STOCHASTIC PROPERTIES OF CROSS-SECTIONAL FINANCIAL DATA

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I. Introduction

Financial data may be viewed as random variables which imperfectly reveal information about a firm's financial and operational activities. In order to infer that information from cross-sectional financial data, we need to identify characteristics about the underlying stochastic processes generating these data.

Cross-sectional financial data can be affected by many systematic factors. For example, a firm's current assets generally depend on its size, its industry's credit practice, the nature of its output market, its current debts, etc. As a result, cross-sectional financial data are not identically and independently disbributed (IID). If the data are not IID, the underlying stochastic processes cannot be directly identified. However, certain statistical tools can be applied to abstract important systematic factors, in which case the refined data should be approximately IID.[1] This will then allow us to identify the underlying stochastic process from the refined data.

This paper has two objectives. First, I explore the underlyiing stochastic process of financial data to build a statistical foundation for financial statement analysis. Second, following the lead of Lev and Sunder (1979), I then develop regressional approaches to achieve control over the systematic factors affecting cross-sectional financial data.[2]

The remaining sections of the paper are as follows. In Section II, I further discuss the relationship between IID and normality of cross-sectional financial data and then suggest four models which might achieve factorial control for financial statement analysis. Section III provides a description of the methodology and the data for the tests. Section IV contains the empirical findings; the conclcuding remarks are in Section V.

II. Models of Factorial Control

Deakin (1976) first tested for normality of cross-sectional financial ratios, but did not explore why we should expect normality from cross-sectional financial data. Moreover he did not make a distinction between normality of cross-sectional financial data and normality of the stochastic process that generates financial data. He simply collected a large group of samples and tested their normality assumptions. The null hypothesis of normality was consistently rejected from sample to sample. But since the cross-sectional financial ratios in Deakin's study were not IID, his tests of normality were not very meaningful.

In this section I derive four models of factorial control. If these models properly control the important systematic factors, the refined data will be approximately IID. A normality test applied to these data then reveal the underlying stochastic properties of cross-sectional financial data.

Let the random variable $\tilde{Y}_i$ be the financial data of firm i, where a tilde "~" indicates randomness. Assume that $\tilde{Y}_i$ is generated by a joint stochastic process with a set of random variables, $\tilde{S}_i$, and that the intersect of $\tilde{S}_i$ for all i is neither an empty set nor equal to the union set. Hence $\tilde{Y}_i$ are not IID across all i. Let $\tilde{\eta}_i$ be the random noise so that

$$(1) \qquad \tilde{\eta}_i = \tilde{Y}_i - E(\tilde{Y}_i \mid \tilde{S}_i)$$

where $E(\cdot \mid \cdot)$ is the conditional expectation. If all the systematic factors in the set S_i have been properly controlled for, the random noises $\tilde{\eta}_i$ will, by definition, be IID.[3]

The accounting data in financial statements are aggregates in the sense that each item in the income statement summarizes numerous independent accounting entries occurring during an accounting period and each item in the balance sheets represents the integration of many entries over the life of the business entity. According to the Central Limit Theorem, therefore, the random noise in the accounting numbers should be approximately normal, in which case $\tilde{\eta}_i$ should be approximately IIDN (IID and normal) for all i.

Unfortunately, we do not have full knowledge about all of the elements in each set of $\tilde{S}_i$. So in practice we can only work with a subset $\tilde{H}_i \subset \tilde{S}_i$. Also since we cannot specify the functional form of the conditional expectation $E(\tilde{Y}_i \mid \tilde{S}_i)$, we must estimate it with some factorial control model $\hat{E}(Y_i \mid H_i)$. Let $\tilde{\theta}_i$ be the ex post refined data such that

(2) $$\tilde{\theta}_i = \tilde{Y}_i - \hat{E}(Y_i \mid H_i).$$

Then unless (1) the factorial control model is reasonably correct, (2) the subset $\tilde{H}_i$ includes most of the important systematic factors, and (3) the data in Y_i and H_i are large enough to afford a good estimate, the refined data $\tilde{\theta}_i$ will not be IID. As a result, the test of normality on

$\tilde{\theta}_i$ could be rejected even if the underlying stochastic process were Gaussian. Hence the validity of the test of normality depends on the quality of factorial control. The test of normality is a joint test of IID of the refined cross-sectional financial data and the normality of the underlying stochastic process.[4]

Size and industrial identification seem to be the two important proxy variables for systematic factors. The effect of firm size on accounting numbers is rather obvious in that the magnitude of accounting numbers of a larger firm should be greater. Beyond that obvious effect, size can also stand for leadership, economy of scale, market power, political influence, etc. Although we do not precisely understand the theoretical underpinnings, empirical evidence does indicate that size matters.[5] Similarly, there are numerous commonalities in accounting techniques, production technologies, and socio-economic constraints for firms in the same industry. Hence, we need some industrial factor as a proxy for all those variables. The control for industrial factors is attained here through a mechanical blocking. The industrial "block" is grouped according to its Standard Industrial Classification (S.I.C.).

