

CAPITAL BUDGETING UNDER UNCERTAINTY:

THE ISSUE OF OPTIMAL TIMING

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The Issue of Optimal Timing

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### Abstract

In a capital budgeting decision, timing is often an important part of the decision-maker's opportunity set. The ability of the manager to choose when to undertake an investment has similarities to the ability of the holder of a securities option to choose when to exercise that option. This paper uses formulas developed for the valuation of securities options to evaluate the timing option and to derive decision rules for optimal investment timing. The paper provides examples of application in the cases of plant and equipment replacement, marketing of new products, and real estate development.

## I. Introduction

In capital budgeting decisions, the manager often has some freedom to choose the timing of project implementation.<sup>1</sup> For example, before old equipment is totally worn out, the manager has choices on the timing of its replacement.<sup>2</sup> This choice of timing is not a free good. Managers may have to expend resources to acquire this timing option. For example, to defer the purchase of new equipment, the manager needs to increase the maintenance budget on the existing equipment. Hence the valuation of this timing option should be of great interest to managers. This paper makes use of the literature on securities options to develop two stochastic models for the valuation of the timing option and for the determination of optimal timing.

The paper is organized as follows. Section II states the assumptions, defines the problem and derives the stochastic models. Three examples are considered in Section III where our two models are applied to decide the optimal timing of plant and equipment replacement, marketing a new product, and real estate development. In Section IV, we provide concluding remarks.

## II. Models of Optimal Timing Decision

We consider an investment project whose present value  $V$  is generated by a stochastic process. In the specific applications which appear in Section III, the investment project will be interpreted as the replacement of a capital asset, the inauguration of a new product and the development of real estate. The project generates a stream of cash flows and the present value of the project represents the time and risk-adjusted value of these cash flows.<sup>3</sup>

The manager has the option to implement the project at any point within the time horizon  $(0, T)$ . We have three alternative specifications for the present value of the project,  $V$ , and the cost of the project,  $X$ . Based on these three specifications, we develop two stochastic models to evaluate the timing option and to determine optimal timing.

#### A. Black Model

We assume that the present value of the investment project,  $V(z, t)$  is completely specified by the state variable  $z$ , and time  $t$ . The present value changes in the time interval  $(t, t + dt)$  by :

$$(1) \quad \frac{dV}{V} = \mu dt + \sigma dz ,$$

where  $\mu$  is the instantaneous expected rate of return on the project,  $\sigma$  is the instantaneous standard deviation of return on the project, and  $z$  is generated by a Wiener process.<sup>4</sup>

The cost of project,  $X$ , is assumed to be given and constant. However, this assumption is not crucial to our analysis. Adopting the valuation models developed by Fischer (1978) and Margrabe (1978), we can incorporate the stochastic process of a random  $X$  into our analysis. The Specification 3, which will be discussed in the Roll-Whaley Model, allows  $X$  to change by a known amount.

The opportunity of implementing an investment project within the time horizon  $(0, T)$  is analogous to an American call option on a security which pays no dividends. Merton (1973) showed that the value of an American call option without dividend  $C(V, 0, T, X)$  is equal to the value of a European call option  $c(V, 0, T, X)$ , where the upper case  $C$  denotes American call option and the lower case  $c$  denotes European call option. Hence the value of this deferrable investment opportunity can be derived by the Black and Scholes (1973) formula:

$$\begin{aligned}
(2) \quad & C(V, 0, T, X) \\
& = c(V, 0, T, X) \\
& = VN(d_1) - X e^{-rt} N(d_2)
\end{aligned}$$

where

$$\begin{aligned}
d_1 & = \{ \ln(V/X) + (r + 0.5\sigma^2) T \} / \sigma \sqrt{T} , \\
d_2 & = d_1 - \sigma \sqrt{T} ,
\end{aligned}$$

$r$  is the risk-free interest rate, and  $N(d)$  is the univariate cumulative normal density function with upper integral limit  $d$ .

The value of the timing option is the difference between the value of the deferrable investment opportunity when the timing option is "alive" and when the timing option is "dead." When the firm decides to implement the investment opportunity now, the value is:

$$(3) \quad C(V, 0, 0, X) = \max(V_0 - X, 0) .$$

where  $V_0 - X$  is the net present value (NPV) of the investment project at time 0. Hence, the value of the timing option,  $P(0, T)$ , is:

$$\begin{aligned}
(4) \quad P(0, T) & = C(V, 0, T, X) - C(V, 0, 0, X) \\
& = \min [C(V, 0, T, X) , C(V, 0, T, X) - (V_0 - X)] \geq 0
\end{aligned}$$

Eq. (4) indicates that the relationship between the value of timing option,  $P(0, T)$ , and the value of the deferrable investment opportunity depends on the NPV of the investment project at time 0.

