EXPECTATIONS OF REAL INTEREST RATES AND AGGREGATE CONSUMPTION: SYNTHESIS AND TESTS

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1. Introduction

The relation of equilibrium interest rates to aggregate consumption is a relevant topic for a wide range of interests in economics. Optimal consumption, investment and output are determined simultaneously with interest rates and all other asset prices in general equilibrium. Therefore, determination of the response of consumption or interest rates to shocks in the economy generally requires specification of production equilibrium and exchange equilibrium in the securities markets. Important papers in financial economics have applied intertemporal portfolio theory to the pricing of bonds under uncertainty [e.g. Merton (1973), Cox, Ingersoll and Ross (1978)] but, the integration of modern asset pricing theory and its rigorous microeconomic foundations with the study of important macroeconomic variables like aggregate consumption, interest rates and real output, represents a significant and important research challenge. Several recent studies(1) have moved toward this goal by modeling production economies embedding the partial, exchange equilibrium conditions which are the core of financial valuation models.

A fundamental implication of exchange equilibrium in models with perfect markets is that real interest rates should be higher *ceteris paribus*, when anticipated real consumption growth is higher. This condition is necessary for general equilibrium in a class of models given a perfect securities market. Interest rates should reflect, in equilibrium, consensus rational forecasts of future aggregate consumption changes.

This paper analyzes a discrete time version of the consumption-based intertemporal asset pricing model relating consumption changes to interest rates. Expectations of economic agents are modeled as linear functions of pre-determined information, and the theoretical relations between consumption growth and interest rates are shown to imply proportional parameters across equations in a multivariate regression model. These nonlinear restrictions are estimated and the models are tested, producing estimates of aggregate risk aversion.

Section Two discusses the theoretical relation between interest rates, prices and anticipated real consumption changes in perfect markets, employing a discrete-time analogue for Breeden's (1979) multi-period consumption-based asset pricing model with many consumption goods and time-additive state independent utility. Assuming joint lognormality of price levels and marginal utility of aggregate real consumption, Section Three presents linear closed-form solutions which include as special cases the models of Rubinstein (1976), Breeden and Litzenberger (1978), Hall (1978), and Grauer and Litzenberger (1979).

Section Four discusses the econometric methodology used to test the bond pricing model with constant absolute and constant relative risk aversion utility functions. The methodology is more widely applicable in models with linear relations among rational expectations of economic variables and has some advantages over approaches more traditional in financial economics. The approach does not suffer from bias due to "missing variables," measurement error in "betas" nor, in principle, from the unobservability of a "market portfolio." By focusing directly upon changes in expectations and employing auxilliary information with cross-section of assets, similar methods allow tests of asset pricing models without observing the relevant "hedging" portfolios. This is an important advantage for application to financial models [see, e.g. Gibbons and Ferson (1981)].

Section Five describes the data on United States aggregate consumption, real interest rates, and the predetermined instruments used to model conditional expectations. Section Six presents tests of the consumption models and estimates of aggregate risk aversion. Section Seven concludes with some interpretations of the results and their implications for future research.

2. The Theoretical Relation of Consumption Growth and Interest Rates

The present study focuses on exchange equilibrium relations and their implications for securities prices; in particular the relation of default-free interest rates to optimal consumption changes.

The central hypothesis is there should exist (in a perfect financial market) a positive relation between the level of interest rates and contemporaneous expectations for aggregate consumption changes. This is because consumer-investors behaving optimally will equate (under fairly general assumptions) their marginal rates of substitution for consumption at different points in time with price ratios of traded financial claims which promise future consumption payouts. Consider, for example, a hypothetical default-free discount bond which pays one unit of "consumption" at future date $t=2$. The value at time $t=1$ of this future payout will be positively related to the "social" marginal utility of date-2 consumption relative to that of the current numeraire. If marginal utility declines monotonically in the level of real consumption, times of higher real consumption levels are times when real consumption payouts are valued less highly relative to times of lower real consumption. Therefore one should find real bond prices are lower (real interest rates higher), other things held equal, when the anticipated increment between current and future levels of consumption is greater.

Perhaps the simplest and most fundamental model of competitive market equilibrium under uncertainty is the state preference model of Arrow (1964) and Debreu (1959). Formally, the model assumes more about trading opportunities (e.g., complete markets) than is required to establish a positive relation between consumption changes and interest rate levels; but it will serve as a useful starting point and example. In this model, interior equilibrium may be characterized by market clearing, feasibility, and the requirement that each expected utility maximizing agent equate marginal rates of substitution between current and state-contingent future consumption with prices of primitive claims. A primitive claim for a state pays off one unit of consumption if and only if that state obtains. The first order conditions for optimality at date t imply equation (1) holds for each individual k.

$$\begin{aligned} &\pi\big(s(T),k,t\big)u_c\big(C^k_{s(T)},k,T\big) = \lambda_k\Phi\big(s(T),t\big) \\ &u_c\big(C_t,k,t\big) = \lambda_k, \quad \forall\, T > t \ , \end{aligned} \tag{1}$$

where $\pi\big(s(T),k,t\big)$ is the probability, as currently assessed by individual k, that state of the world s(T) will occur at future date T. $u(\cdot)$ is the state independent, concave utility function for consumption at the indicated date and $u_c(\cdot)$ is its partial derivative with respect to the quantity of consumption. C_s is the number of units of consumption chosen for state s, C_t the quantity of current consumption and λ_k is a Lagrange multiplier for individual k's budget, which constrains expenditures for current and future consumption claims to equal current wealth. $\Phi\big(s(T),t\big)$ is the current market price of one unit of consumption to be delivered at date T if and only if state s(T) has obtained. Combining the first order conditions yields

$$\Phi\big(s(T),t\big) = \frac{\pi\big(s(T),k,t\big)u_c\big(C^k_{s(T)},k,T\big)}{u_c\big(C^k_t,k,t\big)} \ . \tag{2}$$

In a perfect market, the value of a linear combination of securities must equal the linear combination of values to avoid arbitrage opportunities. Therefore, if a security is characterized by time-state contingent payouts of consumption $\{\rho_{sT}\}sT$; $T > t$, it must have time-t price

$$P_t = \sum_{(T>t)} \ \sum_{(s\varepsilon S(T))} \Phi(s,t)\rho_{sT} \ . \tag{3}$$

The term structure of real interest rates is defined through prices of default-free discount bonds of various maturities, which pay one unit of consumption at a specified date regardless of state. Let $R_j(t)$ be the j-period continuously compounded return (spot rate) at date t for a real bond which matures at time t+j. Equation (3) implies the bond price may be written in a perfect market as

$$\begin{aligned} e^{-R_j(t)j} &= \sum_{(s\varepsilon S_{t+j})} \left[\pi\big(s(t+j),k,t\big) \frac{u_c\big(C^k_{s(t+j)},k,t+j\big)}{u_c\big(C^k_t,k,t\big)}\right] \\ &\equiv E^k_t \left[\frac{u_c\big(C^k_{t+j},k,t+j\big)}{u_c\big(C^k_t,k,t\big)}\right] . \end{aligned} \tag{4}$$

For each individual, equilibrium real bond prices vary with maturity as the expectation of future marginal utility of consumption at the maturity date varies relative to that of current consumption. The shape of the equilibrium term structure at any date thus reflects currently anticipated changes in future consumption levels.

Let $C_{s(t+j)} - C_t \equiv \Delta C_{sj}$, dropping the k superscript. ΔC_{sj} is the j-period change in optimal consumption if state s(t+j) obtains. Using the implicit function theorem and equation (4) with $C_t + \Delta C_{sj}$ substituted for $C_{s(t+j)}$, it is easy to show that $\frac{\partial}{\partial \Delta C_{sj}} \left[R_j(t)\right] > 0$ if the utility function u(C,k,t+j) is increasing and concave in C. Equilibrium real interest rates are higher in this model when individuals anticipate greater increments in real consumption over a horizon terminating at the bond maturity date. High anticipated growth of consumption may occur because current consumption is relatively "low," because relatively high future consumption is anticipated, or some combination of the two. Risk of uncertain future consumption also affects default-free real bond prices. For fixed current and expected future (time t+j) consumption, risk averse consumers will have lower expected utility and thus higher expected marginal utility for guaranteed delivery at t+j of a marginal unit when uncertainty about future consumption is greater. Therefore, the expected marginal rate of substitution and thus the time-t price of a j-period bond will be higher (the real interest rate lower), other things equal, if consumption at t+j is more uncertain.(2) Equilibrium real interest rates for a risk averse consumer are therefore increasing in anticipated real consumption increments and decreasing in consumption risk at each maturity date.

In the special case of risk neutral preferences, marginal utility is independent of the level of future consumption. The real term structure in this model would be invariant to both the rate of consumption growth and to consumption risk in a world populated with risk-neutral investors.

Without further restrictions on individuals' probabilities and utility functions, analysis of this simple model offers little more than intuition. If some individuals' consumption prospects improve when others decline, no guarantee exists that these results would hold in an aggregate sense.

Interpretation of individual equilibrium results for aggregate quantities derives from the imposition of market clearing. The finance literature contains numerous examples, using various combinations of restrictions on the forms of individuals' marginal utility functions and probability assessments [Grauer and Litzenberger (1979) provide a review]. The assumption that individuals' utilities display hyperbolic absolute risk aversion, while not necessary for aggregation, has played an important role in this literature.(3) This paper does not address the aggregation issue directly (a representative individual model is simply assumed), but examines empirically closed-form expressions which derive when specific HARA utilities are assumed in a consumption model.

The model employed is a discrete-time analogue to the consumption based asset pricing model of Breeden (1979). Similar models have recently appeared in papers by Grossman and Shiller (1981), Hall (1981), Hansen and Singleton (1981), Hansen, Richard and Singleton (1981), Richard (1980), and Shiller (1981), among others. The version described here is developed in more detail in Ferson (1980, 1982).

Competitive consumer-investors trade at each date G non-storeable consumption goods, $\underline{c}$ (one chosen as numeraire), and shares of N assets. Markets may be incomplete but are perfect, so that equilibrium allocations of the traded claims are Pareto efficient. The representative Von Neumann-Morgenstern utility function is time additive and state independent:[4]

$$\sum_t e^{-ht} u\big(g(\underline{c})\big) \; . \tag{5}$$

In expression (5), h is a rate of pure time preference, $g(\cdot)$ is an ordinal indicator of utility of consuming $\underline{c}$ and $u(\cdot)$ is a positive concave transformation. Following Breeden (1977, 1981), if $\underline{c}^*(C_n, P_c)$ maximizes $g(\underline{c})$ given nominal consumption expenditures $C_n = \underline{c}' P_c$ (P_c is the commodity price vector), then "real" consumption expenditures may be meaningfully defined as $C \equiv C_n / I(C_n, \underline{P}_c) \equiv g(\underline{c}^*)$. $I(C_n, P_c)$ is an expenditure deflator, inversely proportional to maximum direct utility per unit expenditure at prices P_c. This "price level" index may depend on C_n, P_c and the form of $g(\cdot)$.[5] Individuals plan consumption and investment to maximize expected lifetime utility of consumption, given their available information and wealth. They take account of possible shifts in future consumption, inflation, and investment opportunities when forming current demands. Indirect utility of real consumption may depend on all prices, wealth, and the state of the economy as reflected in the joint distribution of future asset payoffs and goods price levels. Provided consumption is positive and wealth is not entirely liquidated for current consumption, intertemporal optimization requires at each stage of a dynamic programming solution, that the marginal utility of expenditure for current consumption equal the indirect marginal utility of investment for future consumption.(6) The basic asset pricing model derives from the first order conditions for optimality; the time-t price of a claim to random future amounts of the numeraire $\{\tilde{\rho}_{t+j}\}_j$ is given as

$$P_t(\underline{\tilde{\rho}}) = \sum_{j>0} e^{-hj} E_t^*\{\tilde{\Phi}_{t+j}\tilde{\rho}_{t+j}\} \; ;$$
$$\tilde{\Phi}_{t+j} \equiv \left[\frac{u_c(\tilde{C}_{t+j})}{u_c(C_t)} e^{-j\tilde{i}(t,t+j)}\right] , \tag{6}$$

where $u_c(C)$ is the marginal utility of real consumption per capita and $E_t^*(\cdot)$ is the consensus rational (i.e. mathematical conditional) expectation given information Ω_t used by the market in setting prices at time t. Markets are assumed to be efficient in the sense that Ω_t includes at least the information set subsequently employed in tests of the model and the true values of stationary parameters. The j-period average inflation rate in equation (6) is $i(t,t+j) \equiv (1/j)\ln(\frac{I^*_{t+j}}{I^*_t})$, where I* equals by definition $I[1 - \frac{C_n}{I} I_{C_n}]^{-1}$. I_{C_n} denotes the partial derivative with respect to nominal expenditures of $I(C_n, P_c)$. I* reduces to I in the special case of homotheticity ($I_{C_n} = 0$), for which equation (6) has been derived by Grauer and Litzenberger (1979) in a two-period state preference model.