The financial ratio is the most parsimonious way of controlling for systematic factors. The ratio is often neutral with respect to size, and there is often a close correlation between the numerator and denominator. For example, the optimal inventory level is closely related to targeted sales, so the inventory turnover ratio can indicate a firm's inventory policy and at the same time be homogeneous across firms.

Model 1 asserts that the financial ratios in each industry are approximately IID.

(3) $$\tilde{\theta}_i = (Y_i \tilde{/} X_i), \quad \text{Model 1.}$$

According to Lev and Sunder (1979), we can regard X_i in Model 1 as the control variable for Y_i. Then Model 1 implies that the systematic factors embodies in X_i have variance-shifting as well as mean-shifting effects on the stochastic process. If the systematic factors embodied in X_i have only mean-shifting effects, then Model 2 would be a better approach to factorial control.

(4) $$\tilde{\theta}_i = \tilde{Y}_i - [a + b\ \tilde{X}_i], \quad \text{Model 2,}$$

where the term within parentheses represents the systematic effect. Model 3 incorporates features of both Models 1 and 2.

(5) $$\tilde{\theta}_i = (Y_i \tilde{/} X_i) - [c + d\ (1 \tilde{/} X_i)], \quad \text{Model 3.}$$

Since the denominator of a financial ratio controls for other things in addition to size, it usually does not provide a perfect control for size. Hence, we can select variable Z_i (e.g., assets) as a size variable and have Model 4.

(6) $$\tilde{\theta}_i = (Y_i \tilde{/} Z_i) - [e + f\ (X_i \tilde{/} Z_i)], \quad \text{Model 4.}$$

Model 4 implies that the size effect is multiplicative and the effect embodied in X_i is additive. Without a strong theoretical basis, I will proceed in manner of moving from the simplest to the more complicated models in order to test the hypothesis that the refined data would be approximately IIDN. More formally, the hypothesis to be tested is:

<u>Hypothesis</u>: $\tilde{\theta}_i \sim$ IIDN, for all $i \varepsilon I$, where I is the class of firms in the same S.I.C. industry.[6]

Since a test of normality in cross-sectional financial data will be a joint test of the quality of factorial control and the normality of underlying stochastic process, rejection of the null hypothesis of normality will require that we further improve the factorial control and repeat the tests. If the null hypothesis of normality in cross-sectional financial data is consistently rejected over all possible models that control for systematic factors, then we can infer that the underlying stochatic process is not Gaussian.

III. Methodology and Data

In this section, I briefly describe the statistical methods of the tests of homoscedasticity and normality. The nature of sample data are also discussed here.

Test for heteroscedasticity may be classified into two groups: nonconstructive tests and constructive tests. Nonconstructive tests are primarily designed to establish the presence or absence of heteroscedasticity without providing information about the nature of the heteroscedasticity. Constructive tests are designed to determine the particular specification of heteroscedasticity. Since I am more concerned with the presence than the nature of heteroscedasticity, I adopted a widely used nonconstructive test--the Goldfeld-Quandt F-test.[7]

The Goldfeld-Quandt (1965) F-test is based on separate regressions of the subsamples after omitting a specified number c of the middle observations.[8] The test involves the following steps: (1) order the independent variables, e.g., X_i in eq. (4), $(1 / X_i)$ in eq. (5), and (X_i/Z_i) in eq. (6), from low to high; (2) omit 15% of the middle observations; (3) fit the equation to the two subsamples; and (4) let S_1 and S_2 denote the sum of the squared residuals from the two regressions. The test statistic f is:

$$f = \frac{S_2}{S_1} \qquad (7)$$

Under the null hypothesis of homoscedasticity, f is generated by an F-distribution with $(n-c-4)/2$ and $(n-3-4)/2$ degrees of freedom, where n is the number of items in the total sample.

The Kolmogarov-Smirnov (K-S) test was used to examine normality. An alternative method would have been the chi-square test. According to Massey (1951) and Lilliefors (1967), the K-S test has at least two major advantages over the chi-square test:

(1) It can be used with small sample sizes, where the validity of the chi-square test would be questionable;

(2) It often appears to be a more powerful test than the chi-square test for any sample size.[9]

The K-S test procedure determines the maximum difference between the theoretical cumulative distribution $F(\theta)$ and the sample cumulative distribution $S(\theta)$, i.e.,

$$D = \operatorname{Max}_{\theta} \mid F(\theta) - S(\theta) \mid , \qquad (8)$$

where θ's are the sample residuals from each ordinary least-square estimation of eqs. (4)-(6). The theoretical normal distribution is derived from the sample mean (always zero) and sample standard deviation of θ's. If the value of D exceeds the critical value in the table, the hypothesis of normality is rejected.

The validity of the K-S test depends on homogeneity and independency (i.e., IID) of the refined data. If the nature of any dependency of the refined data is known, it is possible to develop a better factorial control to eliminate dependency. In the case of time series, sample autocorrelation functions can be used to estimate intertemporal dependency. Weiss

(1978) demonstrates that with knowledge of sample autocorrelation functions, one can empirically derive corrections for the K-S statistics.

