BIASES IN COMPUTED RETURNS: AN APPLICATION TO THE SIZE EFFECT

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Previous estimates of a "size effect" based on daily returns data are biased. Several properties of quoted closing prices impart an upward bias to computed returns on individual stocks. Returns computed for buy-and-hold portfolios largely avoid the bias induced by closing prices. Based on such buy-and-hold returns, the full-year size effect is half as large as previously reported, and all of the full-year effect is, on average, due to the month of January.

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1. INTRODUCTION

Recent empirical work in finance reports that average risk-adjusted returns on stocks of small firms exceed those of large firms, where size is measured as the market value of outstanding common equity.[1] This "size effect" is particularly pronounced in the studies that use daily returns data, but we show that, due to a statistical bias, those studies significantly overstate the magnitude of the size effect. Although we empirically analyze the bias in the context of the size effect, the same bias could potentially occur in any study using closing prices to compute returns, particularly daily returns.

Using daily returns for stocks on both the New York and American Stock Exchanges, Reinganum (1982a) finds that, during the 1964-1978 period, the average return for firms in the lowest market-value decile exceeds the average return for firms in the highest decile by more than 0.1 percent per day--over 30 percent per year. He also finds that various methods of risk adjustment contribute little towards explaining such impressive differences.[2] Keim (1982) reports that almost half of the annual difference between returns on small and large firms occurs in January.

This study reexamines the size effect using daily returns for NYSE and AMEX stocks. We find that (1) the average size effect over the entire year is about 0.05 percent per day--only half as large as reported by Reinganum and Keim--and (2) virtually all of this full-year average is attributable to January. In other words, the size effect averages about 0.60 percent per day in January and roughly zero in the remainder of the year. The sample contains all firms listed on the New York and American Stock Exchanges, and the time period covers 1963 through 1980. Thus, our study uses essentially the same data as many prior studies.

The difference in results arises from the method used to compute average returns. Reinganum and Keim compute the arithmetic average of daily returns on stocks of firms within each market-value decile. The portfolio strategy implicit in this averaging is one of daily rebalancing to equal weights. Section 2 shows that returns on individual stocks computed with recorded closing prices contain an upward bias, and this bias may be nontrivial for daily returns on stocks of small firms. Since the computed return on a rebalanced portfolio is merely an arithmetic average of the computed returns of individual stocks, such a computed portfolio return is also upward biased. However, due to a "diversification" effect, the computed return on a buy-and-hold portfolio contains virtually no bias.

Section 3 presents empirical results for both rebalanced portfolios and buy-and-hold portfolios. The differences between returns on the two strategies are negligible for large-firm portfolios. In contrast, for the portfolio of the lowest-market-value firms, the rebalanced return exceeds the buy-and-hold return by an average of 0.05 percent per day, which is approximately half of the magnitude of the average size effect reported in previous studies. The analysis in section 4 finds that the difference between the rebalanced and buy-and-hold returns varies inversely with share price, holding market value constant, and bears no significant relation to market value, holding share price constant. This finding is consistent with the analysis of the bias presented in section 2.

2. COMPUTING RETURNS WITH CLOSING PRICES

2.1 A Model of Closing Prices

Define the true price of a security as the price at which, aside from transactions costs, a share of stock can be both bought and sold at a given time by placing a market order. On the Exchange, the true price can be viewed as the price at which (nearly) simultaneous public market buy and sell orders would "cross" on the floor. Denote the true price of stock i at time t as $P_i(t)$.

The Center for Research in Security Prices (CRSP) provides daily returns for stocks listed on the New York and American Stock Exchanges, and these returns are computed with "closing" prices. The <u>closing price</u> is the price at which the last transaction occurred prior to the close of trading.[3] Let $\hat{P}_i(t)$ denote the reported closing price of stock i for the period ending at time t.

There are various reasons why the closing price, $\hat{P}_i(t)$, can deviate from the true price, $P_i(t)$. First, the last transaction may reflect a public market order on only one side. For example, a market sell order might be matched with a limit buy order or bought by the specialist on his own account. Denote the price recorded for such transactions a "bid" price, and note that such a price is most likely less than the true price due to the "spread"--a charge for immediacy of execution. Similarly, a market buy order that is not crossed on the floor results in the recording of an "ask" price, which is likely to be greater than the true price. We refer to this property of closing prices as the <u>bid-ask effect</u>.