Non-homotheticity ($I_{C_n} \neq 0$) admits an additional source of interaction between the "cost of living" and expenditure level: a poor man grown rich will no longer simply expand his consumption mix proportionately at given prices, nor will the rich person who encounters a wealth reduction, consume proportionately smaller amounts of each good.(7) The index I* reflects these effects and their impact on "inflation" (this is a distinction we do not make at the empirical level, however). $\tilde{\Phi}_{t+j}$ may be interpreted as an uncertain marginal rate of substitution of nominal consumption expenditures between dates t and t+j(8); thus equation (6) implies that the marginal utility of the cost of investing in any asset today must equal the expected marginal utility gained from holding the asset. With specific assumptions on preferences, equation (6) provides multiperiod valuation formulae which specify how cash flows at all future dates may be "discounted" to present values.(9) The ability to price securities in this way is convenient for the study of discount bond prices, because knowledge that a given nominal payment will occur after several discrete periods, is formally incorporated into the pricing model. Models stated in terms of the characteristics of single period returns and "betas" do not lend themselves as directly to valuation of discount bonds and other long-lived assets.(10)

3. Closed-Form Solutions for Nominal Bonds

Deriving closed-form expressions for the relation between the term structure and expectations of future consumption changes, requires further assumptions. Define the time-t price of a unit nominal bond (one paying a single numeraire unit with certainty) which matures at t+j, to be $e^{-jr_j(t)}$, where $r_j(t)$ is the nominal j-period spot rate. From equation (6) it is apparent that bond prices will depend in general upon all conditional co-moments of "inflation" with consumption, through aggregate marginal utility. Yet, it is possible to characterize a family of preference-distribution assumptions which permit simple expressions.

Let $\tilde{K}(C) \equiv -\ln u_c(\tilde{C})$ and assume that conditional on information (Z_t) available <u>to the econometrician</u> at time t, the joint distribution of $\{K(\tilde{C}_{t+j}) - K(\tilde{C}_t), \tilde{i}(t,t+j)\}$ is bivariate normal (equivalently, the joint conditional distribution of price levels and marginal utility of real consumption is lognormal). Application of this assumption and the normal moment generator to equation (6) results in the following closed form solutions for nominal spot rates, where the expectations are conditioned on Z_t.

$$
\begin{aligned}
r_j(t)-h &= (1/j)E_t\{K(t+j)-K(t)\} + E_t\{i(t,t+j)\} - \tfrac{1}{2}\,\psi_{tj}, \\
\psi_{tj} &\equiv j\left[\operatorname{var}\left\{\left(\frac{K(t+j)-K(t)}{j}\right) + i(t,t+j)\right\}\right] .
\end{aligned}
\tag{7}
$$

When there is no uncertainty about future inflation rates, equation (7) reduces to

$$
R_j(t)-h = (1/j)\ E_t\{\tilde{K}(t+j)-K(t)\} - \left(\frac{1}{2j}\right)\operatorname{var}\{\tilde{K}(t+j)-K(t)\} , \tag{8}
$$

where $R_j(t) = r_j(t) - i(t,t+j)$ is the (known) j-period real default-free interest rate. Equation (8) also gives the rate under uncertainty on an "indexed bond" which pays out $(\tilde{I}^*_{t+j}/I^*_t)$ on date t+j. In each case, since there is no money in the model, bonds are denominated in units of a numeraire good. Note that $K'(C) = -u_{cc}/u_c > 0$ is the coefficient of absolute risk aversion. Any particular choice of utility function implicitly specifies a conditional distribution assumption for real consumption if marginal utility is conditionally joint lognormal with price levels.

Equation (8) includes models for the real term structure derived under constant relative risk aversion and lognormal future consumption [e.g., Rubinstein (1976) and Breeden and Litzenberger (1978)]. It also includes analogous closed-form results for constant absolute risk aversion and normal consumption (and, in principle, a variety of other unspecified cases).

Equation (7) states that the term structure of nominal spot interest rates is related at each point in time, positively to expected growth rates of real consumption and expected rates of inflation over the interval to bond maturity. Spot rates are negatively related to conditional variances of consumption growth, inflation and the covariance between them.

Rubinstein (1976), Cox-Ingersoll-Ross (1978) and Breeden (1981), among others, have noted the opposing influences of mean consumption growth versus variability on the level of real spot rates. Depending on the parameter values, this model produces term structures which may be positively sloped, negatively sloped or take on a variety of shapes. Holding mean growth constant in equation (8), if the variance of real consumption changes grows less than proportionately longer forecast intervals, a tendency for positively sloped real term structures is implied. Holding uncertainty fixed, positively sloped term structures suggest that economic growth rates are expected to be higher in the more distant future relative to near term growth (vice-versa for negatively sloped real term structures). If successive changes in K(C) are independent and identically distributed or if consumers are risk neutral [K(C) constant], the real term structure will be flat. This is consistent with results of LeRoy (1981) and others. The nominal term structure will not in general be flat or deterministic, due to the influences of uncertain inflation rates, even if consumers are risk neutral. Nominal term premiums (the difference between current forward and expected future spot rates) are determined in this model by three effects: (1) the behavior of consumption risk with futurity; (2) a pure Jensen's inequality effect depending on the variance of future inflation rates and (3) a risk premium which depends on risk aversion and the covariance of surprises in inflation with those of real consumption.

Equation (7) also has a "Fisher equation" interpretation. Much of the literature on the Fisher equation under uncertainty has addressed the question of whether the interest rate on a nominal bond should vary one-for-one with expected inflation (commonly called the "Fisher effect"). Fisher himself (1930, p. 43) suggested that this effect should not hold under uncertainty. Mundell (1963), Treynor (1980) and Fama (1981), among others, have argued that nominal rates may change by less than the amount of a change in expected inflation. Darby (1975) argues for a greater than one-for-one adjustment because of taxation in nominal terms. Levy and Makin (1978, 1979) include in their model a labor market, taxes on nominal interest, and uncertain inflation. They argue that changes in expected inflation can produce real output and employment effects in addition to Mundell's "real balance" effect and Darby's tax effect.

The asset pricing models examined here concur that the impact of expected inflation on nominal rates could exceed or be less than unity. That is, differentiating equation (7) implies

$$\partial r_j(t)/\partial E_t\{i(t,t+j)\} = 1 + (1/j)\,\frac{\partial E_t\{K(t+j)-K(t)\}}{\partial E_t\{i(t,t+j)\}} - (1/2)\,\frac{\partial x_{tj}}{\partial E_t\{i(t,t+j)\}}\;;$$

the "Fisher hypothesis" is that the right hand side equals unity. In this model, dependence of expected real interest rates on (or persistent association with) the level of expected inflation may arise through two opposite-acting terms. The first term reflects "real effects" of expected inflation changes, association with changing mean growth of aggregate real per capita consumption. The second term becomes important if uncertainty about marginal utility of real consumption or inflation changes with the level of inflationary expectations. The net effect could be less than or exceed the value of unity which the "Fisher effect" hypothesizes.

Specific utility functions are examined for empirical tests of equation (7); the constant relative risk aversion and the constant absolute risk aversion utility functions. In Figure (1) these cases are tabulated; in each case $\{K(C),i\}$ is assumed to be conditionally bivariate normal.

Case (iii) in the figure (log utility) obtains as constant relative risk aversion tends to unity in case (ii). The risk-neutral linear utility function has both constant absolute and relative risk aversion, with a=0. The distribution of consumption is irrelevant to this utility, provided the mean exists.

These particular combinations of assumptions about utility functions and probability distributions are motivated by at least three considerations. First, they provide a manageable set of simple, linear solutions in terms of real per capita consumption. Each case has been a central feature of several important studies.(11) Second, some assumption about parameter stability is required if a measure of consumption is to be used in estimation. Assuming

Figure (1)

Some Utility Functions and Distribution Assumptions Included in Equation (7)

	utility function	marginal conditional probability distribution assumed for per capita real consumption
(i)	$u(C) = (1/a)\exp\{-aC\}$ (constant absolute risk aversion equal to "a")	$C \sim$ Normal
(ii)	$u(C) = (1-a)^{-1}C^{(1-a)}$ (constant relative risk aversion equal to "a")	$C \sim$ Lognormal
(iii)	$u(C) = \ln(C)$	$C \sim$ Lognormal

stability of risk aversion, a basic parameter of the utility functions, allows intuitive interpretation of the estimates. The alternative to specifying a utility function often includes assuming stability of some reduced-form parameter [an alternative ably criticized by Lucas (1979)]. Here, one could examine formulations based on consumption "betas" and expected returns. Significant difficulties with this alternative arise from nonstationarity of consumption betas.(12)

Third, the assumption that $\{\Delta K(C),i\}$ is conditionally normal implies, in the econometric methodology described in the next section, that error terms in the resulting multivariate regression system are normal. Thus, theoretically convenient and popular assumptions also provide a rigorous foundation for the test methods.

An example to emphasize the usefulness of assuming stable risk aversion compares this formulation to traditional tests of the permanent income hypothesis. The tests on consumption and interest rates presented here may be viewed as implications of particular versions of the permanent income hypothesis, because they are derived from models where optimal consumer behavior is consistent with that theory. Furthermore, these tests are developed without requiring some of the restrictive assumptions employed in previous tests. Thus, we admit states of nature where the permanent income theory could be the correct model, but which previous studies have ruled out by assumption.

Permanent income may be defined, assuming rationality, as the expectation given currently available information of an individual's lifetime resources. In a model with perfect marketability, permanent income is the (value equivalent annuity of) current wealth of an individual. According to the simple permanent income hypothesis, current optimal consumption is proportional to permanent income.

Hakansson (1970) has shown that the optimal consumption of a consumer-investor with time additive utility will be proportional to current wealth, in models with constant absolute or relative risk aversion and perfect markets. The fraction of permanent income consumed will depend in general on risk aversion and a vector of state variables which parameterize the distribution of future investment and consumption opportunities (for the logarithmic utility, the proportionality factor is independent of state). Changes in current income would affect consumption in this model as hypothesized in the permanent income theory; by inducing fluctuations in current wealth or by signalling changes in the state variables which influence the proportionality factor.

Tests of the permanent income hypothesis have traditionally focused on the implied proportional relation of consumption to permanent income [e.g., the recent studies of Barro (1978), Hall (1978), Hayashi (1979), Hall and

Mishkin (1980), Blinder (1981), or Flavin (1981)]. In so doing, most studies have assumed a stable proportionality factor [there are exceptions; e.g. Weber (1970)], and are thus joint tests of the permanent income model and stationarity. A joint test might reject the null hypothesis when the permanent income theory is valid, because of nonstationarity. In this study, it is not assumed that the marginal propensity to consume out of permanent income is stable. Instead, a parameter of the utility function is assumed constant. One could interpret this formulation as testing permanent income theory under the assumption that the propensity to consume is nonstationary. When the real rate of interest is changing over time, there is no reason to expect that the portion of permanent income optimally consumed will be a stationary parameter.

This approach to modeling consumption and interest rates may also be viewed as an extension of previous results from Hall (1978). When individuals maximize utility of consumption over time, the conditional expectation of future marginal utility of consumption should depend on current consumption, Hall argued, and to a good approximation consumption itself should follow a random walk with drift.

The present models extend Hall's (1978) argument to a world with changing interest rates, finding that in general neither optimal consumption nor marginal utility of optimal consumption will follow a random walk.(13) The current level of consumption and the expected real interest rate on a bond maturing at the forecast horizon, together describe expected future marginal utility of consumption. Given these variables, no other economic information would improve consumption forecasts, but current consumption will not in general be sufficient to predict future consumption. K(C) may be thought of as a random walk with drift, but the drift for a given horizon depends at each point in time on the current expected real rate of interest on a bond with matching maturity (and on risk). To the extent that lagged variables are useful in modeling conditional expectations, this model predicts they should have explanatory power for future consumption given current consumption.

4. Econometic Test Methods

This section discusses stochastic implications of the consumption models in equation (7), specialized to the cases in Figure (1). The empirical tests consider only a single forecast horizon and treasury bill maturity, corresponding to the three month sampling interval. The models are restated in more efficient notation in equation (9), setting the maturity subscript equal to unity.

$$\text{(a)}E_t\{\tilde{y}_{t+1}\} = \alpha_1 + E_t\{\tilde{x}_{t+1}\}, \tag{9}$$

$$\alpha_1 = a^2[(\tfrac{1}{2})\ \sigma_v^2] + [(\tfrac{1}{2})\ \sigma_u^2 - h] + a\sigma_{uv}\ ,$$

$$\sigma_u^2 = E_t\{(\tilde{x}_{t+1} - E_t(\tilde{x}_{t+1}))^2\}\ ,$$

$$\sigma_v^2 = E_t\{(\tilde{y}_{t+1} - E_t(\tilde{y}_{t+1}))^2\}\ ,$$

$$\sigma_{uv} = E_t\{(\tilde{y}_{t+1} - E_t(\tilde{y}_{t+1}))(\tilde{x}_{t+1} - E_t(\tilde{x}_{t+1}))\}\ ,$$

where

$$\tilde{y}_{t+1} \equiv C_{t+1} - C_t\ ,$$

and

C_t = aggregate real consumption per capita (constant absolute risk aversion model) or its logarithm (constant relative risk aversion model).

