GENERAL EQUILIBRIUM PROPERTIES OF THE TERM STRUCTURE OF INTEREST RATES

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ABSTRACT

In a sequential general equilibrium with a single representative risk-averse consumer, stationary uncertainty, a one-period lag between investment and production, and concave production functions, we show that the forward price of a one-period real default-free bond one period hence is less than the expected price of the bond, if markets are locally complete and utility is state-independent. Thus the real term structure premium is always positive. This result is consistent with the "Liquidity hypothesis". However it is not based on any assumptions about the nature of risk or on time-dependent consumption preferences. The term structure is positive because long-term bonds turn out to be a poor wealth hedge, because of the way consumers allocate consumption and investment over time.

The results hold for real interest rates in complete markets, with whatever pattern of (possibly time-dependent) discount factors the consumer has. In incomplete markets the results will also hold, as long as the utility function exhibits either constant or increasing absolute risk aversion. The nominal term structure is also explored for a class of money demand specifications. The value and sign of the term structure premium critically depend on changes in the supply of money. Thus the nominal term structure premium may be negative even if the real term structure premium is positive.

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I. INTRODUCTION

In this paper we examine the properties of the real and nominal term structure premia in a model of uncertainty. We show that in a stationary economy with complete markets and concave production functions the real term structure premium is always positive. In other words, the implicit forward real interest rate is an upwards-biased predictor of the future real short-term interest rate. We also show that even if the utility function is time-dependent the term premium is always positive. We then examine the real term structure in a stationary economy with incomplete markets, and we show conditions under which the term premium could be negative.

The results for the nominal term structure premium are less clear-cut, because they depend fundamentally on monetary policy and to some extent on the nature of the demand for money in the economy. We analyse the benchmark case of a fixed quantity of money and we give conditions under which the nominal term structure premium could be negative, even if the real term structure premium is positive.

There is a large body of theoretical literature that deals with the existence and the nature of the term structure premium. The Liquidity Preference hypothesis, Hicks (1939), the Market Segmentation hypothesis, Culbertson (1957), and the Preferred Habitat hypothesis, Modigliani and Sutch (1966), all rely to varying degrees on an analysis of consumer and firm preferences under certainty to arrive at conclusions about the term structure premium under uncertainty. The hypotheses advanced by these early works are not formally worked out. More recently Stiglitz (1970) explores the implications of consumers' preferences under uncertainty on asset holdings and interest rates in a partial-equilibrium setting.

A second strand of literature that developed mainly in the 1970's is based on the notion that asset prices are set by arbitrage. The fundamental insight of this literature is that in a complete market (in the Arrow-Debreu sense) the state prices must determine the values of all assets, including the prices of bonds of differing maturities. As Beja (1979) shows, all term structure propositions may be derived from relations among the equilibrium state prices. Applications of the arbitrage approach are found in Benninga and Protopapadakis (1983), Breeden (1983), Cox, Ingersoll, and Ross (1978, 1981), Dothan (1978), Long (1974), and Richard (1978).

While the arbitrage approach is capable of yielding general term structure relationships, the determination of the size and sign of the term structure premium depends upon the equilibrium of the economy. Any attempt to go beyond the arbitrage propositions depends on being able to make statements about the general equilibrium allocation of investment and consumption. To date, the main effort in this direction has been that of Cox, Ingersoll, and Ross (1978), who showed that, for a class of stochastic economic models with linear production technologies and a representative consumer having a logarithmic utility function, an equilibrium allocation may be calculated.

This paper is grounded in the arbitrage tradition. We characterize the equilibrium when uncertainty is stationary (defined precisely in the text) for an extremely broad class of concave utility functions and concave production functions. We show that the term structure premium is positive under these conditions. Our results hold for all time-separable and state-independent utility functions and for all concave production functions which can be ordered by a relatively trivial dominance condition, while Cox, Ingersoll and Ross derive results only for logarithmic utility functions and linear production functions. However, our specification of the underlying uncertainty of the model is somewhat less general than that of Cox, Ingersoll and Ross. While their model allows any Markovian specification of state probabilities, our model is restricted to a class of stationary models in which the conditional probabilities of the states remain the same over time.

In Section II we introduce the model and summarize results that exist in the literature. In Section III we explore the real term structure premium in a stationary world where the utility function is state-independent and markets are complete. In Section IV we explore the implications of time-dependency in the utility function, and in Section V we relax the assumption of complete markets. In Section VI we introduce money in the model and we offer some results on the nominal term structure. The concluding section provides some comparisons of our results with those in the literature.

II. THE MODEL, FIRST-ORDER CONDITIONS, AND SOME PRELIMINARY RESULTS

We consider an economy in which the equilibrium is determined by a representative consumer who has access to a set of production technologies. These technologies allow him to span the economy's uncertainty. There are three dates at which consumption takes place. At Date 1 one of two possible states of the world can occur, and from each of these two states of the world another two states are possible at Date 2. Uncertainty thus follows a "tree-like" structure, which is illustrated in Figure 1. The π's on each branch denote state probabilities: Thus π_1 and π_2 denote the probability that states 1 and 2 will occur respectively at Date 0. Similarly π_3 and π_4 denote the probability that states 3 and 4 will occur at Date 2, if state 1 occurred at Date 1. π_5 and π_6 denote the probability that states 5 and 6 will occur at Date 2 if state 2 occurred at Date 1 ($\pi_1+\pi_2 = \pi_3+\pi_4 = \pi_5+\pi_6 = 1.0$).[1]

The representative consumer maximizes the discounted expected utility of consumption in each state. We assume initially that the utility function is time-additive and state-independent. Denote the consumer's utility function by $U(\cdot)$ and the consumer's pure time preference factor by δ ($\delta < 1$). The consumer maximizes

$$(1) \qquad V(c_0, c_1, \ldots, c_6) = U(c_0) + \delta\{\pi_1 U(c_1) + \pi_2 U(c_2)\} + \delta^2 \pi_1 \{\pi_3 U(c_3) + \pi_4 U(c_4)\} + \delta^2 \pi_2 \{\pi_5 U(c_5) + \pi_6 U(c_6)\}.$$

The investment opportunities available to the representative consumer are state-specific. At any date, the consumer can invest amount z_n in technology n that yields output only if state n occurs. These technologies produce output $\alpha_n h(z_n)$, and they differ only by the value of the efficiency coefficient α_n. Thus, for example, if the consumer finds herself in state 2, Date 1, she will invest z_5 and z_6 in the technologies that yield output in states 5 and 6, respectively, at Date 2. The output will be $\alpha_5 h(z_5)$ and $\alpha_6 h(z_6)$ in states 5 and 6.[2,3]

The consumer has an initial endowment $\bar{c}$, and she maximizes expected utility shown in equation (1), subject to the budget constraints shown in equation (2) below.