If it is profitable to implement the project now, i.e.,  $V_0 - X > 0$ , then the value of the timing option is equal to the value of this deferrable investment opportunity minus the NPV. If the NPV is nonpositive, then this investment is not worth implementing now; the value of the deferrable investment opportunity is totally attributable to the value of the timing option.

So far, the optimal timing issue is trivial; the firm will not undertake the investment project until the last moment, i.e., time  $T$ . When the firm finally implements this investment project, the value of this investment opportunity is:

$$(5) \quad C(V, T, T, X) = \max (V_T - X, 0) .$$

Hence we have the traditional result: the firm simply follows the NPV rule of capital budgeting at the moment when it can no longer postpone the investment decision.

The reason for the triviality of the optimal timing issue is that  $V$  is assumed to be generated by a Wiener process; hence, the expected present value of the investment project is not affected by the deferment, namely, there is no cost to waiting. Now let us introduce a cost to waiting,  $D$ . Without the cost of waiting, the present value of the investment project that is implemented right now, denoted by  $V$ , would be the same as that which will be implemented in the future, denoted by  $S$ . With the cost of waiting, the present value of the investment project that will be implemented in the future,  $S$ , is equal to the present value of the investment project that is implemented right now,  $V$ , minus the present value of waiting cost,  $D$ . Let  $S_k$  be the present value of the investment project that will be implemented at time  $t_k$  and  $V_0$  be the present value of the investment project that is implemented right now, then, we have:

$$(6) \quad S_k = V_0 - \sum_{i=1}^{k-1} D_i e^{-rt_i},$$

where  $0 \leq t_i \leq T$  and  $D_i$  is the discrete waiting cost incurred at time  $t_i$ .

In this paper we develop three specifications on variables  $(V, S, D)$ . In Specification 1, we set  $dV/V$  to be generated by an Ito process and  $dS/S$  to be generated by an Ito process with discrete jump.<sup>5</sup> In Specifications 2 and 3, we set  $dS/S$  to be generated by an Ito process and  $dV/V$  to be

generated by an Ito process with discrete jump. The Black model of option valuation can be applied to Specification 1. The Roll-Whaley model of option valuation can be applied to Specifications 1 and 2.

### Specification 1

- i. The movement of  $V$ , the present value of the investment project that is implemented at time 0, is described by eq.(1).
- ii. If the investment project is not implemented at time 0, the present value of the project periodically falls by a known amount  $D_i$  at time  $t_i$  where  $i = 1, 2, \dots, n$ .
- iii. If the project is implemented at time  $t_k$ , then  $S_k$ , its present value at time 0, can be described by eq. (6), for  $k = 2, 3, \dots, n$ .

An example for the Specification 1 is the introduction of a new product. Before a new product is introduced to the market and takes up a dominant market share, the barrier to entry of competitive product would be small. The introduction of a similar product by a competitor throughout the waiting period will reduce the present value of the given new product.

The discrete waiting cost,  $D_i$ , is equivalent to dividend in an American call option. According to Black (1975) and Smith (1976), the American call option holder may benefit from exercising early, just prior to the ex-dividend instant. Hence, the firm will initiate the project at time  $t_k$  prior to the decline of  $D_k$ . Consequently,  $D_k$  is not included in the calculation of  $S_k$ .

Black (1975) recommends an approximate value of American call equal to the higher of the value of a European call where the stock price net of the present value of the escrowed dividend is substituted for the stock price and a European call where the time to ex-dividend is substituted for



the time to expiration. Therefore, the value of the investment opportunity under Specification 1 can be approximated by eq. (7).

$$(7) \quad C(V, 0, T, X) \\ = \quad \text{Max} [\{c(S_k, 0, t_k, X) \mid k = 2, \dots, n\}, c(V, 0, t_1, X)].$$

The Black formula as described by eq. (7) gives rise to a downward biased approximation.<sup>6</sup>

Because the movement of project present value follows a stochastic process, the planned optimal timing of project implementation can be different from the ex post optimal timing. At time 0, when the manager looks over the entire decision horizon (0, T), the planned optimal timing,  $t^*$ , is determined by eq. (8).

$$(8) \quad t^* = \underset{t}{\text{argmax}} [\{c(S_k, 0, t_k, X) \mid k = 2, \dots, n\}, c(V, 0, t_1, X)].$$

Since  $c(V, 0, t_1, X) \geq c(V, 0, 0, X)$ , the firm will wait at least until the incipient moment of first decline to implement the project.