$E_t\{\cdot\}$ = denotes conditional expectation given information Z_t used by the econometrician in testing the model.

a stands for the risk aversion parameter corresponding to the definition of C_t employed (a=1 for log utility, zero for risk neutrality).

$x_{t+1} \equiv r_1(t) - i(t,t+1)$ is the <u>ex post</u> real rate on a single-period nominal discount bond purchased at date t.

Note in equation (9) that conditional variances and covariances of the forecast errors are assumed to be stationary. This assumption is maintained throughout.

A basic implication of the bond pricing model (9) is that conditional expectations for consumption changes and "real rates" on nominal bonds are linearly related. Assume the coefficients in this linear relation are constant over time; then if conditional expectations of the two variables change, they must shift proportionately. By modeling conditional expectations explicitly and imposing the requirement of proportional changes, the test methodology allows consistent and efficient estimation of the utility parameter and the parameters of the forecasting model in a constrained multivariate regression. An important feature of this methodology is that it allows an econometrician to construct valid tests with access to only a subset of the information used by agents in forming their expectations.(14)

Extensions of this basic technique may be employed to test a wide class of models which predict linear relations among rational expectation variables. A particular advantage for financial models is that a model may be tested on a cross-section of assets without requiring observability of the relevant "hedging portfolios."(15) For example, valid tests of the Capital Asset Pricing Model [Sharpe (1964), Lintner (1965)] have been elusive, because the market portfolio of all traded assets may be difficult to adequately measure. Roll (1977) strongly criticized traditional methodology along these lines.(16)

The advantages of this approach do not come without cost, for some assumption about the form of conditional expectations is necessary. The assumption maintained is that conditional expectations may be modeled as linear functions with constant coefficients on the information set, Z_t. The assumption of constant coefficients, although not strictly required, greatly simplifies discussion and application of the technique.

Assuming linear expectation rules is helpful (and quite common) in developing econometric tests; but is not terribly restrictive. For prespecified information, the assumption is in principle testable using standard specification tests for linear regression. Fairly general dependencies are allowed, because arbitrary transformations of information variables may enter the conditional expectation linearly. Indeed, by Taylor's expansion, any differentiable nonlinear expectation rule may be approximated linearly with arbitrary accuracy. The key requirement is linearity in the parameters, a characteristic of many interesting economic models.(17) Conditions sufficient for both the closed-form linear theoretical model and linearity of conditional expectations (with constant coefficients), are that inflation rates and the log of marginal utility of consumption are jointly (covariance stationary) normally distributed with the information.

The tests are conditional on the following stochastic specification:

$$
\begin{aligned}
&\tilde{y}_{t+1} = \alpha_y + \beta_y' Z_t + \tilde{v}_{t+1} \\
&\tilde{x}_{t+1} = \alpha_x + B_x' Z_t + \tilde{u}_{t+1} \\
&E(\tilde{v}_{t+1}|Z_t) = E(\tilde{u}_{t+1}|Z_t) = 0 \; ; \; t = 0, \ldots, T-1 \\
&E\{\binom{v_{t+i}}{u_{t+i}}(v_{t+j}, u_{t+j})|Z_t\} = \begin{cases} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \text{if } i \neq j \\ \begin{bmatrix} \sigma_v^2 & \sigma_{uv} \\ \sigma_{uv} & \sigma_u^2 \end{bmatrix} & \text{if } i = j \, . \end{cases}
\end{aligned}
\tag{10}
$$

In (10) the unexpected changes in consumption and inflation may be contemporaneously correlated given Z_t. Rationality implies the standard econometric assumption that the error terms display no cross-correlation at different lead times.[18]

Implications of the theoretical model may be applied to the stochastic model in (10) to derive testable parameter restrictions. The theoretical model, equation (9), implies when first differenced, that changes in conditional expectations are proportional:

$$a\Delta E(y_{t+1}|Z_t) = \Delta E(x_{t+1}|Z_t) \ . \tag{11}$$

Taking conditional expectations given Z_t of the stochastic model (10), differencing and substituting the resulting expressions into equation (11) implies

$$(\beta_y' - \frac{1}{a}\beta_x')\Delta Z_t = 0 \ . \tag{12}$$

Since this must be true for all realizations of ΔZ_t,

$$\beta_y = (\frac{1}{a})\beta_x \ . \tag{13}$$

Equation (13) will be called the "proportionality restriction."
Writing (9) again and substituting for the conditional expectations implies

$$\alpha_1 = aE(y_{t+1}|Z_t) - E(x_{t+1}|Z_t) = (a\beta_y - \beta_x)'Z_t + (a\alpha_y - \alpha_x) \ ,$$

which, using (13), implies

$$\alpha_1 = a\alpha_y - \alpha_x \ . \tag{14}$$

Relation (14) may be considered an identifying restriction for the pure time preference parameter, h, in terms of α_1 and the generating process parameters. Substituting the definition of α_1 from (9) into (14) implies

$$h = a^2\left(\frac{\sigma_v^2}{2}\right) + a\left[\sigma_{uv} - \alpha_y\right] + \alpha_x + \left(\frac{\sigma_u^2}{2}\right) . \tag{15}$$

Since (13) reduces by one half the number of separate slope parameters in the forecasting equations (10), it is possible to estimate these jointly with increased precision under the assumption the model holds. Simply stated, if expectations for x and y are known to move together, then information relevant for predicting one variable is relevant for predicting the other.

Asset pricing models are typically stated in terms of expectations conditioned on "the market's" information set, Ω_t. The market is assumed here to be efficient in the sense that the model parameters and the information used by the econometrician to estimate expectations, Z_t, is a proper subset of Ω_t. Maintaining the assumption that conditional expectations for x_{t+1} and y_{t+1} given Z_t are linear, the corresponding "partial information" coefficients β_x and β_y, are estimated by regressing x_{t+1} and y_{t+1} on Z_t. These coefficient estimators will be biased for their corresponding elements in the "true" coefficient vectors (say, γ_x and γ_y), unless the information omitted from Ω_t in choosing Z_t happens to be uncorrelated with Z_t. Because of "left-out-variables" bias, the β coefficients will not properly measure the marginal influence of Z_t on the "market's" expectations. However, the focus here is to test parameter restrictions implied by the theory, not estimate the marginal effect of particular information. This allows valid tests based on the vector Z_t. Essentially, because the "true" coefficient vectors γ_x and γ_y would satisfy the proportionality restriction under the null hypothesis, bias in the β_x and β_y due to missing information will be offsetting, so that proportionality is preserved [see Ferson (1981) or Gibbons and Ferson (1981) for proof and further discussion].

When the parameter restrictions (13) and (14) are imposed on the stochastic model (10), the result is a restricted bivariate generating process:

$$
\begin{aligned}
\tilde{y}_{t+1} &= \alpha_y + \beta_y' Z_t + \tilde{v}_{t+1} , \\
\tilde{x}_{t+1} &= -\alpha_1 + a\left(\alpha_y + \beta_y' Z_t\right) + \tilde{u}_{t+1} ,
\end{aligned}
\tag{16}
$$

where "a" is the risk aversion coefficient relevant to the model being tested and α_1 is defined in equation (9).

The model may be tested under the maintained assumptions by estimating the unconstrained system (10) and separately the system (16). Imposing the constraints allows estimation of the risk aversion parameter, a. Comparing statistical fit across the two models allows one to test whether the implied restrictions are significantly inconsistent with observed data. The risk aversion coefficient is identified if Z_t is at least a one dimensional vector and an intercept is included. In that case, the utility parameter would be just identified: $a = \beta_x/\beta_y$. It would be possible to estimate the parameter, but no restrictions would be left over. Increasing the dimensionality of the Z vector provides overidentifying restrictions which may be tested. Tests on smaller subsets of information utilize fewer overidentifying restrictions.

The unconstrained bivariate system (10) is a version of Zellner's (1962) seemingly unrelated regression model. Since the predictor variables are the same in each equation, ordinary least squares (OLS) equation by equation is an efficient estimation procedure.

Under the null hypothesis, OLS is asymptotically inefficient relative to a joint nonlinear multivariate regression, because the number of slope parameters to be estimated is reduced from 2K (in the unconstrained system) to K+1 (under the null hypothesis), where K is the dimension of the Z vector.

Nonlinear joint generalized least squares (JGLS) estimation is employed for system (16). This procedure utilizes the residual covariance across equations in the estimation. Consistent and efficient estimates of the regression coefficients and utility parameter are obtained simultaneously.[19]

Actual calculations employ a modified Gauss-Newton procedure, which approximates the nonlinear normal equations with a Taylor expansion evaluated at initial parameter estimates and iterates until the residual sum of squares converges. A nonlinear JGLS procedure employing this numerical method and consistent initial estimates, which does not iterate on the residual covariance matrix, produces estimators with the same asymptotic properties as using full iterated covariance methods, at a considerable savings in computation costs.[20]

Estimation of (10) and (16) produces two sets of estimates for the coefficient vectors β_x and β_y, one restricted to proportionality by the null hypothesis and the other unrestricted. The objective is to use these estimates to test the joint hypothesis generating the restriction. If the null hypothesis were true, the unrestricted estimates would approach their true proportional values as the sample size increased and the proportionality restriction would not be binding, asymptotically. This implies that the "fit" of the multivariate regressions (10) and (16) would be asymptotically equivalent under the null hypothesis.

This paper employs the likelihood ratio test statistic, which tests the null hypothesis by comparing the maximized likelihood function of the sample, when unrestricted versus restricted under the null hypothesis. Minus two times the natural log of this ratio is asymptotically chi-square distributed, with degrees of freedom equal to the difference between the dimensionality of the parameter space under the alternative hypothesis versus the null [see, e.g. Silvey (1975)]. Under the null hypothesis

$$T \ln\{|\hat{\Sigma}_r|/|\hat{\Sigma}_u|\} \sim \chi^2(K-1) , \tag{17}$$

where T is the sample size and K the number of variables in the information vector, Z (recall that imposing the proportionality restriction reduces by K-1 the number of parameters to be estimated). $\hat{\Sigma}_r$ and $\hat{\Sigma}_u$ are the estimated restricted and unrestricted residual covariance matrices respectively, and $|\cdot|$ denotes determinant.

If the data were perfectly in accord with the model, then generalized unexplained variance should not be increased by restricting the parameters across the two equations, i.e. $|\Sigma_r| = |\Sigma_u|$. The test statistic would take values close to zero. The model is rejected when significantly large values of the sample statistic are observed, indicating $|\Sigma_r| > |\Sigma_u|$.

5. The Data

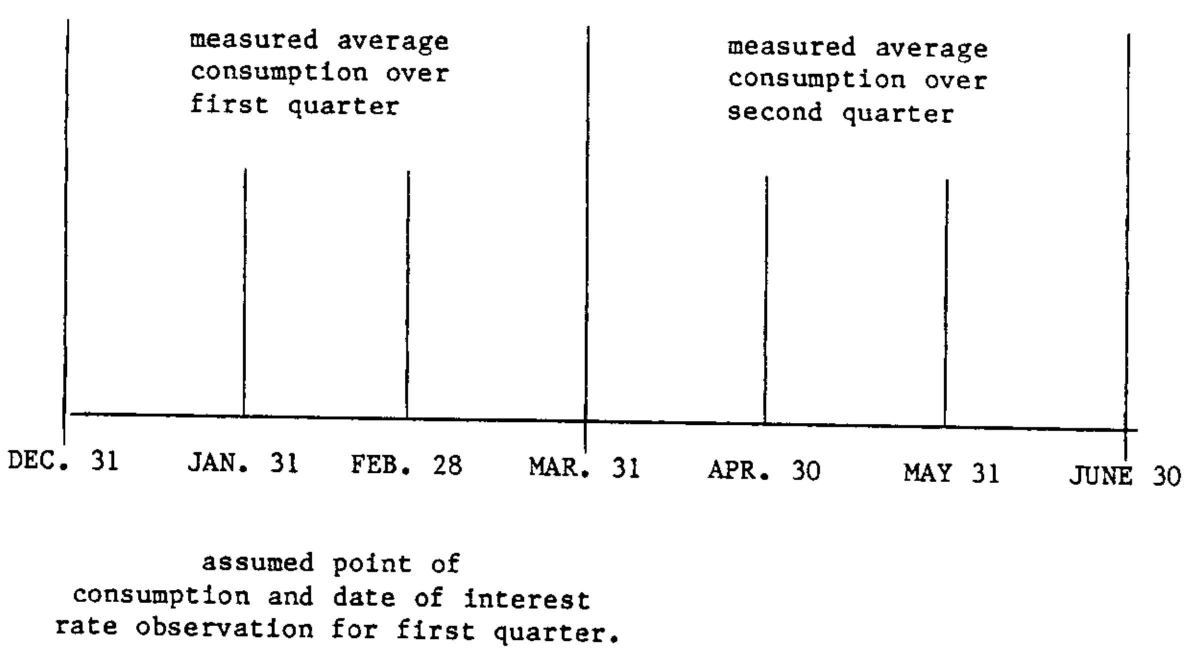
The basic data are quarterly observations (1947 through 1980) on measures of United States real aggregate consumption per capita, expenditures for consumer durables, inflation rates, default-free interest rates and real returns on the Standard and Poors Composite Index of common stocks. The data and sources are described in the appendix, where various sample statistics are presented. Two measures of consumption are included: consumer nondurables and nondurables plus services. These department of commerce data have been employed to measure consumption changes by many previous writers.(21) To conserve space, complete results are only reported for the more inclusive measure (they were similar, except where noted). The implicit price deflator for total Personal Consumption Expenditures (PCE) was chosen to measure inflation, instead of a Consumer Price Index (CPI). The PCE index employs aggregate expenditure weights, updated quarterly, whereas CPI uses weights updated at infrequent intervals. Current weights should better reflect the fact that consumers may change their expenditures shares on various goods as prices change. The consumption and PCE data are seasonally adjusted, at annual rates.(22)

