AN EMPIRICAL EXAMINATION OF THE IMPLICATIONS OF ARBITRAGE PRICING THEORY

By

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1. Introduction

The two key implications of arbitrage pricing theory (APT), as developed by Ross (1976,1977) and subsequently tested by Roll and Ross (1980) and many others, are first that only covariance measures of risk (beta coefficients on different factors) are relevant to the relative pricing of risky assets, and second that the constant term or intercept in the linear relationship of expected returns on these risky assets to their covariance measures (or factor loadings) is either the risk-free rate of return or zero-beta rate.[1] These two implications characterize all modern capital asset pricing theory, including the well-known capital asset pricing model (CAPM).

This paper will present a comprehensive set of tests of both of these implications of the APT, which lead to substantially different conclusions from those drawn by RR. We find a very limited relationship between the expected returns and the covariance measures of risk (factor loadings). Furthermore, unique variance measures of risk, while generally making only small contributions to the explanation of asset returns, turn out to be significant about as frequently as the covariance measures of risk -- which is inconsistent with the APT model. The intercept tests are more mixed but provide only limited support to the APT.[2]

Before discussing in detail our tests of the covariance and intercept implications of the APT, we should note that the procedures followed in these tests, like those used in virtually all previous tests of the APT, are subject to several basic limitations in the application of factor analysis which we have described in an earlier paper (see Dhrymes, Friend and Gultekin (1983)). As demonstrated in that paper, there is a general non-equivalence of factor analyzing small groups of securities (customarily limited by computer software to 30 or so) and factor analyzing a group of securities sufficiently large for the APT model to hold. As a result, it is found that as one increases the number of securities in the groups to which the APT/factor analytic procedures are applied, the number of "factors" determined increases. Moreover, this increase in the number of "factors" with larger security groups cannot readily be explained away by a distinction between "priced" and "non-priced" risk factors. As our earlier paper shows, it is generally impermissible to carry out tests on whether a given "risk factor is priced" though such tests are invariably found in the standard factor analytic models used to test the APT.

It is interesting to note that just as our earlier paper found that the number of "factors" determined increases with the number of securities in the groups to which the APT/factor analytic procedures are applied, the analysis in this paper indicates that the number of "factors" increases with the number of time-series observations used to estimate factor coefficients.

Our paper is organized in five sections. In Section 2, we briefly explain the empirical procedures. Section 3 presents the significance tests for the risk premia and for unique variance as a measure of risk. Section 4 is devoted to intercept tests, and Section 5 provides summary and conclusions of our results.

2. The APT Model and Empirical Tests[3]

The APT model, originated by Ross (1976) and extended by Huberman (1982), starts by postulating the return-generating process for securities

$$r_{t\cdot} = E_{t\cdot} + f_{t\cdot}B + u_{t\cdot} \tag{1}$$

where $r_{t\cdot}$ is an m-element row vector containing the observed rates of returns at time t for the m-securities under consideration; $E_{t\cdot}$ is similarly an m-element row vector containing the expected (mean) returns at time t. Furthermore,

(2) $$v_{t\cdot} = f_{t\cdot}B + u_{t\cdot}$$

represents the error process at time t, and the APT model assumes that the error process has two components: the idiosyncratic component

$$u_{t\cdot} \quad , t = 1,2,...$$

and the common component

$$f_{t\cdot}B \quad .$$

Finally, the two components are assumed to have the following properties,

$$\{u'_{t\cdot} : t = 1,2,...\}$$

is a sequence of independent identically distributed (i.i.d.) random vectors with

(3) $$E(u_{t\cdot}) = 0 \ , \ Cov(u'_{t\cdot},f'_{t\cdot}) = 0 \ \text{ and } \ Cov(u'_{t\cdot}) = \Omega \quad ,$$

Ω being a diagonal matrix. Regarding the common component, we specify that[4]

$$\{f'_{t\cdot} : t = 1,2,...\}$$

is a sequence of k-element i.i.d. random vectors with

(4) $$E(f'_{t\cdot}) = 0 \ , \ Cov(f'_{t\cdot}) = I \quad .$$

It is a consequence of the assertion above that

$$\{(r_{t\cdot} - E_{t\cdot})' : t = 1,2,...\}$$

is a sequence of i.i.d. random vectors with

$$(5)\qquad E\left[(r_{t\cdot} - E_{t\cdot})'\right] = 0 \;,\; \mathrm{Cov}\left[(r_{t\cdot} - E_{t\cdot})'\right] = B'B + \Omega = \Psi$$

where B is a kxm matrix and Ψ is a diagonal matrix of size m. Ross shows that, if the number of securities (m) is sufficiently large, there exists a (k+1)-element row vector, $c_{t\cdot}$, such that

$$(6)\qquad E_{t\cdot} = c_{t\cdot}B^* \quad , \quad t = 1,2,\ldots,T$$

where

$$(7)\qquad B^* = \begin{matrix} e' \\ B \end{matrix} \quad ,$$

e being an m-element column of ones.[5] The result in (6) characterizes the no-arbitrage condition of the APT model, and we can rewrite (1) with any desired degree of approximation as

$$(8)\qquad r_{t\cdot} = c_{t\cdot}B^* + f_{t\cdot}B + u_{t\cdot} \quad , \quad t = 1,2,\ldots,T \quad .$$

The empirical tests of the APT initiated first by Roll and Ross (1980) are based upon a two-step factor analytic approach. Factor analytic methods, in effect, are suggested by the formulation in (1) and the composition of the covariance matrix (5). In the first step, one determines the number of factors (k) and estimates the elements of B. If T, the number of trading days, is sufficiently large, using factor analysis we estimate B, say by $\tilde{B}$, and Ω by $\tilde{\Omega}$. We thus implicitly estimate

$$(9)\qquad \tilde{\Psi} = \tilde{B}'\tilde{B} + \tilde{\Omega} \quad .$$

In the second stage, using $\tilde{B}$ as "independent variables," one estimates the vector $c_{t\cdot}$, whose elements have the interpretation that c_{ti} is the "risk premium" attached to the i^{th} factor, $i = 1,2,\ldots,k$, while c_{to} is the risk-free rate or possibly the return on a zero-beta asset. Thus in the second step for each t we may estimate[6]

$$(10) \qquad \tilde{c}'_{t\cdot} = (\tilde{B}^*\tilde{\Psi}^{-1}\tilde{B}^{*\prime})^{-1}\tilde{B}^*\tilde{\Psi}^{-1}r'_{t\cdot} \quad , \quad t = 1,2,\ldots,T \quad .$$

If the underlying process is normal and if T is sufficiently large, it can be shown that (approximately)

$$(c_{t\cdot} - c_{t\cdot})' \sim N[0,(B^*\Psi^{-1}B^{*\prime})^{-1}] \quad .$$

Thus, we may view the $\{\tilde{c}'_{t\cdot} : t = 1,2,\ldots,T\}$, approximately, as drawings from a multivariate normal distribution with mean

$$c'_{t\cdot} \quad : \quad t = 1,2,\ldots,T$$

and covariance matrix

$$(B^*\Psi^{-1}B^{*\prime})^{-1} \quad .$$

There are two crucial testable hypotheses implied by the APT model. First, the intercept term c_{to} in (10) is the risk-free rate.[7] Second, there is a linear relation between the risk measures embodied in B and the expected returns. As we indicated in our earlier paper, one cannot test unambiguously the "significance" of individual risk premia but one can test unambiguously the null hypothesis

$$c^*_{t\cdot} = 0$$

where

$$c_{t\cdot} = (c_{to}, c^*_{t\cdot}) \quad .$$

In addition to the two testable hypotheses implied by the APT model, general tests also involve the introduction of other explanatory variables and a test of the hypothesis that the corresponding coefficients are zero. The respecified model in this context is

$$r_{t\cdot} = c_{t\cdot}B^* + d_{t\cdot}P + v_{t\cdot} \quad , \tag{11}$$

where P is a matrix of "extraneous" variables. Recall that the restriction on empirical evidence implied by (8) is that no other (relevant) economic-financial variables have any effect on the determination of expected rates of return. This implies that $d_{t\cdot}$ should not be significantly different from zero.

In the next section, we investigate whether the risk premia are priced, i.e. whether $d_{t\cdot}$ is a null vector and whether higher-order moments of returns when introduced as "extraneous" variables as in (11) are priced. The significance tests on the intercept term c_{to} are presented in Section 4.

3. Empirical Results: Contribution of Covariance Measures Vs. Unique Variance to Asset Returns

The time series of security returns used in this paper (like RR) consists of daily returns for the July 3, 1962 - December 31, 1972 period for each of 1260 New York and American Stock Exchange stocks, providing a sample size (number of observations) ranging from 2509 to 2619 daily returns per security. The first step factor analysis and the second step cross-section analysis are based on 42 groups of 30 stocks each, with the stock selection in these groups determined from an alphabetical ordering.