The sample sizes under 4-digit S.I.C. codes in Compustat Files can be small. The ten industries selected for this study were chosen in order to have at least 25 observations for each. Financial institutions were omitted since some of the ratios used here were not apporpriate for that class of firms. Table 1 lists the ten industries selected and the sample size for each.

Insert Table 1 Here.

The data base was drawn from the Compustat Expanded Annual Industrial File for the five years: 1961, 1965, 1970, 1975 and 1980. The sample size in each industry grew over the last two decades, so the lower bound of the sample size range in Table 1 often turned out to be the sample size of 1961, and the upper bound that of 1980. The "Expanded" file has less survivorship bias than the alternative, Merged Annual Industrial File. Because homogeneity is an important condition for normality to be detected in cross-sectional financial data and "survivorship bias" implies a higher degree of homogeneity, the choice of the Expanded file runs against the null hypothesis of normality. According to Dun and Bradstreet (1976), the failure rate of operating concerns is less than one percent for all industries, so the choice of data file should have little effect on any resulting conclusions.

Five ratios were chosen for analysis: (1) current asset/current liabilities, (2) cash flow/total debt, (3) total debt/total assets, (4) quick assets/net sales and (5) net income/net sales. These five ratios are popular among practitioners and researchers.[10] Ratios (1), (2) and (4) relate to short term liquidity, whose optimal level is a well studied subject in Economics [e.g., Tobin (1958)]. Although we may not know the optimal levels of these three ratios, we generally admit some optimal level does exist. Ratio (3) is a reflection of long term liquidity and (5) is an index of profitability.

IV. Empirical Results

A. Factorial Control and Tests of Normality

The empirical work begins with data of minimum refinement and moves toward models which increase factorial control. After each stage of control design, I test the null hypothesis of normality in the cross-sectional data. If there is a clear trend that increased control leads to innoreased normality of the refined data, then the underlying stochastic process should be approximately Gaussian.

Since Deakin (1976) did an extensive study of normality of financial ratios, his empirical findings serve as a starting point of my exploration.[11] Deakin applied the chi-square test to a set of five financial ratios for each of the 19 years between 1954 and 1972. His null hypothesis was that the cross-sectional financial ratios of the U.S. economy are distributed normally. The null hypothesis was rejected 79 times out of 94 tests.[12] All 15 exceptions occurred with one ratio, total debt/total assets. He also tested for normality after transforming the ratios into their square roots and logarithms and got similar results. This empirical evidence can be attributed either to (1) the lack of appropriate factorial control, or (2) the absence of normality in the underlying stochastic process, or (3) both. Using the more powerful K-S test for this study, I repeated his tests using five years of data (1961, 1965, 1970, 1975, 1980).

The results of Deakin's and my tests are reported in the second and third columns, respectively, of Table 2. All of my 25 tests rejected the null hypothesis of normality, indicating that, the K-S test is indeed more powerful than Deakin's chi-square tests. The 15 "exceptions" in Deakin's study did not stand up to the K-S test.[13]

Insert Table 2 Here.

Proceeding, I attempted to control industrial factors first. As a trial run, I applied the tests to the data of 1975. Factorial control was assessed at two levels, first using the less stringent S.I.C. 2-digit code and then the more stringent 4-digit code. A total of 22 S.I.C. 2-digit industries and 10 S.I.C. 4-digit industries were chosen. Although more S.I.C. 4-digit industries exist than S.I.C. 2-digit industries, most of them had to be eliminated because of small sample size.[14] The result of K-S tests are reported in the fourth and fifth columns of Table 2. Although normality is still absent from the evidence, the data do move in that direction as control is increased. This is somewhat encouraging and suggests further benefit from a reiterative testing process.

After blocking the data according to the S.I.C. 4-digit code, the data were then refined according to the four models described in Section III. Models 1, 2 and 3 use univariate factorial control and Model 4 uses bivariate factorial control, where the size effect is specifically controlled by the variable Z_i. Models 1 and 2 are mutually exclusive, whereas Models 3 and 4 add more factorial control to Model 1. Consequently, we would expect to better detect the presence of normality from the data refined by Model 3 and 4 than from those refined by Model 1. Since

Model 1 can control both mean-shifting and variance-shifting factors, but Model 2 can control only for mean-shifting factors, the performance of Model 2 may be less than that of Model 1.[15] The empirical evidence in Table 3 strongly supports these conjectures.

Insert Table 3 Here.

Each entry in Table 3 indicates the number of rejections at the 5% significant level of 50 tests (for five years and ten industries). Three versions of Model 4 were employed, using total assets, net sales, and total debts as control variables for size in 4-a, 4-b and 4-c respectively. Total assets is the best stock measurement of size, and net sales is the best flow measurement; total debts are inferior to both. If the size effect is a crucial systematic factor, then the data refined by either Model 4a or 4b should outperform those refined by Model 4c. This is supported by the evidence in Table 3. Model 4a is not applicable to the ratio of total debts/total assets, because when total assets stands for both X_i and Z_i in eq.(6), Model 4a reduces to Model 1. For similar reasons, four other "n.a.'s" appear in Table 3.

