A NOTE ON "WHY DO COMPANIES PAY DIVIDENDS?"

By

Isik Inselbag

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THE WHARTON SCHOOL

University of Pennsylvania

Philadelphia, PA 19104

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In a recent article in this review, Feldstein and Green (F-G) present a model which they claim provides an explanation why companies pay dividends despite the unfavorable taxation of dividend income. At the end of their paper, F-G suggest that a possible extension of their model would be "to recognize that both corporations and portfolio investors can also borrow and that corporations as well as investors can earn the risk-free return." They also indicate that such an extension "would weaken the link between dividends and real corporate investment," however, "such a link between dividend policy and real corporate investment would persist, contrary to Modigliani-Miller theorem."[1]

Modigliani and Miller (M-M) have shown that, when there are no market imperfections like transactions costs or differential taxation of different investors and different sources of income, given a firm's investment policy, the current price of its shares is independent of its dividend policy.[2] F-G explicitly introduce differential taxation of investors, and differential taxation of income from dividends and capital gains into their model, find a functional relationship between firm dividend policy and its current share price, and derive an optimal dividend policy for the firm from this relationship. The model, however, is restricted to a special case where the source of financing real investments of firms is exclusively retained earnings. The difficulty with this special case is that, as M-M note in their original paper, the optimality of dividend policy is "inevitable if one works exclusively with the assumption, explicit or implicit, that funds come only

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[1]See Feldstein and Green (1983), p. 29.

[2]See Miller and Modigliani (1961).

from retained earnings. For in that case dividend policy is indistinguishable from investment policy."[3] In order to find the independent dividend effect on current share price, optimal real investment level for each firm should be taken as fixed. Given the optimal real investment level, the firms may then be allowed to finance those investments by retaining earnings and/or selling new shares, or alternatively, by retaining earnings and/or borrowing. Derivation of the optimal dividend policy with such financing alternatives within an environment of differential taxation can properly show the unique dividend effect on current share price independent of the real investment effect.

The intent of this note is to show that the optimal dividend policy that F-G derive is due to their treating retained earnings as the sole source of financing. It will be shown that, taking the level of optimal real investments as given, if riskless corporate borrowing and lending is allowed, a non-corner solution for the optimal dividend payout policy does not exist, as expected due to the original M-M analysis and contrary to the F-G assertion.

F-G Model with Corporate Lending and Borrowing

F-G consider two classes of investors. First is the households, denoted by H, with a flat tax rate, θ, on dividend income, none on capital gains; and the second is institutions, denoted by I, with no tax obligations.[4]

There are two firms undertaking production and making investment and financing decisions. The behavior of investors and firms are analyzed using a

[3]Ibid, p. 424.

[4]In the formulation that follows, interest income of households is also assumed to be taxed at the flat rate θ.

one period model. Investors try to maximize the expected utility of their terminal wealth, firms try to maximize the current share price of their stock.

At the beginning of the period, the two types of investors own the following numbers of shares of the two firms: $\bar{s}_{H1}$, $\bar{s}_{H2}$, $\bar{s}_{I1}$, and $\bar{s}_{I2}$, where subscripts 1 and 2 indicate the two firms. The total number of shares of each company is normalized and fixed at 1. After the companies announce their dividend policies, trade takes place where the investors can sell their shares and can buy other shares at market prices. Investors can also place some of the proceeds of their share sales in a riskless asset. Households pay a θ percent tax on their interest income of $R - 1$, while the interest income of institutions is not taxed. Both types of investors prefer present dollars to future dollars; one present dollar (obtained either as after-tax dividends or from the sale of shares) is worth R dollars <u>before taxes</u> at the end of the period, or $R_H = R - \theta(R - 1)$ dollars <u>after taxes</u> for households and R dollars for institutions.