At each instant,  $t_h$ , just before the known present value decline,  $D_h$ , we can calculate the trigger-point present value,  $V_h^*$ , from the solution that satisfies eq. (9):

$$(9) \quad c(S_k^*, t_h, t_k^*, X) = \max [c(S_k, t_h, t_k, X) \mid n \geq k > h],$$

$$c(S_k^*, t_h, t_k^*, X) = V_h^* - X,$$

$$S_k^* = V_h^* - \sum_{i=t_h}^{t_k^*} D_i e^{-r(t_i - t_h)}.$$

In eq. (9), we have three sub-equations to determine solutions for three variables,  $S_k^*$ ,  $t_k^*$ , and  $V_h^*$ . The first subequation in eq.(9) derives the value of a deferrable investment opportunity and last subequation demonstrates the relationship between the present values of the project

that is implemented at time  $t_k$  and that is implemented at  $t_h$ . The second subequation derives the condition under which the value of timing option over the horizon  $(t_h, t_k)$  is equal to zero. The trigger-point present value,  $V_h^*$ , is what makes the timing option worthless at time  $t_h$ . Whenever the present value of the project implemented at time  $t_h$  is no smaller than the trigger point present value, i.e.,  $V_h \geq V_h^*$ , we have  $P(t_h, T) = 0$ . The deferrable investment opportunity is worth more "dead" than "alive"; the project should be implemented immediately at time  $t_h$ .

#### B. The Roll-Whaley Model

The following two specifications, which are based on Roll (1977) and Whaley (1982), lead to the same closed form solutions.

##### Specification 2

- i. If the project has not yet been implemented, the project present value,  $S$ , moves through time according to the following stochastic differential equation:

$$(10) \quad \frac{dS}{S} = m dt + s dz ,$$

where the specification of  $m$ ,  $s$ ,  $z$  are the same as those in eq. (1).

- ii. If the project is implemented before time  $t$ , the project generates a known extra cash flow,  $D$ , at time  $t$ .<sup>7</sup>

##### Specification 3

- i. Same as Specification 2.
- ii. If the project is implemented after time  $t$ , the cost of the project increases by the amount  $D$  at time  $t$ .

These two specifications make the Roll-Whaley model somewhat more restrictive than the Black Model, but they give rise to a closed form solution while the Black Model can only provide a biased approximation.

An example for Specification 2 is to open a department store in Los Angeles before the opening of the 1984 Olympics. The rate of return of the department store is generated by a "Ito process" with a blip during the 1984 Olympics if the store is open by then.

An example for Specification 3 is to replace an old car before the January snow blizzard. Otherwise, the cost of replacement is increased by a large repair bill, D.

Under either specification, we can adopt the solution derived by Whaley (1981) to calculate the value of the timing option and the optimal timing of project implementation. The value of the deferrable investment opportunity,  $C(S, 0, T, X)$ , is :

$$(11) C(S, 0, T, X) = S [N_1(b_1) + N_2(a_1, -b_1; -t/T)] \\ - X e^{-rt} [N_1(b_1)e^{-r(T-t)} + N_2(a_2, -b_2; -t/T)]. \\ + D e^{-rt} N_1(b_2)$$

where

$$a_1 = \{ \ln(S/X) + (r + 0.5 \sigma^2) T \} / \sigma T,$$

$$a_2 = a_1 - \sigma T,$$

$$b_1 = \{ \ln(S/S_t^*) + (r + 0.5 \sigma^2) t \} / \sigma t,$$

and  $N_2(a, b; \rho)$  is the bivariate cumulative normal density function with upper integral limits  $a$  and  $b$ , and correlation coefficient  $\rho$ .  $S_t^*$  is the trigger-point value at time  $t$ , the incipient moment before the waiting cost  $D$  is realized.

