.993 (.049)	.998 (.005)	.010 (.004)	-.009 (.004)	45	.10 (1.44)	.01 (1.45)
	-.001 (.001)	-- --	.999 ‡ (.001)	-- --	.001 (.001)	-- --		.33 (1.38)	.33 (1.39)

TABLE C-4 (Cont'd Page 5)

Results from Selected LOP Test Regressions
(2nd Order Adjustment and Myopic Expectations)

Commodity	α_0	α_1	β_1	β_1^*	β_2	β_2^*	No. of Obs.	H-Statistic and D.W. Coefficient[c] Dom.	For.
	.251	.945	.019 ‡	.061 ‡	.011	-.051		.02	2.70 ‡
	(.399)	(.118)	(.036)	(.036)	(.036)	(.036)		(2.02)	(1.89)
Lead Spot							425		
	.053	--	.037 ‡	--	-.010	--		-.02	2.87 ‡
	(.041)	--	(.021)	--	(.022)	--		(1.99)	(1.93)
	.119	.981	.090 ‡	.370 ‡	.112 ‡	-.243 ‡		.68	.81
	(.049)	(.013)	(.021)	(.024)	(.026)	(.024)		(2.01)	(1.84)
Tin Spot							419		
	.049	--	.220 ‡	--	-.030	--		-.10	.73
	(.004)	--	(.014)	--	(.020)	--		(1.81)	(2.09)
	.669	.835	.066 ‡	-.069 ‡	-.051	.064†		-.21	.70
	(1.408)	(.413)	(.055)	(.032)	(.052)	(.031)		(1.95)	(1.96)
Zinc Spot							424		
	.081	--	-.007 ‡	--	.017	--		-.11	-.90
	(.138)	--	(.019)	--	(.019)	--		(2.12)	(1.82)

Notes for Table C-4

[a]In the first regression reported for each commodity, $\alpha_2=\alpha_1$. In the second regression, $\alpha_1=\alpha_2=1.00$, $\beta_1=\beta_i^*$. n-statistics are reported in parentheses below the coefficients. These statistics are asymptotically normally distributed. The significance notation for α_1 is with respect to the null hypothesis, $\alpha_1 = 1.00$ (Long run LOP). The significance notation for β, β^* is with respect to the null hypothesis, β_1, $\beta_1^*=1.00$ and β_2, $\beta_2^*=0.00$.

[b](†) 95 percent significance level.

(‡) 99 percent significance level.

[c]The H-statistics reported are approximate, and they are calculated by regressing the lagged values of the residuals and all the other regressors on the current value of the residuals. The value reported is the t-statistic of the coefficient of the lagged residual. The number in parentheses below the H-statistic is the Durbin-Watson coefficient of the reported regression.

APPENDIX C

C.2. Second-order Adjustment:

In order to account for the autocorrelation of the residuals of equations incorporating a first-order adjustment process, we develop equations that assume a second-order adjustment mechanism. Assume that actual prices adjust to their deviation from their long-run values and also to past deviations from past long-run values.

(C.2-1) $$\tilde{p} - p_{-1} = \beta_1(\hat{p} - p_{-1}) + \beta_2(\hat{p}_{-1} - p_{-2}) ,$$

$$\tilde{p}^* - p^*_{-1} = \beta^*_1(\tilde{p}^* - p^*_{-1}) + \beta^*_2(\tilde{p}^*_{-1} - p^*_{-2}) ,$$

which yields

$$\tilde{p} = \beta_1\hat{p} + \beta_2\hat{p}_{-1} + (1-\beta)p_{-1} - \beta_2 p_{-2} + \tilde{v},$$

$$\tilde{p}^* = \beta^*_1\tilde{p}^* + \beta^*_2\tilde{p}^*_{-1} + (1-\beta^*_1)p^*_{-1} - \beta^*_2 p^*_{-2} + \tilde{v}^* .$$

It is expected that $0 \leq \beta_1 < 1$, $0 \leq \beta_2 < 1$. As β_1, β^*_1 approach unity, and as β_2, β^*_2 approach zero, actual prices gets closer to "desired" prices. Substitute equations (3) and (5) from the text into (C.2-1) to get:

(C.2-2) $$\tilde{p} = \alpha_0(\beta_1+\beta_2) + \alpha_1\beta_1 p^* + \alpha_2\beta_1\pi + \alpha_1\beta_2 p^*_{-1} + \alpha_2\beta_2\pi_{-1}$$

$$+ (1-\beta_1)p_{-1} - \beta_2 p_{-2} + \tilde{w} ,$$

$$\tilde{p}^* = -\frac{\alpha_0}{\alpha_1}(\beta^*_1+\beta^*_2) + \frac{\beta^*_1}{\alpha_1}\tilde{p} - \frac{\alpha_2\beta^*_1}{\alpha_1}\pi + \frac{\beta^*_2}{\alpha_1}p_{-1} - \frac{\alpha_2\beta^*_2}{\alpha_1}\pi_{-1}$$

$$+ (1-\beta^*_1)p^*_{-1} - \beta^*_2 p^*_{-2} + \tilde{w}^* .$$