(2) $$c_0 = \bar{c}-z_1-z_2,$$

$$c_1 = \alpha_1 h(z_1)-z_3-z_4,$$

$$c_2 = \alpha_2 h(z_2)-z_5-z_6,$$

$$c_n = \alpha_n h(z_n), \; n = 3, \ldots, 6.$$

The first-order conditions of the consumer's maximization problem are sufficient to determine the prices of any financial asset which may be traded in the economy. To derive these conditions, write

(3) $$\partial V/\partial z_m = 0, \; m=1, \ldots, 6.$$

One representation of the resulting conditions is:

(4-a) $$\frac{\delta\pi_1 \, \partial U/\partial c_1}{\partial U/\partial c_0} = \frac{1}{\alpha_1 h'(z_1)},$$

(4-b) $$\frac{\delta\pi_2 \; \partial U/\partial c_2}{\partial U/\partial c_0} = \frac{1}{\alpha_2 h'(z_2)},$$

(4-c) $$\frac{\delta\pi_m \partial U/\partial c_m}{\partial U/\partial c_1} = \frac{1}{\alpha_m h'(z_m)}, \qquad m = 3, 4.$$

(4-d) $$\frac{\delta\pi_n \partial U/\partial c_m}{\partial U/\partial c_2} = \frac{1}{\alpha_m h'(z_m)}, \qquad m = 5, 6.$$

We use the following notation and assumptions:

$$\frac{\partial U(c)}{\partial c} \equiv U'(c), \quad \frac{\partial^2 U(c)}{\partial c^2} = U''(c).$$

$$U'(c_i) > 0, \ U''(c_i) < 0, \ U'(0) \to \infty.$$

$$\frac{\partial h(z)}{\partial z} \equiv h'(z), \quad \frac{\partial^2 h(z)}{\partial z^2} \equiv h''(z).$$

$$h'(z) > 0, \ h''(z) < 0, \ h(0) = 0.$$

These assumptions guarantee that in equilibrium $c_m > 0$, $z_m > 0$, $m=0, \ldots, 6$.

Denote the economy's <u>real state prices</u> by $q_1, \ldots, q_6$, where

(5) $$q_1 = \delta\pi_1 \frac{U'(c_1)}{U'(c_0)}, \ q_2 = \delta\pi_2 \frac{U'(c_2)}{U'(c_0)},$$
$$q_3 = \delta\pi_3 \frac{U'(c_3)}{U'(c_1)}, \ q_4 = \delta\pi_4 \frac{U'(c_4)}{U'(c_1)}, \ q_5 = \delta\pi_5 \frac{U'(c_5)}{U'(c_2)}, \ q_6 = \delta\pi_6 \frac{U'(c_6)}{U'(c_2)}.$$

The interpretation of the state prices is as follows: q_1 and q_2 are the Date 0 prices of an asset which delivers one unit of the commodity in states 1 and 2 (at Date 1) respectively. q_3 and q_4 are the state 1 (Date 1) prices of an asset which delivers one unit of the commodity in states 3 and 4 (at Date 2) respectively. Similarly q_5 and q_6 are the state 2 (Date

1) prices of an asset which delivers one unit of the commodity in states 5 and 6 (at Date 2), respectively.

Given the equilibrium and the consequent state prices, the interest rate of a one-period real default-free bond sold at Date 0 will be r_0 such that

(6) $$1/(1+r_0) = q_1+q_2.$$

If state 1 occurs the interest rate on a one-period real default-free bond will be r_1 such that

(7-a) $$1/(1+r_1) = q_3+q_4.$$

Similarly, the interest rate on such a bond if state 2 occurs will be r_2,

(7-b) $$1/(1+r_2) = q_5+q_6.$$

A two-period default-free real bond sold at Date 0 (i.e., a bond which promises to deliver one unit of consumption at Date 2, regardless of which state occurs) will have an interest rate R such that

(8) $$1/(1+R) = q_1(q_3+q_4) + q_2(q_5+q_6).$$

Theorem 1 gives the relation between the price of a long-term real bond, $\frac{1}{1+R}$, the price of a short-term real bond, $\frac{1}{1+r_0}$, and the Date 0 expected price of a real short-term bond that will be available at Date 1, $E(\frac{1}{1+\tilde{r}^1})$, where $\tilde{r}^1$ refers to the distribution of the one-period interest rate that will obtain at Date 1.

Theorem 1:

$$(9) \qquad 1/(1+R) = \frac{E(\frac{1}{1+\tilde{r}^1})}{1+r_0} + \mathrm{Cov}(\frac{\tilde{q}^1}{\tilde{\pi}^1}, \frac{1}{1+\tilde{r}^1})$$

where the covariance term refers to the covariance between Date 1 probability-normalized state prices and Date 1 prices of the one-period real bonds (i.e., the covariance between the pairs q_1/π_1 and q_3+q_4, q_2/π_2 and q_5+q_6), and $E(\frac{1}{1+\tilde{r}^1}) \equiv \pi_1(\frac{1}{1+r_1}) + \pi_2(\frac{1}{1+r_2})$.

Proof:

$$\frac{1}{1+R} = q_1(q_3+q_4) + q_2(q_5+q_6)$$

$$= \pi_1 \frac{q_1}{\pi_1}(q_3+q_4) + \pi_2 \frac{q_2}{\pi_2}(q_5+q_6).$$

Writing this as $E[\frac{\tilde{q}^1}{\pi_1}(\frac{1}{1+\tilde{r}^1})]$, gives

$$\frac{1}{1+R} = E[\frac{\tilde{q}^1}{\tilde{\pi}^1}]\, E\,[\frac{1}{1+\tilde{r}^1}] + \mathrm{Cov}(\frac{\tilde{q}^1}{\tilde{\pi}^1}, \frac{1}{1+\tilde{r}^1}),$$

$$= \frac{\pi_1/(1+r_1) + \pi_2/(1+r_2)}{1+r_0} + \mathrm{Cov}\,(\cdot).$$

Q.E.D.

The covariance term of Theorem 1 may be interpreted as measuring the risk of the long-term discount bond. To see this, suppose that the covariance is negative: This means that when the probability-adjusted state price is high (i.e., the probability-adjusted value of a unit of consumption at Date 1 is high) the value of the two-period discount bond at Date 1 tends to be low, and vice versa. Since a high state price is indicative of a high marginal utility of consumption (and hence wealth), negative covariance in (9) means that the two-period bond is a poor hedge against bad states, because the bond's value is lower when wealth is greatly desired, and higher when wealth is less desired. When the covariance in (9) is positive, on the other hand, the bond is a good hedge because its value is positively correlated with the marginal utility of consumption.

Variants of the result in Theorem 1 have been derived in different contexts by various authors.[4] The implications of Theorem 1 for the term structure premium can best be seen by comparing the forward interest rate with the expected short-term interest rate. Let the forward price of the short-term real bond be given by $\frac{1}{1+r_f}$. Then arbitrage assures that,

$$(10) \qquad 1+R = (1+r_0)(1+r_f).$$

Define the expected short-term real interest rate r^e such that $\frac{1}{1+r^e} \equiv E(\frac{1}{1+\tilde{r}})$. This definition of the expected short-term rate avoids the Jensen inequality problem that Cox, Ingersoll and Ross (1981) discuss. From Theorem 1 it follows that,

$$(11) \qquad \frac{1}{(1+r_0)(1+r_f)} = \frac{1}{(1+r^e)(1+r_0)} + \text{cov}(\cdot),$$

and

$$(12) \qquad r_f - r^e = -(1+R)(1+r^e)\text{cov}(\cdot).$$

If $\text{cov}(\cdot) < 0$ the forward real rate overstates the expected short-term real rate (and vice versa). We call the expression on the right hand side of equation (12) the real term structure premium.