Consumption is of course a continuous process. The theory relates a bond price observed at a specific date to changes in consumption rates between two discrete points in time. The data, however, refer to average consumption over an interval. Differences of average consumption measured over short intervals should approximate more closely true point-to-point changes than differences of averages for longer intervals. Therefore, changes in consumption were measured as quarterly differences using data for the shortest available observation interval (prior to 1959, the shortest available interval is one quarter; monthly data are available for later periods). The measured average consumption over an interval was used to estimate the consumption rate at the midpoint of the interval, when the bond price observation occurs. This is illustrated in Figure 2. For example, the difference between average measured consumption in February and May is assumed to represent the change between mid-February and mid-May (from 1947 through 1958, quarterly averages are used). Continuously compounded inflation rates are measured from similar differences in the PCE deflator.

It is important that lagged values used as predetermined information to estimate ex ante expectations actually be predetermined with respect to the variables they are used to forecast. There is some overlap in the measurement interval, for example, between the current treasury bill used to compute a real interest rate and the previous lagged value of measured consumption or

FIGURE 2

TEMPORAL ALIGNMENT OF CONSUMPTION
AND INTEREST RATE DATA



inflation. To minimize any spurious correlation in the forecasting equations, only past values at lags of two or more quarters were employed as predetermined variables.(23)

Two alternative sets of predetermined instruments are used for estimation and tests. Results discussed in the preceding section permit a certain flexibility in choosing the predetermined variables; any subset (with at least two elements) of the information used by economic agents forming linear expectations will generate testable restrictions. Of course, this does not justify "dredging" the data until some set of Z's which produce regressions in accordance with the experimenter's prior are discovered.

The instruments were chosen based on results of previous theoretical and empirical work, ready data availability and the desire to keep the data base relatively small while attempting to capture a range of factors which might influence expectations. Both sets of instruments include enough lagged values to display annual patterns in the data. They differ in that one set excludes lagged dependent variables while the other set consists solely of lagged dependent variables.

The first set of Z's consists of lagged per capita real expenditures on consumer durable goods, real returns on stocks and inflation rates. Expenditures on consumer durable goods should be directly relevant for consumption flow in future periods. The "stock-adjustment" models of consumer durable expenditures have long emphasized this relation [see, e.g. Ferber's (1973) review]. A change in the level of durables expenditures should therefore signal future changes in consumption.

Past values of inflation have been employed in several recent empirical studies [see, e.g., Hess and Bicksler (1977), Crockett (1980), Mishkin (1980) or Wakeman and Bhagat (1980)] to measure inflationary expectations. Many theoretical studies (a few mentioned in the last section) have suggested that expected real returns on bonds should vary with the level of expected inflation. Stock market prices have long been regarded as efficient aggregators of economic information [e.g.: Cootner (1964), Fama (1970)]. If stock prices represent discounted expected future values then changes in stock prices (or returns) should reflect revisions of agents' expectations and thus provide a proxy for relevant new information about economic growth rates [see also Fama (1981)]. Predetermined values of the real market return will have predictive power, assuming rational expectations, only if the mean real return is changing. Recent studies [e.g., Fama and Schwert (1977), Fama (1981), Merton (1980), Hall (1978, 1981), French (1980), Gibbons and Hess (1981), Gibbons and Ferson (1981), Hansen and Singleton (1981) and Klem (1981)] are uncovering evidence that expected stock returns are not constant and may be in part predictable.

The second set of Z's, consisting solely of lags of the dependent variables, follows a long tradition in the macroeconomics literature of modeling expectations in terms of the times series behavior of the variables to be forecast [see, e.g., Fisher (1930), Friedman (1957), Houtakker and Taylor (1970), Sargent (1979), Hall and Mishkin (1980), or Flavin (1981)].

6. Empirical Results

One situation consistent with the consumption model is the case where expected real rates on nominal bonds are constant, as assumed in Fama's famous (1975) article. This would occur if aggregate preferences are risk neutral, regardless of patterns in mean consumption growth. Alternatively, if aggregate marginal utility of consumption follows a random walk with trend [as suggested by Hall (1978)], then constant expected real rates can obtain in the presence of risk aversion.

If expected consumption changes and real rates are constant, the test methodology discussed in Section Four would not produce overidentifying restrictions because the relevant predetermined information variable would be a scalar constant (the test would be based on unconditional means). Table One examines separately the null hypotheses that expected real interest rates and expected changes in real consumption are intertemporal constants. In each case, rational expectations is a maintained assumption and the alternative hypothesis is that lagged own values are useful in modeling changes in expectations.(24)

If rational conditional expectations of a variable were constant, then no variable included in the conditioning information would be useful in predicting its ex post realized values. All of its observed variation would be independent forecast error.

Fama (1975) studied monthly ex post real returns on treasury bills (using CPI inflation rates) for 1953 to 1971 to see if lagged real returns were useful in prediction. The first panel of Table One examines the autocorrelations for quarterly data (based on the PCE deflator as an index of price changes) for the same period. Consistent with Fama's findings, the sample autocorrelations of the ex post real rate are not large relative to

TABLE 1

MEANS, STANDARD DEVIATIONS AND SAMPLE AUTOCORRELATIONS OF INTEREST RATES, INFLATION, EX POST REAL RATES AND GROWTH RATES OF REAL CONSUMPTION EXPENDITURES PER CAPITA FOR NONDURABLES PLUS SERVICES

(quarterly data at annual rates)

variable	mean (%)	std. dev. (%)	sample autocorrelations: LAG 1			LAG 4	LAG 5			LAG 8	Q	$\left(\frac{Q-E(Q)}{\hat{\sigma}(Q)}\right)$
First quarter 1953 to second quarter 1971												
NOMINAL INTEREST RATE	3.47	1.62	.94	.86	.78	.70	.61	.53	.47	.43	301	65.1
INFLATION RATE	2.31	1.61	.48	.48	.40	.37	.37	.45	.16	.21	93	18.9
EX POST REAL RATE	1.16	1.32	.21	.17	.05	.17	.18	.34	-.02	.13	22.0 (18.1)	3.13 (2.7)
First quarter 1953 to fourth quarter 1980												
NOMINAL INTEREST RATE	4.64	2.54	.89	.83	.77	.69	.58	.49	.42	.38	403	91.9
INFLATION RATE	3.88	3.00	.82	.76	.72	.62	.55	.51	.41	.38	355	80.7
EX POST REAL RATE	0.76	1.57	.35	.30	.27	.31	.26	.28	.04	.21	67.0 (52.2)	13.7 (11.3)
second quarter 1947 to fourth quarter 1980												
NOMINAL INTEREST RATE	4.08	2.62	.91	.85	.80	.74	.65	.58	.53	.49	566	131.3
INFLATION RATE	3.80	3.27	.65	.60	.53	.39	.32	.23	.19	.22	203	45.9
EX POST REAL RATE	0.28	2.54	.40	.34	.24	.12	.06	-.06	-.08	.06	51.0 (28.4)	10.1 (5.5)
CONSUMPTION GROWTH	1.80	3.03	.02	.00	.08	.06	-.05	-.03	.04	-.07	2.5	1.3

NOTES: The sample autocorrelations are Box Jenkins estimates with approximate standard errors in the first, second and third panels equal to .12, .10 and .09 respectively. Q is the adjusted Box-Pierce statistic for the first 8 autocorrelations (statistics in parentheses omit the first order autocorrelation). Under the null hypothesis that the true autocorrelations are zero, the expected value of Q is 8 (7 for the statistics in parentheses). The last column shows the number of approximate standard errors by which the sample Q statistic differs from its expected value under the null hypothesis.

their approximate standard error for the first eight autocorrelations. The Box-Pierce statistic appears significant at the 10% level; but when the first lag is not considered (statistics in parentheses) the Q statistic does not attain the 10% level. The real rate is apparently much closer to a non-autocorrelated time series than either inflation or the nominal rate.

Many empirical studies have noted significant autocorrelation in ex post real treasury bill returns when Fama's sample period is extended to include more observations.[25] Results in the second panel of Table one for the 1953-1980 period also find evidence of changes in the mean real return. The second order sample autocorrelation alone is approximately three standard errors from zero. The Box-Pierce statistic for seven sample autocorrelations is 52.2; this rejects the null hypothesis that the true autocorrelations are zero at any conventional significance level. The ex post real rate also shows significant autocorrelation for the full 1947 to 1980 period. This is reported in the third panel of Table One.

These tests reject the hypothesis of constant expected real interest rates when the sample period extends outside the relatively "flat" (1953-71) period. Fama (1975) chose to omit the post-1971 period, arguing that Nixon-era wage and price controls systematically distorted the CPI, since it did not reflect the full cost of goods (including queuing). These errors would also be reflected in the PCE deflator, of course, suggesting that part of the autocorrelation of the post-1971 measured real rates could be spurious. To the extent that spurious autocorrelation is a problem for real rates, it should also influence the observed autocorrelation of the measured changes in real consumption, because both series employ the PCE deflator to measure inflation. The observation, in Table One, that real rates display significant autocorrelation but consumption changes do not suggests the hypothesis that spurious autocorrelation in the inflation measure has exaggerated the appearance of autocorrelation in real interest rates may be inadequate.(26) Sample autocorrelations in Table One do not reject the hypothesis that the expected change in consumption is constant. Similar results for other measures on consumption are presented in the appendix, Table A1. The Box-Pearce statistic for the first eight autocorrelations is not significantly different from its value under the null hypothesis of zero autocorrelation for any of the measures of consumption examined. Vector autocorrelations of real rates and consumption changes for the later 1959-1980 period produced similarly mixed results. The autocorrelation structure of the real interest rate appears more significant than that of consumption growth and lagged cross correlations are generally smaller than two approximate standard errors (see the appendix, Table A3). However, autocorrelation tests have limited power in rejecting constant expectations. They ignore any information not contained in own lags which may be useful in detecting changes in expectations.

Table Two extends the investigation to include more predetermined information than own lagged values. Using the two sets of instruments described in the last section, the unconstrained ordinary least squares estimation of system (10) is summarized for the constant relative risk aversion model, where consumer nondurables plus services is the measure of consumption.(27) Each set of information variables appears significant in explaining variation in ex post real interest rates at the five or six percent level, as measured by the F statistic for the null hypothesis of a zero coefficient vector in a particular equation. However, none of the consumption change regressions attain significance at ten percent. The fifth and sixth columns of Table Two test the joint hypothesis across the two forecasting equations that all the coefficients are zero. This amounts to a bivariate

TABLE 2

Summary of Unrestricted Least Squares Estimation of Equation (10) for the Constant Relative Risk Aversion Model Using Quarterly Data for 1947–1980. Number of Observations is 127 (128 in the First Panel). The Bivariate Regression Model Has the Form

$$\tilde{y}_{t+1} = \alpha_y + \beta_y' z_t + \tilde{v}_{t+1}, \quad \tilde{x}_{t+1} = \alpha_x + \beta_x' z_t + \tilde{u}_{t+1} \quad \forall\ t = 0, \ldots, T-1$$

where $y_{t+1} \equiv$ change in the logarithm of per capita real expenditures for consumer non-durables and services,

$x_{t+1} \equiv$ realized real return on 3-month treasury bill and

$z_t \equiv$ vector of predetermined variables known at date t.