The value of timing option,  $P(0, T)$  is:

$$(12) P(0, T) = \min [C(S, 0, T, X), C(S, 0, T, X) - (V_0 - X)].$$

The trigger-point present value is determined by:

$$(13) c(S_t^*, t, T, X) = S_t^* + D - X.$$

If at the instant, time  $t - \epsilon$ , before  $D$  is realized, the present value of the project that will be implemented immediately after  $D$  is realized is no smaller than the trigger-point present value, i.e.,  $S_t \geq S_t^*$ , the project should be implemented at time  $t - \epsilon$ , where  $\epsilon$  is an infinitesimal value.

### III. Applications

#### A. Replacement of Plant and Equipment

One of the most common capital budgeting decisions is the replacement of plant and equipment. Let  $V$  and  $X$  be respectively the present value and the cost of replacement. The existing equipment has remaining life of  $T$  years. If the firm keeps the equipment in operation, it will face a major overhaul or other cost at time  $t$  with costs in the amount of  $D$ .

The firm operates in a competitive market. Hence the risk and time-adjusted present value of stream of future cash flows can be described by eq. (1) and eq. (10).<sup>8</sup> The capital goods supplier generally keeps a stable price policy with price adjustments announced well in advance.

This example fits the Specification 3 of Section II. The variable  $D$  can be interpreted as a planned major overhaul on the existing equipment or a pre-announced price hike on the new equipment. We can apply Roll-Whaley Model to evaluate the timing option and to calculate the optimal timing of replacement. At time  $t$ , the gross value of timing option  $P(t, T)$  is:

$$(14) P(t, T) = c(S, t, T, X) - (S_t - X),$$

and the cost of timing option is  $D$ . Hence, the trigger-point calculation in eq. (13) can be interpreted as a trade-off between the benefit and the cost of the timing option. Since the value of timing option is non-negative, unless the cost of the timing option is positive, a manager would keep this timing option alive and not implement the project until time  $T$ .

The uncertainty of the project can also affect the optimal timing of implementation. We can measure the uncertainty of the project by the standard deviation of project return,  $\sigma$ . Then, we have a proposition about the effect of uncertainty on the optimal timing of implementation and the value of timing option.

Proposition 1 For a given present value, when the uncertainty of project return increases (decreases), then the value of the timing option increases (decreases) and the probability of delay in implementation increases (decreases).

Proof: According to the Theorem 15 of Merton (1975), we have:

$$\frac{\partial P(t, T)}{\partial \sigma} = \frac{\partial c(S, t, T, X)}{\partial \sigma} \geq 0. \quad \text{Q.F.D.}$$

The validity of this proposition does not depend on the risk preference of the manager.

Before a firm commits its resources to a given investment project, it can pull out the planned project without penalty should the NPV in the future fall below zero. However, if the NPV in the future increases, the firm can then implement the project to take advantage of that increase. The larger the standard deviation,  $\sigma$ , the more widely spread the NPV in the future will be. Since the timing option affords the firm all the benefit of NPV increase but limits the cost of the NPV decline, the value of timing option on a more uncertain project would be larger. The firm would be more inclined to delay a project with higher uncertainty.

Proposition 2 is about the relationship between the timing decision and the size of the project.

Proposition 2 If the distribution of project returns is independent of the size of the project (measured in terms of present value of the

project), then the value of the timing option  $P(t, T)$  is homogeneous of degree one in the present value  $V$  and cost  $X$ .

Proof:

According to Theorem 9 of Merton (1973),  $c(V, t, T, X)$  is homogeneous of degree one in  $V$  and  $X$ . From eq. (14), we can see that  $P(t, T)$  is homogeneous of degree one in  $V$  and  $X$ .

Therefore the value of the timing option per dollar of investment is independent of the size of the project. Hence, the optimal timing decision is independent of the size of project, i.e., when the value of  $V$ ,  $X$  and  $D$  all increase proportionally, the timing decision is unaffected.

#### B. Introduction of New Products

When a firm's R and D department develops a new project, the firm acquires the deferrable investment opportunity to introduce that product. In theory, the time horizon,  $T$ , for this opportunity can be infinite. In practice,  $T$  is finite and depends on the introduction of similar products by the firm's competitors.

The cost of introduction, such as plant, equipment, and marketing cost, is given as  $X$ . If the product is introduced and quickly commands dominant market share, the present value of this new product is  $V$  as described by eq. (1). Suppose other firms are developing similar products and are close to the market introduction stage. If the given firm has not introduced the product and pre-empted the market, other firms will do so at time  $t_i$ , where  $i = 1, \dots, n$ . When a competitor introduces a new product at  $t_k$ , the present value of the given yet-to-be introduced product will lose value of  $D_k$  at time  $t_k$ .