Closing prices can also deviate from true prices due to an <u>infrequent trading effect</u>. Thus, the closing price may be that of a transaction earlier in the period.

A model of both the bid-ask and infrequent trading effects is the following:

$$\hat{P}_i(t) = [1 + \delta_i(t)]P_i(t - s_{i,t}) \quad (1)$$

$$= P_i(t - s_{i,t}) + \varepsilon_i(t) , \quad (1a)$$

where $E\{\delta_i(t)\} = 0$, $\delta_i(t)$ is independently distributed across t, and $\delta_i(t)$ is independent of $P_i(\tau)$ for all τ. The multiplicative disturbance $\delta_i(t)$ represents the bid-ask effect. Also, the mathematical analysis below requires the plausible assumption that closing prices are greater than zero but less than twice the true price, that is, $-1 < \delta_i(t) < 1$. At some points in the discussion, it will be convenient to use (1a), which is restated with an additive disturbance $\varepsilon_i(t)$, defined as $\delta_i(t)P_i(t - s_{i,t})$. The period of nontrading prior to t is denoted by $s_{i,t}$, and $s_{i,t}$ is assumed to be independent and identically distributed over time as well as independent of all $\delta_i(t)$'s. In the absence of a bid-ask effect, $\delta_i(t) = 0$; in the absence of infrequent trading, $s_{i,t} = 0$.

It is well-known that both the bid-ask and infrequent trading effects produce negative first-order autocovariance in recorded price changes for individual stocks.[4] It is shown here that both effects also impart an upward bias to computed rates of return for individual stocks.[5] Single-period returns are first analyzed in detail, primarily because most empirical studies employ single-period returns—often averaged either cross-sectionally or over time. Multiperiod compounded returns are then briefly analyzed in terms of the negative autocovariance arising from the bid-ask and infrequent trading effects.

2.2 Single-Period Returns

The true return for security i for the single period ending at t is defined, assuming no dividends for the period, as

$$r_i(t) = \frac{P_i(t)}{P_i(t-1)} - 1 \ . \tag{2}$$

The <u>computed return</u> is, using (1), defined as

$$\begin{aligned}\hat{r}_i(t) &= \frac{\hat{P}_i(t)}{\hat{P}_i(t-1)} - 1 \\ &= \frac{[1 + \delta_i(t)]P_i(t - s_{i,t})}{[1 + \delta_i(t-1)]P_i(t - 1 - s_{i,t-1})} - 1 \ .\end{aligned} \tag{3}$$

Considering the case where there is only a bid-ask effect, with $s_{i,t} = s_{i,t-1} = 0$, and combining (2) and (3) yields

$$\hat{r}_i(t) = \frac{1 + \delta_i(t)}{1 + \delta_i(t-1)} [1 + r_i(t)] - 1 \ . \tag{4}$$

Taking expectations of both sides of (4) gives

$$E\{\hat{r}_i(t)\} = E\{\frac{1 + \delta_i(t)}{1 + \delta_i(t-1)}\}[1 + E\{r_i(t)\}] - 1 \ . \tag{5}$$

By Jensen's inequality, $E\{[1 + \delta_i(t)]/[1 + \delta_i(t-1)]\} > 1$. Therefore, $E\{\hat{r}_i(t)\} > E\{r_i(t)\}$. The bias can be approximated by taking a Taylor series expansion as follows:

$$\begin{aligned}E\{[1 + \delta_i(t)]/[1 + \delta_i(t-1)]\} &= E\{1 + \delta_i(t)\}E\{1/[1 + \delta_i(t-1)]\} \\ &= 1 \cdot E\{1 - \delta_i(t-1) + [\delta_i(t-1)]^2 - \dots \} \\ &\approx 1 + \sigma^2\{\delta_i(t-1)\} \ ,\end{aligned} \tag{6}$$

where $\sigma^2\{\ \}$ denotes the variance. Combining (5) and (6) yields, after dropping the cross-product term,

$$E\{\hat{r}_i(t)\} \approx E\{r_i(t)\} + \sigma^2\{\delta_i(t-1)\} . \tag{7}$$

If the third and higher-ordered odd moments of $\delta_i(t-1)$ are zero, then the variance term in (7) provides a lower bound for the bias induced by the bid-ask effect.

