The Pricing of Treasury Bond Futures:
The Quality Variation Option

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I. Introduction

Like many other futures contracts, the Treasury Bond (T-Bond) futures contract allows the holder of a short position to satisfy the contract by delivering one of a variety of T-Bonds. Accordingly, the traditional approach to pricing such contracts has concentrated almost exclusively on the concept of "cheapest to deliver," which asserts that the price of the futures contract will be determined by the minimal cost of carrying any deliverable bond to the maturity of the contract. Several recent papers (Garbade and Silber (1981), Gay and Manaster (1983)) have presented alternative approaches that explicitly incorporate the option inherent in the short position of switching from one quality of deliverable goods to another at the time of maturity. Specifically, if at the time the contract is purchased, knowledge of which of the deliverable assets will be cheapest to deliver is uncertain, then the quality option will have value. This option is referred to as the "quality variation" option by Cox, Ingersoll and Ross (CIR) (1981). The greater the value of this option, the lower will be the *ex ante* holding period returns for delivery of a given T-Bond.

The quality variation option inherent in a T-Bond futures is a complicated security that does not fit the American or European call option model, nor is it adequately described by the two asset exchange option model developed by Margrabe (1978).[1] Despite the lack of an explicit pricing formula, empirical evidence on the existence and determinants of such an option in the T-Bond futures market may provide insight into developing such a framework and in determining appropriate financial futures prices. Gay and Manaster (1983), using Margrabe's framework, find empirical support in the CBOT wheat market for a quality variation option having positive value. These authors argue that such an option is likely to be extremely important in financial futures since the differences among securities in the deliverable set are large and variable, both characteristics that would tend to make the quality variation option more valuable.

The purpose of this paper is to determine if the quality variation option inherent in T-bond futures contract is valued by the market and, hence, affects futures prices. This paper is divided into four sections. In Section II, we discuss the delivery process of the T-bond futures contract and the holding period returns received by traders in T-Bonds. Section III discusses the quality variation option inherent in T-Bond futures contracts. Section IV examines the pricing of T-Bond futures contracts, holding period returns and the value of the quality variation option. We find a significant option value, positively related to both the price variability of the T-bond and the length of the holding period. Section V contains our summary and conclusions.

II. Treasury Bond Futures, Delivery and Holding Period Returns

The standard delivery vehicle of the T-Bond futures contract is an 8% T-bond with at least 15 years to first call. Such bonds may be delivered against the T-Bond futures contract at par on any day of the contract month. Thus, for example, if the June 1984 T-Bond futures price is 70.02 (70 and 2/32) on February 8, 1984, then the short may deliver any 8% bond with at least 15 years to call against this contract throughout June for $70,062.50 (.700625*100,000) plus accrued interest on the bond. If a different bond is delivered against the contract, a conversion factor that takes into consideration the actual coupon rate and maturity of the T-Bond being delivered is used to adjust the settlement price.

By simultaneously purchasing a deliverable T-Bond and shorting a T-Bond futures contract, a hedger can lock in a guaranteed rate of return on his investment. To illustrate, consider a short who on February 8, 1984 decides to undertake such a strategy by purchasing a 10 3/4 May 2003 T-Bond for purposes of delivery on June 29, 1984 (the last delivery day in June) and simultaneously shorting a June T-Bond futures at the above price. On February 8, the delivery bond cost 91-25 plus accrued interest, bringing its purchase price to $94,291. On June 29, the delivery bond has 18 3/4 years to maturity (rounding down to the nearest quarter) and, accordingly, a conversion factor of 1.2645. If, on June 29, the short chose to deliver this bond against the futures contract, his cash flow would be:

DATE	CASH FLOW	EXPLANATION
February 8	-94,291	Price of bond plus accrued interest
May 15	5,375	Interest payment on bond
June 15	89,880	(Conversion factor times futures price) plus accrued interest

That is, for a cash outlay (or borrowing) on February 8 of 94,291, the trader can guarantee receiving by June 29 95,255. The internal rate of return of this cash flow is 2.75% annually (continuously compounded). We denote this return the holding period return (HPR).[2]

The surprising fact about the holding period returns of the various deliverable T-Bonds is that they vary widely. Table 1 reports the holding period returns for all deliverable T-Bonds on February 8, 1984 assuming delivery against the June, September and December 1984 T-Bond futures on the last delivery day of these respective delivery months. Note that of these 33 holding period returns, six are negative and none exceed the risk free T-Bill rate over the holding period.[3] Also note that the HPR rises with the more distant contracts, an observation that we conjecture is due to the increased importance of marking to market in determining the difference between forward and futures prices the further into the future is the contract month.[4]

III. The Quality Variation Option

The quality variation option can be illustrated by considering the strategy described in the previous section. In that strategy and example, the investor held the deliverable bond until the delivery month, delivered this bond against the futures contract and received the corresponding holding period return. This is not, however, the investor's only choice: the investor can at any time before the ultimate date of delivery, sell the bond he currently holds and buy some other deliverable security. There is no limit to the number of times this exchange can be made. At each point in time, the investor can calculate exactly the profit or loss involved in selling the bond he holds and purchasing another bond to hold for delivery.