The results of the second step of the analysis, testing whether (any of) the risk premia are priced for the standard five-factor model used by RR and others, are shown in Table 1.[8] Columns (1)-(6) show the means of the intercept term and of the vector of risk premia obtained from the "cross-sectional" GLS regressions for each t for 42 groups separately. Columns (7) and (8) show the significance tests for the intercept term and the vector of risk premia respectively. Row numbers correspond to the forty-two 30-security groups arranged alphabetically. The test statistic for the significance test for the risk premia for each group is given by (for simplicity, we omit the group superscript below)

$$(12) \qquad T\bar{c}^{*}W^{-1}\bar{c}^{*\prime} \sim \chi^2_5 \quad ,$$

where

$$\bar{c}^{*} = \frac{1}{T}\sum_{t=1}^{T} \tilde{c}^{*}_{t\cdot}$$

and

$$W = \frac{1}{T}\sum_{t=1}^{T} (\tilde{c}^{*}_{t\cdot} - \bar{c}^{*})'(\tilde{c}^{*}_{t\cdot} - \bar{c}^{*}) \quad .$$

The test statistic is asympototically chi-square with 5 degress of freedom. Column (8) of Table 1 shows that, for the standard five-factor model used by RR and others, in only six (30-security) groups out of 42 (about 14%) is the risk premium vector "significantly" different from zero (3 groups at the 5% level and 3 groups at the 10% level). The evidence of Table 1 suggests a very substantial failure for one of the crucial implications of the APT model. Thus, just how much explanation for asset returns is afforded by the APT model in the RR context is questionable and certainly at variance with the results they present.[9]

Turning now to the question as to whether the use of the (five) factor model exhausts the "explanation" of the factor return process, the relevant results are in Table 2. Previous writers have considered the ability of unique variance to add to the explanation of asset returns provided by common factors to be a critical test of the APT (e.g., RR (1980)).

To determine whether unique variance is priced (or any other variable which is extraneous from the viewpoint of the APT), the obvious procedure which we (and others) have followed is to add that variable to the factor coefficients (loadings) to determine whether it adds in the cross-section explanation of asset returns. The rationale for pursuing this analysis, in spite of the dependence of the results of factor analysis on the size of the groups selected for analysis, is that while it is not possible to determine the true number of factors in the universe of assets from relatively small samples it may still be possible to determine the relative importance of common factors vs. unique variance or other "extraneous" risk variables in explaining asset returns. There is at least no clear bias in favor of either the factor or other risk variables, and there is no obvious way of avoiding this complication.

In column 12 in Table 2, we give the relevant statistics for testing the null hypothesis of zero-risk premia, when the factor premia are estimated in conjunction with other extraneous variables' coefficients--in this instance standard deviation and skewness of own returns. The skewness of returns has been added to these regressions as another "extraneous" variable because of the well-known complication in estimating the relation between mean returns and standard deviation which may be caused by skewness in the returns' distribution. In only two (out of forty-two) groups is the null hypothesis of zero risk premia rejected at conventional significance levels.[10]

Interestingly enough, column 13 of Table 2, which gives the relevant statistics for the test that the coefficients of standard deviation and skewness are zero, shows that the null hypothesis is rejected at the five percent level in three cases (groups) and at the ten percent level in five additional cases (groups). Thus, in thirty-four out of forty-two groups standard deviation and skewness cannot be said to have any perceptible influence on the return generating process, while a similar statement can be made in forty out of forty-two groups for factor risk premia.

Overall, the implications of the test results reviewed in this section are not very favorable to the APT model. Our results, also, do not fully accord with those of RR; in testing for extraneous variables, however, they did not use a sample of contiguous days. This is not likely to explain the difference in our results but further investigation of this issue may be warranted.

To minimize the problem of "spurious" correlation, RR order their daily returns chronologically and estimate mean returns from data for days 1, 7, 13, 19,..., factor loadings from days 3, 9, 15, 21,..., and standard deviations from days 5, 11, 17, 23,.... Following their procedure, we obtain the results presented in Table 3, which indicate that the null hypothesis of zero-risk premia is rejected at the ten percent level in only nine of the 42 groups (five at the .05 level), while the hypothesis of no effect of own standard deviation on return is rejected in only six groups (four at the .05 level).[11] The moderate difference in results from Table 2 may represent the use of non-contemporaneous rather than contemporaneous measures of risk and returns, the cut in the number of observations by a factor of about 85% is a reflection of the skipped dates caused by the use of non-contemporaneous measures, and the omission of a skewness variable. There are, however, some peculiarities in the results that should be noted. For example, when we use factor loadings (without the standard deviation) as independent variables using the non-contiguous observations, the risk premia are "priced" in only six groups.[12] Similarly, when we use the standard deviation as a single independent group, again six groups have significant coefficients.[13] On the other hand, when both factor loadings and standard deviation are included as explanatory variables, the risk premia are priced in nine groups while standard deviation is significant in six groups. The findings in Table 3, therefore, do not provide much evidence to support the argument that factor loadings rather than standard deviation represent the appropriate measure of risk.

To make a more direct comparison of the results of the analysis of contemporaneous and non-contemporaneous measures of returns and risk, the regressions in Table 4 include skewness as an additional explanatory variable, thus duplicating the form of the Table 2 regressions. However, as in Table 3, non-contemporaneous dates are used, with mean return estimated from days 1, 9, 17, 25,..., factor coefficients from days 3, 11, 19, 17,..., standard deviation from days 5, 13, 21, 29,..., and skewness from days 7, 15, 23, 31,.... Now, with eight days skipped between successive measures of return and risk, the number of observations is again reduced by about 25% from that covered by Table 3 and by close to 90% from Table 2. In this analysis, the hypothesis of zero-risk premia is rejected at the 10 percent level in only four of the 42 groups (with one group just below the 10 percent level) while the hypothesis of both zero standard deviation and skewness effects is not be rejected in any group (with five groups just below the 10 percent level).

As we pointed out earlier, however, some care should be exercised in interpreting this set of results. For example, when we use the factor loadings as "independent" variables in a similar fashion to the GLS regression presented in Table 1 while using non-contemporareous measures of returns as in Table 4 (i.e. mean returns are estimated from days 1, 9, 25 etc. and factor loadings from 3, 11, 19, 27 by skipping eight days between successive measures of return and risk measures), we find only three groups where risk premia are "priced": groups 4, 8 and 18. Interestingly enough, when standard deviations (estimated using the days 5, 13, 21, etc.) are used, the null hypothesis of zero-risk premia is rejected in three groups (groups 8, 17 and 18) while the hypothesis of no effect of standard deviation is rejected in two groups (groups 4 and 14).

Combining these results, there is some but not very strong evidence that both common risk factors and residual or unique risk do affect asset returns, without much basis for differentiating between the relative importance of the common and unique risk factors. There is some difference in the results based on the longer time-series of observations and contemporaneous returns and risk measures, and those based on the much smaller number of observations associated with non-contemporaneous measaures of returns and risk, but it is not clear which set of results is the more reliable.

An even more interesting difference in results occurs when we carry out the usual kind of factor analysis, assuming a maximum of five factors as RR and many others have done, and obtain the results presented in Table 5 based on the same 404-434 observations per group of stocks covered in Table 3. Table 5 presents chi-square tests on one to five factors for each of the 42 groups covered. These results differ substantially from those obtained in Table 1 of our earlier paper based on 2618 observations. It should be pointed out, as noted in our earlier paper, that with 2618 observations we find a larger number of groups for which the five-factor decomposition is inadequate than do RR, which may be attributable either to the very large number of missing observations for some securities in the RR sample or to the greater precision of our computer software (SAS) or both. However, in any case, in a recent independent analysis, Cho, Elton and Gruber (1982) found results similar to ours.

The sensitivity of the factor results to the number of time-series observations used to estimate mean returns and the different risk measures for each stock included in the analysis is indicated by the tabulation below which shows the number of times (out of 42) that the null hypothesis of at most k (1-5) factors is accepted at the 5% significance level based on 404-434 non-contiguous observations contrasted with the number of times for 2618 contiguous observations. (The non-contiguous observations cover the same time period as the contiguous observations but include only data for days 1, 7, 13, 19... or roughly one-sixth of the total number of daily observations.)

Number of Times Null Hypothesis is Accepted at 5% Significance Level

	k=1	k=2	k=3	k=4	k=5
404-434 non-contemporaneous observations	11	20	30	38	42
2618 contemporaneous observations	0	1	11	29	36

Obviously, more factors are indicated by the analysis based on the larger number of observations. The reason for this disparity in results is not clear but one possible explanation is that sampling error is so large that only with a very large number of observations can we adequately cope with it.[14] Another possible explanation is the type of stationarity assumption implicit in the estimations of factors from two different sets of days. The stationarity assumption is likely to be a less important basis for explaining the difference in results than the size of the sample of observations because both sets of days cover the same overall period. A more definitive determination of the relative importance of these two different explanations is provided by the summary data in Table 6, which presents an analysis of the number of factors obtained from daily observations covering (1) every other day for the entire period, (2) every day in the first half of the period, (3) every day in the second half of the period, (4) the first 400 or so days in the period, and (5) the last 400 or so days in the period. Clearly, as the number of observations increases from about 400 to 1309 to 2618, the number of "factors" estimated increases markedly. Even with a sample of observations as large as 1309, the number of "factors" is appreciably lower than with 2618 observations. On the other hand, with the number of observations held constant, the time period covered seems to have less influence on the number of factors estimated. The results are as sensitive to the number of observations used in estimating mean returns and risk as they are to the number of securities used in each group (the latter having been shown in our earlier paper (DFG (1983)).

3. Testing the Intercept in the Cross-Section APT Pricing Relationship

Another important implication of the APT, which it shares with other capital asset pricing models, is that the "constant" term in the relation

$$r_{t\cdot} = c_{t\cdot}B^* + f_{t\cdot}B + u_{t\cdot}$$

i.e., the term c_{to}, corresponds to the risk-free rate, or at least a zero beta asset. As we pointed out in our earlier paper, the operational procedure for obtaining $c_{t\cdot}B^*$ is time invariant, which would argue strongly that $c_{t\cdot}$ is time invariant. In turn this could imply that the "risk-free" rate is also time invariant--which is rather far-fetched and questionable. Despite this and the other reservations expressed earlier regarding the testability of the APT, we proceeded to carry out a test of this particular set of implications. We felt it particularly appropriate since RR and others carry out a test on the equality of intercept terms for adjacent groups only, instead of a test on equality of intercept terms for all groups.

