THE EFFECT OF A REDUCTION IN CORPORATE TAXES ON INVESTMENTS IN RISKFREE AND RISKY ASSETS

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This short note utilizes a highly-stylized model to ascertain the effect of a reduction in corporate tax rates upon the value and quantity of risky capital assets -- both immediately and after full adjustment has taken place. It is not obvious what the effect will be since a reduction in corporate taxes would affect both the expected return of an investment and the risk characteristics of that investment. Depending upon how individuals weight these two characteristics of an investment, a reduction in the corporate tax could lead to an immediate increase or decrease in the value of risky capital assets. After full adjustment to a reduction in the corporate tax rate, the proportion of wealth invested in risky assets might increase or decrease and the savings rate might be larger or smaller than before. The reason for this ambiguity stems from the explicit introduction of risk into the model.

Consider the following highly-stylized model: The government issues short-term riskfree assets and taxes investors and firms at rates τ_p and τ_c respectively. Neither firms nor investors obtain any benefit from government expenditures. We shall assume that the riskfree rate is constant over time and that the inflation rate is zero or that the covariance between the rates of return on risky assets and the rate of inflation is close to zero. Otherwise, the variables in the model could be interpreted as nominal variables under a money illusion assumption.

Investors are characterized by multiperiod utility functions which display constant proportional risk aversion, though not necessarily the same value for every investor. There is no labor income, and all corporate taxes are borne by investors. Investors can invest in the riskfree liabilities of the government and can borrow at the riskfree rate from the government. The market for these liabilities is perfect in the sense that there are no transaction costs and that investments are infinitely divisible. Finally, we shall assume that investors evaluate the propects of all firms in the same way -- the so-called homogenous expectation assumption.

Firms are all equity-financed and their liabilities constitute the only risky assets in the economy. Firms in turn own productive machines which produce consumption goods. When placed in operation, each machine costs the same amount in terms of consumption goods and is identical in its productive capabilities as measured by the probability distributions of the real productive output. Consumption goods can be coverted without costs into productive machines, but productive machines cannot be coverted back to consumption goods. While it would be possible to assume an explicit depreciation schedule, it will serve the purposes of this paper to assume that, after a fixed amount of time, the machine has no further productive capability and is therefore valueless, but up to that point produces in the same way as a new machine. Thus, the scale of firms can be adjusted over time either upward or downward.

This paper will first examine the immediate impact of a reduction in corporate taxes and then the long-term impact. To be precise, we shall assume in measuring the immediate impact that investors have already made their consumption/investment decision and that firms have already purchased their productive machines for the period. Thus, the aggregate physical amount available for consumption as well as the probability distribution of productive output for the next period are fixed. In the long term, we shall allow the scale of firms to adjust to the levels desired by investors. In the short term, the number of machines will be assumed fixed, but in the long run fully variable.

The Immediate Effect: In the long run, the value of all productive machines will be their replacement value, but in the short term there is no guarantee that the market value of these machines will be the same as their relacement value. If, for instance, the change in the tax law were to make the number of machines in existence greater than desired, the value of a machine would be less than its replacement cost; if less than desired, the reverse.

It can be shown[1] under the assumptions given above that the following expression relating the before-corporate tax return to firms, r_m, and the before-personal tax return on riskfree assets, r_f, must hold at the macro-level at every point in time:

$$\frac{E(r_m)(1 - \tau_c)(1 - \tau_p) - r_f(1 - \tau_p)}{(1 - \tau_c)^2(1 - \tau_p)^2 \mathrm{Var}(r_m)} = \alpha C \quad , \tag{1}$$

where τ_c is the corporate tax, $(1 - \tau_p)$ is an appropriate aggregate of personal tax rates, C is a simple harmonic mean of the investors' coefficients of relative risk, and α is the ratio of the value of risky assets to the aggregate value of all assets held by investors.[2]

The stochastic return r_m is defined as the ratio of the stochastic total output from the productive machines, say R, to the market value of these machines, say V_m. If the number of productive machines is exactly equal to the desired number, V_m will be the replacement value. However, an

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1 Irwin Friend and Marshall Blume, "The Demand for Risky Assets," American Economic Review, (December 1975).

2 It is assumed that all taxes are symmetric in gains and losses.

unanticipated change in tax law may make the existing number of machines non-optimal, which would cause V_m to deviate from the replacement value. The following calculus will examine how V_m varies with changes in τ_c holding the distribution of R constant.

Equation (1) can be rewritten as:

$$E(r_m) = \frac{r_f}{(1 - \tau_c)} + (1 - \tau_c)(1 - \tau_p)\sigma^2(r_m)\alpha C \quad . \tag{2}$$

Defining V_f as the net value of all riskfree assets outstanding, one can rearrange (2) as:

$$E(R) = \frac{r_t}{(1 - \tau_c)} V_m + (1 - \tau_c)(1 - \tau_p)\sigma^2(R) \frac{C}{V_m + V_f} \quad . \tag{3}$$