It is this ability to exchange deliverable bonds that gives rise to the quality variation option. More specifically, the quality variation option reflects the opportunity inherent in the hedged position described by a long position in a deliverable T-Bond and a short position in the T-Bond futures contract, to make a series of trades by which the investor can improve the holding period rate of return derived from delivering the bond ultimately held against the T-bond future purchased at the initial point in time.[5]

As an example, consider the following two bonds and their price data on February 8, 1984:

	Bond A	Bond B
Coupon	10 3/4	14 1/4
Maturity	May 2003	February 2002
Price	91-25	118-12
Yield to Maturity (YTM)	11.85	11.77
HPR	2.75%	-4.13%

where the HPR is calculated assuming a short position in the June, 1984 T-Bond futures and delivery on June 29, 1984. Note that despite having virtually identical yields, the HPR for the two bonds is very different and that of Bond B is, in fact, negative. Our argument is that this difference can at least be partially explained by the quality variation option.

To see this, assume that the trader purchases Bond B and sells a June T-Bond futures contract for purposes of delivery on June 29. Suppose that immediately after this transaction, the yield to maturity on both bonds changes such that the discount factor, defined as 1 + YTM, changes by one percent. In this case, the price of Bond B will rise to 126-04, or an increase in price of 7-24. The price of Bond A rises to 98-04, or an increase in price of 6-11. Subsequent to the drop in YTM, the investor can sell bond B and purchase bond A for an effective price of 98-04 - (126-04 - 118-12) = 90-12. If bond A is then used as the delivery instrument on the June futures contract, the locked in HPR after the bond swap exceeds 2.75%.

Although this example is stylized in that the imposed interest rate change assumes a flat yield curve and uniform shifts in the term structure, the illustration makes it clear that there is an exchange option in T-bonds that can change guaranteed HPR. To the extent this is the case, we would expect the option to be valued by the market and, ceteris paribus the higher the value of the option associated with a given T-bond, the lower the guaranteed HPR from delivering that T-bond should be.

One issue, then, is what determines the value of the exchange or quality variation option. We posit that a primary factor that affects the option's value will be the price sensitivity of the deliverable T-Bond to changes in interest rates. Specifically, for a given change in interest rates, the larger the change in the price of the bond the more likely an exchange of deliverable bonds is to be profitable. Accordingly, the more price sensitive the deliverable T-Bond, the greater the value of the quality variation option and the lower the HPR. This can be seen in the previous example: the greater the price change in bond B when interest rates fell, the greater the HPR that could be locked in by selling bond B and buying bond A. Thus the quality variation option and the price sensitivity of the T-Bond are directly related, which implies an inverse relation between HPR and price sensitivity.

IV. The Quality Variation Option and Holding Period Returns: Empirical Evidence

Ideally, we would be able to directly determine both the existence and value of the quality valuation option. Unfortunately, the lack of an explicit pricing formula makes this impossible. Given the above discussion, however, we can infer the existence of a positively valued quality variation option by looking at holding period returns of delivering a variety of T-bonds against the short futures position and regressing these HPR on the price sensitivity of the delivery instrument. Specifically, since price sensitivity and the option value are positively related, we expect that in a regression of HPR on a bond's price sensitivity (PS), the coefficient on PS should be negative and significant. To capture the marking-to-market component of HPR (see footnote 3) we also estimate regressions that include the variable DAYS, which is defined as the number of days from the initiation of the hedged position to the actual delivery day. We expect the coefficient on DAYS to be positive.

To estimate the regression, we identified all T-Bonds that on Wednesday, February 8, 1984 qualified as deliverable against the June, September and December T-Bond futures contracts of that year. 11 bonds, which correspond to those listed in Table 1, qualified as deliverable. Given these 11 bonds and 3 delivery dates, we were able to calculate 33 HPR for February 8. The necessary data were obtained from the Wall Street Journal and the calculations were performed, as before, assuming delivery on the last day of the delivery month.[6] For every Wednesday for the next nine weeks (through April 11), we use these same 11 bonds and delivery assumptions to calculate HPR. In total, 330 HPR are calculated. The distribution of these HPR for the entire sample and broken down by contract month are reported in Table 2. As can be seen, there is a wide range of values, including a good proportion of negative values.[7]