Recall from the last section that once the matrix B_i of factor loadings for the i^{th} group has been estimated on the basis of a five-factor model, we obtain GLS estimators by

(13) $$\tilde{c}_{t\cdot}^{(i)\prime} = (\tilde{B}_i^*\tilde{\Psi}_{ii}^{-1}\tilde{B}_i^{*\prime})^{-1}\tilde{B}_i^*\tilde{\Psi}_{ii}^{-1}r_{t\cdot}^{(i)\prime} \qquad i = 1, 2, \ldots, 42$$

$$\text{where } \tilde{B}^* = \begin{vmatrix} e' \\ \tilde{B}_i \end{vmatrix} , \quad \tilde{\Psi}_{ii} = \tilde{B}_i'\tilde{B}_i + \tilde{\Omega}_i \qquad t = 1, 2, \ldots, T$$

If daily returns are normal and if the sample size T on the basis of which $\tilde{B}_i$ and $\tilde{\Omega}_i$ are estimated is large--which it is in the present context--then we would expect that, approximately,

(14) $$\tilde{c}_{t\cdot}^{(i)\prime} \sim N\left[c_{t\cdot}', \, (\tilde{B}_i^*\tilde{\Psi}_{ii}^{-1}\tilde{B}_i^{*\prime})^{-1}\right]$$

The important thing to realize here is that the covariance matrix is time invariant. Hence, we can treat the $\tilde{c}_{t\cdot}^{(i)}$ as "observations" from a population with mean $c_{t\cdot}$ and a constant covariance matrix. Hence, defining

(15) $$z_{t\cdot}^{(i)} = \tilde{c}_{t\cdot}^{(i)} - \tilde{c}_{t\cdot}^{(1)} , \qquad i = 2, 3, \ldots, 42$$

we have that under the APT model, approximately,

(16) $$z_{t\cdot}^{(i)\prime} \sim N(0, K_{ii})$$

where K_{ii} is an appropriate time invariant covariance matrix. Extracting the first element therefore, we find

$$z_{to}^{(i)} \sim N(0, K_{oo,i}) , \qquad i = 2, 3, \ldots, 42$$

In general, $z_{to}^{(i)}$ is correlated with $z_{to}^{(j)}$ but their covariance is also time invariant. Thus, let

$$z_{to}^* = \left(z_{to}^{(2)}, z_{to}^{(3)}, \ldots, z_{to}^{(42)}\right)$$

and observe that

$$z_{to}^{*\prime} \sim N(0, Q_o) ,$$

where Q_o is an appropriate time invariant covariance matrix. Clearly we can estimate the mean vector and covariance matrix by

(17) $$\bar{z}^{*\prime}_o = \frac{1}{T}\sum_{t=1}^{T} z^{*\prime}_{to} \,, \qquad \tilde{Q}_o = \frac{1}{T}\sum_{t=1}^{T} (z^{*\prime}_{to} - \bar{z}^{*\prime}_{to})(z^{*}_{to} - \bar{z}^{*}_o)$$

and employ the test statistic

(18) $$T\bar{z}^{*}_o\tilde{Q}_o^{-1}\bar{z}^{*\prime}_o \sim \chi^2_{41}$$

to test the hypothesis that the intercepts of the 42 groups are equal.[15]

In this instance, the test statistic turns out to be

$$T\bar{z}^{*}_o\tilde{Q}_o^{-1}\bar{z}^{*\prime}_o = 34.4$$

and thus the hyopthesis is accepted. This accords with the results of RR who interpret this finding as an endorsement of the APT model. In this connection we should point out that applying the same tests for the equality of constant terms in a one-factor model yields the statistic

$$\chi^2_{41} = 43.30$$

which similarly implies acceptance. The same conclusion is reached when we use a two, three or four factor model.

Now acceptance of such a hypothesis while confirming an implication of the APT model does not tell us very much; for example, this hypothesis would be accepted even if all or nearly all intercepts were zero. Such a situation would cast some doubts on the usefulness of the APT. Thus, next we tested the hypothesis that all intercepts are zero. This is done through the statistic

(19) $$T\bar{c}^{*}_o\tilde{W}_o^{-1}\bar{c}^{*\prime}_o \sim \chi^2_{42} \,,$$

where

$$(20) \qquad \bar{c}_o^* = \left(\bar{c}_o^{(1)}, \bar{c}_o^{(2)}, \ldots, \bar{c}_o^{(42)}\right), \qquad \bar{c}_o^{(i)} = \frac{1}{T}\sum_{t=1}^{T} \tilde{c}_{to}^{(i)}$$

$$(21) \qquad \tilde{W}_o = \frac{1}{T}\sum_{t=1}^{T} (\tilde{c}_{to}^* - \bar{c}_o^*)'(\tilde{c}_{to}^* - \bar{c}_o^*), \qquad \tilde{c}_{to}^* = (\tilde{c}_{to}^{(1)}, \tilde{c}_{to}^{(2)}, \ldots, \tilde{c}_{to}^{(42)})$$

The test statistic in this case is

$$T\bar{c}_o^*\tilde{W}_o^{-1}\bar{c}_o^{*\prime} = 67.8$$

and thus, the hypothesis is rejected.[16]

However, rejection of such a hypothesis only means that there is at least one coefficient which can be said to be non-zero. To clarify this issue, we examine Table 1, which gives the mean intercepts and the corresponding "t-ratios" in the 42 groups. Even a casual perusal of the table shows that at the .1 level of significance only 13 of the intercepts can be said to be non-zero with a bilateral test. Using a unilateral test we find 24 "significant" intercepts. Thus, we are not really violating the meaning of the empirical evidence if we state that at best (from the point of view of the APT model) the evidence is ambiguous and at worst that one of the implications of the model is contradicted by the empirical evidence. The (mean) risk free rate computed from the 7th root of weekly Treasury Bill rates is clearly positive and its standard deviation does not support the hypothesis of a zero daily risk free rate. In fact, the mean weekly rate on Treasury Bills over this period is .00084166 with standard deviation .0002344, which is clearly significantly different from zero. The associated mean daily rate computed as the 7th root of one plus the weekly rate minus one is .0001204 and is also significantly different from zero.[17]

The intercept tests so far are generally consistent with the "zero beta" version of the APT model and our results are quite similar to that generally found in tests of the CAPM, where it has generally been concluded that a zero-beta must be substituted for the risk-free rate to explain the empirical results. However, it should be noted that the zero-beta is not a very satisfactory substitute for the risk-free rate since it is simply a statistical construct until economic theory is brought to bear on its meaning. To a substantial extent, this has been done for the CAPM which has been reformulated theoretically to adjust in a statistically testable manner for the effects of inflation and other variables (e.g., human wealth, differences in lending and borrowing rates, etc.), which would be expected to affect the relation between returns and risk, including the intercept. This has not been done for the APT and it is difficult to see how it could be. It is, in any case, not clear how much importance one should attribute to the intercept test results even if they are consistent with a zero-beta version of the APT model since the risk premia vector has empirically been shown in the previous section to yield few significant results.

4. Conclusions

In a previous paper, we pointed out that the factor analytic procedures which have been used to test the APT are seriously flawed for a number of reasons, including notably the dependence of the number of "factors" found on the number of assets included in the group which is factor-analyzed and the inability to directly test whether a given "factor" is priced. As a by-product of the present paper, which attempts to test the two key implications of the APT -- the irrelevance of unique variances (and other variables uniquely affecting a particular asset) to the explanation of asset returns, and the risk-free or zero-beta interpretation of the intercept in the cross-section mean return-risk relationships, we have found that the number of factors determined is a positive function not only of the number of assets factor-analyzed but also of the number of observations used to estimate mean returns and risk. These basic problems create doubt on the testibility of the APT by proper econometric procedures given the present state of the literature.

Setting aside these basic objections, we adopted the general methodology used in previous empirical tests of the APT and sought through more extensive statistical analysis to test one of the important implications of the model, viz., that the intercept terms $c_{to}^{(i)}$ are, on average, the same in all groups which would be true if the intercepts were either the risk-free or zero-beta rates of return. This implication is not rejected by the empirical evidence; on the other hand, the same evidence suggests that on average $c_{to}^{(i)}$ is insignificantly different from zero for most groups. This, of course, runs contrary to the interpretation the APT model places on this coefficient.

Second, and more importantly, we find that the risk premia vector is not priced in most groups (at least 36 out of 42), indicating a lack of linear relationship between the expected rates of return and the measures of risk parameters implied by the APT model. Furthermore, when (own) standard deviation or (own) standard deviation and skewness are introduced into the asset-return function, they turn out to be "significant" as frequently as the factors suggested by RR and other authors, although generally yielding insignificant coefficients. We conclude that the evidence on the usefulness of the APT model is at best mixed. Further work and a different methodology is needed to probe more deeply into its implications.

Footnotes

1 Under certain conditions, Ingersoll (1982) argues that the intercept in the APT could be a "zero beta" asset even though a risk-free asset exists. However, this would seem to imply that the market does price risk other than common or factor risk and that arbitrage pricing theory, unlike the CAPM, cannot explain the basic risk premium between risky and risk-free assets.

2 The unique variance and intercept results in our APT tests are consistent with a number of previous tests of the CAPM (e.g., see Friend and Westerfield (1981) and Blume and Friend (1973)). However, while virtually all previous tests of the CAPM find an intercept which corresponds to a zero-beta rather than risk-free rate, they differ in their implications for the relative importance of covariance and unique variance measures of risk.

3 The APT model and its empirical implications are explained in detail in our previous paper, Dhrymes-Friend-Gultekin (1983). The APT model and empirical methodology is summarized here in order to establish our notation and to make the paper as self-contained as possible.

4 Note that neither $f_{t\cdot}$ nor B are directly observable. The specification in (4) is to eliminate this problem. B, however, is identified only up to left multiplication by an orthogonal matrix.