This example fits the Specification 1 of Section II. We can derive several interesting implications from our model:

1. A firm generally delays introducing of a new product until its major competitor is ready to do so.
2. If there is no major competitor in sight, a firm may hold off the introduction of a new product forever.
3. If there is no major competitor in sight, the value of the timing option is equal to the cost of introduction.

$$\begin{aligned}
 \text{Proof: } P(0, \alpha) &= c(V, 0, \alpha, X) - (V_0 - X) \\
 &= V_0 - (V_0 - X) \\
 &= X.
 \end{aligned}$$

According to Theorem 3 of Merton (1973):

$$C(V, 0, \alpha, X) = V_0.$$

4. When the uncertainty of the new product's return increases, i.e.,  $\sigma$  is higher, the value of timing option increases and the firm will postpone the introduction of the new product.

### C. Real Estate Development

In any major city, we can observe prime real estate left vacant. Leaving property vacant is equivalent to holding a timing option on the real estate development. In theory, the time horizon of this timing option can be infinite.

The cost of real estate development is  $X$  and the present value of the real estate development is  $V$ . If the real estate development has not been implemented, the present value of future development is  $S$  which is described by eq. (10).

Consider the case in which if the real estate is developed before time  $t$ , say, the opening of the 1984 Los Angeles Olympics, the present value at time  $t$  would be  $S_t + D$  and if it is not developed before time  $t$ , the present value is  $S_t$ . Then the case fits the Specification 2 of

Section II. We can apply eqs. (11) and (12) to calculate the value of the timing option. We can also calculate when the real estate should be developed.

During 1973-74, the energy crisis set off wide-spread land speculation in Taiwan. While the prices of real estate skyrocketed, many landlords held their land from development. To encourage real estate development, the Taiwanese government reduced the sales tax on real estate transactions. However, this policy did not dampen the fever of land speculation. Consequently the Taiwanese government shifted the policy from sales tax reduction to a new tax on lots that were kept vacant for more than a year. After the new policy, the land speculation gradually died out. We can apply our model to explain this episode.

From Proposition 1, the value of the timing option,  $P(0, T)$ , on real estate development is an increasing function of uncertainty on real estate return. The energy crisis increased the uncertainty, hence it increased the value of the timing option. From Proposition 2, the value of  $P(0, T)$  is homogeneous of degree one in  $V$  and  $X$ . The sales tax reduction increased the value of  $V$  while keeping  $X$  constant. Hence, although the sales tax reduction made real estate development more attractive, it also increased the value of the timing option on real estate development, thus encouraging further land speculation.

The new tax on vacant lots was equivalent to the increase of development cost by the amount of  $D$  at time  $t$ . This case fits the Specification 3 of Section II. When the tax on vacant lots,  $D$ , is large enough, the land speculator would "kill" the timing option.



#### IV. Concluding Remarks

Timing is an important factor in capital budgeting decision, but it is generally ignored in the literature. We develop three specifications and two stochastic models to determine the value of the timing option and the optimal timing of project implementation.

Our valuation models depend crucially on the specifications of the present values of the investment project that is implemented right now and that will be implemented in the future, i.e., the variables,  $V$  and  $S$ . However, the general economic implications, for example, Propositions 1 and 2, are rather robust; they can generally survive without those specifications of present value. If the cost of investment project,  $X$ , is random, we can modify eqs. (2) and (9) by respecifying  $\sigma$  to include the standard deviation and covariance related to  $X$ . Our general results are not affected.

This paper has two major limitations. Although the Black model can deal with the optimal choice among multiple timing alternatives, there is no closed form solution to the valuation and decision formula. While there is a closed form solution to the Roll-Whaley model, it can only deal with the optimal choice of two timing alternatives. However, this paper provides a useful analytical tool to conceptualize the optimal timing issue in a capital budgeting decision under uncertainty.

