THE SPEED OF ADJUSTMENT OF FINANCIAL RATIOS:
AN ERROR-IN-VARIABLE PROBLEM

Chi-Wen Jevons Lee*

The Wharton School
University of Pennsylvania
Philadelphia, PA 19104

First Draft	April 1982
Second Draft	November 1982
Third Draft	May 1983
Fourth Draft	June 1983

*Associate Professor, University of Pennsylvania. This paper was written when I was a Visiting Assistant Professor at University of Chicago.
Baruch Lev taught me a lot about Accounting research and Nick Dopuch help me write a better paper. All the remaining errors are mine.

The Speed of Adjustment of Financial Ratios =
An Error-In-Variable Problem

I. Introduction

The traditional literature of financial statement analysis suggests that firms adjust in financial ratios to predetermined targets, such as industry-wide averages.[1] Lev (1969), using Koyck-Nerlove partial adjustment model, offered evidence that financial ratios slowly adjusted to their industrial means.

The purpose of this paper is to show that Lev's early empirical work may not provide an unambignous answer regarding the question of how fast financial ratios adjust to their industrial means. I do so by developing a rigorous stochastic model which shows the difficulty of making inferences on the speed of adjustment. The difficulty is one to the inherent "error-in-variable" problem of financial data. In the next section, I briefly summarize Lev's work. A model of Markov process of financial ratio will be developed in Section III. A brief comment and summary is given in Section IV.

II. Lev's Study

To investigate the speed of adjustment of financial ratios, Lev employed a Koyck-Nerlove adaptive adjustment model (Nerlove, 1958) in equation (1):

$$(1) \qquad Y_t - Y_{t-1} = \alpha + \beta\ (Y_t^* - Y_{t-1})$$

where Y_t is the financial ratio at time t , Y_t^* is the desired financial ratio, and β is the speed of adjustment. When $\beta = 1$, then the adjustment is instantaneously completed. Since Y_t^* is unobservable, Lev need the industrial norm $\bar{x}_t$ as a proxy:

$$(2) \qquad Y_t - Y_{t-1} = \alpha + \beta\,(\bar{X}_t - Y_{t-1})$$

However, if the speed of adjustment is not instantaneous, $\bar{X}_t$ could be a very noisy proxy. Moreover, if the speed of adjustment is not constant with respect to Y_t, then $\bar{X}_t$ could also be a biased proxy.

Lev examined the data of six financial ratios and 245 firms. His empirical evidence indicated that the mean estimate of 245 β's for each ratio varied from 0.3 to 0.51. He concluded that the speeds of adjustment of financial ratios for most firms were significantly less than one. However, in equation (2) Lev did not explicitly consider the stochastic nature of variables Y_t, Y_{t-1}, and $\bar{X}_t$. Neither does he pay attention to the random disturbance in equation (2). There are two random noises in accounting data warranting attention: (1) the measurement errors on data collection and (2) the implementation errors of the underlying business activity.[2] In this paper, I demonstrate that the "speed of adjustment" of financial ratio in the Koyck-Nerlove-Lev sense may not necessarily reflect the actual speed of adjustment of underlying business activity. To illustrate, I will use a scenario in which the speed of adjustment of the underlying business activity is instantaneous but the financial ratio shows a sluggish "adjustment" in the Koyck-Nerlove-Lev model. That sluggish "adjustment" may be attributed to the stochastic properties of measurement errors in accounting numbers.

III. Koyck-Nerlove Model with Markov Process

To explicitly examine the stochastic properties of equation (2), consider first the measurement errors of accounting numbers. Let Y_t be the observed financial ratio, θ_t be the underlying business activity to be measured, a_t be the measurement error, and a tilde (~) indicate random variable, I have:

$$(3) \qquad \tilde{Y}_t = \tilde{\theta}_t + \tilde{a}_t$$

Most financial ratios use the stock variables on the Balance Sheet. Because the stock variables are the integral of thousands of accounting entries over the whole history of a business entity, the measurement error a_t is the summation of random shocks over this whole history.