To calculate the price sensitivity of each bond, we start with the concept of duration. As is well known, the duration of a bond, denoted D, is equal to the price elasticity of a bond with respect to the discount factor. The standard duration formula is given by the expression

$$D = \sum_{t=1}^{n} t \left| \frac{\dfrac{C_t}{(1 + r)^t}}{\sum_{t=1}^{n} \dfrac{C_t}{(1 + r)^t}} \right| \tag{1}$$

where r is the interest rate, n is the number of time periods to maturity, C is the cash flow of the bond and t denotes the time period. This formula assumes a flat yield curve and parallel shifts in r. Although these assumptions do not accurately capture changes in r, empirical studies have indicated that there is little gain from using more complex duration formulas. The elasticity implied by the duration formula implies that one percent change in 1 + r leads to a dollar price change in the bond of D times the initial price of the bond, P_0. Assuming r = YTM, we define the price sensitivity as the dollar price change for a one percent change in 1 + YTM, or

$$PS = DP_0/(1 + YTM) \ . \tag{2}$$

The distributional characteristics of PS are reported in the last row of Table 2. As with the HPRs, these price sensitivities show considerable variation.

The results of estimating the regressions

$$HPR_{it} = a + \beta PS_{it} + \varepsilon_{it} \tag{3}$$

and

$$HPR_{it} = a' + \beta' PS_{it} + \gamma' DAYS + \varepsilon'_{it} \tag{4}$$

for hedge i at time t are reported in the first two rows of Table 3. As can be seen, the coefficient on PS_{it} is negative and significant at the one percent level in both specifications. This suggests that the quality variation option does have market value and that this option affects T-Bond futures prices. The magnitude of β suggests that an increase in PS of $100 decreases the HPR by approximately .4 percent. Given the range of HPR, this represents a fairly large change. We view this as further evidence of the importance of the quality valuation option and its relevance in futures pricing. Additionally, the coefficient DAYS has the expected positive sign.

We also divided the sample into three parts, based on the delivery month and estimated equations (3) and (4) for each subsample. The results of this estimation for the June, September and December contracts are reported in rows 3 through 8, respectively of Table 3. In each case, the coefficient on PS is negative and significant at the one percent level. Finally, we divided the sample into 30 parts based on the initial day of the hedge and the three futures contract delivery months and estimated equation (3) for each subsample.[8] To conserve space, in Table 4 we report only the coefficient and t-statistic on PS from these regressions. The results are quite strong. In 26 of the 30 regressions, the coefficient on PS is negative and significant at the one percent level. In all but one case, this coefficient is negative and significant at the ten percent level. The results from these subsamples provide added support for the assertion that the quality variation option is valued by the T-Bond futures market and that this valuation affects futures prices.

V. Summary and Conclusions

This paper has attempted to evaluate whether the quality variation option inherent in the T-Bond futures contract is valued by the market and hence can affect futures prices. Since there is no explicit pricing formula, we examine this issue indirectly by looking at holding period returns for a variety of deliverable bonds. These returns should be inversely related to the value of the option. Using price sensitivity to proxy for the value of the option, we find a strong negative relationship between this variable and holding period returns. This suggests that the quality variation option may be an important feature in futures pricing. At a minimum, these results may be useful in developing theoretical models of futures pricing and developing or examining differences in trading strategies.

Table 1

Holding Period Returns, Deliverable T-Bonds
Against June, September and December Contracts,
February 8, 1984

Bond		Holding Period Returns, by Contract Month		
Coupon	Maturity	June	September	December
11 3/4	Feb., 2001	-3.92	.64	2.82
13 1/8	May, 2001	-4.12	.53	2.75
13 3/8	Aug., 2001	-3.89	.69	2.87
15 3/4	Nov., 2001	-8.56	-2.24	.79
14 1/4	Feb., 2002	-4.13	.54	2.77
11 5/8	Nov., 2002	.83	3.56	4.91
10 3/4	Feb., 2003	2.56	4.62	5.68
10 3/4	May, 2003	2.75	4.73	5.75
11 1/8	Aug., 2003	2.65	4.67	5.72
11 7/8	Nov., 2003	2.40	4.53	5.61
10 3/8	Nov., 2007	7.03	7.37	7.64

Table 2

Distribution Characteristics,
HPR and PS

Variable	Delivery Month	Mean	Standard Deviation	Fractile Distribution				
				.10	.25	.50	.75	.90
HPR	All	.4346	7.61	-9.556	-2.462	1.700	4.632	7.519
HPR	June	-3.717	10.206	-17.666	-10.381	-3.312	.149	9.688
HPR	September	1.719	4.222	-4.182	-1.855	2.110	3.832	7.243
HPR	December	3.849	4.670	.414	1.607	4.146	5.395	7.516
PS	All	6562	689.4	5835	6052	6408	6876	7740