5 Ross's original formulation of the APT relies on "diversified or efficient" portfolios. Huberman (1982) provides a more precise definition of the arbitrage condition and shows that the relation in (6) is an approximation. In both approaches, one relies on the strong law of large numbers. Recently, however, a number of authors introduced models under the generic name of "Arbitrage Pricing Model or Theory". These authors, see for example Connor (1981) and Dybvig (1983), attempt to derive Ross's APT model in a finite economy. In doing so, however, these authors rely on much more restrictive assumptions than Ross's model. An attractive feature of Ross's model is the minimal assumptions required as in developing demand theory through revealed preferences.

6 In the remainder of the paper, we shall group the securities into 42 groups of 30 securities as RR did. In this context, equations should have a superscript indicating the group number. For example, (10) should be rewritten as

$$\tilde{c}^{(i)\prime}_{t\cdot} = (\tilde{B}^*_i \tilde{\Psi}^{-1}_{ii} \tilde{B}^{*\prime}_i)^{-1} B^*_i \tilde{\Psi}^{-1}_{ii} r^{(i)\prime}_{t\cdot} \quad , \quad i = 1,2,\ldots,42 \quad .$$

We omit the group superscript for simplicity whenever there is no ambiguity.

7 The implications of the "zero-beta" version are discussed further in Section 3.

8 The first stage of the analysis, i.e. the determination of number of factors and estimation of factor loadings, and the problems associated with dividing the universe of asset are discussed in great detail in our previous paper. While we carried out most of the analysis presented in this paper using one to five factor models separately, we only present the results for the five factor model for the sake of brevity and to be comparable to those results reported by RR and others for the five factor model. Most conclusions regarding a five factor model in this paper are, however, also true for one to four factor models.

9 In the RR paper, there is no counterpart to Table 1, so a direct comparison is not possible. RR estimated the risk premia by

$$\bar{c} = (\tilde{B}^*\tilde{\Psi}^{-1}\tilde{B}^*)^{-1}\tilde{B}^*\tilde{\Psi}^{-1}\bar{r}' \quad ,$$

where $\bar{r}$ is the mean of rates of returns, i.e. $\bar{r} = (1/T)\sum_{t=1}^{T} \tilde{r}_t$. and use the statistic

$$T\bar{c}^*\bar{W}^{-1}c^{*\prime} \sim \chi^2_5$$

where

$$\bar{W} = \tilde{B}_i\tilde{\Psi}_{ii}^{-1}[\tilde{\Psi}_{ii} - \delta_i ee']\tilde{\Psi}_{ii}^{-1}\tilde{B}_i$$

and

$$\delta_i = 1/(e'\tilde{\Psi}_{ii}^{-1}e) \quad .$$

In the RR approach, one relies heavily on the "truth" of one's assertions relating to the distributional aspects of the cross-sectional GLS estimated coefficients. The approach we employ is more robust to departures from underlying RR procedures.

We did, however, repeat the analysis presented in Table 1 using the mean returns as shown above and using the test statistics employed by RR. Our results show again that risk premia are priced only in six groups. These groups, using the same group numbers in Table 1, are 4, 12, 22, 23, 30 and 39. In Table 1, on the other hand, groups with priced risk premia are 4, 12, 18, 22, 23 and 29 with an overlap of four groups between the two methods. This result is again in sharp contrast to those by RR who reported that in 88.1% of the groups at least one factor is priced.

Also note that the above test statistic is appropriate when there are no extraneous variables; when there are, as in equation (11), simply replace in the bracketed equation

$$\tilde{\Psi}_{ii} - \delta_i ee'$$

by

$$\tilde{\Psi}_{ii} - P^{*\prime}(P^*\tilde{\Psi}_{ii}^{-1}P^{*\prime})^{-1}P^*$$

where

$$P^{*} = \begin{matrix} e' \\ P \end{matrix}$$

and p is the matrix of observations on the extraneous variables shown in equation (11).

10 Incidentally, when we introduce standard deviation as the only extraneous variable, the null hypothesis that the coefficient of standard deviation is zero in the GLS regressions is rejected for 13 groups while the hypothesis that the risk premia vector is null is rejected in 7 groups. It is not clear whether the changes in these results are due to the dependency among higher moments because of the deviations of the daily stock returns from normality or due to the (high) correlation among standard deviation, skewness and factor loadings when they are all used together as "independent" variables.

11 RR use mean returns as dependent variables instead of running the GLS cross-sectional regressions for the trading day $t = 1, 7, 13, \ldots$. If we follow this procedure and employ the test statistics shown in footnote 6, we obtain the following result: Risk premia are priced only in two groups (groups 13 and 29) while standard deviations are "priced" in eight groups (groups 4, 14, 20, 23, 24, 29, 34 and 39).

12 These groups generally do not correspond to the same groups with significant risk premia when we use contiguous data shown in Table 1. When we use non-contiguous data, groups 13, 14, 18, 22, 34 and 36 have significant risk premia. In Table 1, using contiguous data, however, groups with significant risk premia are 4, 12, 18, 22, 23 and 29.

13 These groups are 4, 13, 14, 26, 34 and 39.

14 It might also be noted that the number of Heywood cases (i.e., degenerate cases where one of the "factors" is perfectly correlated with a security) increases to 22 groups when factor loadings are estimated from non-contiguous data, resulting in the major reduction in the number of observations. When we use all 2618 contiguous observations, we encounter 11 Heywood cases. In order to avoid Heywood cases, we substituted new securities whenever a Heywood case appeared. Results presented so far are materially the same whether one includes the Heywood cases in the regressions or not. Also note that there are no Heywood cases when one extracts only one factor and the number of Heywood cases are far fewer with a smaller number of factors extracted.

15 Note that the mean of the daily regression coefficients $c_{t\cdot}^{(i)}$ can easily be estimated using mean return data. If one strictly relies on the model and assumes time stationarity, one can also obtain the covariance matrix of the estimators from mean returns. We have chosen to work with the daily regression coefficients, however, since this represents a procedure that is more robust to departures from stationarity.

16 Since most readers are most familiar with the normal distribution, one may use the normal approximation

$$(x_r^2 - r)/\sqrt{2r} \quad f \quad N(0,1) \quad .$$

This would yield, in the present instance,

$$(67.8 - 42)/\sqrt{82} = 2.8 \quad ,$$

while in the previous case we have

$$(34.4 - 41)/\sqrt{82} = -.72 \quad .$$

Thus, in the first case we reject and in the second case we accept at the 10% significance level.

17 We also test directly the hypothesis that the intercept is the risk-free rate using the weekly T-bill rate. The test is identical to the one in equation (15) except we subtract the seventh root of the (one plus) weekly Treasury bill yield observed every Thursday from the intercepts. We have mixed results. There is a problem in choosing a representative daily risk-free rate among possible contenders such as Federal funds rates, daily T-bill yields, etc. Daily T-bill rates contain serious measurement errors due to infrequent trading.

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Table 1: TESTS OF SIGNIFICANCE FOR INTERCEPTS AND RISK PREMIA FOR 5-FACTOR MODEL USING ALPHABETICALLY RANKED GROUPS
7/12/1962 - 12/31/1972

Group	Statistic								
	$\bar{c}_o$ a/	$\bar{c}_1$	$\bar{c}_2$	$\bar{c}_3$	$\bar{c}_4$	$\bar{c}_5$	$t(c_o)$ b/	χ^2 c/	Number of Observations
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	.00031	.02906	-.00777	-.01885	.01955	.03965	1.56	1.987	2468
2	.00033	.01406	.02373	.04012	.01230	-.01387	1.44	2.551	2571
3	.00015	.02496	.02278	.03118	.07034	.04462	.86	4.594	2568
4	.00033	.04338	.00014	.02896	.08462	-.07534	1.30	9.869†	2575
5	.00022	.05741	.05473	.04039	.01538	-.00236	.76	6.122	2554
6	.00046**	.01674	.02822	-.01043	-.00628	-.07789	2.79	4.169	2405
7	.00023	.06399	.04183	-.03783	-.04399	-.01042	.91	8.109	2540
8	.00043*	.03022	-.04610	-.03291	.01554	-.02364	1.77	4.432	2553
9	.00043**	.02555	-.01606	.02111	.04431	.04152	2.21	3.935	2559
10	.00017	.07112	.03367	.04216	-.05987	-.00789	.70	8.859	2540
11	.00030*	.04828	-.01387	.01289	-.03399	.00652	1.67	4.095	2512
12	.00022	.01358	.07201	-.04364	-.00894	.06428	.88	11.959††	2575
13	.00035	-.01068	.04991	-.03986	.02182	-.03180	1.46	4.471	2536
14	.00015	.09109	-.04646	-.01407	.02251	.02635	.91	8.929	2547
15	.00017	.00330	.06522	-.03195	.00244	.00603	.58	4.329	2560
16	.00060**	-.00024	.09049	-.00043	-.02093	-.00129	2.06	5.970	2565
17	.00041	.01352	.00119	.03840	.00874	.04897	1.60	2.259	2547
18	-.00011	.01109	.15558	.01691	.05566	-.08433	-.29	10.737†	2416
19	.00033	.04910	.01900	.01450	-.00472	-.02273	1.53	2.517	2562
20	.00024	.03398	.04584	.05198	.03330	-.01970	1.15	4.963	2527
21	.00041**	.03279	-.00245	.03490	.00535	-.04920	2.20	3.588	2506
22	.00067**	-.01277	-.07062	.10889	-.08044	.06094	2.41	9.928†	2559
23	.00004	.01680	.10448	.04674	-.06551	-.08662	1.15	14.250††	2504
24	.00028	.02564	.01924	.02261	.01928	-.06094	1.61	3.426	2575
25	.00018	.04813	.01171	-.04854	.02135	-.01703	.86	3.202	2563
26	.00056**	.03205	-.07018	.01329	.06546	.02552	2.48	7.448	2561
27	.00014	.06678	-.00318	-.01892	.01963	-.03929	.72	5.830	2510
28	.00046**	.03424	-.00961	-.00822	.00370	-.07811	2.51	6.057	2561
29	-.00014	.12947	-.03723	.06784	.00819	.03151	-.50	12.022††	2544
30	.00008	.06412	-.00678	.02018	.00510	.02009	.42	3.846	2545