Throughout the paper we shall consider the term structure premium in a <u>stationary environment</u>. We assume that $\alpha_1=\alpha_3=\alpha_5=\alpha$ and that $\alpha_2=\alpha_4=\alpha_6=\beta$, where (with no loss in generality) $\alpha < \beta$; the state probabilities likewise are stationary: $\pi_1=\pi_3=\pi_5$ and $\pi_2=\pi_4=\pi_6$. The model then has the following interpretation: At any Date, the representative consumer finds herself faced with a subsequent "bad" state of the world (the α state) and a subsequent "good" state of the world (the β state). The environment is stationary in that both "bad" and "good" states have the same probability of occurrence at each Date. Thus, the technological possibilities replicate themselves through time.

The following result shows that if production exhibits constant returns to scale, the term structure premium is <u>zero</u>.

Theorem 2: Suppose that $h''(\cdot) = 0$. Then in a stationary environment, the covariance in (9) is zero, and the expectations hypothesis holds, i.e., $r_f = r^e$.

Proof:

By assumption $h(\cdot)$ is linear. Letting $b=h'(\cdot)$, it follows from (5) and (4) that $q_1=q_3=q_5$ and $q_2=q_4=q_6$. Thus $q_3+q_4 = q_5+q_6$, and the covariance is zero. If follows immediately that $r_f = r^e$. Q.E.D.

The linear technology case has been considered by other authors, most notably Cox, Ingersoll, and Ross (1978, 1981).[5]

Throughout the remainder of this paper, we shall consider concave production technologies (i.e., $h''(\cdot) < 0$). In the following three sections of the paper we analyse the determinants of the real term structure premium. In particular we show that in a stationary environment with complete markets, the covariance term in (9) is always negative and the risk premium is always positive.

III. THE REAL TERM STRUCTURE PREMIUM IN A STATIONARY WORLD

In this section we explore the term structure premium in a stationary environment when production functions are concave. The main result is that the real term structure premium is always positive. In order to simplify the analysis we assume initially that all state probabilities are equal. We then show that the results carry through even if probabilities are unequal.

An intuitive interpretation of the theorems we prove below may help clarify the characteristics of the economy we describe. The representative consumer is given an initial endowment $\bar{c}$ at Date 0, and he decides on the allocation of his consumption, given technology and uncertainty. The investment opportunities consist of one technology that produces output only in the bad state, and one technology that produces output only in the good state, at Date 1. The same investment opportunities are again available at Date 1 regardless of which state obtained. The theorems we prove below say that a consumer faced with this environment will allocate his resources so that he consumes more in the good states (state 2, Date 1, states 4 and 6 at Date 2) than in the comparable bad states (Theorem 3). At the same time he will invest in the available technologies in such a way that the marginal product of capital ($\alpha_i h'(z_i)$) of the technology for the bad state ($\alpha_i=\alpha$) is always lower than the marginal product of capital of the technology for the comparable good state. However, the investment in the technology of the bad state can be more or less than the investment in the technology of the good state (Theorem 4).

Once Date 1 uncertainty reveals itself (i.e., the consumer is at state 1 or 2) the consumer invests in the "good-state" and "bad-state" technologies that will produce at Date 2. We show that the representative consumer invests more in both Date 2 technologies when he is at the good Date 1 state (state 2) than when he is at the bad Date 1 state (state 1) (Theorem 3). The result about the real term structure premium that we prove in Theorem 5 is a direct consequence of these allocation decisions of the representative consumer, because the state prices are inversely related to the marginal product of the production function for each state.

III.a. Equal State Probabilities

We suppose first that $\pi_j = 0.5$, $j=1, \ldots, 6$. We start by dividing (4-b) by (4-a): This gives

$$(13) \qquad \frac{U'(c_2)}{U'(c_1)} = \frac{\alpha h'(z_1)}{\beta h'(z_2)}.$$

Holding z_2 constant, the left-hand side (LHS) of (13) is an upward-sloping function of $z_1 - z_2$, and the right-hand side (RHS) of (13) slopes downward. The intercept of the RHS is α/β (See Figure 2).

We now repeat the above exercise by dividing one of the equations (4-c) into one of the equations (4-d). Doing this for m=3 and m=5, respectively, gives,

$$(14) \qquad \frac{U'(c_5)}{U'(c_3)} \frac{U'(c_1)}{U'(c_2)} = \frac{h'(z_3)}{h'(z_5)},$$

where we note that the π's and α_3 and α_5 cancel out, since by the assumptions of this section $\pi_3=\pi_5=0.5$ and $\alpha_3=\alpha_5=\alpha$. Holding all z's constant except for z_3, the LHS of (14) is upward sloping and the RHS is downward sloping as functions of z_3-z_5. The intercept of the RHS is 1.0, and the intercept of the LHS is $U'(c_1)/U'(c_2)$. This is graphed in Figure 3.

Note that by changing the indices (i.e. dividing (4-d) when m=6 by (4-c) when m=4), we obtain

$$\frac{U'(c_6)}{U'(c_4)}\frac{U'(c_1)}{U'(c_2)} = \frac{h'(z_4)}{h'(z_1)}. \tag{15}$$

We now prove the following theorem:

Theorem 3: In the economy described in this section, the following must hold:

$$c_1<c_2,\ c_3<c_5,\ c_4<c_6, \text{ and } z_4<z_6,\ z_3<z_5. \tag{16}$$

Proof:

Suppose that $c_1 \geq c_2$. Then it follows from Figure 3 that $z_3 \geq z_5$ and that $z_4 \geq z_6$. Since $\alpha < \beta$, it must be that $z_1>z_2$. But by Figure 1, $z_1>z_2$ only if the intercept of the left-hand side is below α/β, and this can only be so if $c_1< c_2$. We thus obtain a contradiction, and the ordering of the consumptions is described by the Theorem. The ordering of the investments follows directly from the above argument. Q.E.D.

Theorem 4: In the economy described in this section $q_1> q_2$, and $\alpha h'(z_1) < \beta h'(z_2)$.

Proof:

$q_1 > q_2$ follows directly from $c_2 > c_1$, since $\frac{U'(c_2)}{U'(c_1)} = \frac{q_2}{q_1} < 1$.

And since $q_1 = \frac{1}{\alpha h'(z_1)}$, $q_2 = \frac{1}{\beta h'(z_2)}$, $\alpha h'(z_1) < \beta h'(z_2)$. Q.E.D.

We can now determine the sign of the real term structure premium:

Theorem 5: In a stationary economy with equal probabilities the real term structure premium is always positive.

Proof:

It follows from Theorem 4 that $q_3+q_4<q_5+q_6$. But $q_1>q_2$ since by equation (5), $q_1/q_2 = U'(c_1)/U'(c_2)$ and by Theorem 3, $c_1<c_2$. Thus the covariance term in Theorem 1 is negative, and the term structure premium is positive.

Q.E.D.

III.b. **Unequal State Probabilities**

We are now ready to generalize Theorem 5 by allowing state probabilities to be unequal. We label the probability of the "bad" states (i.e., states 1, 3, 5) as π_1; the probability of the good states (2, 4, 6) is $\pi_2 = 1-\pi_1$.