Each firm has an initial amount of one dollar per share available for distribution and retention. Investment opportunities of the two firms are fixed as follows: the cost per share of all acceptable real investments for plant and equipment is represented by ϕ_1 for firm 1 and by ϕ_2 for firm 2 such that:

$$0 < \phi_1, \phi_2 < 1 .$$

The returns from these projects at the end of the period are subject to uncertainty. The two firms invest in these projects through retaining earnings and/or by borrowing at the riskless after corporate tax cost of R_1. (R_1 is equal to principal plus after tax interest payments.) Besides investments into plant and equipment, the firms can also invest in a riskless asset which generates an after corporate tax return of R_1. The after tax returns that firm i generates and distributes as liquidating dividends to its stockholders at the end of the period can be written as:

$$\tilde{D}_i = \phi_i \tilde{r}_i + (1 - \phi_i - d_i) R_i \qquad i = 1, 2$$

where $\tilde{D}_i$ is the after corporate tax liquidating dividends of firm i, $\tilde{r}_i$ is the after tax return on per dollar real investment and d_i is the dividends that firm i distributes to each share at the beginning of the period. The expected value and variance of the liquidating dividends for firm i, and its covariance with the liquidating dividends of the other firm are respectively equal to:

$$D_i^e = \phi_i r_i^e + (1 - \phi_i - d_i) R_i \qquad i = 1, 2$$

$$\mathrm{Var}(\tilde{D}_i) = \phi_i^2 \sigma_{ii} \qquad i = 1, 2$$

$$\mathrm{Cov}(\tilde{D}_1, \tilde{D}_2) = \phi_1 \phi_2 \sigma_{12}$$

where r_i^e and σ_{ii} are the expected value and variance of $\tilde{r}_i$, and σ_{12} is the covariance between the after tax returns per dollar real investment in the two firms.

The initial dividend per share should be:

$$0 \leq d_1, d_2 \leq 1 .$$

Note that firm i will be a borrower if $d_i > 1 - \phi_i$ or an investor in a riskless asset if $d_i < 1 - \phi_i$.

It is assumed that investors pay no taxes on the liquidating dividends they receive. Investors' demand functions for the shares of the two companies are derived by maximizing their expected utility of terminal wealth subject to their budget constraint on the use of their initial wealth.

The budget constraints for the households and institutions, given the market prices p_1 and p_2 of the two firms, can be written as:

$$
\begin{aligned}
p_1\bar{s}_{H1} + p_2\bar{s}_{H2} &= p_1 s_{H1} + p_2 s_{H2} + z_H \\
p_1\bar{s}_{I1} + p_2\bar{s}_{I2} &= p_1 s_{I1} + p_2 s_{I2} + z_I
\end{aligned}
\tag{1}
$$

where s_{Hi}, s_{Ii}, $(i = 1, 2)$, represents the shares owned of each company after trade by households and institutions respectively, and z_H and z_I are the investments of the two types of investors in the riskless asset.

With dividend payouts of d_1 and d_2, the households' total after tax funds at the beginning of the period are $(1 - \theta)[d_1 s_{H1} + d_2 s_{H2}] + z_H$. Additional funds to be received from firms at the end of the period are the uncertain amount $s_{H1}\tilde{D}_1 + s_{H2}\tilde{D}_2$. Finding the future after tax value of the former and adding it to the latter gives the terminal wealth of the households:

$$
\tilde{W}_H = R_H(1 - \theta)[s_{H1}d_1 + s_{H2}d_2] + R_H z_H + s_{H1}\tilde{D}_1 + s_{H2}\tilde{D}_2 \; . \tag{2a}
$$

Terminal wealth for the institutions can be written as:

$$
\tilde{W}_I = R[s_{I1}d_1 + s_{I2}d_2] + Rz_I + s_{I1}\tilde{D}_1 + s_{I2}\tilde{D}_2 \; . \tag{2b}
$$