The total differential of (3) is:

$$dE(R) = \frac{r_t}{(1 - \tau_c)} dV_m + \frac{r_t V_m}{(1 - \tau_c)^2} d\tau_c - (1 - \tau_p)\sigma^2(R)C \left[\frac{(1 - \tau_c)}{(V_m + V_t)^2} dV_m + \frac{1}{V_m + V_t} d\tau_c \right] \quad , \tag{4}$$

which, upon setting $dE(R)$ to be zero, yields

$$dV_m/d\tau_c = - \frac{\left[\frac{r_t V_m}{(1 - \tau_c)^2} - \frac{(1 - \tau_p)\sigma^2(R)C}{V_m + V_f} \right]}{\left[\frac{r_t}{(1 - \tau_c)} - \frac{(1 - \tau_p)(1 - \tau_c)\sigma^2(R)C}{(V_m + V_f)^2} \right]} \quad . \tag{5}$$

Finally, by factoring out the expression $(V_m + V_f)/(1 - \tau_c)$ from the numerator of (5), $dV_m/d\tau_c$ can be written as

$$dV_m/d\tau_c = - \frac{\left[\frac{r_f}{(1 - \tau_c)} - (1 - \tau_c)(1 - \tau_p)\sigma^2(r_m)C\ \alpha\right]\left[\frac{\alpha(V_m + V_f)}{(1 - \tau_c)}\right]}{\left[\frac{r_f}{(1 - \tau_c)} - (1 - \tau_c)(1 - \tau_p)\sigma^2(r_m)C\ \alpha^2\right]} \quad . \tag{6}$$

Since the term in the second brackets in the numerator of (6) is positive, and since α and τ_p will be between 0 and 1, $dV_m/d\tau_c$ would be negative if one of the two following conditions held:

$$r_f > (1 - \tau_c)^2(1 - \tau_p)\sigma^2(r_m)C\ \alpha \tag{7a}$$

or

$$r_f < (1 - \tau_c)^2(1 - \tau_p)\sigma^2(r_m)C\ \alpha^2 \quad . \tag{7b}$$

Under condition (7a), both the numerator and demominator of (6) would be positive; and under condition (7b) both would be negative. For realistic values of the relevant variables, such as $\tau_c = .45$, $\tau_p = .25$, $\sigma(r_m) = .15$, $\alpha = .85$, $C = 2$ and $r_f = .02$, condition (7a) would hold, implying that V_m would increase when the corporate tax is decreased.[1] If the r_f and r_m are measured in nominal terms, it is even more likely that condition (7a) would hold. Thus, the before-tax cost of capital would be expected to fall under a reduction in corporate taxes stimulating firms to undertake additional investment.

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[1] It should be noted that $r_f = .02$ is at the low end of plausible values, even for the real riskfree rate. Higher values of r_f would reinforce the applicability of condition 7(a). C may be higher than 2, and τ_c and τ_p somewhat lower than the above assumption, but not by enough to change the conclusion drawn.

If condition (7a) holds, it is easily shown that the rate of return required by investors after corporate taxes has in fact increased even though the firm's before-tax cost of capital has decreased. The required return after taxes is $(1 - \tau_c)E(R)/V_m$. Taking the derivative of this quantity, one obtains

$$\frac{d\,\dfrac{(1-\tau_c)E(R)}{V_m}}{d\tau_c} = \left[(1-\tau_c)E(R)\,\frac{dV_m^{-1}}{d\tau_c} - \frac{E(R)}{V_m}\right]$$

$$= E(r_m)\left[-\frac{(1-\tau_c)}{V_m}\cdot\frac{dV_m}{d\tau_c} - 1\right] \quad . \qquad (8)$$

The first term in the brackets is, from (6), equal to

$$\frac{(r_f/(1-\tau_c)) - (1-\tau_c)(1-\tau_p)\sigma^2(r_m)C\,\alpha}{(r_f/(1-\tau_c)) - (1-\tau_c)(1-\tau_p)\sigma^2(r_m)C\,\sigma^2} \quad . \qquad (9)$$

Under condition (7a), (9) will be between 0 and 1, so that the derivative given by (8) is negative.

Thus, under plausible assumptions about the parameters of the model, it would appear that the immediate effect of lower corporate tax rates would be a reduction in a firm's before-tax cost of capital and an increase in the after-corporate tax return required by investors.

The Long Run: In the long run, the stock of productive machines becomes variable and the rates of return on these machines will be commensurate with their replacement values. Thus, in the long run, it may well be that $E(r_m)$

and $\sigma^2(r_m)$ will be constant; what will change is the proportion of assets invested in risky ventures, namely α.

The effect of α of a change in τ_c, holding constant $E(r_m)$ and $\sigma^2(r_m)$, is readily derived from (2). From (2), the total differential of $E(r_m)$ is

$$dE(r_m) = \frac{r_f}{(1 - \tau_c)^2} d\tau_c - (1 - \tau_p)\sigma^2(r_m)\alpha C d\tau_c$$

$$+ (1 - \tau_c)(1 - \tau_p)\sigma^2(r_m) C d\alpha \quad . \tag{10}$$

Setting (10) to zero, one obtains:

$$\frac{d\alpha}{d\tau_c} = \left[\frac{-r_f + (1 - \tau_c)^2(1 - \tau_p)\sigma^2(r_m)\alpha C}{(1 - \tau_c)^3(1 - \tau_p)\sigma^2(r_m)C}\right] \quad , \tag{11}$$

which is likely to be negative for reasonable values of the parameters involved.

Thus, a decrease in corporate taxes would under this model probably lead to an increase in the proportion of wealth devoted to investment in risky assets. Whether there would be any increase in the total investment in risky assets, as distinct from the proportion of wealth in risky assets, would depend upon the subsequent savings plans of investors and the impact of these plans upon the level of wealth.