Insert Table 4 Here.

Table 4 reinforces the empirical evidence in Table 3. A regressional approach to factorial control can perfectly account for the ex post mean-shifting effect since the refined data (i.e., the regression residuals) will always have a zero mean by construction. Therefore, the relative strength of factorial control of the five regression models will be based on their ability to control for the systematic effect on variance--i.e., their ability to bring about homoscedasticity. The joint evidence in Tables 3 and 4 strongly indicates that the issues of homoscedasticity and normality are closely related. By attaining homoscedasticity best, the data refined by Model 4a also show the strongest evidence of normality. In contrast, Model 2, which can only control for the mean-shifting effect, leaves the refined data with strong heteroscedasticity and hardly any trace of normality.

Insert Table 5 Here.

Since Model 3 is more general than Model 1, we would expect the data refined by Model 3 to show stronger evidence of normality than those refined by Model 1. Although the results in Table 3 are consistent with that assertion, there is not much difference between these two sets of refined data. Indeed, a test of the marginal contribution of factorial control of Model 3 over Model 1 for explanation, where d is the coefficient of $(1/X_i)$ in Model 3, resulted in no significant improvement in over 50% of the 250 cases - see Table 5. Although the bivariate regressional control outperforms the simple ratio control design in terms of data refinement, parsimony still makes financial ratio analysis useful. Financial ratio analysis is simple to apply and easy to understand; however, when a more sophisticated analysis is needed, bivariate regressional control, such as Model 4a, will achieve better results.

B. Absence of Normality

Whereas the above tests indicate that the better the data are refined, the more they demonstrate normality, the disturbing fact remains that 22.5%

of the tests rejected the null hypothesis of normality even after the data were refined by the best model--i.e., Model 4a. I now investigate the possible reasons for the absence of normality.

One possibility is the artificiality of the industrial blocking used in this paper. The S.I.C. code is based on the nature of the product-output instead of the nature of the firm. As a result, the industrial blocking may not adequately control industrial effects in all cases. Firms which sell similar products may still adopt different accounting methods, be under different regulations, and may engage in different markets, etc. One way to test for the impact of these factors is to determine whether the results of the normality tests are independent of the industrial categories. Table 6 provide evidence for this explanation.

Insert Table 6 Here.

Table 6 cross-tabulates the results of normality tests in terms of industry and financial data. It demonstrates that the industrial blocking by S.I.C. 4-digit codes is more approriate to some industries than to others. If S.I.C. codes were equally appropriate to all industrial blockings, then the chance of rejecting normality should be independent of industrial blocks. This statement applies to the data refined by any of Model 1 to 4c because none of these models deals with the industrial factors. Consequently, the cross-industrial presence of normality in data refined by Model 1 should be independent of those refined by Model 4a (4b for TD/TA).[16] On the other hand, if S.I.C. codes are more appropriate for some industrial blockings than for others, as we suspect to be the case in Table 6, then for those heterogeneous industries the probability of rejecting normality is high for both Model 1 and Model 4, and for those homogeneous industries it is low for both Model 1 and Model 4. Consequently, there should be a high correlation between the rejections of normality from the data refined by Model 1 and the data reined by Model 4. The Pearson Correlation between the last two columns is 0.877, which is significant at 0.1% level. I also derive a similar conclusion from the ANOVA test. The details of ANOVA are given in the Appendix.

Another possible explanation for the absence of normality is that the underlying economic activities of accounting numbers may be different across firms. Hence the refined data are not IID. The Pearson Correlation between Model 1 and Model 4 according to the last row of Table 6 is 0.839. The evidence suggests that the quality of factorial control varies with the nanture of accounting numbers. The Pearson Correlation between Model 1 and Model 4 according to the 50 entries (5 X 10) of Table 7 is 0.547. The evidence indicates that the results of normality tests depend on the joint effect of inappropriate industrial blocking and the different nature of accounting numbers.

Insert Table 7 Here.

The last possible explanation for the rejection of over 22% of the normality tests is the instabiliity of the economic environment. The economy of the 1960s was characterized by stable growth; the annual inflation rate was hovering about zero with small variances, the real GNP grew at a steady rate of 4% per annum, the unemployment rate was steady at 4%, and the supply of natural resources was relatively certain. The economic harmony and tranquility of the '60s turned into chaos and tumult in the '70's. The inflation rate surged to double digits, the unemploymnet rate soared to as far as 9%, and the supply of nature resources was no longer certain. The drastic change of economic environment forced firms to undertake a series of adjustments.[17] The effect of these factors is shown in Table 7. The evidence indicates less universal presence of an equilibrium financial and operational activities across firms in the tumultuous '70s than in the tranquil '60s. The Pearson Correlation of the 1st two columns in Table 7 is 0.906, which indicates that the normality of accounting numbers strongly depends on the general economic environment.