Using the F-G expected utility function:[5]

$$
E[U(\tilde{W}_H)] = E(\tilde{W}_H) - 0.5\ \gamma\ \mathrm{var}(\tilde{W}_H) \tag{3}
$$

where $\gamma > 0$ is a measure of risk aversion and where $E(\tilde{W}_H)$ and $\mathrm{var}(\tilde{W}_H)$ are now given by:

$$
E(\tilde{W}_H) = R_H(1 - \theta)[s_{H1}d_1 + s_{H2}d_2] + R_H z_H + s_{H1}D_1^e + s_{H2}D_2^e \tag{4}
$$

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[5]The expected utility function used by F-G is questioned by Hasbrouck and Friend, 1983. We continue to use the same function, however, to show that even within the same F-G framework, the nature of the dividend policy changes dramatically when corporate borrowing and lending is allowed.

and

(5) $$\mathrm{var}(\tilde{W}_H) = s_{H1}^2\phi_1^2\sigma_{11} + s_{H2}^2\phi_2^2\sigma_{22} + 2s_{H1}s_{H2}\phi_1\phi_2\sigma_{12}$$

The market clearing share price functions that would correspond to any combination of dividend policies is found by first deriving the share demand functions of households and institutions and then by equating the total share demands for each stock to the fixed-share supplies. Following the F-G procedures, given equations (1)-(5), we find the following price functions:[6]

(14) $$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \frac{1}{R_H + R}\begin{bmatrix} [R_H(1-\theta) + R]d_1 + 2[\phi_1 r_1^e + (1-\phi_1-d_1)R_1] - \gamma[\phi_1^2\sigma_{11} + \phi_1\phi_2\sigma_{12}] \\ [R_H(1-\theta) + R]d_2 + 2[\phi_2 r_2^e + (1-\phi_2-d_2)R_2] - \gamma[\phi_2^2\sigma_{22} + \phi_1\phi_2\sigma_{12}] \end{bmatrix}.$$

As in the F-G model, the price of each share is positively related to the expected return on per dollar real investment, and negatively related to the variance of that return and its covariance with the return on per dollar real investment of the other share. However, the two models differ in the pattern of relationship between share prices and the three parameters cited above. The following table shows the partial derivatives of the price functions with respect to expected value, variance and covariance in the two models.

TABLE 1

Relevant Partial	In F-G Model	In This Model
$\frac{\partial p_i}{\partial r_i^e}$	$\frac{1}{2R}[2(1-d_i)]$	$\frac{1}{R_H + R}[2\phi_i]$
$\frac{\partial p_i}{\partial \sigma_{ii}}$	$-\frac{\gamma}{2R}[(1-d_i)^2]$	$-\frac{\gamma}{R_H + R}[\phi_i^2]$
$\frac{\partial p_i}{\partial \sigma_{12}}$	$-\frac{\gamma}{2R}[(1-d_1)(1-d_2)]$	$-\frac{\gamma}{R_H + R}[\phi_1\phi_2]$

[6]For the derivation of the price functions, see the Appendix.

In our model, the marginal effects on share prices of expected value, variance and covariance of returns on real investments are independent of firm dividend policies and are determined by the level of real investments. In the F-G model, although those marginal effects seem to be functions of dividend policy, since they have d_i terms, this is totally due to the fact that $1 - d_i$ represents the level of real investments. Consequently, their dividend policy is indistinguishable from investment policy.[7]

We follow the F-G assumption that each firm selects its dividend payout rate to maximize its share price. To find these maximization conditions, we take the partials of each price with respect to the dividends, and find that:

$$\frac{\partial p_i}{\partial d_i} = \frac{R_H(1 - \theta) + R - 2R_i}{R_H + R} \lesseqgtr 0 \qquad i = 1, 2 . \tag{15}$$