To assess roughly the potential magnitude of the bid-ask bias, consider a stock with a true price of \$5 per share, but with a bid price of \$4 7/8 and an ask price of \$5 1/8, and assume that the closing price is either a bid or an ask with equal probability.[6] Then δ_i is plus or minus 0.025, and $\sigma^2\{\delta_i\} = 0.63 \times 10^{-3}$. As shown later, the average computed daily return for "small" stocks--those in the lowest decile of equity capitalization--is approximately 1.41×10^{-3}. Thus, for the typically low-priced stocks of small firms, a significant portion of the computed daily returns may reflect a bid-ask bias.

The magnitude of the bid-ask bias can also be assessed by comparing returns on "rebalanced" portfolios to returns on "buy-and-hold" portfolios. Assume that a set of N securities is used to implement two alternative portfolio strategies, where both strategies initially invest equal amounts in each security at time 0. The rebalanced portfolio maintains the equal weights by rebalancing at the end of each period, whereas the buy-and-hold portfolio performs no further transactions.

The return on the rebalanced portfolio for a given period is an arithmetic average of the individual security returns. For the true rebalanced return, $r_{RB}(t) = \frac{1}{N} \Sigma\, r_i(t)$. The computed return, $\hat{r}_{RB}(t) = \frac{1}{N} \Sigma\, \hat{r}_i(t)$, simply contains the average bid-ask bias in the individual returns. That is, multiplying (4) by $\frac{1}{N}$, summing over i, and repeating precisely the same steps leading to (7) yields

$$E\{\hat{r}_{RB}(t)\} \approx E\{r_{RB}(t)\} + \overline{\sigma^2\{\delta_i(t-1)\}} , \tag{8}$$

where a bar indicates an average over i.

To simplify notation in analyzing the buy-and-hold portfolio, assume all N stocks have the same price at time 0 and that no dividends are paid from time 0 through time t. Then the true return for the buy-and-hold portfolio is given by

$$r_{BH}(t) = \frac{\sum_{i=1}^{N} P_i(t)}{\sum_{i=1}^{N} P_i(t-1)} - 1 . \tag{9}$$

The computed buy-and-hold return is, using the additive representation in (1a),

$$\hat{r}_{BH}(t) = \frac{\sum_{i=1}^{N} \hat{P}_i(t)}{\sum_{i=1}^{N} \hat{P}_i(t-1)} - 1 = \frac{\sum_{i=1}^{N} P_i(t) + \sum_{i=1}^{N} \varepsilon_i(t)}{\sum_{i=1}^{N} P_i(t-1) + \sum_{i=1}^{N} \varepsilon_i(t-1)} - 1 . \tag{10}$$

Rewriting (10), using (9), yields

$$\hat{r}_{BH}(t) = \frac{1 + r_{BH}(t) + \dfrac{\overline{\varepsilon_i(t)}}{\overline{P_i(t-1)}}}{1 + \dfrac{\overline{\varepsilon_i(t-1)}}{\overline{P_i(t-1)}}} - 1 . \tag{11}$$

Taking expectations of both sides of (11), again using a Taylor series expansion, gives

$$E\{\hat{r}_{BH}(t)\} = [1 + E\{r_{BH}(t)\}]E\{\frac{1}{1 + \dfrac{\overline{\varepsilon_i(t-1)}}{\overline{P_i(t-1)}}}\} - 1$$

$$= [1 + E\{r_{BH}(t)\}]E\{1 - \frac{\overline{\varepsilon_i(t-1)}}{\overline{P_i(t-1)}} + [\frac{\overline{\varepsilon_i(t-1)}}{\overline{P_i(t-1)}}]^2 - \ldots\} - 1$$

$$\approx E\{r_{BH}(t)\} + \sigma^2\{\frac{\overline{\varepsilon_i(t-1)}}{\overline{P_i(t-1)}}\} . \qquad (12)$$