C. Ergodicity of Financial Data: A Digression

The sample size for each statistical test in this paper is rather small, varying from 26 to 95. In practice, factorial control by statistical blocking tends to result in small sample problems. One way to increase the sample size so as to increas the power of statistical inference is to pool the time-series and cross-sectionall data together. A meaningful pooling requires the ergodicity in data.[18] A necessary condition for ergodicity is that the autocorrelation approaches zero at a sufficiently fast rate when the time lag increases. Table 8 examines this necessary condition for ergodicity.

Insert Table 8 Here.

Since the data refined by Model 4a have the highest probability of presenting normality, I used Model 4a to refine the data for the test of ergodicity. The second cloumn of Table 8 shows the autocorrelations at a lag of five years, and the third column shows a lag of ten years. The refined data are autocorrelated even after a long time lag. Hence, pooling the time-series and cross-sectional data is not feasible for my study.

This high autocorrelation is attributable to the nature of accounting numbers on the balance sheet. Each item on the balance sheet is a stock variable which is an integration of flows over the whole history of the business entity. Another possible explanations the sluggishness of the adjustment of the underlying economic activities. In my other paper (1983a), I develop a rigorous stochastic model to discuss this issue.

VI. Concluding Remarks

The test of normality in cross-sectional financial data is a joint test of the normality in the underlying stochastic process and the homogeneity and independency of the cross-sectional data. Hence a meaningful test requires an effort to refine the cross-sectional data so as to attain the properties of IID.

I developed a set of models to control important systematic factors. After the cross-sectional financial data were refined by each model, I tested normality of the refined data focusing on whether a higher degree of IID gives rise to a stronger presence of normality. If homogeneity and independency of the cross-sectional financial data is attained, the presence of normality in the cross-sectional data would imply the presence of normality in the underlying stochastic process.

The empirical evidence displayed a clear pattern that the better the control over the systematic factors, the more the refined data approach normality. This control was achieved using industrial blocking and adjusting for size. Unfortunately, the results also indicated that a substantial amount of the tests (22+%) rejected normality, suggesting the need to control other factors.

Perhaps the most promising approach would be to control general economic conditions since these can affect the homogeneity of cross-sectional financial data. In a turbulent economic environment, firms must scramble for optimal financial and operational policies in order to cope with the resulting changes. Unless adjustment costs and information costs are trivial, all firms will not adopt the same financial and operational activities. With heterogeneous financial and operational activities, the refined cross-sectional financial data will still not be IID and will not demonstrate the presence of normality. My empirical evidence shows that cross-sectional financial data demonstrate less presence of normality in the turbulent '70s than in the tranquil '60s.

TABLE 1

SAMPLE DATA

S.I.C. Code		Sample Size Range
1311	Crude Petroleum and Natural Gas	28-95
2200	Textile Mill Products	27-60
2300	Apparel and other Textile Products	33-87
2911	Petroleum Refining	35-53
3310	Blast Furnace and Basic Steel Products	45-54
3679	Electronic Components	29-65
3714	Motor Vehicle Parts and Accessories	34-49
4511	Certified Air Transportation	26-30
4911	Electric Services	63-64
5311	Department Stores	28-49

Data Source: Compustat Expanded Industrial File.

TABLE 2: THE NORMALITY OF FINANCIAL RATIOS

$$H_0: \quad (\widetilde{Y_i/X_i}) \sim N(\mu^*, \sigma^2), \quad \text{for all } i \in I.$$

Financial Ratios[a/]	Deakin's Result[b/]	K-S test Result[c/]	S.I.C. 2-digit Industries[d/]	S.I.C. 4-digit Industries[e/]
CA/CL	19 (100%)	5 (100%)	21 (95%)	8 (80%)
CF/TD	n.a.	5 (100%)	22 (100%)	8 (80%)
TD/TA	4 (21%)	5 (100%)	22 (100%)	9 (90%)
QA/NS	19 (100%)	5 (100%)	21 (95%)	7 (70%)
NI/NS	n.a.	5 (100%)	21 (95%)	4 (40%)

a/ CA: Current Assets , CL: Current Liabilities , CF: Cash Flow , TD: Total Debt , TA: Total Assets , QA: Quick Assets , NS: Net Sales , NI: Net Income .

b/ Each entry in this table indicates the number and the proportion (in brackets) of tests that reject the null hypothesis of normality. All the tests in this paper are reported at 5% significance level. Deakin applies chi-square test to the whole Compustat data of 1954 to 1972 (19 years). He does not study CF/TD and NI/NS. However, he examines another two ratios: CA/TA and NI/TA which are not reported here. Except for 15 years of TD/TA data, all Deakin's tests have rejected the null hypothesis.

c/ We apply the K-S tests to the whole Compustat data for each of five years (1961, 1965, 1970, 1975, 1980).

d/ We apply the K-S tests to the data of 22 S.I.C. 2-digit industries in 1975.

e/ The K-S tests are applied to the data of 10 S.I.C. 4-digit industries in 1975.