It is clear from (15) that, when the effects of dividend policy are separated from the effects of real investments, share prices will be either independent of dividend policy, or the optimal payout of each firm will be a corner solution, i.e., $d_i = 0$ or 1. In fact, when we substitute the values used by F-G in their example or other plausible values for the risk-free rate and the tax rates on corporate and personal income, we find that $\partial p_i/\partial d_i$ is negative. This result holds even if we ignore the taxation of the households' interest income like F-G do. Given the tax structure in the model, corporate investment into real and financial assets works as a tax shelter for the

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[7]The other difference between the marginal effects in Table 1 is that in the denominators F-G have 2R, while our model has $R_H + R$. This is due to F-G's not taking into account differential taxation of interest income of households and institutions. Since we have assumed that the interest income of households is taxed at the flat rate θ, the after tax return on investments into the riskless asset is R_H for households and is R for tax exempt institutions.

dividend income of the households. This suggests that, in an environment where dividend income is unfavorably taxed, firms should distribute no dividends in order to maximize their current share prices.

Given these results of extending the model in the direction F-G suggested, it seems that the question that has to be asked still remains "Why Do Companies Pay Dividends?"

APPENDIX

The households' demand for shares is found by maximizing (3) subject to (1). The first order conditions are given by:

(6) $$0 = R_H[(1 - \theta)d_1 - p_1] + D_1^e - \gamma[s_{H1}\phi_1^2\sigma_{11} + s_{H2}\phi_1\phi_2\sigma_{12}]$$

and

(7) $$0 = R_H[(1 - \theta)d_2 - p_2] + D_2^e - \gamma[s_{H2}\phi_2^2\sigma_{22} + s_{H1}\phi_1\phi_2\sigma_{12}] \ .$$

The first order conditions can also be written as:

(8) $$\gamma \begin{bmatrix} \phi_1^2\sigma_{11} & \phi_1\phi_2\sigma_{12} \\ \phi_1\phi_2\sigma_{12} & \sigma_2^2\sigma_{22} \end{bmatrix} \begin{bmatrix} s_{H1} \\ s_{H2} \end{bmatrix} = \begin{bmatrix} R_H(1 - \theta)d_1 + D_1^e \\ R_H(1 - \theta)d_2 + D_2^e \end{bmatrix} - R_H \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$$

or, in matrix notation

(9) $$\gamma A s_H = a_H - R_H p \ .$$

Assuming that A is nonsingular, we can solve these equations for s_H which is the vector of demand functions for households:

(10) $$s_H = \frac{1}{\gamma} A^{-1}[a_H - R_H p] \ .$$

Maximization of the expected utility of terminal wealth for institutions subject to their budget constraints yield the following demand functions:

(11) $$s_I = \frac{1}{\gamma} A^{-1}[a_I - Rp]$$

where

$$a_I = \begin{bmatrix} Rd_1 + D_1^e \\ Rd_2 + D_2^e \end{bmatrix} \ .$$

In equilibrium the sum of the demands of households and institutions for the shares of the two firms is equal to their fixed share supplies. Since the supply of shares of the two firms are fixed at unity, this equality is written as:

(12)
$$s_H + s_I = e$$

where e is a column vector of ones. Substituting into (12) from (10) and (11) we have:

(13)
$$\frac{1}{\gamma} A^{-1}\left[a_H + a_I - (R_H + R)p\right] = e \ .$$

Solving (13) for the price vector yields:

(14)
$$p = (R_H + R)^{-1}\left[a_H + a_I - \gamma Ae\right]$$

which can also be written as equation (14) in the text.

REFERENCES

Feldstein, Martin and Green, Jerry, "Why Do Companies Pay Dividends?" The American Economic Review, March 1983, 73, 17-30.

Hasbrouck, Joel and Friend, Irwin, "Why Do Companies Pay Dividends?: Comment," Working Paper No. 7-83, Rodney L. White Center for Financial Research, 1983.

Miller, Merton and Modigliani, Franco, "Dividend Policy, Growth and the Valuation of Shares," Journal of Business, October 1961, 34, 411-433.