Consequently, we can model the measurement error a_t as if it were generated by a moving average stochastic process of infinite order, or

$$(4) \qquad \tilde{a}_t = \tilde{b}_t + \phi_1 b_{t+1} + \phi_2 b_{t-2} + \phi_3 b_{t-3} + \dots + \phi_i b_{t-i} + \dots$$

where b_{t-i} are the past random shock (the errors in previous accounting entries). Since many of the past random shocks are corrected and adjusted in later accounting entries, the impact of earlier random shocks on the measurement error of financial data should be smaller than the most recent random shock. Hence it is reasonable to assume smoothly declining parameters for equation (4), i.e.,

$$(5) \qquad \phi_i = \phi^i < 1 \text{ for all } i \ .$$

From equations (4) and (5) we get equation (6), a Markov process:

$$(6) \qquad \tilde{a}_t = \phi a_{t-1} + \tilde{b}_t$$

Therefore the time series of measurement errors in financial data can be modelled as a Markov process.

The second random factor considered is the implementation errors in business activities. In a dynamic economy with limited information, overshooting or undershooting a business target is inevitable. Let $\tilde{c}_t$ be the random implementation errors, I then have a Koyck-Nerlove adaptive model of business activity in equation (7):

$$\tilde{\theta}_t = \theta_{t-1} + r(\theta_t^* - \theta_{t-1}) + \tilde{c}_t \ . \tag{7}$$

The Markov process of measurement error in accounting numbers as stated in equation (6) is larger due to the adoption of historical cost accounting. Under historical cost accounting, the accounting number in financial statement is an aggregation of all relevant accounting entries throughout the life of the business entity. These could be reduced to "white noise" if firms adopted current cost accounting and there existed an efficient asset market to acquire information about current cost.[3]

I do not have any basis for presuming a particular stochastic process on the implementation errors. However, assuming a rational management requires that no business policy is systematically and consistently misimplemented, the implementation error $\tilde{c}_t$ can be an intertemporally independent white noise. To dramatize the effect of measurement errors on the estimation of a Koyck-Nerlove model, I assume that the adjustment toward θ_t^* can be completed in one period and that the implementation errors are intertemporally independent.

(8) $$\tilde{\theta}_t = \theta^*_t + \tilde{c}_t,$$

$$\text{Cov } \{\tilde{c}_t, \tilde{c}_{t-1}\} = 0, \text{ for all } t.$$

Lev's study implicitly assumed a changing optimal level θ^*_t. I postulate that the optimal level θ^*_t is changed by a constant h:[4]

(9) $$\theta^*_t = h + \theta^*_{t-1} .$$

The time-series of θ^*_t can follow any deterministic or stochastic process since the conclusion will not depend on the specification of equation (9).

Since the variables $\tilde{\theta}_t$, θ_{t-1} and θ^*_t in equation (7) are unobservable, we employ $\tilde{Y}_t$, Y_{t-1} and $\tilde{\bar{X}}_t$ correspondingly as proxy variables which are are defined in equation (3) and (10).

(10) $$\tilde{\bar{X}}_t = \theta^*_t + \tilde{d}_t,$$

where $\tilde{d}_t$ is the measurement error of the desired financial ratio. Because the industrial mean of financial ratios is adopted as a proxy for θ^*_t, the measurement error $\tilde{d}_t$ can conceivably follow a Markov Process. However, the stochastic property of $\tilde{d}_t$ is inconsequential to my conclusion. Therefore I do not further discuss the stochastic properties of $\tilde{d}_t$. From equation (6), we have:

(11) $$\tilde{\theta}_t + \tilde{a}_t = (1 - \phi)\ \tilde{\theta}_t + \phi\ (\tilde{\theta}_t - \theta_{t-1}) - (1 - \phi)\ (\theta_{t-1} + a_{t-1}) + (\theta_{t-1} + a_{t-1}) + \tilde{b}_t .$$

From equations (8) and (9), we have:

(12) $$\tilde{\theta}_t - \theta_{t-1} = h - c_{t-1} + \tilde{c}_t$$

If there is no measurement error in accounting numbers, then equation (12) can be rewritten as:

(13) $$\tilde{Y}_t - Y_{t-1} = h - c_{t-1} + \tilde{c}_t .$$

Consequently if (1) there were no measurement error in accounting numbers, (2) the change in optimal level was deterministic and (3) the adjustment of business activity could be completed within one period, then the accounting numbers Y_t can be modelled as an order 0, 1, 1 ARIMA Process.