Table 1 (continued)

Group	Statistic $\bar{c}_o$ a/	$\bar{c}_1$	$\bar{c}_2$	$\bar{c}_3$	$\bar{c}_4$	$\bar{c}_5$	$t(c_o)$ b/	χ^2 c/	Number of Observations
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
31	.00037*	.03452	.02932	-.02035	-.04517	-.07711	1.67	6.574	2557
32	.00015	.08805	-.02079	.03852	-.03136	-.01862	.85	7.339	2492
33	.00031	.03286	-.00259	.02204	.01301	.00448	1.53	.855	2574
34	.00064**	.00090	-.01326	.05227	.00233	-.01953	3.04	1.976	2511
35	.00028	.06050	.00472	-.03068	.02146	-.09301	1.38	7.608	2546
36	.00037*	.01042	.03715	-.02369	-.04576	-.04488	1.85	3.265	2555
37	.00027	.04540	.05001	-.02108	-.06003	.04478	1.01	2.959	2076
38	.00012	.07880	.03941	-.01249	.01153	-.02106	.68	6.797	2563
39	.00011	.07281	-.00797	-.02980	-.07269	.06763	.38	7.304	1917
40	.00044**	.02028	-.01039	.01125	.00507	.06808	2.81	3.444	2508
41	.00032	.02727	.03385	.01141	.06001	.02950	1.61	3.476	2542
42	.00027*	.01926	.04634	-.00777	.05096	-.04919	1.50	5.401	2546

a/ $\bar{c}_o$ through $\bar{c}_5$ are the arithmetic means of the daily cross sectional regression estimates using the GLS model $\tilde{c}'_{t\cdot} = (\tilde{B}^*\tilde{\Psi}^{-1}\tilde{B}^{*\prime})^{-1}\tilde{B}^*\tilde{\Psi}^{-1} r'_{t\cdot}$. $B^{*\prime}$ is $[e:B]$, which is the augmented matrix of factor loadings with unit vector.

b/ $t(c_o)$ is the "t-ratio" for the intercept term. "t-ratio" is given by $\sqrt{T}(\bar{c}_o/s_o)$, where $\bar{c}_o = \frac{1}{T}\sum_{t=1}^{T}\tilde{c}_{to}$ and $s_o = [\frac{1}{T}\sum_{t=1}^{T}(\tilde{c}_{to} - \bar{c}_o)^2]^{1/2}$. * and ** indicate intercept terms which are significantly different from zero at 10 and 5 percent levels respectively. In case of a "t-test," this refers to a bilateral test; if a unilateral test is desired the critical points are 1.65 and 1.31 respectively for the 5 and 10 percent level of significance tests.

c/ χ^2 is the test statistic to test the null hypothesis that none of the risk premia is priced. The test statistic is distributed as chi-square with 5 degrees of freedom. The critical values for chi-square distribution with 5 degrees of freedom at 10 and 5 percent significance levels are 9.24 and 11.10 respectively. † and †† indicate groups for which the null hypothesis is rejected at 10 and 5 percent levels of significance respectively.

Table 2: SIGNIFICANCE TESTS OF STANDARD DEVIATION AND SKEWNESS OF DAILY SECURITY RETURNS AS AN ALTERNATIVE HYPOTHESIS TO 5-FACTOR APT MODEL
7/12/1962 - 12/31/1972

Group	$\bar{c}_o$ a/ (1)	$\bar{c}_1$ (2)	$\bar{c}_2$ (3)	$\bar{c}_3$ (4)	$\bar{c}_4$ (5)	$\bar{c}_5$ (6)	$\bar{c}_6$ (7)	$\bar{c}_7$ (8)	$t(c_o)$ b/ (9)	$t(c_6)$ (10)	$t(c_7)$ (11)	χ^2_c c/ (12)	$\chi^2_{\sigma,k}$ d/ (13)	Number of Observations (14)
1	.00001	-.03783	-.07908	-.01371	-.00420	.01427	.03485	.00010	.04	1.41	.40	2.456	4.192	2468
2	-.00017	.00235	-.00961	.00857	-.05965	-.03645	.03865*	.00009	-.49	1.89	.41	2.178	5.932†	2571
3	-.00013	.03656	.00776	-.01452	.06596	.01809	.02348	.00015	-.56	1.57	1.02	4.332	3.931	2568
4	.00016	-.03912	-.02668	-.02482	.04715	-.05703	.02790*	.00024	.58	1.75	1.32	2.738	5.349†	2575
5	.00009	.00147	.02431	-.00108	.00853	-.06110	.03392	-.00027	.27	1.38	-1.21	2.245	2.663	2554
6	.00046**	.01532	.02302	-.01061	-.00669	-.07785	.00296	-.00003	2.10	.13	-.23	3.513	.054	2405
7	.00007	.03959	.01366	-.04117	-.04679	.01282	.01653	-.00005	.19	.81	-.29	3.954	.686	2540
8	.00035	.02473	-.03681	-.03594	.03054	-.02378	.00187	.00019	1.12	.08	.78	3.429	.783	2553
9	.00049**	.05007	-.00188	.02049	.04386	.04296	-.01380	.00009	2.19	-.83	.88	4.573	1.041	2559
10	.00016	.03880	.01806	.03115	-.06601	-.00243	.01694	-.00019	.57	.87	-1.11	3.969	1.412	2540
11	.00011	.00515	-.03769	.00571	-.03735	-.01791	.02288	.00004	.53	1.47	.31	1.936	3.427	2512
12	.00032	.02547	.10158	-.02315	-.00638	.07869	-.01650	.00000	1.07	-.77	.01	9.546†	.594	2575
13	.00006	-.01529	.04487	.00331	-.00123	.00330	.00935	.00023	.21	.51	1.50	1.005	4.807†	2536
14	.00008	-.07285	-.05287	-.04776	-.03221	.03541	.02288	-.00045	.36	.86	-1.46	6.761	2.251	2547
15	.00017	-.00678	.03384	-.04442	-.00291	-.00194	.01229	-.00007	.58	.61	-.25	2.023	.380	2560
16	.00028	.02331	.05987	.01510	-.01345	-.01167	-.00049	.00028	.67	-.02	1.12	1.745	1.390	2565
17	.00047	.03402	.03268	.05450	.01245	.05608	-.01680	.00022	1.53	-.79	1.33	2.578	2.213	2547
18	-.00073*	-.03615	.05279	-.01240	-.05942	-.12489	.07972**	-.00036	-1.65	2.51	-1.39	8.416	6.311††	2446
19	.00022	.05679	.02509	.02881	-.01251	-.03451	-.00182	.00019	.93	-.09	1.21	2.803	1.578	2562
20	.00004	.02563	.03358	.03710	.01617	-.04285	.01242	.00014	.15	.78	.73	3.753	1.350	2527
21	.00028	.01006	-.02361	.01270	.01619	-.05447	.02188	-.00017	1.29	1.40	-.65	2.393	2.006	2506
22	.00065*	-.01383	-.07108	.10832	-.07522	.05933	.00049	.00004	1.88	.02	.16	6.974	.029	2559
23	-.00005	.00615	.07042	.03423	-.06484	-.08953	.01824	-.00011	-.21	1.08	-.64	8.751	1.272	2504
24	.00015	.00961	.01333	.02624	.00522	-.04448	.00899	.00010	.66	.56	.39	1.323	.751	2575
25	-.00006	.02460	.00649	-.07010	-.01240	-.02746	.01814	.00006	-.19	.70	.28	2.675	.922	2563
26	.00051	.09075	-.06659	-.00845	.10648	-.00803	-.02824	.00051**	1.53	-1.02	2.10	11.349††	4.519	2561
27	.00007	-.02385	-.06570	-.03161	.01895	-.02430	.03354*	-.00001	.32	1.87	-.08	2.605	3.868	2510
28	.00024	.00765	-.07675	-.03080	.00530	-.08602	.03259	.00009	1.07	1.53	.66	5.927	5.926†	2561
29	-.00011	.01278	-.00711	-.01440	.05113	-.01208	.03532*	.00001	-.39	1.69	.05	1.892	3.722	2544
30	-.00016	.03900	-.03632	.03584	-.00207	-.01693	.01691	.00017	-.57	.72	.46	1.091	1.366	2545

Table 2 (continued)