The bid-ask bias for the buy-and-hold return is reduced by a diversification effect. Note that $\sigma^2\{\delta_i(t-1)\}$, the approximate individual-security bias shown in (7), can also be written as $\sigma^2\{\frac{\varepsilon_i(t-1)}{P_i(t-1)}\}$. The buy-and-hold bias, shown in (12), is of the same form, except that bars appear over ε_i and P_i.[7] The computed buy-and-hold portfolio return behaves like that of a security whose closing price deviates by $\bar{\varepsilon}_i$ from its true price $\bar{P}_i$.

Consider the expected difference in computed returns on the two portfolios. Using (8) and (12), with $\sigma^2\{\delta_i(t-1)\} = \sigma^2\{\frac{\varepsilon_i(t-1)}{P_i(t-1)}\}$,

$$E\{\hat{r}_{RB} - \hat{r}_{BH}\} \approx E\{r_{RB} - r_{BH}\} + [\overline{\sigma^2\{\frac{\varepsilon_i}{P_i}\}} - \sigma^2\{\frac{\bar{\varepsilon}_i}{\bar{P}_i}\}] , \qquad (13)$$

where time indexes are suppressed to ease notation. The expected computed difference between rebalanced and buy-and-hold returns equals (i) the expected difference in true returns plus (ii) the difference in the bid-ask biases. Under rather weak conditions, $E\{r_{RB} - r_{BH}\}$ is negative, and the quantity is closer to zero the lower is the cross-sectional dispersion in true expected security returns.[8] The difference in bid-ask biases is essentially an average variance minus a variance of an average, and the latter becomes small for large portfolios. For a portfolio of 100 stocks, with the occurrences of bids or asks independent across stocks, the average variance, or average individual stock bias, is about 100 times as large as the variance of the average, the buy-and-hold bias. In general, (13) indicates that the expected difference in computed returns, $\hat{r}_{RB} - \hat{r}_{BH}$, provides a lower bound for the average bid-ask bias for an individual security, and the approximation is best for large portfolios whose securities have identical expected true returns.

Whether or not the bid-ask effect is present, infrequent trading imparts a further upward bias to computed returns for individual securities. Assume that the "true" price, $P_i(t)$, follows an infinitely divisible lognormal process, with $\alpha_i = \ln[E\{P_i(t)/P_i(t-1)\}]$.[9] Taking expectations of (3), with the s_i's no longer set to zero, gives

$$E\{\hat{r}_i(t)\} = E\{\frac{1 + \delta_i(t)}{1 + \delta_i(t-1)}\}E\{\frac{P_i(t - s_{i,t})}{P_i(t - 1 - s_{i,t-1})}\} - 1 . \tag{14}$$

The first expectation on the right of (14) represents the bid-ask bias analyzed above. To evaluate the second expectation, first condition on $s_{i,t}$ and $s_{i,t-1}$, which yields

$$E\{\frac{P_i(t - s_{i,t})}{P_i(t - 1 - s_{i,t-1})} \mid s_{i,t}, s_{i,t-1}\} = e^{\alpha} e^{\alpha(s_{i,t-1} - s_{i,t})}$$

$$= E\{1 + r_i(t)\}e^{\alpha(s_{i,t-1} - s_{i,t})} . \tag{15}$$

Next take the unconditional expectation of (15), and observe $E\{e^{\alpha(s_{i,t-1} - s_{i,t})}\} > 1$ by Jensen's inequality. Thus, infrequent trading increases, by a multiplicative factor, any bias already arising from the bid-ask effect.