From equations (8), (9), (10), and (12) we obtain

(14) $\tilde{\theta}_t = \tilde{\bar{x}}_t + \tilde{c}_t - \tilde{d}_t$, and

from equations (3) and (11),

(15) $\tilde{Y}_t - Y_{t-1} = (1 - \phi)\ (\tilde{\theta}_t - Y_{t-1}) + \phi(\tilde{\theta}_t - \theta_{t-1}) + \tilde{b}_t.$

From equations (12), (14) and (15), we can derive:

(16) $\hat{\tilde{Y}}_t - Y_{t-1} = \phi h + (1 - \phi)\ (\hat{\tilde{x}}_t - Y_{t-1}) + \tilde{e}_t,$

where $\tilde{e}_t = b_t + \tilde{c}_t + \phi c_{t-1} - (1 - \phi)\tilde{d}_t$ is the random disturbance of the Koyck-Nerlove-Lev regression function. It is obvious that $\tilde{e}_t$ is serially correlated. Comparing equations (2) and (16), we note:

(17) $\alpha = \phi h,$

$\beta = 1-\phi.$

Consequently, the estimated "speed of adjustment" in Lev's study may simply reflect the parameter of Markov Process of the measurement errors in accounting data.

IV. Conclusion

I have shown that Lev's measures of the "speed of adjustment" of financial ratios may be tainted due to the Markov process of measurement errors inherent in accounting data. As a result, even if the underlying business activity could be instantaneously adjusted, the estimated "speed of adjustment β" from a Koyck-Nerlove model will be significantly less than one. Moreover, empirical evidence indicates that the regression residuals from the Koyck-Nerlove model are often serially correlated. Equation (16) demonstrates that even if implementation errors were serially independent, the regression residual in Lev's Model would still be serially correlated.

The errors-in-variable problem is one of statistical identification. When errors-in-variables exist in a regression study, there are alternative structures with different parameter values that will produce the same expected moment matrix of observable variables. If we can place sufficient restrictions on (1) the parameters, (2) the covariance matrix of the unobservable variables, and (3) the covariance matrix of the disturbance, then it may be possible to find only one structure that is consistent with the observed information and the restrictions.[5] Future research is needed to help us identify the structure, or otherwise we cannot unabiguously interpret Lev's empirical results.

Footnotes

1. See Lev (1969) and Foster (1978, Ch. 5) for more detailed elaboration.

2. The measurement errors arise from valuating, recording, calculating and cheating throughout the course of transaction. For example, in an era of price fluctuation, the historical cost of fixed assets bears little resemblance to their market value. This discrepancy is the measurement error. In a dynamic economy with limited information, overshooting or undershooting a business target is inevitable. The implementation errors can be attributed to the noise in communication, deficiency in control system, random exogenous influence and goal incongruence. Although my conclusion depends on the stochastic properties of measurement errors and implementation errors, but it is independent of their magnitudes. Even if this magnitudes were small, but when they are primary sources of data variability, their stochastic properties would influence our statistical inference.

3. The "efficient" asset market is defined in Fama's (1970) sense.

4. If there is no change in the optimal level, then the question of adjustment of business activities becomes meaningless.

5. For detail analysis of error-in-variable problem, see Griliches (1974) and Judge, Griffiths, Hill and Lee (1980, Chapter 13).

References

Beaver, W. and D. Morse, "What Determines Price-Earning Ratio?" Financial Analyst's Journal. July, 1978. pp. 65-76.

Box, G.E.P., and G.M. Jenkins, Time Series Analysis. San Francisco: Holden Day, Inc., 1976.

Griliches, Z., "Distributed Lags: A Survey." Econometrica, January, 1967. pp. 16-49.

Griliches, Z., "Errors in Variables and Other Unobservables." Econometrica, September, 1974, pp. 971-998.

Fama, E.F., "Efficient Capital Markets: A Review of Theory and Empirical Work," The Journal of Finance, May, 1970, pp. 383-417.

Foster, G.T., Financial Statement Analysis, Englewood Cliffs, NJ: Prentice-Hall, 1978.

Judge, G.G., W.E. Griffiths, R.C. Hill and T. Lee, The Theory and Practice of Econometrics. New York: John Wiley & Sons, Inc., 1980.

Lee, C.J., "Homogeneity and Normality of Cross-Sectional Financial Data." Working Paper No. 96, Center for Research in Security Prices, Graduate School of Business, University of Chicago, May, 1983.

Lev, B., "Industry Averages as Targets for Financial Ratios." Journal of Accounting Research, Autumn 1969, pp. 290-299.

Nerlove, M., Distributive Lags and Demand Analysis, Agriculture Handbook No. 141, U.S. Department of Agriculture, Washington, D.C., 1958.