Group	Statistic $\bar{c}_o$ [a/] (1)	$\bar{c}_1$ (2)	$\bar{c}_2$ (3)	$\bar{c}_3$ (4)	$\bar{c}_4$ (5)	$\bar{c}_5$ (6)	$\bar{c}_6$ (7)	$\bar{c}_7$ (8)	$t(c_o)$ [b/] (9)	$t(c_6)$ (10)	$t(c_7)$ (11)	χ^2_c [c/] (12)	$\chi^2_{\sigma,k}$ [d/] (13)	Number of Observations (14)
31	.00005*	-.01467	.00712	-.07649	.02892	-.12455	.02882*	.00021	.20	1.66	1.34	8.555	7.076††	2557
32	.00007	-.05440	.01843	-.00050	-.03638	-.01186	.00961	.00021	.35	.48	1.15	2.980	2.013	2492
33	.00004	-.05363	-.07487	-.00650	.01200	.07720	.03483	.00030	.14	1.47	1.40	3.160	6.073††	2574
34	.00039	-.04330	.01769	.00395	.01076	-.02522	.02099	.00022	1.51	1.01	.98	1.089	3.995	2511
35	.00016	.00121	-.03797	-.04815	-.01024	-.12319	.01615	.00026	.55	.62	1.07	8.427	3.541	2546
36	-.00001	.00225	.02570	-.01805	-.02191	-.07575	.02822*	.00004	-.05	1.86	.61	3.042	4.697†	2555
37	.00010	.02402	.04291	-.04103	-.06405	.05175	.01228	.00010	.35	.46	1.15	3.438	3.420	2076
38	-.00002	.05644	.02344	.00550	.02246	-.01185	.01913	-.00014	-.09	1.10	-.76	2.314	1.418	2563
39	.00011	.07328	.00209	-.02705	-.08171	.00608	-.00272	.00010	.28	-.09	.39	7.674	.219	1917
40	.00024	-.00121	-.01832	.00615	.00497	.03024	.01898	-.00001	.98	1.06	-.02	.650	1.267	2508
41	.00020	.00574	.03139	.00415	.05180	.00824	.01689	-.00010	.78	.98	-.55	1.477	1.166	2542
42	.00023	.01782	.03529	-.02173	.04517	-.04707	.00405	.00004	1.21	.18	.19	3.733	.087	2546

a/ $\bar{c}_o$ through $\bar{c}_7$ are the arithmetic means of the daily cross sectional regression estimates using the GLS model $\tilde{c}'_{t.} = (\tilde{B}^{**}\tilde{\Psi}^{-1}\tilde{B})^{-1}\tilde{B}^{**}\tilde{\Psi}^{-1}r'_{t.}$. $B^{**'}$ is $[e{:}B'{:}\sigma{:}k]$ which is the augmented matrix of factor loadings with unit vector, (e), standard deviation (σ), and skewness (k) of the securities over the sample period (i.e., for the i^{th} security, $\sigma_i = [\frac{1}{T}\sum_{t=1}^{T}(r_{ti} - \bar{r}_i)^2]^{1/2}$), c_o is the intercept term, c_1 through c_5 are the regression coefficients for the factor loadings, and c_6 and c_7 are the coefficients for the standard deviation and skewness.

b/ $t(c_o)$ is the "t-ratio" for the intercept term and $t(c_6)$ and $t(c_7)$ are the "t-ratios" for the coefficients of the standard deviation and skewness, respectively; the "t-ratio" for the i^{th} coefficient is given by $\sqrt{T}(\bar{c}_i/s_{ii})$, where $\bar{c}_i = \frac{1}{T}\sum_{t=1}^{T}\tilde{c}_{ti}$ and $s_{ii} = [\frac{1}{T}\sum_{t=1}^{T}(\tilde{c}_{ti} - \bar{c}_i)^2]^{1/2}$. * and ** indicate regression coefficients which are significantly different from zero at 10 and 5 percent significance levels respectively. In the case of "t-test" this refers to a bilateral test. If a unilateral test is desired, the critical points are 1.65 and 1.31 respectively for the 5 and 10 percent levels of significance tests.

c/ χ^2_c is the test statistic to test the null hypothesis that none of the risk premia is priced. The test statistic is distributed as chi-square with 5 degrees of freedom. The critical values for chi-square distribution with 5 degrees of feedom at 10 and 5 percent significance levels are 9.24 and 11.10 respectively.

d/ $\chi^2_{\sigma,k}$ is the test statistic to test the null hypothesis that c_6 and c_7 are not different from zero. It is distributed as chi-square with 2 degrees of freedom. The critical values for chi-square distribution with 2 degrees of freedom at 10 and 5 percent significance levels are 4.61 and 5.99 respectively. † and †† indicate the groups for which the null hypothesis is rejected at 10 and 5 percent significance levels respectively.

Table 3: SIGNIFICANCE TESTS OF STANDARD DEVIATION OF DAILY SECURITY RETURNS AS AN ALTERNATIVE HYPOTHESIS TO 5-FACTOR APT MODELS
NON-CONTEMPORANEOUS DATA: 7/12/62 - 12/31/72

Group	Statistic $\bar{c}_o$ a/	$\bar{c}_1$	$\bar{c}_2$	$\bar{c}_3$	$\bar{c}_4$	$\bar{c}_5$	$\bar{c}_6$	$t(c_o)$ b/	$t(c_6)$	χ^2 c/	Number of Observations
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	
1	.00099*	-.02411	.00670	-.05770	.15431	-.08796	.00439	1.77	.13	5.306	415
2	.00060	-.00454	.01327	.01530	-.03833	.01432	-.00991	.84	-.30	.398	432
3	.00045	-.17124	.02814	-.12179	.03573	-.02036	.04864	.96	1.61	6.907	431
4	-.00054	-.02213	.09559	.07184	-.11728	-.04389	.02279	-.86	.53	5.704	432
5	.00015	-.09791	.09584	-.11564	.07680	-.04102	.04145	.26	1.32	7.836	429
6	.00007	.00437	.04252	.02140	.02987	.09399	.01407	.14	.35	1.386	404
7	.00089	-.04200	-.04707	-.04845	-.01184	-.10839	-.00322	1.14	-.09	5.212	428
8	.00017	-.02944	.05188	-.05772	.02201	-.00618	.01863	.30	.43	1.967	429
9	.00053	-.07900	-.08816	.01476	.08874	.04990	.00939	1.26	.28	4.987	428
10	.00034	.03434	-.02655	.05775	.06443	-.05139	-.00121	.81	-.04	3.276	425
11	.00020	-.03672	.07978	.06514	.02431	.02649	.00802	.42	.26	2.818	422
12	.00018	-.03591	.05075	.05685	.03812	-.08505	.01022	.41	.29	4.273	432
13	.00003	.12780	.12752	.05378	.04074	.10661	.00283	.07	.10	9.652†	425
14	-.00011	-.11654	.07841	.09985	-.12782	-.05442	.08869	-.21	2.10**	10.767††	428
15	-.00017	-.09831	-.01429	-.03814	.02788	-.10313	.03393	-.31	.85	5.145	426
16	.00052	-.07376	.03307	-.03441	-.15271	.01455	.01728	.90	.49	4.779	433
17	.00058	-.02186	.04008	-.09331	-.02177	-.07517	-.00294	.94	-.09	4.325	433
18	.00044	.08343	.40931	.18353	.15576	.03324	-.10599	.47	-2.26**	17.404††	432
19	-.00021	-.05289	-.02693	-.00581	.00910	-.05837	.04438	-.37	1.10	2.139	433
20	-.00129	-.03907	.03057	-.11749	-.11754	-.12559	.07009	-.22	2.10**	9.117	428
21	.00057	.06336	-.02380	-.02260	-.01822	-.01598	-.00826	1.09	-.24	2.436	424
22	.00073	-.07700	-.09576	.11271	.03082	-.17165	.02698	1.38	.68	11.874††	427
23	-.00050	-.04523	-.06315	.00428	-.02717	-.08207	.07216	-.71	1.66*	1.928	419
24	-.00069	-.07787	.11208	-.02357	.02810	.02198	.05678	-1.37	1.92*	5.660	432
25	-.00015	.04040	-.12453	.01534	-.05517	-.01148	.06603	-.23	1.34	6.495	433
26	.00189**	-.05467	.00439	-.02094	-.02889	.06853	-.05180	2.49	-.99	1.592	430
27	.00013	.05224	.02934	.07143	.11874	-.03272	-.00395	.30	.11	3.743	420
28	.00075	-.03774	.00948	.01466	.09488	.00169	-.00753	1.39	-.19	3.403	428
29	.00018	-.13478	-.13379	-.00940	.03946	.08805	.06431	.45	2.04**	10.367††	429
30	.00019	-.04442	-.02895	-.03361	-.02834	-.01248	.03596	.04	1.01	.930	434

Table 3 (continued)

Group	Statistic $\bar{c}_o$ [a/] (1)	$\bar{c}_1$ (2)	$\bar{c}_2$ (3)	$\bar{c}_3$ (4)	$\bar{c}_4$ (5)	$\bar{c}_5$ (6)	$\bar{c}_6$ (7)	$t(c_o)$ [b/] (8)	$t(c_6)$ (9)	χ^2 [c/] (10)	Number of Observations
31	.00038	.00842	.00563	-.01885	.10888	.08047	.00268	.73	.07	3.464	427
32	.00003	-.03827	.02999	-.05944	.02771	.00270	.03946	.05	1.25	1.684	422
33	.00111*	-.19078	.01458	-.02095	.04010	-.05740	.03513	1.99	.93	6.305	431
34	-.00044	-.00491	-.03774	-.07499	-.02103	.21029	.05335	-.83	1.53	10.649†	422
35	.00071	-.06688	.04559	.04169	.02055	-.00387	.02911	1.36	.89	2.303	430
36	.00098*	-.01521	-.01726	.11693	.03626	-.01267	.00765	1.90	.22	10.730†	431
37	.00074	.07372	-.03011	.04755	-.01725	.00506	-.03559	.95	-.87	1.214	433
38	.00051	-.09809	-.03131	-.06356	-.00175	-.14972	.02985	1.07	.83	9.464†	435
39	.00100	-.01016	-.14150	.03102	-.14756	.04998	-.01082	1.07	-.18	4.887	433
40	-.00004	.02641	.01499	.02034	.03701	-.01879	.00973	-.08	.32	.601	418
41	.00022	-.14077	-.14295	-.05267	-.03433	.03332	.05317	.38	1.51	10.130††	422
42	.00080*	-.12312	-.00619	.04561	.01546	-.05611	.02817	1.71	.63	3.041	431

a/ $\bar{c}_o$ through $\bar{c}_6$ are the arithmetic means of the daily cross sectional regression estimates using the GLS model $\tilde{c}'_{t\cdot} = (\tilde{B}^*\tilde{\Psi}^{-1}\tilde{B}^{*\prime})^{-1}\tilde{B}^*\tilde{\Psi}^{-1}r'_{t\cdot}$. $B^{**\prime}$ is $[e{:}B'{:}\sigma]$ which is the augmented matrix of factor loadings with unit vector, (e), and standard deviation, σ, of the securities over the sample period (i.e., for the i^{th} security, $\sigma_i = [\frac{1}{T}\sum_{t=1}^{T}(r_{ti} - \bar{r}_i)^2]^{1/2}$), c_o is the intercept term, c_1 through c_5 are the regression coefficients for the factor loadings, and c_6 is the coefficient for the standard deviation.