Rederiving equations (13) and (15) gives:

(17) $$\frac{U'(c_2)}{U'(c_1)} = \frac{\alpha\pi_1 h'(z_1)}{\beta\pi_2 h'(z_2)},$$

and

(18) $$\frac{U'(c_5)}{U'(c_3)}\frac{U'(c_1)}{U'(c_2)} = \frac{h'(z_3)}{h'(z_5)}.$$

These equations are graphed in Figures 4 and 5 respectively. We now consider two cases:

Case 1: $c_1 > c_2$. When $c_1 > c_2$ the intercept of the LHS in Figure 5 is less than 1.0 It then follows from Figure 5 (equation 17) that $z_3 > z_5$ and $z_4 > z_6$. By the first-order conditions for production (4a-d),

$$(19) \qquad \frac{q_1/\pi_1}{q_2/\pi_2} = \frac{U'(c_1)}{U'(c_2)} < 1.$$

Using the same conditions gives

$$(20) \qquad q_3 = \frac{\pi_1 U'(c_3)}{U'(c_1)} = \frac{1}{\alpha\delta h'(z_3)} > \frac{1}{\alpha\delta h'(z_5)} = \frac{\pi_1 U'(c_5)}{U'(c_2)} = q_5.$$

By a similar argument $q_4 > q_6$. Thus $q_1/\pi_1 < q_2/\pi_2$ and $q_3 + q_4 > q_5 + q_6$. Hence the covariance is negative and the real term structure premium is positive.

Case 2: $c_1 < c_2$. In this case it follows from Figure 5 that $z_3 < z_5$ and $z_5 < z_6$, so that $q_3 + q_4 < q_5 + q_6$. Furthermore, since $c_1 < c_2$, it follows that $U'(c_1) > U'(c_2)$. It follows from equation (17) that $q_1/\pi_1 > q_2/\pi_2$. Therefore the covariance is negative and the real term structure premium is positive.

III.c. The Real Term Structure in Complete Markets: Summary

We have shown that in a stationary environment characterized by concave production functions the consumption/investment decisions of risk-averse consumers (with state-independent utility functions) result in a distribution of state prices that gives rise to systematic risk in two-period real default-free bonds. The covariance of Date 1 state prices and Date 1 prices of short-term real bonds, that captures this systematic risk is always negative. This negative covariance means that the two-period discount bond is a poor wealth hedge in the first period; hence the structure premium is positive.

IV. THE REAL TERM STRUCTURE PREMIUM WHEN THE UTILITY FUNCTION IS TIME-DEPENDENT

In the previous section we considered a stationary world in which the utility function of the representative consumer is time-independent. However, traditional hypotheses on the term premium often rely on consumers preferring assets of certain maturities.[6]

In this section we modify the model to capture the notion of differential time preference and we explore its effects on the term structure premium. We consider the following form of a time-dependent utility function:

$$G(c_0, c_1, \ldots, c_6) = U(c_0) + \pi_1 V(c_1) + \pi_2 V(c_2) + \pi_1\{\pi_3 W(c_3) + \pi_4 W(c_4)\} + \pi_2\{\pi_5 W(c_5) + \pi_6 W(c_6)\}. \tag{21}$$

The form of (21) is the most general expected utility function that includes time-dependence but that is separable and not state-dependent. Equation (21) would be identical to equation (1) if $V(\cdot) = \delta U(\cdot)$, and if $W(\cdot) = \delta^2 U(\cdot)$, but in this section we make no such restrictions, requiring only that U, V, and W be concave and increasing in consumption. The first-order conditions for the new problem are given below:

$$(22)\qquad q_1=\pi_1\frac{V'(c_1)}{U'(c_0)},\qquad q_2=\pi_2\frac{V'(c_2)}{U'(c_0)},$$

$$q_3=\pi_3\frac{W'(c_3)}{U'(c_1)},\qquad q_4=\pi_4\frac{W'(c_4)}{V'(c_1)},$$

$$q_5=\pi_5\frac{W'(c_5)}{V'(c_2)},\qquad q_6=\pi_6\frac{W'(c_6)}{V'(c_2)}.$$

We now show that generalizing the utility function in this way does not affect the sign of the real term structure premium: The results of the previous section hold even if the utility function is of the form (21).

Theorem 6: The sign of the real term structure premium is independent of of U, V, and W in equation (21).

Remark:

As shown in the previous section, we may--with no loss in generality--assume that the state probabilities are equal. To prove Theorem 6 it is sufficient to show that none of the first-order conditions used in the proofs of Theorems 3 through 5 are affected by the form of (21); it is not actually necessary to reprove the theorems themselves. The intuitive reason that the theorem holds is that the sign of the term premium is a function of the distribution of consumption across states of the same Date, and it does not involve the distribution of consumption over different Dates.

Proof:

Consider equations (13) and (14). The former becomes:

$$(23)\qquad \frac{V'(c_2)}{V'(c_1)}=\frac{\alpha h'(z_1)}{\beta h'(z_2)}.$$

and equation (14) becomes:

$$\frac{W'(c_5)}{W'(c_3)} \cdot \frac{V'(c_1)}{V'(c_2)} = \frac{h'(z_3)}{h'(z_5)}. \tag{24}$$

Since the inequalities of consumption, investment, and state prices in Theorems 3, 4, and 5 derive only from the concavity of the utility and the production functions, the results will hold also for (23) and (24).

Q.E.D.

It is clear from the proof above that the only assumption needed to get a positive risk premium is that the utility function must be the same across possible states at each Date. If this condition is met, the term premium will remain positive regardless of how the functional form changes over time. Naturally, different utility configurations will change the distribution of consumption over time, and they will also change the size of the term structure premium, but not its sign.

If, on the other hand, the time preference factor of the utility function varies across states for the same Date, the term structure premium could be positive or negative.

V. THE REAL TERM STRUCTURE IN INCOMPLETE MARKETS

The analysis in the preceding section assumes complete markets, in which the consumer can allocate consumption among states of the world as he desires, given available technology and an initial endowment. We ensure market completeness by assuming the existence of two "primitive" technologies. These technologies are orthogonal to each other, in that each produces in one state only.

In this section we drop the assumption of primitive technologies and assume instead that the consumer has available a number of "complex" technologies, which produce output in both states of the world given an input in the preceding state. The following result is well-known: Suppose that in equilibrium the consumer uses at least two "complex" technologies, and suppose furthermore that these technologies are independent (i.e., the output vectors are independent vectors in a two-dimensional Euclidean space). Then the resulting equilibrium is equivalent to one in which the consumer invests in orthogonal "primitive" technologies which span the states of the world.

The assumption of "complex" technologies thus adds nothing to the model unless the optimal solution is one where the consumer's choices do _not_ span the state space. By the result stated in the previous paragraph, this is formally equivalent to assuming that the consumer has available only one complex technology in which to invest. We explore this one complex technology case in this section. With only one complex technology and two states the economy is characterized by _incomplete_ markets, because the output outcomes are constrained since one investment decision determines the output in both states.

In this section we analyse the real term structure premium when markets are incomplete. We show that the term structure premium in the incomplete market economy can be negative only if absolute risk aversion decreases with consumption. We retain the basic three-period structure of the economy of Figure 1, and we retain also the assumption made in Section III, that states have equal probabilities (i.e., $\pi_i = .5$, $i=1, \ldots, 6$).

We shall, however, restrict the production technology in the following way: We assume that only one technology, $H(\cdot)$, is available for investment in each state. The consumer invests x_i (x_0 at Date 0, x_1 or x_2 in state 1 or 2, Date 1 respectively) in the technology. The output the consumer receives is $\alpha H(x_i)$ if a "bad" state obtains, and $\beta H(x_i)$ if a "good" state obtains. Markets are incomplete because the ratio of output between the "good" and the "bad" states is fixed at β/α.