Predetermined Independent Variables	Dependent Variable	$\bar{R}^2$(a)	F(b)	p-value	Two-Equation(c) F Statistic	p-value	Durbin H Statistic	Residual Autocorrelations
Past values of inflation rates, real stock market returns and per capita expenditures for consumer durable goods at lags 2, 3 and 4.	y_{t+1}	.01	1.27	>.10	1.74	.085	0.11	.00, -.04, .02
	x_{t+1}	.04	1.91	.06			3.68	.30, .17, .15
Past values of the two dependent variables at lags 2, 3, 4 and 5.	y_{t+1}	.04	1.53	>.10	1.91	.070	0.45	.04, .00, .01
	x_{t+1}	.09	2.24	.05			2.23	.19, -.01, -.01
Lags 2 through 5 of the two dependent variables and three past lag values of the inflation rate, real stock market returns and durable goods expenditures.	y_{t+1}	.11	1.46	>.10	2.03	.010	-0.80	-.07, -.01, -.03
	x_{t+1}	.15	1.67	.05			1.81	.14, -.04, -.06

Notes: (a) $\bar{R}^2$ is the coefficient of determination adjusted for degrees of freedom.
(b) F statistic for the significance of the regression; the null hypothesis is that the true coefficient vector equals zero for a particular equation
(c) The two-equation F statistic is for the null hypothesis that $\beta_y = \beta_x = 0$ in the bivarate regression.

test for constant expectations, consistent with the models of both Fama (1975) and Hall (1978). The joint hypothesis is rejected with p-values smaller than 0.10 in each of the three panels.

Although the proportion of variation in the dependent variables left unexplained by the forecasting equation is quite high (the coefficients of determination adjusted for degrees of freedom range from one to fifteen percent), it is important to recognize that this does not imply that the predetermined variables contain no information used by agents in forming expectations. First, the conservative approach of using only second and higher order lagged values to minimize the risk of spurious association has probably cost something in terms of R^2.(28) Second, the F tests have little power to reject the null hypothesis of an insignificant regression if pure forecast errors account for a substantial fraction of the variability of the dependent variable, even if the model does a good job of explaining changes in expectations per se.(29)

Individual t ratios for the explanatory variables are not shown; it would be difficult to correctly infer the relative importance of the predictors using them. In addition to the usual ambiguity in a multiple regression when predictors are correlated, there is no guarantee that t ratios are independent across the equations. Also, the Durbin H statistics indicate the presence of mild positive residual autocorrelation in the real rate regres-ions; largest when no lagged dependent variables occur on the right hand side.(30) A general impression from the first stage regressions was that lagged inflation rates (in the first and third panels) were the most important predictors for both independent variables. Consistent with other empirical studies, the association was always negative for the lowest order lag.(31) In general, lagged consumption changes were the least significant predictor in all the equations.

Other studies have reported apparently significant information in past consumption. For example, Hansen and Singleton (1981) find that monthly real stock returns are significantly correlated with past consumption changes at up to six lags. At least three possible explanations might reconcile this difference in results. On one hand, monthly data might more accurately measure the temporal correlation properties of true consumption changes than quarterly data. On the other hand, monthly consumption data might reflect spurious correlation that is not representative of true consumption. A significant fraction (about 40%) of the data in the monthly PCE deflator, for example, is not covered directly by monthly observations; it is interpolated based on quarterly and annual figures. The apparently greater significance of lagged consumption based on monthly data could be, in part, an artifact of these sampling procedures. A third explanation relates to the fact that Hansen and Singleton use common stocks as their real return variable, while this study employs nominally riskless treasury bills. In both cases stationarity of the relevant forecast error second moments is assumed in developing the test methodology. For nominal bills, this requires that the "real consumption beta" of inflation be constant. For common stocks, the assumption of constant consumption betas implies stable relationships among a wider range of risks. As Cornell (1981) has emphasized, consumption betas will generally be state dependent when the productive opportunity set underlying stock returns is stochastic. Assuming constant betas for stocks is probably less tenable than for treasury bills where the only relevant risk presumably is inflation. It is possible that lags of consumption are representing, in Hansen and

Singleton's (1981) results, state dependency of stock betas which does not affect the real returns on treasury bills.

Table Three presents the likelihood ratio tests of the proportionality restriction, equation (13). The three panels show results for three combinations of predetermined information variables, for both the constant absolute and constant relative risk aversion models. Estimates of the risk aversion parameters and their asymptotic standard errors are also displayed (the risk aversion estimates are discussed below).

The null hypothesis is rejected for the constant relative risk aversion model with p-value .04 using lagged dependent variables as instruments (second panel), but using the set of instruments with no lagged dependent variables the chi-square statistic is only asymptotically significant at 30%. The p-value using the combined set of predictors lies between that of the first two, at 20%.

When constant absolute risk aversion is assumed, the likelihood ratio rejects the null hypothesis with greater significance. The p-value of the chi-square statistic for constant absolute risk aversion in the second and third panels is particularly impressive (less than .001%). Excluding lagged dependent variables, the asymptotic significance rises to 16%. Although not significant at standard levels, the p-value is still much smaller than the corresponding value assuming constant relative risk aversion.

Other statistics produced in the estimation include the sample covariance matrix of the residuals and the intercepts (which were not restricted). These may be combined with equation (15) to produce an indirect estimate of the time preference parameter, h. Without prior information on h this equation does not imply a parameter restriction for a given set of Z's; and since covariance restrictions involving the equation were not imposed in the estimation, no

TABLE 3

Likelihood Ratio Tests of the Proportionality Restriction [equation (13)] and Estimates of Risk Aversion Parameters.*

Predetermined Independent Variables[a]	Estimated Coefficients[b] when Consumption Variable Corresponds to Assumption of Constant:		χ^2	df[d]	p-value
	Relative Risk Aversion	Absolute Risk Aversion[c]			
Past values of inflation rates, real stock market returns and per capita expenditures for consumer durable goods at lags 2, 3 and 4.	5.38 (13.80)		9.6	8	0.30
		2.68 (2.35)	11.8	8	0.16
Past values of dependent variables: consumption changes and real interest rates at lags 2, 3, 4 and 5.	-0.38 (0.24)		15.9	7	0.04
		1.84 (1.10)	57.4	7	<0.001
Past values of inflation rates, real stock market returns and durables expenditures at lags 2 through 5 and the two dependent variables at lags 2 through 4.	-1.41 (0.51)		29.5	16	0.20
		-2.79 (2.66)	40.6	16	<0.001

Notes: * Quarterly data for 1947-1980. Number of observations is 127 (128 in the first panel).

[a] The measure of consumption is per capita real expenditures for consumer nondurables and services. Quarterly consumption changes are measured in the levels for the constant absolute risk aversion model and in the natural logarithms for constant relative risk aversion.

[b] Asymptotic standard errors of the risk aversion estimates are in parentheses.

[c] Absolute risk aversion estimates and their standard errors have been scaled by the ratio of average total per capita real expenditures for consumer nondurables plus services to the sum of these plus estimated consumption from the stock of durables plus real government expenditures for goods and services (from annual Commerce Department data). Units are basis points per (1972) dollar change in consumption per capita, at annual rates.

[d] Degrees of freedom for the chi-square statistic.

standard error for the resulting estimate of h is easily obtained. The six estimates (2 choices of risk aversion × 3 choices of Z's) implied by the residual covariances, intercepts and utility parameters were all positive. Some of the point estimates however would be judged "too large" to be reasonable. The range of these estimates for time preference was 3.4 to 36.3 percent per annum.

Table Four compares the risk aversion estimates produced in the constrained estimation with previous results reported by other writers. Estimates from Ferson (1982) are included which are based on the same data as the constrained estimates of Table Three, but which employ simpler techniques. The first of these simply compares subperiod sample means of the two series to estimate risk aversion,[32] and so does not rely on the validity of the forecasting equations on which the constrained estimates are based. The second approach provides an asymptotic bound for the coefficient based on the contemporaneous simple and partial correlations of the real rate and changes in consumption [see Ferson (1982) for details].

No previous study of which I am aware has attempted to estimate a coefficient of constant absolute risk aversion using consumption, so only estimates for constant relative risk aversion are compiled in the table.

It could be misleading to compare the estimates because of differences in the assumptions and methods behind them. Friend and Blume (1975) and Friend and Hasbrouck (1980) estimate risk aversion for wealth, not consumption. The two are only equivalent in general for single-period models.

Grossman and Shiller (1981) assume a perfectly predictable stock market price path in a consumption model. Hall (1981) estimates (1/a) using annual consumption with annual stock and (3 month) treasury bill returns, where Hansen and Singleton (1981) estimate (1-a) with monthly stock returns and

TABLE 4

Estimates of Constant Relative Risk Aversion

Estimate For Relative Risk Aversion	Source*	Methodology Employed
2	Friend and Blume (1975)	Cross sectional survey data. Single period model assuming common stocks represent all risky wealth.
6	Friend and Hasbrouck (1980)	Modification of Friend and Blume to include human capital in wealth.
4	Grossman and Shiller (1981)	Perfect foresight model in consumption to rationalize observed time series variation in stock market prices.
1.7	Hall (1981)	Constrained nonlinear estimation of consumption model. Annual time series data on nondurable goods expenditures and annualized real returns to 3-month treasury bills.
26	Hall (1981)	Same as above except based on annual real returns on common stock.
-13	Hall (1981)	Same as above except using Bayesian estimation procedure for forecasting equations.
.07 to .62	Hansen and Singleton (1981)	Range of 12 point estimates from constrained nonlinear estimation of consumption model using monthly consumption data (nondurables and nondurables plus services) and real stock market returns (two indices).
1.2 to 1.6	Ferson (1982)	Consumption model implication for unconditional means in split sample: 1947-63 versus 1964-80.
less than 5.3	Ferson (1982)	Asymptotic upper bound from consumption model estimated by sample covariances and assuming true consumption beta of inflation is smaller than unity.
-1.4 to 5.4	Table Three*	Constrained nonlinear estimation of consumption model using quarterly treasury bill data (described in text).

*Notes: Estimates denoted with asterisks are results of the present study. The first two estimates in the table refer to relative risk aversion for wealth. The remaining estimates are risk aversion for consumption.

consumption data. Both of these studies used econometric techniques similar to those employed here.

To obtain a feel for the estimates in the table, one can approximately interpret the magnitudes of relative risk aversion in terms of timeless gambles for real consumption to which a hypothetical individual would be indifferent if he behaved as if he had the assumed utility function with that parameter value. Pratt (1964) showed that an individual would be indifferent between a coin toss over α fraction of current consumption versus paying $\frac{1}{2} a\alpha^2$ of current consumption to avoid the gamble, where "a" is the coefficient of relative risk aversion. For example, with a = .05 a person would pay up to .00025% of current consumption to avoid an even wager over 1%. With a = 1.0 (the log utility), a consumption penalty of .005% would be endured to avoid the 1% gamble. With a = 2, the number rises to one basis point; and a constant relative risk averse individual with a = 10 would be indifferent between a .05% penalty in current consumption versus the fair gamble over 1%.

7. Summary and Conclusions

7.1

Conclusions of this study are presented in three parts. The focus is to interpret implications of the results for current and future research. Following a review of the empirical findings, Section 7.2 discusses implications for theory and methodology assuming inferences drawn from the statistical analyses are correct. Section 7.3 takes the contrary view, focusing on inadequacies in the data and methodology which could have influenced the results, and offers suggestions for future research addressing these problems.

The results suggest the existence of changes in rational expected real returns to nominal bonds, but we do not reject that expected real per capita consumption growth is constant. Evidence from quarterly data was presented initially, conditioned on past realizations of the two time series. The first stage, unrestricted regression results extend the evidence to a broader set of conditioning information. Constancy of the expected real interest rate was rejected at standard significance levels under ordinary least squares assumptions, although there was some evidence of misspecification in the regressions. None of the consumption change regressions, however, attained significance at the 10% level.

A multivariate F test of the joint hypothesis that the means are constant, consistent with both the models of Fama (1975) and Hall (1978) (and implying no overidentifying restrictions to test), was rejected at the ten percent level.

The likelihood ratio statistic for the significance of cross-equation restrictions produced different results for the constant relative versus constant absolute risk aversions models. Constant absolute risk aversion was more significantly rejected conditioned on either set of instruments. In two out of three cases, the p-value was less than .001. For constant relative risk aversion the most significant of three p-values was .04, the least .30. In general, the restrictions were less significant when the information set did not include the past values of the dependent variables.

Estimates of aggregate relative risk aversion produced under the null hypothesis were presented and compared with results of other studies. Estimates of constant absolute risk aversion were also produced, which suggest a value for the aggregate coefficient of less than 10 basis points for each (1972) dollar change in real consumption per capita. Each of the constrained estimates were within two asymptotic standard errors of zero (a zero value means we do not reject that expected real interest rates were insensitive to changes in consumption because bonds are priced as if consumers are risk neutral). The range of estimates for constant relative risk aversion suggests a number not much greater than five and probably much smaller. The risk neutral value of zero was within three standard errors of each of the constrained estimates. Two out of these three estimates were negative and included zero, but not values as high as for the logarithmic utility within a three standard error confidence interval. Overall, the range of estimates is large, although it should be noted that the range of estimates found in the literature is also quite broad.