b/ $t(c_o)$ is the "t-ratio" for the intercept term and $t(c_6)$ is the "t-ratio" for the coefficient of the standard deviation; the "t-ratio" $\sqrt{T}(\bar{c}_o/s_o)$, where $\bar{c}_o = (1/T)\sum_{t=1}^{T}\tilde{c}_{to}$ and $s_o = [(1/T)\sum_{t=1}^{T}(\tilde{c}_{to} - \bar{c}_o)^2]^{1/2}$. * and ** indicate intercept terms which are significantly different from zero at 10 and 5 percent levels respectively. In the case of the "t-test", this refers to a bilateral test; if a unilateral test is desired the critical points are 1.65 and 1.31 respectively for the 5 and 10 percent levels of significance tests.

c/ χ^2 is the test statistic to test the hypothesis that none of the risk premia is priced. The test statistic is distributed as chi-square with 5 degrees of freedom. The critical values for chi-square distribution with 5 degrees of freedom at 10 and 5 percent significance levels are 9.24 and 11.10 respectively. † and †† indicate the groups for which the null hypothesis is rejected at 10 and 5 percent significance levels respectively. † and †† indicate the groups for which the null hypothesis is rejected at 10 and 5 percent significance levels respectively.

Table 4: SIGNIFICANCE TESTS OF STANDARD DEVIATION AND SKEWNESS OF DAILY SECURITY RETURNS AS AN ALTERNATIVE HYPOTHESIS TO 5-FACTOR APT MODELS
NON-CONTEMPORANEOUS DATA: 7/12/62 - 12/31/72

	STATISTIC												
Group	$\bar{c}_o$ a/ (1)	$\bar{c}_1$ (2)	$\bar{c}_2$ (3)	$\bar{c}_3$ (4)	$\bar{c}_4$ (5)	$\bar{c}_5$ (6)	$\bar{c}_6$ (7)	$\bar{c}_7$ (8)	$t(c_o)$ b/ (9)	$t(c_6)$ (10)	$t(c_7)$ (11)	χ^2_c c/ (12)	$\chi^2_{\sigma,k}$ d/ (13)	Number of Observations (14)
1	-.00008	-.04619	.00425	-.00876	.05141	-.03852	.02977	-.00023	-.13	.77	-.81	.852	1.446	312
2	-.00048	-.04117	-.07897	.09634	.01139	-.02757	.02444	.00010	-.63	.59	.30	4.902	.449	328
3	-.00013	-.01304	.01808	-.03945	-.04506	.04618	.02409	.00012	-.21	.68	.71	1.413	.937	326
4	-.00165**	.11777	.03226	.13202	.21100	-.12113	.05347	.00030	-2.51	1.60	.96	9.687*	4.225	324
5	-.00061	.06664	.00465	-.03694	.10578	-.06371	.05383	-.00044	-.68	1.48	-1.16	3.220	3.055	324
6	.00041	.03194	-.03190	-.06846	-.13760	-.07362	.01978	.00005	.74	.44	.15	5.824	.294	305
7	-.00012	.13684	.03990	.02068	.11361	-.00602	-.01237	-.00039	-.17	-.35	-1.28	4.740	2.010	318
8	.01538**	.10100	-.04631	-.15065	-.09078	-.25432	-.04594	.00046	2.17	-.86	1.25	13.312**	1.945	324
9	.00061	-.06532	.06192	.02783	.07496	-.04868	.00750	-.00033	1.16	.24	-1.20	4.125	1.444	325
10	.00061	.06904	-.10191	.03406	.03583	.06961	-.03086	.00017	.87	-.58	.47	2.441	.387	319
11	.00040	.13101	-.01571	.13536	.10147	-.11766	-.04237	.00043	.61	-.90	1.44	6.715	2.219	319
12	-.00013	.01798	.06065	-.05358	-.15187	.03213	.02995	-.00030	-.22	.64	-.96	5.908	1.086	326
13	.00011	-.01842	.03414	.04750	-.02283	.02149	-.00276	-.00028	.22	-.10	-1.02	1.014	1.037	324
14	-.00086	-.02420	.00600	-.20458	-.02669	-.04640	.06839	-.00025	-1.63	1.64*	-.65	7.512	3.060	324
15	-.00004	.09089	.06216	.01531	.02127	.12839	-.03221	-.00019	-.06	-.58	-.53	4.263	1.064	324
16	.00008	-.07677	-.10831	.00788	-.02945	-.16867	.04318	.00030	.11	1.14	1.04	5.045	2.153	325
17	-.00025	.15655	-.04481	.00365	.10033	-.15934	.06459	.00069	-.34	-1.56	1.77	10.343*	4.564	321
18	.00070	-.12471	-.16435	.02881	.01429	.07976	.02621	-.00089	.52	.46	-1.92	10.166*	4.354	186
19	.00000	-.01752	.05483	.05131	.09073	.05009	-.01578	-.00026	.00	-.37	-.82	2.877	1.553	325
20	.00034	-.00331	-.00904	.00191	-.08533	.00699	-.00273	.00007	.45	-.07	.30	1.036	.102	320
21	.00051	-.01681	.01832	-.00501	.03339	.07314	-.00394	-.00012	.88	-.10	-.59	1.347	.365	318
22	.00039	.06499	-.01569	.00563	.07549	-.01108	-.04399	.00019	.52	-.82	.96	1.461	1.140	323
23	-.00044	-.10119	.04108	-.01166	.03031	.00147	.03960	-.00004	-.65	1.06	-.07	3.658	1.128	315
24	-.00059	-.06228	-.01757	-.05986	-.04411	-.04452	.03906	.00024	-.94	1.01	.45	1.703	1.179	327
25	.00091	-.07330	-.00213	.11166	-.04898	-.04000	-.04826	.00059	1.34	-1.03	1.97	4.822	4.016	325
26	.00051	.14328	.06086	.07361	.02990	.11212	-.03829	-.00012	.67	-.79	-.48	4.840	1.005	322
27	.00002	-.14939	.01273	.02261	-.09082	-.09675	.07225	-.00047	.03	1.71	-1.56	4.576	3.532	316
28	.00005	-.00125	-.09576	-.05941	.12718	-.07014	.01191	-.00009	.09	.32	-.23	6.560	.166	324
29	.00021	.11317	.14355	-.10060	.04146	.20776	-.04586	-.00011	.45	-1.23	-.39	8.926	1.531	321
30	-.00024	.00421	.02324	-.07133	.02542	-.10280	.03537	-.00079	-.38	.91	-1.82	3.054	4.001	323

Table 4 (continued)

Group	$\bar{c}_o$ a/ (1)	$\bar{c}_1$ (2)	$\bar{c}_2$ (3)	$\bar{c}_3$ (4)	$\bar{c}_4$ (5)	$\bar{c}_5$ (6)	$\bar{c}_6$ (7)	$\bar{c}_7$ (8)	$t(c_o)$ b/ (9)	$t(c_6)$ (10)	$t(c_7)$ (11)	χ^2_c c/ (12)	$\chi^2_{s,k}$ d/ (13)	Number of Observations (14)
31	.00032	-.07273	.00911	.04404	.07365	-.04567	.02882	-.00025	.55	-.11	-1.36	2.752	1.952	324
32	.00086	-.13213	-.02647	-.02951	.00247	.03567	.00776	-.00026	1.46	.18	-.48	2.347	.228	313
33	.00104	.00044	-.00911	-.09850	.01887	.04268	.03215	-.00009	1.49	-.84	-.33	2.371	1.026	326
34	.00054	.06780	-.00275	.11000	.01987	.15964	.01819	-.00023	.80	-.41	-1.03	5.619	1.332	316
35 e/						it is not possible to estimate five factors								
36	-.00009	-.06722	-.00949	-.11132	.03269	-.10251	.04246	.00013	-.15	.90	.45	3.603	1.371	321
37	.00026	-.01413	-.14027	.12121	-.14495	-.01567	.01260	-.00065	.40	.30	-1.24	7.648	1.591	261
38	.00048	.09599	-.07723	.03458	-.04648	.10490	-.04151	.00008	.83	-1.10	.27	5.154	1.242	324
39	-.00002	-.20141	.06519	-.12187	.08570	-.00921	.05975	-.00008	-.03	1.24	-.46	6.145	1.703	243
40	.00054	.03192	.02794	-.01567	-.11384	.03250	.01600	-.00017	.98	.51	-.57	2.763	.532	320
41	.00081	-.02393	-.06348	-.05060	-.10806	.10338	.00334	-.00030	1.11	.07	-.82	3.622	.687	321
42	-.00003	.11003	-.08287	-.07542	.03196	.10678	.04398	-.00033	-.06	1.22	-1.20	6.717	2.490	322

a/ $\bar{c}_o$ through $\bar{c}_7$ are the arithmetic means of the daily cross sectional regression estimates using the GLS model $\tilde{c}'_{t\cdot} = (\tilde{B}^{**}\Psi^{-1}\tilde{B})^{-1}\tilde{B}^{**}\Psi^{-1}r'_{t\cdot}$. $B^{**'}$ is $[e:B':\sigma:k]$ which is the augmented matrix of factor loadings with unit vector, (e), standard deviation (σ), and skewness (k) of the securities over the sample period, i.e., for the i^{th} security, $\sigma_i = [\frac{1}{T}\sum_{t=1}^{T}(r_{ti} - \bar{r}_i)^2]^{1/2})]$, c_o is the intercept term, c_1 through c_5 are the regression coefficients for the factor loadings, and c_6 and c_7 are the coefficients for the standard deviation and skewness.