More formally, we shall reexamine the utility function considered in Sections II and III:

$$\text{(1)} \qquad \text{Max } V(c_0, c_1, \ldots, c_6) = U(c_0) + \delta\{\pi_1 U(c_1) + \pi_2 U(c_2)\} + \delta^2 \pi_1 \{\pi_3 U(c_3) + \pi_4 U(c_4)\} + \delta^2 \pi_2 \{\pi_5 U(c_5) + \pi_6 U(c_6)\},$$

subject to (note the change in notation for the production function):

$$c_0 = \bar{c} - x_0,$$

$$c_1 = \alpha H(x_0) - x_1, \quad c_2 = \beta H(x_0) - x_2,$$

$$\text{(25)} \qquad c_3 = \alpha H(x_1), \quad c_4 = \beta H(x_1),$$

$$c_5 = \alpha H(x_2), \quad c_6 = \beta H(x_2),$$

$$x_0, x_1, x_2 \geq 0, \quad 0 < \alpha < \beta.$$

A comparison of the budget constraints (25) with the equivalent constraints (2) in the complete markets case reveals the essential difference between the two cases. In complete markets the consumer may allocate inputs separately for consumption in any given state of the world. In incomplete markets, however, the consumer has only one production technology available at each decision node, and this technology gives a pattern of returns in the subsequent Date, which is fixed.

In the remainder of this section we show the conditions under which the results in Section III hold. The first-order conditions for the maximization of (1) subject to (25) are:

$$\alpha q_1+\beta q_2 = \frac{1}{\delta h'(x_0)}$$

(26)
$$\alpha q_3+\beta q_4 = \frac{1}{\delta h'(x_1)}$$

$$\alpha q_5+\beta q_6 = \frac{1}{\delta h'(x_2)}$$

where the q's are defined as in (5). Since the budget constraint equations preclude the possibility that $c_1^* > c_2^*$ and $x_1^* > x_2^*$ there are three categories of equilibria.

Category 1:

In the optimal solution, $c_1^* > c_2^*$ and $x_1^* < x_2^*$. In this case, since $c_1^* > c_2^*$, and since the state probabilities are equal, $q_1 < q_2$, since $q_1/q_2 = U'(c_1)/U'(c_2)$. Furthermore $\frac{q_3}{q_5} = \frac{U'(c_3)}{U'(c_5)} \cdot \frac{U'(c_2)}{U'(c_1)} > 1$, so that $q_3 > q_5$. Similarly, it is easy to show that $q_4 > q_6$. Thus the term structure premium is positive for this case.

Category 2:

In the optimal solution, $c_1^* < c_2^*$, $x_1^* > x_2^*$. Using logic similar to that in Case 1, we can show that $q_1 > q_2$, $q_3 < q_5$, and $q_4 < q_6$. Thus the term structure premium is positive for this case as well.

Category 3:

$c_1^* < c_2^*$, $x_1^* < x_2^*$. This case is the most difficult to solve. For this case we prove the following Theorem:

Theorem 7: Suppose the utility function of the representative consumer exhibits either constant absolute risk aversion or increasing absolute risk aversion. Then if $c_1^*<c_2^*$, $x_1^*<x_2^*$ the term structure premium is positive.

Proof: Since $c_1^* < c_2^*$, it follows that $q_1>q_2$. Furthermore it follows from the first-order conditions (26) that

$$\alpha q_3+\beta q_4 < \alpha q_5+\beta q_6. \tag{27}$$

Now suppose that

$$q_3+q_4 \geqslant q_5+q_6. \tag{28}$$

We shall show that this leads to a contradiction if the absolute risk aversion is constant or increasing. To see this, note first that (27) and (28) together imply that $q_4<q_6$:

$$q_4 = q_3 \frac{U_4}{U_3} < q_5 \frac{U_6}{U_5} = q_6. \tag{29}$$

Note that with the assumptions about risk aversion,

$$\frac{U_6\ U_3}{U_5\ U_4} \cong \frac{1 + (c_5-c_6)ARA(c_5)}{1 + (c_3-c_4)ARA(c_3)} < 1, \tag{30}$$

where $\cong$ denotes that first-order approximations have been used. It now follows from (27) that

$$q_3 < q_5 \frac{U_6\ U_3}{U_5\ U_4} < q_5. \tag{31}$$

Since $q_3 < q_5$ and $q_4 < q_6$, it follows that $q_3+q_4 < q_5+q_6$, which is a contradiction. Q.E.D.

To sum up: in incomplete markets, the only case for which it is possible to have a negative real term structure premium is the case where $c_1^* < c_2^*$, $c_3^* < c_5^*$, $c_4^* < c_6^*$ (i.e., consumption is always less in "bad" states than it is in corresponding "good" states) and when the absolute risk aversion is a decreasing function of wealth.

VI. SOME OBSERVATIONS ON THE NOMINAL TERM STRUCTURE

Our results on the real term structure premium show that generally the premium will be positive. The economically interesting exception is that if the state probability distribution is sufficiently skewed, the real term structure premium could become negative.

However, most of the discussion in the literature, and most of the empirical work in this area deals with nominal, rather then real, interest rates. In this section we provide some analysis of the nominal term structure premium and we show conditions under which it could be negative even when the real term structure premium is positive.

The difference between the real term structure and the nominal term structure is that nominal prices are substituted for real prices in equation (9) (Theorem 1). We state without proof[7]

$$(32)\qquad \frac{1}{1+I} = \frac{E\left(\frac{1}{1+\tilde{i}^1}\right)}{1+i_0} + \operatorname{cov}\left(\frac{\tilde{\sigma}^1}{\tilde{\pi}^1}, \frac{1}{1+\tilde{i}^1}\right)$$

where i_1, i_2 are the nominal short-term interest rates that obtain at states 1 and 2 (Date 1), respectively, for a nominal default-free bond; I is the interest rate for a two-period nominal default-free bond at Date 0;

$p_n (n = 0, 1 \ldots 6)$ are the \$ prices of a unit of consumption good in each state, and $\tilde{\sigma}^1$ represents the nominal state prices $\frac{q_1 p_0}{p_1}, \frac{q_2 p_0}{p_2}$. Equation (32) reduces to:

$$(33) \qquad i_f - i^e = -(1+I)(1+i^e)\text{cov}(\cdot),$$

where $E(\frac{1}{1+\tilde{i}^1}) \equiv \frac{1}{1+i^e}$, and $\frac{1}{1+i_f}$ is the forward price of the one-period nominal default-free bond.

We call the right-hand side of equation (33) the nominal term structure premium. The nominal term structure premium is a function of the negative of the covariance between $\frac{\sigma_1}{\pi_1}$ and $\sigma_3+\sigma_4$, $\frac{\sigma_2}{\pi_2}$ and $\sigma_5+\sigma_6$, where $\sigma_n = \frac{q_n p_0}{p_n}$ for n = 1,2, $\sigma_n = \frac{q_n p_1}{p_n}$ for n = 3, 4, and $\sigma_n = \frac{q_n p_2}{p_n}$ for n = 5, 6. Just as with the real bonds, $\frac{1}{1+i_1} = \sigma_3+\sigma_4$, $\frac{1}{1+i_2} = \sigma_5+\sigma_6$.