7.2 Implications of the Empirical Findings

A basic result of the empirical analysis, maintaining the assumption of rational expectations, rejects constant expected real returns on three-month treasury bills but does not reject the hypothesis that expected quarterly real per capita consumption growth is constant. This could simply reflect a lack of power in the tests, of course, but we first consider the implications of these results if correct. Problems of data and methodology are discussed in the next section. If the inference is valid, the specification of the joint hypothesis of equation (9) is rejected for all values of the risk aversion parameters. Equation (9) predicts that variations in the means of changes in consumption and real returns are related by aggregate risk aversion. If bond prices are set as if consumers were indifferent to inflation risk (a=0), then constant expected real interest rates and changing mean growth of consumption are consistent with the model. However, constant expected growth of consumption requires in equation (9) that expected real interest rates are constant.

Equation (9) is as a joint hypothesis of the pricing equation (6), stated in terms of an unspecified aggregate utility function, and two additional assumptions: (1) the specific functional forms of preferences with joint lognormality of marginal utility and price levels (see Figure One) used to produce the linear expression (7); and, (2) the assumption that forecast errors for consumption growth and inflation rates have a stationary covariance matrix. If this joint hypothesis is rejected because expected consumption growth is actually constant while expected real bond returns are not, then the basic pricing equation (6) is rejected, unless at least one of these two assumptions is incorrect.

The assumption of stationary forecast error variances may not be strictly accurate, and could be important. If the assumption is relaxed, then possibly the underlying theory—with the assumed preference structure—could be reconciled with the empirical findings. For example, suppose that the variance of inflation is proportional to the level of expected inflation. For some values of the proportionality factor, equation (7) could hold when expected real returns vary with uncertainty even though expected consumption growth is not changing. The possibility that consumption and inflation risks are changing points to an area where refinements of the test methodology could be profitable (discussed below).

Another possibility which preserves the basic pricing theory in the face of changing expected real returns and constant expected consumption growth is that the specific forms of utility functions and/or distributions assumed were incorrect. For example, it is possible that the expected change in consumption is constant but the expected relative marginal utility of consumption is not. Changes in higher moments of consumption beyond the second may also influence marginal utility. If so, the assumption of joint lognormal marginal utility and price levels, used explicitly or implicitly in many papers, is more useful for analytical convenience than for empirical application.

If changing forecast error variances or failure of the assumptions which produce the linear model of equation (7) are insufficient to explain the (assumed correct in this section) empirical findings, then the theory is rejected at the more basic level of equation (6). Some unrealistic simplifying assumptions were used to develop this model, and the results would suggest that future theoretical development could profitably focus on relaxing one or more of these. The main assumptions used in deriving the model of equation (6) were: (1) perfect markets (i.e. no taxes, transactions or information costs and no restrictions on portfolio holdings or other impediments to free marketability of financial claims) and (2) market equilibrium consistent with optimizing an aggregate time-additive and state independent concave utility function for consumption of commodities.

Time separability of the aggregate utility function rules out any complementarity of utility for consumption at different dates; it assumes the utility of what will be consumed tomorrow is independent of what was consumed today or yesterday. Hall (1981), for example, points out that if this assumption is incorrect the risk aversion coefficient is doing "double duty," being influenced not only by risk aversion, but also by any effects of intertemporal complementarity of consumption. The empirical importance of this effect has not yet been demonstrated. Consumption time-complementarity should be indicated by time series behavior of consumption which does not correspond to the one-to-one fluctuations in marginal utility (nor, therefore, to expected real interest rates) as assumed in this model. It seems unlikely that complementarity alone will explain the observed fluctuations in real interest rates if the expected growth of consumption is actually constant.

If one rejects the basic model of equation (6), but does not attribute the rejection solely to inadequacy of the aggregate time additive utility assumption, then perfect markets should be selectively relaxed. The absence of money, income taxes and constraints on marketability of wealth are obviously descriptively inaccurate assumptions. If some assets are not perfectly marketable, individuals behaving optimally will engage in liquidity constrained behavior (an example being consumption this period of less than the unconstrained optimum because of an inability to borrow against the full value of human capital).

An intertemporal optimizer need not face a binding constraint this period in order to have consumption decisions affected by liquidity considerations [Bernanke (1980)]. It is sufficient to face a likelihood of binding constraints in some future period, depending on current investment and consumption decisions. Such behavior may influence the relation between consumption growth and interest rates. A closely related imperfection is asset indivisibility. For example, evidence in the stock adjustment literature [see, e.g. Ferber's (1973) review] suggests that real consumption expenditure levels, especially for durable items, respond negatively to an increase in the real borrowing rate. If consumption from a durable good "flows" in essentially a technologically predetermined pattern over time, liquidity sensitive individuals may postpone or accelerate their purchase of big-ticket durable goods when interest rates rise or fall. This could result in a relation between optimal expected consumption growth pattern and the level of real interest rates different from the implications of perfect market theory.

With perfect marketability of all assets and no money, the model has obviously ignored any affects which government policy may have on expected real bond returns through supply and demand for money or other commodities. If government's demand and supply schedules differ from the simple aggregation of consumer demands assumed here, then deviations from the model could result. Careful treatment of these issues is far beyond the scope of this study.

Another obvious imperfection ignored by the model is taxes. The fact that the return of "consumption" flowing to treasury bill investors is taxed while future consumption flows from a consumer good are not creates an asymmetry. If consumers could take unrestricted positions in durable goods and real bonds of all maturities (assuming certain consumption flows from both assets as an approximation) then no equilibrium prices for the two assets, not allowing riskless arbitrage opportunities, could exist unless all investors were in the same marginal tax bracket [Schaefer (1980a)]. A representative individual equilibrium is of course a model with a single tax bracket. Since the after-tax real bond return must be related to the untaxed consumption return from holding commodities, the before tax equilibrium real bond return must be higher than if untaxed. This suggests that ignoring taxes may have biased the estimates of risk aversion, overstating them by roughly a factor of $1/(1-T)$, where T is a constant "effective" marginal tax rate on bond income [see Brennan (1970) and Schaefer (1980a) for discussions of the problems of taxes].

Research has produced mixed results on the empirical importance of differential taxation of assets. Mishkin (1980), for example, reports no difference in results for his study of real interest rates when after tax returns are used instead of before tax returns. Hansen, Richard and Singleton (1981) and Shiller (1981) also report little sensitivity of their results to before versus after tax returns. Unpublished results of Stephen Schaefer suggest that the prices of the majority of U.S. treasury bonds and notes could be rationalized by an investor in a zero marginal tax bracket. Some studies have measured "tax effects" in common stock returns [e.g. Litzenberger and Ramaswamy (1979), or Hess (1981)] while others have argued there is not a significant effect [e.g. Black and Scholes (1974) or Miller and Scholes (1978)].

7.3 Problems of Data and Methodology

The discussion so far has assumed correct the inference that quarterly expected growth in real consumption per capita is constant and that expectations of real returns to bonds change over time. If this inference is not correct, then little can be said regarding the underlying theoretical proposition until the data and/or methodology are improved. If expectations for consumption growth and real rates are best regarded constant for empirical purposes then justification is provided for the many studies which have implicitly or explicitly made this assumption in formulating tests of theories which do not themselves require constant expectations. There would also be important implications for as-yet-to-be-published tests of the consumption-based asset pricing model of Breeden (1979) if consumption mean growth is stationary. If a continuously-updated portfolio designed to maximize correlation with aggregate consumption has a constant expected return when the mean growth rate of aggregate consumption is constant, then changes in the expected returns on stocks must be associated in Breeden's model with nonstationarities in their consumption betas. Nonstationarity of these betas must therefore be allowed within the empirical test specification if expected returns on stocks change. Cornell (1981) has also emphasized this point on the basis of prior theory.

One hypothesis is that expected consumption growth changes over time but expected real returns on bonds are constant. This requires aggregate risk neutrality of consumers in the consumption model if inflation risk is stationary. Studies of stock market price volatility [e.g. LeRoy and Porter (1981) or Grossman and Shiller (1981)] have rejected the hypothesis of risk neutrality in the stock market.(33) This hypothesis also contradicts substantial previous theory and empirical evidence, including the present empirical results.

A more likely third possibility is that neither expected consumption growth nor expected real returns on bonds are constant. If true, then the tests did not have sufficient power to detect changes in the mean growth rate of consumption, due to inadequacies in either data or the methodology.

If the tests lack power because of inadequate data, the solution could be as simple as finding a better set of predetermined variables for modeling the conditional expectations. Results presented here are based on a limited set which did not have very high explanatory power and certainly do not exhaust the possible sources of variation in expectations of real rates or consumption growth that prior theory would suggest. Alternatively, there are various sources of error in the data relative to theoretical notions of "true" consumption growth and relative price changes. Some of these errors are unavoidably inherent in all macroeconomic data from the processes of collection, compilation, tabulation, etc. We have attempted to minimize the effect of these, but sources of systematic error probably remain. Using quarterly data pretends that individuals base consumption-investment decisions on three-month planning horizons. If the actual decision period differs, temporal aggregation bias may produce inconsistent least squares regression coefficients. If the regression coefficients are inconsistent in the first-stage conditional expectations, then properties of the constrained estimation which rely on initial consistent estimates (e.g. consistency and asymptotic efficiency) are no longer guaranteed.

There may be systematic error in the measures of consumption. Total aggregate consumption is not measured; only certain types of expenditures by consumers, and durable goods have not been properly treated.(34) These are two particularly difficult problems. Even without timing problems, if the relation of the omitted components of consumption to expected real interest rates differs from those included, the relation of "true" consumption may be incorrectly measured. Demand for consumption of different types of goods would grow proportionately only if relative prices did not change and commodity demand functions were homothetic. Departures from either condition mean that measured relations may be influenced. By ignoring the contribution of durable goods to total consumption flows, this study has followed a common "solution" in the empirical literature to the problem of separating current consumption from investment for future consumption. However, the definitions which the Commerce Department uses to discriminate nondurable from durable goods definitely includes among the nondurables and services items not consumed in a single calendar quarter.[35] The ideal model would explicitly recognize a range of durability in consumer goods and model the relation between observable expenditures and the marginal utility of consumption, producing testable reduced-form expressions relating prices to expenditure data.

In addition to errors in measuring consumption, the measurement of inflation rates induces error. For example, although the analytical development does not require an explicit assumption of homothetic consumption preferences, this assumption was implicitly embraced by using changes in the PCE deflator to measure inflation. With general consumption preferences, inflation is measured for local changes in prices [Breeden (1977)] using marginal budget share-weighted averages of commodity price changes. The PCE deflator uses average budget share weights each quarter. This can produce a correct measure of inflation only if preferences are homothetic.

Despite the many sources of error, it is conceivable that the data reflect changes in actual expected real consumption growth and real interest rates which more refined methodology could accurately discern.

If conditional expectations of both variables change, the basic approach of modeling these directly and relating their changes to the implications of theory is sound. The results seem encouraging enough to warrant further development. Extension of the investigation of longer maturities and the term structure could prove fruitful if consumption expectations do change, but slowly or less reliably for shorter observation intervals than for longer horizons. Two specific improvements to the methodology are suggested for future research, (1) incorporation of a model of the covariances of conditional forecast errors and, (2) estimation with restrictions relating the parameters of the covariance matrix to the other parameters of the model. The first extension would permit control for nonstationarity arising, for example, if the variance of inflation changes with its level. With covariance restrictions, it is possible in this model to distinguish and test two different "versions" of the Fisher equation which are otherwise observationally equivalent. Problems of the term structure or with multiple assets, where relationships across equations depend on differences in conditional second moments will also employ covariance restrictions.

FOOTNOTES

(1) See for example, Lucas (1978), Cox, Ingersoll and Ross (1978, 1981), Prescott and Mehra (1980), Long and Plosser (1980), Breeden (1979, 1981), Brock (1981) or Richard and Sundaresan (1981).

(2) To see this, consider a mean-preserving spread of future consumption. That is, let $\tilde{C}_{t+j} = \overline{C}_{t+j} + \sigma_j \tilde{Z}$ where $E\{\tilde{Z}|\overline{C}_{t+j}\} = 0$ and the k superscript is suppressed. Substituting into (4) implies

$$\frac{\partial}{\partial \sigma_j}\left[e^{-R_j(t)j}\right] = \delta_t E_t\{u_{CC}(\overline{C}_{t+j} + \sigma_j \tilde{Z})\tilde{Z}\} ,$$

where $\delta_t > 0$ and $u_{CC}(\cdot)$ is the second derivative of $u(\cdot)$ with respect to the quantity of consumption (other arguments of $u(\cdot)$ are suppressed). If absolute risk aversion of $u(\cdot)$ is everywhere non-increasing, then $U_{CCC} > 0$ [see, for example, Kraus and Litzenberger (1976)], or $U_{CC}(\cdot)$ is strictly increasing with realizations of $\tilde{Z}$ and therefore the expectation of the product above is positive. The implicit function theorem then implies $\frac{\partial}{\partial \sigma_j}[R_j(t)] < 0$.