b/ $t(c_o)$ is the "t-ratio" for the intercept term and $t(c_6)$ and $t(c_7)$ are the "t-ratios" for the coefficients of the standard deviation and skewness, respectively; the "t-ratio" for the i^{th} coefficient is given by $\sqrt{T}(\bar{c}_i/s_{ii})$, where $\bar{c}_i = \frac{1}{T}\sum_{t=1}^{T}\tilde{c}_{ti}$ and $s_{ii} = [\frac{1}{T}\sum_{t=1}^{T}(\tilde{c}_{ti} - \bar{c}_i)^2]^{1/2}$. * and ** indicate regression coefficients which are significantly different from zero at 10 and 5 percent significance levels respectively. In the case of "t-test" this refers to a bilateral test. If a unilateral test is desired, the critical points are 1.65 and 1.31 respectively for the 5 and 10 percent levels of significance tests.

c/ χ^2_c is the test statistic to test the null hypothesis that none of the risk premia is priced. The test statistic is distributed as chi-square with 5 degrees of freedom. The critical values for chi-square distribution with 5 degrees of feedom at 10 and 5 percent significance levels are 9.24 and 11.10 respectively. * and ** indicate the groups for which the null hypothesis is rejected at 10 and 5 percent signficance levels respectively.

d/ $\chi^2_{\sigma,k}$ is the test statistic to test the null hypothesis that c_6 and c_7 are not different from zero. It is distributed as chi-square with 2 degrees of freedom. The critical values for chi-square distribution with 2 degrees of freedom at 10 and 5 percent significance levels are 4.61 and 5.99 respectively. † and †† indicate the groups for which the null hypothesis is rejected at 10 and 5 percent significance levels respectively.

e/ Program does not converge for a five-factor model, therefore it was not possible to estimate the matrix of factor loadings.

Table 5: CHI-SQUARE TEST OF THE HYPOTHESIS THAT k-FACTORS GENERATE DAILY SECURITY RETURNS

GROUP	k=1	k=2	k=3	k=4	k=5
1	453.750 (.0473)	402.043 (.1703)	353.742†† (.4045)	312.763 (.6185)	227.363 (.7622)
2	457.293 (.0370)	392.832 (.2647)	333.713 (.6998)	293.982 (.8580)	257.117 (.9457)
3	441.319 (.1033)	387.114 (.3352)	336.933 (.6549)	292.767 (.8691)	255.158 (.9548)
4	419.850 (.2950)	363.311 (.6713)	312.867 (.9121)	266.314 (.9883)	231.355 (.9975)
5	423.092 (.2579)	374.163 (.5171)	331.430 (.7300)	290.551 (.8879)	259.333 (.9338)
6	498.343 (.0010)	413.097 (.0910)	351.131 (.4429)	303.295 (.7535)	267.543 (.8728)
7	519.254 (.0001)	426.983 (.0355)	376.392 (.1416)	325.004 (.4271)	287.528 (.6113)
8	490.315 (.0023)	436.133 (.0174)	391.226 (.0548)	347.804 (.1455)	306.496 (.3104)
9	462.003 (.0262)	406.805 (.1317)	352.560 (.4218)	312.926 (.6160)	277.505 (.7603)
10	470.477 (.0135)	414.257 (.0846)	366.918 (.2328)	321.703 (.4784)	281.716 (.7012)
11	513.172 (.0002)	458.534 (.0023)	408.658 (.0138)	363.286 (.0519)	315.512 (.1968)
12	583.007 (.0001)	427.877 (.0332)	371.846 (.1816)	324.494 (.4350)	278.030 (.7533)
13	469.107 (.0151)	415.323 (.0791)	364.262 (.2636)	324.473 (.4353)	290.120 (.5693)
14	422.381 (.2658)	354.756 (.7780)	300.538 (.9687)	264.465 (.9906)	228.555 (.9984)
15	483.943 (.0042)	402.347 (.1677)	349.639 (.4652)	309.479 (.6678)	269.182 (.8572)

Table 5 (continued)

GROUP	k=1	k=2	k=3	k=4	k=5
16	413.160 (.3790)	368.261 (.6025)	323.222 (.8256)	278.680 (.9576)	242.120 (.9891)
17	466.258 (.0190)	402.390 (.1673)	347.936 (.4909)	308.706 (.6791)	270.165 (.8473)
18	483.016 (.0046)	424.365 (.0430)	378.034 (.1287)	337.613 (.2511)	296.657 (.4619)
19	437.856 (.1255)	375.290 (.5006)	331.702 (.7265)	296.328 (.8348)	255.297 (.0541)
20	790.796 (.0001)	413.251 (.0901)	330.028 (.7479)	282.079 (.9425)	249.456 (.9746)
21	563.209 (.0001)	485.505 (.0001)	416.897 (.0065)	367.379 (.0379)	315.921 (.1924)
22	412.645 (.3858)	353.812 (.7885)	307.047 (.9443)	268.916 (.9843)	236.094 (.9951)
23	454.095 (.0462)	399.254 (.1962)	354.161 (.3984)	318.036 (.5362)	281.903 (.6984)
24	467.248 (.0175)	407.925 (.1237)	349.453 (.4680)	305.851 (.7194)	272.172 (.8258)
25	490.444 (.0023)	421.278 (.0534)	361.323 (.3002)	315.734 (.5724)	279.839 (.7283)
26	542.929 (.0001)	479.078 (.0002)	422.721 (.0037)	367.148 (.0386)	315.920 (.1924)
27	521.078 (.0001)	431.653 (.0249)	360.941 (.3051)	307.578 (.6953)	266.674 (.8806)
28	481.667 (.0052)	423.573 (.0455)	369.595 (.2040)	317.919 (.5381)	277.105 (.7656)
29	476.338 (.0083)	420.528 (.0562)	370.799 (.1918)	322.841 (.4606)	279.254 (.7365)
30	463.798 (.0229)	408.864 (.1172)	360.331 (.3131)	315.609 (.5744)	274.218 (.8020)
31	507.081 (.0004)	438.487 (.0144)	380.113 (.1138)	341.525 (.2062)	304.734 (.3359)

Table 5 (continued)

GROUP	k=1	k=2	k=3	k=4	k=5
32	521.824 (.0001)	443.379 (.0094)	381.277 (.1060)	338.363 (.2421)	296.901 (.4580)
33	486.613 (.0033)	413.938 (.0863)	357.909 (.3456)	309.374 (.6693)	269.358 (.8555)
34	436.848 (.1326)	377.064 (.4748)	327.255 (.7815)	291.020 (.8841)	248.496 (.9771)
35	420.554 (.2867)	339.520 (.9117)	291.750 (.9872)	251.937 (.9983)	223.479 (.9993)
36	403.948 (.5054)	351.977 (.8081)	309.968 (.9295)	273.635 (.9740)	240.091 (.9916)
37	473.172 (.0108)	407.247 (.1285)	365.407 (.2501)	318.935 (.5221)	285.446 (.6443)
38	495.461 (.0014)	438.274 (.0146)	385.492 (.0810)	337.585 (.2515)	291.758 (.5424)
39	467.352 (.0174)	412.639 (.0936)	369.170 (.2084)	329.619 (.3581)	298.123 (.4382)
40	577.196 (.0001)	488.062 (.0001)	421.972 (.0040)	369.508 (.0320)	275.786 (.7827)
41	489.261 (.0026)	421.790 (.0515)	373.552 (.1658)	327.893 (.3835)	283.900 (.6683)
42	440.180 (.1102)	370.691 (.5676)	321.550 (.8422)	275.815 (.9677)	242.491 (.9886)
Number of Times the Null Hypothesis is Accepted at 5% Significance Level					
	11	20	30	38	42

Number of degrees of freedom for the chi-square test for each group is $\frac{1}{2}[(30-k)^2 - 30 - k]$. Number of securities in each group is 30. Number of observations for each security is 434. No security has more than 110 missing observations. Values in parentheses indicate the p-value associated with the statistic, i.e., the probability that the test statistic (under the null hypothesis) will assume a value at least as large as the statistic obtained in this particular test.

Table 6: Effect of Number of Observations on the Factor Structure

Number of Times Null Hypothesis is Accepted at 5% Significance Level[a]					
	k = 1	k = 2	k = 3	k = 4	k = 5
<u>Contiguous Observations</u>[b]					
All 2618 daily observations for entire period	0	1	11	29	36
1309 observations for first half of period	1	13	26	34	39
1309 observatinos for second half of period	0	9	20	32	38
First 436 observations	7	22	36	38	41
Last 436 observations	16	26	36	41	41
<u>Non-Contiguous Observations</u>					
Every other observation for entire period (1309 observations)	0	9	23	33	38
Every ninth observation for entire period (404-434 observations)	11	20	30	38	42

[a] Hypothesis is that k-factors generate daily security returns. See Table 5 for further explanation of this table.

[b] Missing observations do not exceed 90 for each security for the entire period July 3, 1963 - December 31, 1972.