TABLE 3: TESTS OF NORMALITY[a/],[b/]

$H_0(1)$: $\widetilde{(Y_i/X_i)} \sim N(\mu^*, \sigma^2)$, for all $i \in I$.

$H_0(2)$: $\tilde{\eta}_i \sim N(0, \sigma^2)$, for all $i \in I$.

Financial Ratios	Model 1	Model 2	Model 3	Model 4a	Model 4b	Model 4c
CA/CL	29	48	22	12	18	28
CF/TD	28	47	27	10	17	n.a.
TD/TA	35	47	30	n.a.	22	n.a.
QA/NS	31	46	25	15	n.a.	23
NI/NS	16	43	13	8	n.a.	19
Total	$\frac{139}{250}$ = 55.6%	$\frac{231}{250}$ = 92.4%	$\frac{117}{250}$ = 46.8%	$\frac{45}{200}$ = 22.5%	$\frac{57}{150}$ = 38%	$\frac{70}{150}$ = 46.7%

[a/] Each entry in this table indicates the number of rejections at 5% significance level out of 50 tests (5 years and 10 industries).

[b/] On the regressional approach of factoral control [Eqs. (10) to (12)], Y_i = numerator of the financial ratio, X_i = denominator of the financial ratio, and Z_i = TA, NS, TD, respectively, for models 4a, 4b and 4c.

TABLE 4: TESTS OF HOMOSCEDASTICITY[a]

H_0: $Var(\tilde{\eta}_i) = \sigma^2$, for all i .

Financial Ratios	Model 2	Model 3[b]	Model 4a	Model 4b	Model 4c
CA/CL	50	34	14	24	36
CF/TD	50	31	21	21	n.a.
TD/TA	50	32	n.a.	23	n.a.
QA/NS	50	30	15	n.a.	37
NI/NS	49	13	18	n.a.	35
Total	$\frac{249}{250}$ = 99.6%	$\frac{140}{250}$ = 56%	$\frac{68}{200}$ = 34%	$\frac{68}{150}$ = 45.3%	$\frac{108}{150}$ = 72%

[a] Each entry in this table indicates the number of rejections at 5% significance level.

[b] Because traditional financial ratio (Model 1) is a special case of Model 3, this column also shows the lower-bound of rejections of null hypothesis on Model 1.

TABLE 5: MARGINAL CONTRIBUTION OF FACTORAL CONTROL OF MODEL 3 OVER MODEL 1

H_0: $d = 0$

Financial Ratios	Number of Rejections at 5% Level Out of 50 Tests
CA/CL	24
CF/TD	28
TD/TA	32
QA/NS	19
NI/NS	16
Total	$\frac{119}{250}$ = 47.6%

TABLE 6: S.I.C. 4-DIGIT CODE AND NORMALITY[a]

Industry	CA/CL		CF/TD		TD/TA		QA/NS		NI/NS		Summation	
	Model 1	Model 4a	Model 1	Model 4a	Model 1	Model 4b	Model 1	Model 4a	Model 1	Model 4a	Model 1	Model 4[b]
1311	5	5	5	0	5	4	5	5	2	2	22	16
2200	4	1	2	1	4	3	4	0	1	0	15	5
2300	5	3	5	4	5	3	3	2	2	2	20	14
2911	3	1	2	0	2	4	4	3	4	0	15	0
3310	5	0	3	0	5	3	5	3	3	2	21	8
3679	2	0	4	3	4	1	4	0	1	1	15	5
3714	3	0	1	0	3	3	3	1	1	1	11	5
4511	0	0	1	0	2	0	2	0	2	0	7	0
4911	1	2	2	0	4	0	1	1	0	0	8	3
5311	1	0	3	2	1	0	0	0	0	0	5	2
Total	29	12	20	10	35	21	31	15	16	8	139	66

[a] Each entry in the table indicates the number of ~~rejecting the~~ rejections of normality at 5% significance level out of 5 tests (one for each year).

[b] Applying model 4b for TD/TA and model 4a for the other four ratios.

TABLE 7: TIMING AND NORMALITY[a]

Year	CA/CL		CF/TD		TD/TA		QA/NS		NI/NS		Summation	
	Model 1	Model 4a	Model 1	Model 4a	Model 1	Model 4b	Model 1	Model 4a	Model 1	Model 4a	Model 1	Model 4[b]
1980	3	2	4	3	6	3	5	3	2	0	20	11
1975	0	4	8	2	9	6	7	5	4	2	36	19
1970	8	3	9	2	9	5	6	3	2	3	34	16
1965	6	1	4	3	6	5	7	2	4	2	27	13
1961	4	2	3	0	5	2	6	2	4	1	22	7
Total	29	12	28	10	35	21	31	15	16	8	139	66

[a] Each entry in the table is the number of ~~rejecting~~ rejections of normality at 5% significance level out of 10 tests (one for each industry).

[b] Applying model 4b for TD/TA and model 4a for the other four ratios.