(3) For example, Rubinstein (1974) shows sufficiency of HARA utilities for aggregation of equation (2). Brennan and Kraus (1975) show that the HARA assumption is necessary in complete markets if state prices do not depend on the distribution of initial wealth across consumers. For concave utility, if equilibrium is Pareto efficient and individuals' expectations of events are identical conditional on aggregate consumption outcomes, then there must exist some surrogate social marginal utility so that (2) has an aggregate interpretation [Breeden and Litzenberger (1978)]. This is also true in a multi-good model with homothetic consumption preferences over goods [Grauer and Litzenberger (1979)]. Grossman and Shiller (1981) discuss aggregation without perfect marketability of assets. Breeden (1981) derives aggregate results in a continuous time consumption model assuming concave, time additive, state independent utility functions and homogeneous beliefs.

(4) Similar expressions to the asset pricing model [equation (6) below] may be derived without time additivity, but the expression for the marginal rate of substitution Φ_{t+j} becomes more complex [see, e.g. Ronn (1981) or Richard (1981)].

(5) Samuelson and Swamy (1974) show there exists an exact price index $I(\cdot)$ which is invariant to the level of expenditures ($I_C = 0$), if and only if preferences are homothetic, with unitary expenditure elasticity. In this case $g(\underline{c}^*(C,\underline{P})) = Cg(\underline{c}^*(1,\underline{P}))$ and therefore $I = [g(\underline{c}^*(1,P))]^{-1}$. The index is the inverse of indirect utility. See Grauer and Litzenberger (1979) for examples of commodity price indexes for homothetic preferences.

[6] This intertemporal envelope condition is analogous to the one Breeden (1979) exploited to collapse the "multiple-beta" continuous time asset pricing model [Merton (1973)] into a single beta model relative to aggregate consumption. It is also a central feature which distinguishes this model from earlier discrete-time multiperiod analyses of consumption-investment choice [e.g., Fama (1970), Long (1974) or Friend, Landskroner, and Losq (1976)]. Previous models have expressed asset prices in terms of wealth and many "state variables" (for example, Long's model had one for each consumption good or individual in the economy, whichever is fewer). The present model reflects these state variables in the price level index and the envelope condition, deriving asset prices relative to real consumption and uncertain inflation.

[7] Nonhomothetic preferences also means that "real consumption" as defined here will not in general aggregate properly because the product of a price and its dual quantity index will not be identically equal to the ratio of separate prices multiplied by quantities [Samuelson and Swamy (1974)]. However, Breeden (1977, 1981) shows that aggregate real consumption satisfying this definition is valid without homotheticity in the sense that social welfare can increase (i.e. pareto superior consumption allocations locally exist) if and only if real consumption increases (Breeden employs an additive expenditure-weighted deflator).

[8] Since $\{\frac{\partial}{\partial C_n}[C_n/I(C_n,P_c)]\}^{-1} = I[1 - \frac{C_n}{I} I_{C_n}]^{-1} \equiv I^*(C_n,P_c)$,

$$\tilde{\Phi}_{t+j} = \{\frac{\partial}{\partial C_{n,t+j}}[u(\tilde{C}_{t+j})]\}/\{\frac{\partial}{\partial C_{n,t}}[u(C_t)]\} .$$

[9] See Rubinstein (1974), Kraus and Litzenberger (1975), and Breeden (1979, 1980) for references and other examples of capital budgeting formulae which are consistent with stochastic interest rates and multiperiod valuation.

[10] See Marsh (1980) for discussion and empirical tests based on betas and the relation of bond returns to consumption implied by Breeden's (1979) model. Hansen, Richard and Singleton (1981) have recently argued that the consumption beta return-based formulation ignores restrictions implied by equation (6) and is inherently more difficult to test.

[11] Studies which have made use of constant relative risk aversion and lognormality include Rubinstein (1976), Breeden and Litzenberger (1978), Hall (1981), Grossman and Shiller (1981), Hansen and Singleton (1981), Brennan (1979), Merton (1971, 1973) and Hakansson (1970). Kraus and Litzenberger (1975) and Rubinstein (1974) derive results for the log utility case. Stapleton and Subrahmanyam (1978), Kraus and Sick (1980), Brennan (1979) and Grossman (1976), among others, have discussed pricing models employing the assumptions of constant absolute risk aversion and normal distributions.

[12] Breeden (1979, 1980), Cornell (1981), Stultz (1981) and Hansen, Richard and Singleton (1981) discuss some of the problems with consumption betas.

[13] Hall (1981) also examines a constant relative risk aversion extension of his (1978) work to allow changing expected real interest rates.

(14) This result, an implication of applying the law of iterated conditional expectations to the rational (i.e. mathematical) expectations of the model, has also been emphasized by Grossman and Shiller (1981), Gibbons and Ferson (1981), Ferson (1981) and Hansen, Richard and Singleton (1981).

(15) See Gibbons and Ferson (1981) for a new test of the Capital Asset Pricing Model using this methodology and a discussion of its extension to more general multiple-factor asset pricing models. Tests of the discrete time-consumption CAPM, using similar methodology but different data, have been recently reported in Hall (1981), Hansen and Singleton (1981), and Hansen, Richard and Singleton (1981). Results of these studies are compared with ours below.

(16) See, however, Stambaugh (1981) for an empirical investigation of this issue.

(17) For example, a linear structural model in reduced form will typically have this characteristic, even if expectations of the exogenous variables are formed nonlinearly. Consider the following reduced form model:

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \Phi(B)\begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} \underline{\sigma}'_y \\ \underline{\sigma}'_x \end{bmatrix} \tilde{W}_t + \begin{bmatrix} \tilde{v}_t \\ \tilde{u}_t \end{bmatrix} \; ; \; E\left\{\begin{bmatrix} v_t \\ u_t \end{bmatrix} \Big| Z_{t-1}\right\} = 0 \; ,$$

where $\Phi(B)$ is a matrix polynomial in the backshift operator and W_t a vector of exogenous variables. Suppose $E(\tilde{W}_t|\underline{Z}_{t-1}) = \underline{f}(\underline{Z}_{t-1})$ where $\underline{f}(\cdot)$ is a known nonlinear vector-valued function. Taking expectations conditional on Z_{t-1} and the lagged dependent variables implies:

$$E\left\{\begin{bmatrix} y_t \\ x_t \end{bmatrix} \Big| Z_{t-1}\right\} = \Phi(B)\begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} \underline{\sigma}'_y \\ \underline{\sigma}'_x \end{bmatrix} \underline{f}(Z_{t-1}) \equiv \underline{\gamma}\underline{Z}'_{t-1} \; .$$

Conditional expectations are linear in $\underline{\gamma} \equiv (\underline{\Phi}, \underline{\sigma}_y, \underline{\sigma}_x)$, where

$$\underline{Z}'_{t-1} \equiv (y_{t-1}, x_{t-1}, y_{t-2}, \ldots, \underline{f}(Z_{t-1})) \; .$$

(18) For example, assume $i < j$. Then

$$E(v_{t+i}u_{t+j}|Z_t) = E\{v_{t+i}E(u_{t+j}|Z_{t+i})|Z_t\}$$

$$= E\{v_{t+i} \cdot 0|Z_t\} = 0 \; .$$

(19) [See Gallant (1977), page 73, and appendix, page 87]. Gallant (1977) shows that the JGLS method is consistent almost surely and is asymptotically efficient when asymptotic distributions are normal. Mikhail (1975) presents simulation results which support the superiority of joint estimation methods in small samples, particularly when the residual correlation across equations is large and overidentifying restrictions are present. See Gibbons (1982) for a discussion of the properties of this type of constrained nonlinear

multivariate regression in a different context. Gallant's results are derived assuming no lagged endogenous variables appear on the right hand sides of the equation. In the present context, however, no lagged endogenous variables generated by the system are used in estimation. The realized past values are used strictly as instruments. This fact, plus normality and rationality, is sufficient for Gallant's results to obtain even if Z_t contains past lags of y_t and x_t.

(20)All computations were performed using algorithms supplied by SAS Institute, Inc. Constrained estimation of system (14) did not impose restrictions on the residual covariance matrix implied by the definition of α_1, because this was not computationally feasible using these algorithms.

(21)For example, Hall (1978) and Grossman and Shiller (1981) used nondurables plus services. Flavin (1980) and Hall (1981) used nondurables. Hansen and Singleton (1981a, 1981b) present results for both measures.

(22)Seasonally adjusted data are employed by necessity; not because they are a priori preferred to unadjusted data. Quarterly data on the PCE deflator are available only in seasonally adjusted form; and this study follows Hall (1978, 1981), Flavin (1980), Grossman and Shiller (1981) and Hansen and Singleton (1981a, 1981b) in using adjusted data.

Even if unadjusted data were available, it is by no means clear they should be preferred to data which are seasonally adjusted. On one hand, seasonal adjustment results in "smoother" time series. On the other hand, the sampling and interpolation procedures used in constructing these series may produce spurious seasonal patterns in the data even if no real seasonals exist. One might prefer some type of seasonal adjustment to remove the spurious patterns.

There may be actual seasonals in the mean growth of aggregate consumption. If so, the theory suggests that how seasonal patterns would be reflected in real interest rates depends upon seasonality in consumption preferences. The simple utility functions used here assume no seasonality of preferences and thus the models imply that any seasonal pattern in the mean growth of real consumption corresponds to a seasonal pattern in expected marginal utility of consumption. According to the theory, seasonal patterns in the expected relative marginal utility of consumption should correspond to seasonals in expected real interest rates. This would suggest that unadjusted data might be preferred if "actual" preferences are nonseasonal. Allowing for seasonal preferences, real seasonal patterns in mean consumption growth need not imply corresponding seasonals in marginal utility nor, therefore, in expected real interest rates. Since seasonal preferences are not being modeled, it is conceivable that unadjusted real consumption could provide a noisier measure of marginal utility than seasonally adjusted consumption.

(23)This conservative approach probably results in estimated prediction equations with lower R-squares than might otherwise be obtainable. However, additional reasons for not admitting first order lagged variables relate to publication lag and the sampling procedures employed by the Commerce Department. Quarterly consumption data are not actually published until the second or third week of the quarter succeeding the measurement interval [Schwert (1981) finds that common stocks may react to announcements of the CPI up to a month after the data is collected]. Also, certain items are sampled at intervals longer than three months and interpolated; this procedure could induce spurious correlation between adjacent quarterly observations.

(24)One could maintain the assumption of constant expectations and view the results as tests of rational expectations. Although this view has historical precedent [see, e.g. Fama (1970)], the present study will maintain rationality. Under rationality, if the null hypothesis is rejected, we then have a constructive alternative to offer.

(25)Other empirical studies which have rejected Fama's assumption that the expected real rate is constant include Nelson and Schwert (1977), Carlson (1977), Garbade and Wachtel (1978), Hess and Bicksler (1975), Joines (1977), Fama and Gibbons (1982), Mishkin (1980), Wakeman and Bhagat (1980) and Startz (1981).

(26)There is an alternative interpretation consistent with Fama's (1977) argument that there exist spurious seasonals in the CPI. Suppose there exists a seasonal pattern in accurately measured inflation (or perhaps in expected consumption growth) which is reflected in nominal rates. If seasonal adjustment has removed these patterns from the time series of measured inflation rates and real consumption changes, then the real rate series would display a seasonal pattern because it is the difference between the unadjusted nominal rate and the seasonally adjusted inflation rate. The seasonal autocorrelation in the measured real rate would appear not to be associated with fluctuations in real consumption growth. If this interpretation is correct there should be a seasonal pattern in the measured real rate. Table One provides little evidence for this.

(27)Table A4 in the appendix presents very similar results for the constant absolute risk aversion model.

(28)A comparison with R-squares obtained for real rates by Mishkin (1980), who uses different quarterly data including the first lag, supports this claim.

(29)Nelson and Schwert (1977) note this problem in a discussion of Fama's (1975) study. Mishkin (1980) develops the argument in more detail using expected real interest rates.

(30)The Durbin H statistic is a transformation of the Durbin-Watson designed to avoid a problem of bias in favor of the null hypothesis (zero autocorrelation, corresponding to values of the Durbin-Watson near 2.0) when autocorrelated residuals and lagged dependent variables occur in the regression. It is asymptotically distributed as a normal (0,1) variable under the hypothesis of zero residual autocorrelation. Positive values of H correspond to positive residual autocorrelation. This of course means that OLS standard errors are understated (t ratios overstated).