It is clear that if the nominal state prices bear the same relation to each other as the real state prices (i.e., $\sigma_1 > \sigma_2$, $\sigma_3+\sigma_4 < \sigma_5+\sigma_6$), then by the reasoning of Theorem 5 the nominal risk premium would be positive as well. The nominal risk premium can only be negative if either $\sigma_1 < \sigma_2$ and $\sigma_3+\sigma_4 < \sigma_5+\sigma_6$, or if $\sigma_1 > \sigma_2$ and $\sigma_3+\sigma_4 > \sigma_5+\sigma_6$.

It is possible to design monetary policies that manipulate nominal prices to get such reversals in the relation between nominal and real state prices, and in general it is possible to get any desired nominal risk premium. In that sense, no further generalization is possible. However, it is possible to get some limited insights by analyzing a benchmark case: that of having a fixed supply of money in all the states. It turns out that it is not possible to establish necessary and sufficient conditions on the utility or the production functions under which the nominal risk permium will be negative. Instead, we describe inequalities among dependent variables that will ensure a negative nominal risk premium.

In order to discuss nominal interest rates we must first discuss the demand for money in this economy. Introducing a motive for holding money in such an economy is not simple, because there is no single generally accepted methodology. The literature is replete with models in which real money is in the utility function, or in the production function, and others in which a demand function is assumed, or in which the demand for money arises from trading restrictions, from inability to write certain types of intergenerational contracts, and from legal restrictions. Our purpose here is not to propose an alternative theory to holding money. Therefore we assume that money is held in proportion to consumption. This assumption, generally known as the Clower constraint, has received attention in the literature recently and it is attractive because of its simplicity (see Lucas (1982)). The first-order conditions of the problem are not affected, nor are the budget constraints because any revenue the monetary authority collects must be returned to the representative consumer. It follows immediately that for any two states n, m ($0 \leq n, m \leq 6$) $p_n c_n = p_m c_m$ and prices are inversely proportional to consumption, $p_n/p_m = c_m/c_n$.

First we investigate the condition for which $\sigma_1 < \sigma_2$.

Theorem 8:

$\sigma_1 \lessgtr \sigma_2$ if $z_2 \gtrless z_1$, provided the savings rates in the two states are sufficiently similar.

Proof: We establish necessary and sufficient conditions for $\sigma_2/\sigma_1 > 1$.

Substitute for the σ's to get

$$\frac{q_2}{q_1}\frac{p_1}{p_2} = Q > 1.$$

From $p_n c_n$ = constant it follows that

$$\frac{p_1}{p_2} = \frac{c_2}{c_1}.$$

Thus $\frac{q_2}{q_1}\frac{c_2}{c_1} = Q.$

From the first-order conditions and the budget constraints

$$q_2 = \frac{1}{\beta h'(z_1)},$$

$$q_1 = \frac{1}{\alpha h'(z_1)} = \frac{1}{\alpha[h'(z_2) + (z_1-z_2)h''(z_2)]},$$

$$c_2 = \beta h(z_2)-z_5-z_6, \qquad c_1 = \alpha h(z_1)-z_3-z_4.$$

$$Q = \frac{\alpha}{\beta}\left[1 + (z_1-z_2)\frac{h''(z_2)}{h'(z_2)}\right]\frac{\beta h(z_2)}{\alpha\, h(z_1)}\left[\frac{1-s_2}{1-s_1}\right],$$

where $s_2 = (z_5+z_6)/\beta h(z_2)$, $s_1 = (z_3+z_4)/\alpha\, h(z_1)$; s_1 and s_2 are the savings rates in states 1 and 2, Date 1.

Thus

$$Q = \frac{h(z_2)}{h(z_1)} \cdot \left[1 + (z_1-z_2)\frac{h''(z_2)}{h'(z_2)}\right]\left[\frac{1-s_2}{1-s_1}\right].$$

Assume $\frac{1-s_2}{1-s_1} \approx 1.00$. Then $Q \gtrless 1$ according to $z_2 \gtrless z_1$.

Q.E.D.

The final step is to investigate the conditions under which $\sigma_3+\sigma_4>\sigma_5+\sigma_6$. In order to simplify the discussion we investigate the conditions for $\sigma_3>\sigma_5$ <u>and</u> $\sigma_4>\sigma_6$.

<u>Theorem 9:</u>

$$\sigma_3 \gtrless \sigma_5 \quad \text{if} \quad \frac{h'(z_5)}{h'(z_3)} \frac{h(z_3)}{h(z_5)} \frac{c_2}{c_1} \gtrless 1;$$

similarly

$$\sigma_4 \gtrless \sigma_6 \quad \text{if} \quad \frac{h'(z_6)}{h'(z_4)} \frac{h(z_4)}{h(z_6)} \frac{c_2}{c_1} \gtrless 1.$$

<u>Proof:</u> The proof follows from the definitions of the σ's.

$$\sigma_3 > \sigma_5 \text{ implies } \frac{\sigma_3}{\sigma_5} > 1, \text{ which implies } \frac{q_3}{q_5} \frac{p_5}{p_3} \frac{p_1}{p_2} > 1$$

Substitute the first-order conditions for q_3 and q_5 and use the relations $\frac{p_5}{p_3} = \frac{c_3}{c_5}$, $\frac{p_1}{p_2} = \frac{c_2}{c_1}$ to get

$$\frac{h'(z_3)}{h'(z_5)} \frac{h(z_3)}{h(z_5)} \frac{c_2}{c_1} > 1.$$

Similarly, $\sigma_4 > \sigma_6$ implies

$$\frac{h'(z_4)}{h'(z_6)} \frac{h(z_4)}{h(z_6)} \frac{c_2}{c_1} > 1.$$

Q.E.D.

<u>Lemma 1:</u>

If $z_1>z_2$ then $\frac{c_2}{c_1} > \frac{1}{1 - \frac{\beta - \alpha}{\beta \, RRA(c_2)}}$,

and if $z_1 < z_2$ then $1 < \frac{c_2}{c_1} < \frac{1}{1 - \frac{\beta - \alpha}{\beta \, RRA(c_2)}}$.

where $RRA(c_2)$ is the relative risk aversion at c_2. Thus the greater the RRA the closer c_2 is to c_1.

Proof: The result follows immediately from the Taylor series expansion of the first-order conditions shown in equation (13). If $z_1 > z_2$ then

$$1 + (c_1 - c_2)ARA(c_2) < \alpha/\beta$$

and
$$\frac{c_2}{c_1} > \frac{1}{1 - \frac{\beta - \alpha}{\beta RRA(c_2)}},$$

where
$$RRA(c_2) = -\frac{c_2 U''(c_2)}{U'(c_2)}.$$

If $z_2 > z_1$ then

$$1 + (c_1 - c_2)ARA(c_2) > \alpha/\beta$$

and
$$1 < \frac{c_2}{c_1} < \frac{1}{1 - \frac{\beta - \alpha}{\beta RRA(c_2)}} \qquad \text{Q.E.D.}$$

Theorems 8 and 9 along with Lemma 1 suggest the following possibilities for a negative nominal risk premium. If $z_1 > z_2$, the nominal risk premium can be negative if the RRA is sufficiently low that the conditions of Theorem 9 are satisfied and $\sigma_3 + \sigma_4 > \sigma_5 + \sigma_6$. The reader must keep in mind that this is not necessarily feasible, because a low RRA implies high c_2/c_1 which can only be had with a low z_1/z_2 ratio. For sufficiently low RRA $z_1 > z_2$ is not feasible. If $z_2 > z_1$ the nominal risk premium can be negative as long as the RRA is not so low as to satisfy the conditions of Theorem 9. Heuristically, there seems to be a potential region of RRA's which, combined with "favorable" production technologies and initial endowments, can result in an equilibrium in which the real risk premium is positive but the nominal risk premium is negative. In all other cases both the real and the nominal risk premia will be positive.