TABLE 8: INTERTEMPORAL INDEPENDENCY OF ACCOUNTING DATA

Current Asset/Current Liabilities[a/]

Industry	Pearson Correlation 1980:1975	Pearson Correlation 1980:1970
1311	0.59	0.21*
2200	0.74	0.49
2300	0.60	0.52
2911	0.42	0.49
3310	0.76	0.52
3679	0.61	0.32*
3714	0.65	0.68
4511	0.15*	0.39*
4911	0.31	0.21*
5311	0.79	0.65

[a/] The accounting data have been refined by model 4a. Each entry indicates the Pearson Correlation between the data of two different periods.

[b/] The null hypothesis of no correlation cannot be rejected at 5% significance level.

Appendix

THE EFFECTS OF INDUSTRIAL FACTORS AND ECONOMIC ENVIRONMENT ON THE NORMALITY OF ACCOUNTING NUMBERS: AN ANOVA TEST

In this appendix, we plan to test the following three null hypotheses:

$H_0(1)$: After blocking the data according to the S.I.C. 4-digit code, the presence of normality in the refined data is independent of the industrial block.

$H_0(2)$: Assumptions 2 and 3 apply equally well to all five accounting numbers under study. Consequently, the presence of normality in the refined data is independent of the category of the accounting number.

$H_0(3)$: Assumptions 3 and 4 apply equally well to all the time periods. Consequently, the presence of normality in the refined data is independent of the time period.

In this study, we have done 1,250 tests which are generated by one test each on ten industries, five categories of accounting numbers, five years, and five models.[1] We assume that there are no interactions among these four classifications, since normality is an ex ante property of data and these four classifications are chosen ex post by us. We cannot think of any reasons for the presence of the interaction.

Since the entry in our ANOVA test is the binomial proportion, the variance of each entry depends on the magnitude of the proportion. Consequently, the variance is not constant for all entries. To mitigate this

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[1]There are six models available for factorial control, but not every one of them is applicable to all the accounting numbers. On the average, we have five models for each category of accounting numbers.

problem, we can take an arcsin transformation of the proportion as follows:

$$\text{Angle} = \text{Arcsin} \sqrt{\text{Proportion}} .$$

After the transformation, each entry of our ANOVA test is an angle which has an approximately constant variance. The results of ANOVA tests are given in Tables A3 and A4.

TABLE A1

S.I.C. 4-Digit Code and Normality[a/]

Industry	CA/CL[b/]		CF/TD[c/]		TD/TA[d/]		QA/NS[c/]		NI/NS[c/]		Total	
1311	27	(90%)	16	(64%)	19	(95%)	25	(100%)	14	(56%)	101	(81%)
2200	15	(50%)	13	(52%)	15	(75%)	14	(52%)	9	(36%)	66	(53%)
2300	22	(73%)	21	(84%)	15	(75%)	13	(42%)	16	(64%)	87	(70%)
2911	18	(60%)	11	(44%)	13	(65%)	17	(68%)	12	(48%)	71	(57%)
3310	18	(60%)	13	(52%)	17	(85%)	22	(88%)	17	(68%)	87	(70%)
3679	14	(47%)	17	(68%)	11	(55%)	15	(60%)	6	(24%)	63	(50%)
3714	11	(37%)	12	(48%)	15	(75%)	11	(44%)	10	(40%)	59	(47%)
4511	7	(23%)	5	(20%)	9	(45%)	8	(32%)	5	(20%)	34	(27%)
4911	15	(50%)	10	(40%)	13	(65%)	10	(40%)	5	(20%)	53	(42%)
5311	10	(33%)	11	(44%)	6	(30%)	5	(20%)	5	(20%)	37	(30%)
Total	157	(52%)	129	(52%)	133	(67%)	140	(56%)	99	(40%)	658	(53%)

a/ Each entry of this table indicates the number and proportion of tests rejecting the null hypothesis of normality at 5% significance level.

b/ Number of tests in each entry is 30, 6 (models) × 5 (years).

c/ Number of tests in each entry is 25, 5 (models) × 5 (years).

d/ Number of tests in each entry is 20, 4 (models) × 5 (years).

TABLE A2

Economic Environment and Normality[a]

Year	CA/CL[b]		CF/TD[c]		TD/TA[d]		QA/NS[c]		NI/NS[c]		Total	
1980	25	(42%)	23	(46%)	24	(60%)	27	(54%)	15	(30%)	114	(46%)
1975	41	(68%)	30	(60%)	33	(83%)	33	(66%)	22	(44%)	159	(64%)
1970	43	(72%)	33	(66%)	31	(78%)	29	(58%)	20	(40%)	156	(62%)
1965	25	(42%)	26	(52%)	28	(70%)	26	(52%)	24	(48%)	129	(52%)
1961	23	(38%)	17	(34%)	17	(40%)	25	(50%)	18	(36%)	100	(40%)
Total	157	(52%)	129	(52%)	133	(67%)	140	(56%)	99	(40%)	658	(53%)

[a] Each entry of this table indicates the number and proportion of tests rejecting the null hypothesis of normality at 5% significance level.

[b] Number of tests in each entry is 60, 6 (models) × 10 (industries).