Autocorrelation in the residuals of a given equation also means that t and F ratios are not strictly valid because the sum of squared residuals is no longer distributed as a chi-square in finite samples, nor are the numerator and denominator of the test statistic independent [see, e.g. Johnston (1972) or Thiel (1971)]. Autocorrelation in the real rate residuals could be reflecting various kinds of misspecification; for example omitted autocorrelated predictors (recall that first order lagged predictors are excluded although the real rate has significant first order autocorrelation) or the assumption of linearity in an autocorrelated predictor when the correct relation is nonlinear.

(31)For example, Lahiri (1976), Carlson (1977), Levi and Makin (1979), Pearce (1979), and Mishkin (1980) have documented negative association between real interest rates and predetermined inflation measures.

(32)Hall (1981) and Hansen and Singleton (1982) have also presented relative risk aversion estimates based on unconditional moments.

(33)One could reject risk neutral behavior in the stock market while not rejecting it for the bond market if inflation risk is less important to investors than risk induced by the wider range of uncertainties to which publicly traded firms are exposed.

(34)By measuring only personal consumption expenditures for nondurable goods and services, other sources of consumption are ignored; notably that obtained from consumer durable goods and government-owned assets or activities. For example, of total nominal expenditures for goods and services by individuals, federal, state and local governments in 1972, government expenditures represented about 26% of the total and durable goods 15%. The fraction represented by consumer expenditures for nondurables plus services was about 63% (source: Department of Commerce Data).

(35)For example, the following types of expenditures are included as nondurables or services: clothing and shoes, canned foodstuffs, repair of appliances and autos, household furnishings, and payments for private education and research.

APPENDIX

Quarterly data from 1947 on aggregate personal consumption expenditures and the PCE deflator are published in the National Income and Product Accounts (NIPA) in the Survey of Current Business and Business Statistics supplements. The data employed incorporated revisions through the Bureau of Labor Statistics (BLS) seventh benchmark revision [see the December, 1980, Survey of Current Business for details].

Three month treasury bill rates are computed from discount quotes in Salomon Brothers, Analytical Record of Yields and Yield Spreads. Prior to the first quarter of 1950, Federal Agency securities are used in lieu of Treasury Bills.

Real stock returns are based on the Standard and Poor's (S&P) Composite, total return index as reported in Ibbotson and Sinquefield (1979), and updated through 1980 from the S&P Security Price Index Record. Returns are measured at the end of the first month of each quarter by subtracting the inflation rate from nominal returns, both calculated on a continuously compounded basis.

Figure A1 defines the symbols used in the following statistical tables. Table A1 contains sample means, standard deviations and autocorrelations for each basic series and differenced series. Table A2 is a sample correlation matrix.

Table A3 displays the bivariate autocorrelation structure of ex post real rates and real consumption growth for 1959 to 1980. Correlations with changes in the logarithms of nondurables plus services, denoted "C," are shown (other consumption series produced essentially similar results). The schematic representation denotes by "+" those correlations exceeding two approximate standard errors. Lagged cross-correlations are shown in the off-diagonal positions of the matrices, with rows corresponding to the leading variables and columns to the lagged variables they are correlated with. For example, the correlation of the real rate with the second lag of consumption growth is -0.155; that of consumption with the third lag of the real rate, 0.167.

FIGURE A1

DEFINITION OF SYMBOLS USED IN PRESENTATION
OF STATISTICS

SYMBOL

(A) BASIC DATA SYMBOLS:

NON	Consumer nondurable goods component of real aggregate consumption expenditures per capita.
SERV	Real consumer services expenditures per capita.
NDS	NON + SERV
DX	Real consumer durables expenditures per capita.
RF_t	Nominal riskless rate observed at the beginning of the three month period, continuously compounded annual rates.
INF_t	Ex post inflation rate, continuously compounded annual percentage rate. $INF_t = 400 \ln(I_t/I_{t-1})$; where I_t is the level at time t of the implicit price deflator for total personal consumption expenditures.
RM_t	Continuously compounded total real return over t-1 to t on the S&P 500 stock market index, at annual percentage rate.

(B) DERIVED SERIES:

X	$RF_t - INF_t$; the ex post real return to the nominally riskless bond.

TABLE A1

SUMMARY SAMPLE STATISTICS

quarterly data (1947-1980)
135 observations

SERIES	MEAN	STD DEV	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	Q
NON	1.266	0.175	.98	.96	.95	.92	.89	.87	.85	.83	932.0
Δ NON	0.37	1.53	-.10	-.14	.12	-.02	.02	-.04	-.08	-.06	8.0
Δ ln NON	1.13	4.58	-.13	-.09	.12	-.06	-.01	-.02	-.06	-.10	8.0
SERV	1.26	0.37	.98	.96	.94	.92	.89	.87	.84	.82	924
Δ SERV	0.82	1.00	.02	.23	.02	.27	-.03	.16	.04	.16	25.5
Δ ln SERV	2.48	2.86	.01	.11	-.10	.09	-.14	.05	.04	.13	9.9
NDS	2.53	0.52	.98	.96	.94	.92	.89	.87	.85	.82	930
Δ NDS	1.19	2.08	.08	-.03	.12	.15	.02	-.03	.02	-.01	6.1
Δ ln NDS	1.80	3.03	.02	-.00	.08	.06	-.05	-.03	.04	-.07	2.5
DX	0.39	0.14	.98	.95	.94	.91	.88	.85	.82	.79	899
Δ DX	.033	0.02	-.15	.07	.01	-.07	.05	-.02	-.12	-.18	11.0
RF	4.08	2.62	.91	.85	.80	.74	.65	.58	.53	.49	566
Δ RF	0.90	0.78	.01	-.23	-.02	.25	.03	-.18	-.16	.14	27.2
INF	3.80	3.27	.65	.60	.53	.39	.32	.23	.19	.22	203
Δ INF	0.04	2.73	-.45	.03	.09	-.09	.04	-.08	-.10	.01	32.2
RM	6.40	26.60	.13	.10	-.03	.10	-.07	-.06	-.14	.01	9.3
X	0.28	2.54	.40	.34	.24	.12	.06	-.06	-.08	.06	51
ΔX	0.05	2.75	-.47	.04	.03	-.06	.05	-.06	-.12	.03	34.0

Notes: The sample autocorrelations are Box-Jenkins estimates with approximate standard errors equal to .09. Q is the Box-Pierce statistic for the first 8 autocorrelations. Under the null hypothesis that the true autocorrelations are zero, the expected value of Q is 8 and its standard error approximately equal

TABLE A 2

SAMPLE CORRELATION MATRIX
quarterly data, 1947-1980

	X-2	DX	X-1	RM-1	RM-2	DX-1	INF	RM
X-2	1.000							
DX	0.090	1.000						
X-1	0.409	0.072	1.000					
RM-1	0.017	-0.207	-0.013	1.000				
RM-2	-0.002	-0.209	0.025	0.127	1.000			
DX-1	0.084	0.990	0.054	-0.228	-0.217	1.000		
INF	-0.164	0.647	-0.192	-0.230	-0.252	0.655	1.000	
RM	0.042	-0.215	0.025	0.139	0.104	-0.241	-0.270	1.000
DX-2	0.065	0.983	0.065	-0.253	-0.236	0.990	0.673	-0.237
X	0.365	0.041	0.394	0.015	0.052	0.051	-0.617	0.010
INF-1	-0.206	0.584	-0.651	-0.265	-0.269	0.617	0.656	-0.237
ΔNON	-0.118	0.130	0.205	0.204	0.031	0.052	-0.142	0.152
ΔNDS	-0.077	0.265	0.204	0.144	0.051	0.182	-0.075	0.164
INF-2	-0.653	0.550	-0.212	-0.242	-0.288	0.573	0.615	-0.215
ΔNDS-2	0.164	0.178	0.078	-0.052	0.157	0.168	0.069	-0.098
ΔlnNON	-0.131	0.107	0.248	0.222	0.042	0.033	-0.143	0.150
ΔlnNDS	-0.091	0.175	0.249	0.199	0.079	0.097	-0.120	0.175
ΔlnNON-2	0.147	0.099	0.068	-0.033	0.148	0.089	0.040	-0.057

	DX-2	X	INF-1	ΔNON	ΔNDS	INF-2	ΔlnNDS-2	ΔlnNON
DX-2	1.000							
X	0.038	1.000						
INF-1	0.626	-0.201	1.000					
ΔNON	0.052	0.168	-0.222	1.000				
ΔNDS	0.178	0.157	-0.165	0.890	1.000			
INF-2	0.607	-0.186	0.646	0.036	0.060	1.000		
ΔlnNDS-2	0.174	-0.048	-0.001	-0.085	-0.013	-0.138	1.000	
ΔlnNON	0.036	0.160	-0.257	0.989	0.876	0.042	-0.069	1.000
ΔlnNDS	0.097	0.153	-0.239	0.880	0.972	0.032	-0.003	0.896
ΔlnNON-2	0.103	-0.042	-0.029	-0.112	-0.074	-0.149	0.396	-0.088

	lnNDS	lnNON-2
ΔlnNDS	1.000	
ΔlnNON-2	-0.042	1.000

TABLE A 3

SERIAL AND CROSS-CORRELATIONS OF EXPOST REAL INTEREST RATES AND GROWTH RATES IN AGGREGATE REAL CONSUMPTION PER CAPITA OF NONDURABLES AND SERVICES
QUARTERLY DATA, 1959 TO 1980

LAG=0

	REAL RATE	C
REAL RATE	1.0000000	0.2086276
C	0.2086276	1.0000000

LAG=1

	REAL RATE	C
REAL RATE	0.4429927	0.0004136
C	0.1032774	-0.0425030

LAG=2

	REAL RATE	C
REAL RATE	0.3712776	-0.0771723
C	-0.1553657	-0.0275705

LAG=3

	REAL RATE	C
REAL RATE	0.3437077	0.1677672
C	0.0224151	0.0648899

LAG=4

	REAL RATE	C
REAL RATE	0.3112218	-0.0514474
C	0.0356640	0.1331121

LAG=5

	REAL RATE	C
REAL RATE	0.2811095	0.0489614
C	-0.0201386	-0.0811381

LAG=6

	REAL RATE	C
REAL RATE	0.2504141	-0.0458254
C	-0.0714424	-0.0473608

LAG=7

	REAL RATE	C
REAL RATE	0.1086281	-0.1424681
C	0.0008283	-0.0827714

LAG=8

	REAL RATE	C
REAL RATE	0.2516152	0.0258685
C	0.0472681	-0.1092264

LAG=9

	REAL RATE	C
REAL RATE	0.1808924	-0.0265711
C	0.0558613	-0.0190952

SCHEMATIC REPRESENTATION OF CORRELATIONS

LAG	0	1	2	3	4	5	6	7	8	9	10
	+.	+.	+.	+.	+.	+.	+.	..	+.	..	..
	.+	..	..	..	..	..	..	..	..	..	..

TABLE A4

Summary of Unrestricted Least Squares Estimation of Equation (10) for the Constant Absolute Risk Aversion Model Using Quarterly Data for 1947-1980. Number of Observations is 127 (128 in the First Panel). The Bivariate Regression Model Has the Form

$$\tilde{y}_{t+1} = \alpha_y + \beta'_y z_t + \tilde{v}_{t+1}, \qquad \tilde{x}_{t+1} = \alpha_x + \beta'_x z_t + \tilde{u}_{t+1} \qquad \forall\ t = 0, \ldots, T-1$$

where $y_{t+1} \equiv$ change in per capita real expenditures for consumer non-durables and services,

$x_{t+1} \equiv$ realized real return on 3-month treasury bill and

$z_t \equiv$ vector of predetermined variables known at date t.

Predetermined Variables	Dependent Variable	$\bar{R}^2$(a)	F(b)	p-value	Two-Equation(c) F Statistic	p-value	Durbin H Statistic	Residual Autocorrelations
Past values of inflation rates, real stock market returns and per capita expenditures for consumer durable goods at lags 2, 3 and 4.	y_{t+1}	.01	1.17	>.10	1.22	>.10	0.40	.03, -.11, .03
	x_{t+1}	.04	1.91	.06			3.68	.30, .17, .15
Past values of the two dependent variables at lags 2, 3, 4 and 5.	y_{t+1}	.03	1.40	>.10	1.55	>.10	-0.29	.08, .01, .02
	x_{t+1}	.09	2.20	.05			2.23	.19, -.01, .00
Lags 2 through 5 of the two dependent variables and three past lag values of the inflation rate, real stock market returns and durable goods expenditures.	y_{t+1}	.12	1.81	.025	2.60	<.005	-0.92	-.08, -.03, -.05
	x_{t+1}	.15	1.74	.03			1.32	.11, -.02, -.05

Notes: (a) $\bar{R}^2$ is the coefficient of determination adjusted for degrees of freedom.
(b) F statistic for the significance of the regression; the null hypothesis is that the true coefficient vector equals zero for a particular equation.
(c) The two-equation F statistic is for the null hypothesis that $\beta_y = \beta_x = 0$ in the bivarate regression.

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