VII. CONCLUSION

The results that we get from studying the models proposed in this paper are new and very strong. We show that in a stationary world with complete markets the real term structure premium will always be positive, as long as the utility function of the representative consumer is separable and state-independent. Time-dependency of the utility function does not change this result. If markets are incomplete then the possibility exists that the term structure premium can be negative. Our results for the nominal term structure are weaker, because they depend on assumptions about monetary policy and on the nature of the demand for money.

Our results on the real term structure premium seem to refute formally the Expectations hypothesis and the Preferred Habitat hypothesis, but they seem to confirm the Liquidity Preference hypothesis, elaborated by Hicks (1939).[8] Though the conclusions are similar, our results do <u>not</u> depend on liquidity preference considerations. Our model does not incorporate the notion of liquidity, because transactions costs are assumed zero throughout.[9] Furthermore we show that time-dependent preferences, whether towards or away from current consumption, have no impact on the sign of the risk premium.

The results in this paper are a consequence of the equilibrium consumption and investment allocations established by a risk-averse representative consumer. The consumption/investment opportunities available to our representative consumer lead her to choose a consumption vector such that the value of a two-period riskless real bond is negatively correlated with the marginal rates of substitution between Date 1 and Date 0 consumption. This negative correlation makes the equilibrium expected price of the future one-period real bond higher than its forward price. Thus the term structure premium in our model is the payment to the systematic risk that a consumer takes on if she owns a long-term bond.

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FOOTNOTES

[1]The model can easily be generalized to more states for each Date, and to more Dates.

[2]Our assumption that each technology produces output in only one state is not restrictive. As is well-known, in complete markets any result stated in terms of complex technologies (i.e., technologies that produce some output in every state) may be restated in terms of primitive technologies (i.e., technologies that produce in only one state). We elaborate on this point in Section V.

[3]The production uncertainty in the model corresponds to Diamond's (1967) multiplicative uncertainty. The results of the model hold, however, for any specification of production uncertainty in which the production functions can be ordered by their marginal product of capital. Thus, in our model, it will follow that if $\alpha_1 < \alpha_2$ then $\alpha_1 h' < \alpha_2 h'$; the same term structure results would follow if the production in state 1 were determined by $h_1(z_1)$ and the production state 2 were determined by $h_2(z_2)$ and if $h_1'(z_1) < h_2'(z_2)$ for all z.

[4]See Long (1974), Cox, Ingersoll and Ross (1978, 1981), Beja (1979), Benninga and Protopapadakis (1983).

[5]Cox, Ingersoll and Ross have also solved a non-stationary equilibrium with a logarithmic function when the production process exhibits constant returns to scale. The non-stationary case is much more difficult to solve when production processes are non-linear, and it is in general impossible to get explicit term structure results (see Sundaresan (1984)).

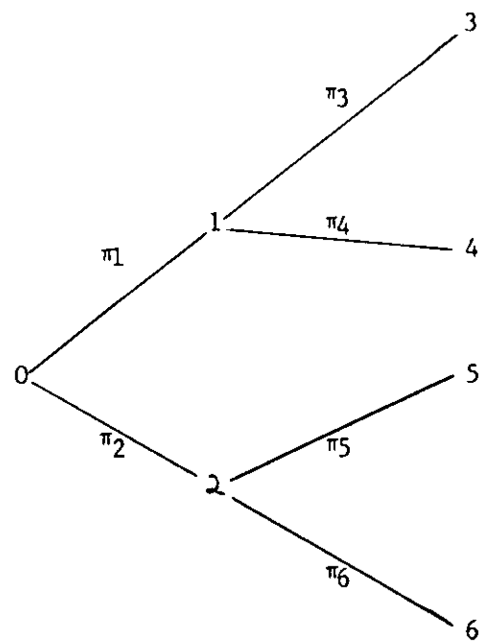
[6]The most general hypothesis along these lines is advanced by Modigliani and Sutch (1966). They state that an investor has "... an n-period habitat if he has funds which he will not need for n-periods and which, therefore, he intends to keep in bonds for n-periods. ... investors can be tempted out of their natural habitat, by the lure of higher expected returns." This hypothesis, which they label Preferred Habitat, encompasses all the term structure hypotheses that are based on differential time preference. The important hypotheses along these lines are Liquidity Preference, Hicks (1939), and market segmentation, Culbertson (1957). This point has been made forcefully by Cox, Ingersoll and Ross (1981).

[7]For a general proof, see Benninga and Protopapadakis (1983).

[8]One statement of the Liquidity Preference hypothesis is given by Hicks: "The forward spot rate will normally exceed the expected spot rate. Equivalently, the expected return on a long bond must exceed that on a short bond by a premium which compensates the lender for assuming the increased risks of price fluctuation."

[9]See Goldman (1974, 1978) for a framework in which liquidity of assets is formally incorporated by introducing differential transaction costs.

FIGURE 1



DATE	DATE	DATE
0	1	2

FIGURE 2

GRAPH OF:

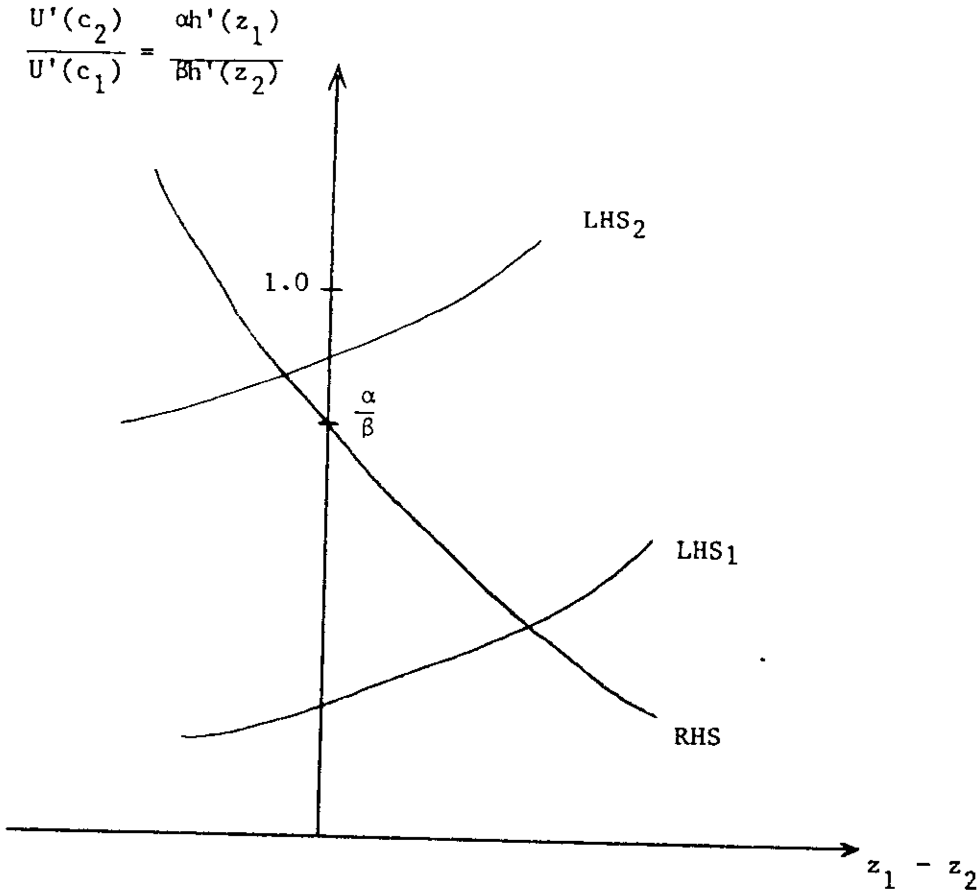
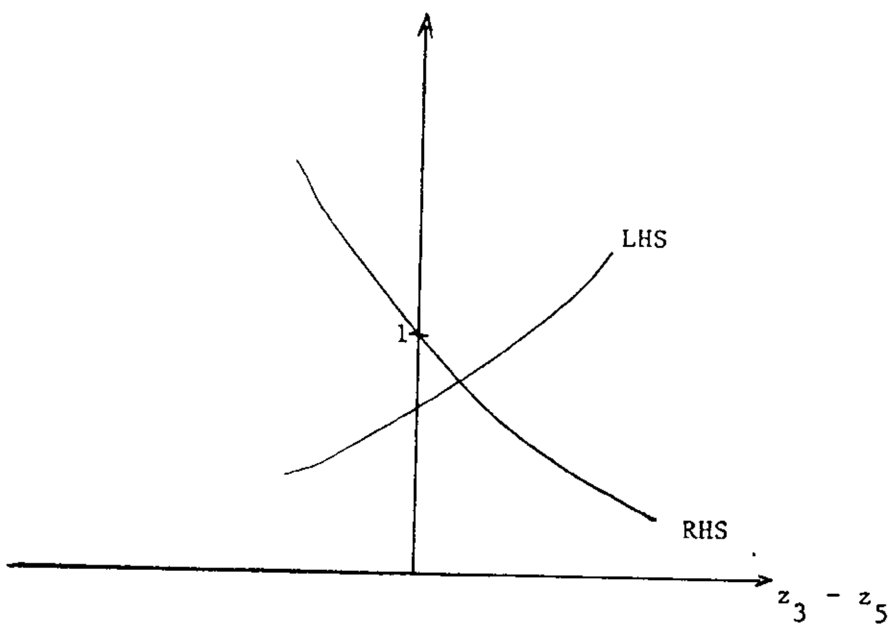


FIGURE 3

GRAPH OF:

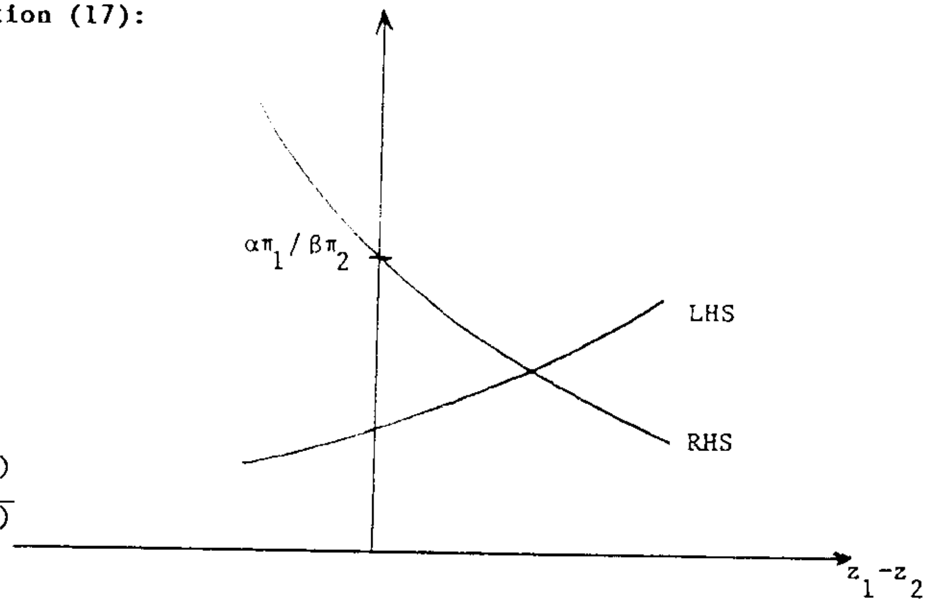
$$\frac{U'(c_5)}{U'(c_3)} \frac{U'(c_1)}{U'(c_2)} = \frac{h'(z_3)}{h'(z_5)}$$



Intercept of LHS is: $U'(c_1)/U'(c_2)$

FIGURE 4

Graph of equation (17):

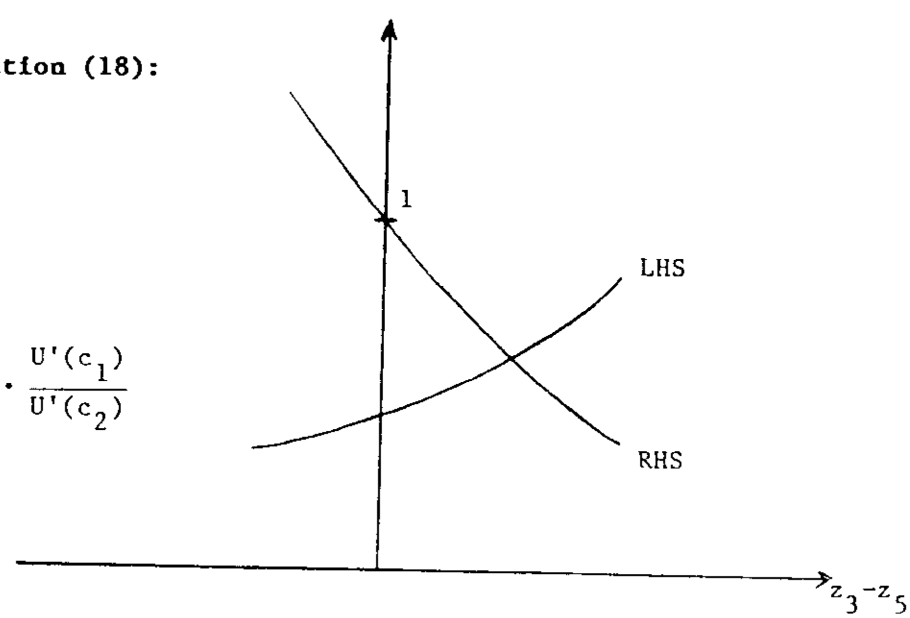


LHS: $\dfrac{U'(c_2)}{U'(c_1)}$

RHS: $\dfrac{\alpha\pi_1 h'(z_1)}{\beta\pi_2 h'(z_2)}$

FIGURE 5

Graph of equation (18):



LHS: $\dfrac{U'(c_5)}{U'(c_3)} \cdot \dfrac{U'(c_1)}{U'(c_2)}$

RHS: $\dfrac{h'(z_3)}{h'(z_5)}$