[c] Number of tests in each entry is 50, 5 (models) × 10 (industries).

[d] Number of tests in each entry is 40, 4 (models) × 10 (industries).

TABLE A3

ANOVA: Industries (Ind) and Ratio

Source of variation	Sum of Squares	df	Mean Square	F	Significance of F
Main effects	7152.395	13	550.184	8.348	0.000
IND	5662.928	9	629.214	9.547	0.000
RATIO	1489.467	4	372.367	5.650	0.001
Explained	7152.395	13	550.184	8.348	0.000
Residual	2372.750	36	65.910		
Total	9525.145	49	194.391		

TABLE A4

ANOVA: Year and Ratio

Source of variation	Sum of Squares	df	Square	F	Significance of F
Main effects	1396.347	8	174.543	8.120	0.000
YEAR	747.830	4	186.957	8.698	0.001
RATIO	648.518	4	162.129	7.543	0.001
Explained	1396.347	8	174.543	8.120	0.000
Residual	343.914	16	21.495		
Total	1740.262	24	72.511		

Footnotes

*Associate Professor, University of Pennsylvania. This paper has benefitted from comments by participants of workshops at the University of Chicago, University of Pennsylvania, Columbia, NYU, MIT, SUNY/Buffalo and the Sixth International Symposium of Multivariate Analysis. John Plaxton has contributed significantly as a research assistant. The incisive and insightful comments of Jim Patell, Nick Dopuch, Nick Gonedes, Richard Leftwich and Ro Verrecchia have greatly improved this paper. I take responsibility for any errors that remain.

[1]If all the systematic factors have been properly controlled, the refined data are white noises which by definition are IID.

[2]Although the spirit of our study is very similar to that of Lev and Sunder (1979), i.e., we all try to find a rigorous methodology to analyze financial data, our focus is rather different. While Lev and Sunder are concerned with the industrial norm properties, I am more interested in the properties of refined individual data. This difference in focus will become clear in Section II.

[3]Hence, the systematic factors are defined as those variables in set $\tilde{S}_i$ which cause the heterogeneity and dependency across $\tilde{Y}_i$.

[4]After all the systematic factors have been controlled, the refined data would be pure white noise and cross-sectionally independent. However, in reality, it is impossible to control all the systematic factors, and the refined financial data can be cross-sectionally and interemporally correlated. When the assumption of independency is violated, the test of normality is not conclusive; the test results should be interpreted descriptivelly, not conclusively. However, because the controls for systematic factors are much more extensively and carefully exerted in this paper than in Deakin's (1976) work, our test of normality should be more conclusive.

[5]For example, all that Watts and Zimmerman (1978) demonstrated was that the size of a firm is an important factor on the advocacy of accounting standard. Their model was one of many possible interpretations of the empirical evidence of size-effect.

Lee and Hsieh (1983) argued that the size effect and the industrial effect on the choice of inventory accounting methods reflect the underlying differences in production-investment opportunity sets.

[6]We assume the industrial effect has been appropriately controlled by the S.I.C. classification.

[7]Bey and Pinches (1980) give a comprehensive survey on the test methodology pertinent to heteroscedascity.

[8]Our sample size varies from case to case. Instead of omitting a constant number of data, we omit the middle 15% of the sample data. Therefore, the value of c depends on the sample size.

[9]In Section IV, we do find that the K-S test is much more powerful than the chi-square test.

[10]For example, see Beaver (1966), Altman (1968), Libby (1975), Abdel-Khalik and Al-Sheshia (1980), and Ohlson (1980).

[11]Deakin is aware of the systematic factor of industrial factors, but he devotes most of his effort to investigating the normality without controlling such an effect.

[12]There are no data for the 1954 test on the ratio of Current Assets/Total Assets. Hence the number of tests is $5 \times 19 - 1 = 94$.

[13]Only three years of our study overlap with Deakin's (1961, 1965, 1970), among which 19654 and 1970 are the "exceptions," i.e., the null hypothesis of normality on Total Debts/Total Assets ratios are not rejected at the 5% significance level in 1965 and 1970.

[14]The 22 S.I.C. 2-digit industries are 13, 20, 22, 23, 27, 28, 29, 30, 33, 34, 35, 36, 37, 38, 39, 49, 50, 51, 53, 54, 60 and 73. A description and list of firms can be found in the Compustat Manual (1981). The ten S.I.C. 4-digit industries have been discussed in Section IV.

[15]A priori, neither model dominates the other. Model 1 does not exert perfect control for either the mean-shifting or the vraiance-shifting factors. If only the mean-shifting factors are present, Model 2 can do a perfect job.

[16]In short, we just call it Model 4 in those analyses related to Tables 6 and 7.

[17]Using Quandt-Likelihood Ratio and Brown-Dubin-Evans test, Lee (1983b) demonstrated significant increase in structural shift in makret equations (CAPM) during the '70s. The financial market structure in the '70s is less stable in the '60s.

[18]For a rigorous account of ergodicity, see Hannon (1970, p. 201). Foster (1978) has also found a high autocorrelation in financial ratios.

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