Footnotes

1. Current literature tends to ignore the issue of optimal timing in capital budgeting analysis. Bierman (1968), and Bierman and Smidt (1980) are among the few exceptions. However, they discuss the issue of optimal timing in the case of certainty. In this paper, we discuss the issue in a stochastic model.
2. Margrabe (1978) developed a model of valuation for an option to exchange one asset for another. Since his model closely follows the Black-Scholes (1973) solution to the call option pricing problem, he also ignores the issue of optimal timing. Moreover, since Margrabe was primarily concerned with the exchange of financial assets, he did not discuss the issues of capital budgeting.
3. Constantinides (1978) developed a rule which reduces the problem of valuation in the presence of market risk to the problem of valuation in a world where the market price or risk is zero. In this paper, we are not concerned with the deviation of the present value of investment project. Readers can refer to Bogue and Roll (1974), Brennan (1973), and Fama (1977) for detailed derivation.
4. A Wiener process is defined as follows: For any partition  $t_0 < t_1 < t_2 < \dots$  of the time interval  $[t_0, \infty]$ , the random variables  $Z(t_1) - Z(t_0)$ ,  $Z(t_2) - Z(t_1)$ ,  $Z(t_3) - Z(t_2)$ , ..., are independent and normally distributed with mean zero and variance  $t_1 - t_0$ ,  $t_2 - t_1$ ,  $t_3 - t_2$ , ... respectively. Therefore, for any infinitesimal time interval, the random disturbance of the state is normally distributed with mean  $M(X, t)dt$  and variance  $\sigma^2(X, t) dt$ , where  $X$  is a state variable and  $t$  is a time variable.

We assume that the functions  $M(X, t)$  and  $\sigma(X, t)$  satisfy the necessary regularity conditions, so that the stochastic process defined by eq.(1) exist. Then the stochastic process that generates  $dV/V$  is called Ito process.

For a heuristic discussion of the Wiener process, see Kushner (1971), Chapter 10.

5. A brief description of Ito proces is given in footnote 4.
6. If the American call is neither a dominant nor a dominated asset, its value is bounded from below by the Black approximation  
 Because  $C(V, 0, T, X) \geq c(S_k, 0, t_k, X)$  for  $k = 2, \dots, n$ ,  
 and  $C(V, 0, T, X) \geq c(V, 0, t_1, X)$ , therefore,  

$$C(V, 0, T, X) \geq \max [ \{ c(S_k, 0, t_k, X) \mid k = 2, \dots, n \}, c(V, 0, t_1, X) ]$$

7. While the Black model can accommodate a case in which there are more than one discrete waiting cost  $D_i$  incurred at time  $t_i$ ,  $i \geq 1$ , the Roll-Whaley model can only accommodate the case in which there is just one discrete waiting cost  $D$  incurred at time  $t$ .
8. In this example,  $V$  and  $S$  are identical variables. Please see Samuelson (1965) for the discussion of why the properly anticipated asset prices fluctuate randomly.

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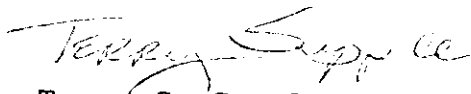
January 25, 1984

Professor Orlin Grabbe  
Finance Department

Dear Dr. Grabbe,

You have been suggested as a reviewer for the enclosed working paper, "Capital Budgeting Under Uncertainty: The Issue of Optimal Timing" by C. Jevons Lee and Christopher R. Petruzzi. The paper has been submitted for the Center's Working Paper Series; please let me know at your earliest convenience whether you think it is appropriate to be published. (Please forward the paper itself and any comments you may have directly to the author.) Thank you for your help.

Sincerely,



Terry S. Supple

# UNIVERSITY of PENNSYLVANIA

The Wharton School  
PHILADELPHIA 19104

*Rodney L. White Center  
for Financial Research*

3253 Steinberg Hall CC

Irwin Friend, *Director*

215-898-7616  
Telex 7106700328

March 23, 1984

Orlin J. Grabbe  
Finance Department  
3410 SH-DH/CC

Dear Orlin:

The working paper series of the Rodney L. White Center is a vital part of research process that goes on in the Finance Department. Our professors depend heavily on the publication of their research papers in the working paper series to solicit responses from their colleagues in the academic community.

It has been brought to my attention that we are waiting for your review and responses on the following paper:

"Capital Budgeting Under Uncertainty: The Issue of  
Optimal Timing"  
by Chi-wen Jevons Lee and C.R. Petruzzi

Please call me at x7616 as soon as you receive this note to let me know the status of the paper.

Thanks for your cooperation.

Cordially,



Janet Cash Wilson

/JCW