Table 3

Regression Results, Entire Sample and by Contract Month[a]

Delivery Month	Intercept	PS	DAYS	R^2	N
All	29.34* (7.94)	-.0044* (7.87)		.158	330
All	22.37* (7.21)	-.0049* (10.59)	.0514* (12.38)	.427	330
June	36.70* (4.31)	-.0061* (4.77)		.174	110
June	19.99* (3.57)	-.0097* (11.22)	.3679* (12.59)	.667	110
September	27.78* (8.84)	-.0040* (8.52)		.402	110
September	12.83* (4.33)	-.0051* (13.35)	.1109* (8.72)	.650	110
December	23.54* (6.12)	-.0030* (5.15)		.197	110
December	1.131* (.218)	-.0039* (7.25)	.099 (5.41)	.371	110

[a] t-statistics in parentheses below coefficients.

*coefficient significant at the one percent level.

Table 4

Regression Coefficients on PS, By Hedge Initiation Date and Contract Month[a]

Month	Day	Contract Month	PS	t(PS)
February	8	June	-.0077	1.71*
		September	-.0042	1.71*
		December	-.0081	1.55
February	15	June	-.0060	3.03**
		September	-.0034	4.13**
		December	-.0024	4.11**
February	22	June	-.0082	2.16*
		September	-.0042	4.51**
		December	-.0029	4.51**
February	29	June	-.0079	4.87**
		September	-.0045	4.88**
		December	-.0031	4.87**
March	7	June	-.0045	4.85**
		September	-.0053	4.84**
		December	-.0037	4.88**
March	14	June	-.0103	5.81**
		September	-.0056	5.61**
		December	-.0038	5.61**
March	21	June	-.0113	5.69**
		September	-.0060	5.83**
		December	-.0041	5.83**
March	28	June	-.0116	5.66**
		September	-.0059	5.66**
		December	-.0039	5.65**
April	4	June	-.0134	5.71**
		September	-.0065	5.70**
		December	-.0043	5.69**
April	11	June	-.0139	5.41**
		September	-.0064	5.39**
		December	-.0042	5.38**

*t-statistic significant at the ten percent level.
**t-statistic significant at the one percent level.

[a] N = 11 for all regressions.

FOOTNOTES

[1] This is because the models cited assume the risk-free rate is constant. Since the value of the quality variation option derives from a fluctuating risk-free rate, these cannot be used to value the quality variation option.

[2] The calculation of the HPR assumes that the futures contract is equivalent to a forward contract. Recent literature (Cox, Ingersoll and Ross (1981) and Jarrow and Oldfield (1981)) shows that when there is no delivery option futures and forward prices will diverge by a term which representes the cost of marking to market the futures contract. For recent empirical evidence on the deviation of futures from forward prices, see Capozzi and Cornell (1979), Cornell and Reinganum (1983), and Mishra, Subrahmanyam and Urich (1984).

[3] On February 8, 1984, the yield on Treasury Bills with an approximate mid-June maturity was 9.48.

[4] Since we have not been able to derive an analytic formulation for futures pricing which includes both marking to market and the quality variation option, we base our conjecture on the following heuristic theory. At time t, the difference between the forward price for a specific commodity for delivery at time T (denoted by H(t, T)) and a futures price for one class of commodities at time T (denoted by G(t, T)) may be decomposed into two components. The first component is the cost of marking to market, which is independent of the specific commodity and depends only on the delivery class of commodities as a whole. The second component is the option value associated with the futures contract and the particular commodity with which a hedge is initiated. In symbols,

$$H_j(t, T) = G_j(t, T) - MTM(t, T) + QV_j(t, T)$$

where $MTM(t, T)$ is the cost of marking to market and $QV_j(t, T)$ is the quality variation option value associated with the contract and specific delivery instrument j.

CIR prove that for financial instruments, MTM is increasing in T, which is reflected in the higher HPR for more distant contracts reported in Table 1.

[5] This quality variation option is similar to what is frequently termed the wild-card option. The wild-card option is used to describe the process by which the short holds a deliverable bond in the delivery month, notifies the clearing house of his intention to deliver, and then uses the two days before delivery occurs to attempt to sell his bond and purchase one which is cheaper to deliver.

[6] Calculations were performed assuming a variety of delivery days, ranging from mid-month to the last day of the month. The results for these different delivery days are qualitatively identical to those reported in the text.

[7] A small proportion of these returns--about one in twenty--exceed the risk-free rate. As discussed in footnote 3, we do not consider these to be pure arbitrage opportunities.

[8] We do not estimate equation (4) for each of these subsamples since the variable DAYS is constant in each subsample.

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