The infrequent trading bias can be approximated, using a Taylor expansion, as

$$E\{e^{\alpha(s_{i,t-1} - s_{i,t})}\} = E\{1 + \alpha(s_{i,t-1} - s_{i,t}) + \frac{\alpha^2}{2}(s_{i,t-1} - s_{i,t})^2 + \dots \}$$

$$\approx 1 + \alpha^2\sigma^2\{s_{i,t}\} \tag{14}$$

Note that the bias depends on the <u>variance</u> of the nontrading periods—not their average length. Compared to the bid-ask bias, though, the infrequent trading bias is quite small. Consider the same \$5 stock from the earlier bid-ask example, assume $\alpha = 1.4 \times 10^{-3}$ per day, and let $s_{i,t}$ take values of either 0 or 1 with equal probability. That is, the stock trades either at the beginning of the period or at the end of the period, but nowhere in between. Even in this extreme case, $\alpha^2\sigma^2\{s_{i,t}\} = 4.9 \times 10^{-7}$, which is over 1000 times smaller than the value of $\sigma^2\{\delta_i(t)\}$ (the bid-ask bias) estimated in the example.

2.3 Compounded Returns

Researchers may occasionally wish to compound single period returns over a number of periods. For individual securities, this compounding simply produces a single-period return for a longer period. The absolute magnitudes of the bid-ask and infrequent trading biases are independent of the length of the return period. Thus, for individual stocks, increasing the return period decreases the biases relative to the expected true return, assuming positive expected returns.

When single-period rebalanced returns are compounded, however, the bias in each single period's return is compounded as well. For such compounded returns, the effects of the closing-price-related biases are most easily expressed in terms of autocovariances.[10] Let $\hat{v}_{RB}$ be the computed product of rebalanced return relatives for two successive single periods. That is,

$$\hat{v}_{RB} = [\frac{1}{N}\sum_{i=1}^{N}\hat{R}_i(t-1)][\frac{1}{N}\sum_{i=1}^{N}\hat{R}_i(t)] \ , \tag{15}$$

where $\hat{R}_i(t) \equiv 1 + \hat{r}_i(t)$. For a buy-and-hold portfolio, the two-period product of return relatives is

$$\hat{v}_{BH} = \frac{1}{N}\sum_{i=1}^{N}\hat{R}_i(t-1)\hat{R}_i(t) \ . \tag{16}$$

Let $E\{\hat{R}_i(t-1)\} = E\{\hat{R}_i(t)\} = \mu_i$. Then

$$E\{\hat{v}_{RB} - \hat{v}_{BH}\} = (\overline{\mu_i})^2 - \overline{\mu_i^2} + \text{cov}\{\overline{\tilde{R}_i(t-1)},\ \overline{\tilde{R}_i(t)}\}$$

$$- \overline{\text{cov}\{\tilde{R}_i(t-1),\ \tilde{R}_i(t)\}}\ , \qquad (17)$$

where bars again indicate that the underlying quantity is averaged over i.

By the Cauchy-Schwartz inequality, the sum of the first two terms on the right in (17) is less than or equal to zero. If $\hat{v}_{RB}$ exceeds $\hat{v}_{BH}$ on average, which occurs in the empirical results presented later, then the sum of the last two terms in (17) must be positive. Both effects described earlier are consistent with such a result. The bid-ask and infrequent trading effects lead to negative values of $\text{cov}\{\hat{R}_i(t-1),\ \hat{R}_i(t)\}$;[11] infrequent (nonsynchronous) trading leads to positive values of $\text{cov}\{\overline{\tilde{R}_i(t-1)},\ \overline{\tilde{R}_i(t)}\}$.

3. EMPIRICAL RESULTS

3.1 The Sample and Portfolio Returns

The assignment of firms to portfolios follows the approach used in numerous studies and is identical to that of Reinganum (1982b).[12] At the beginning of each calendar year, firms are ranked by the total market value of common stock at the end of the previous year and then partitioned into ten portfolios of an equal number of securities each, give or take a security. The sample for each year includes every firm for which the CRSP Daily Master File contains price per share and number of shares outstanding for the end of the previous year.[13] This procedure yields a set of 10 portfolios for each of the eighteen years from 1963 through 1980, and the number of firms in the sample at the beginning of each year ranges from 1456 in 1963 to 2583 in 1976.

For each year, we calculate two series of daily returns on each of the ten portfolios. Both return series assume that an equal amount is invested in each security at the beginning of the year. The first series assumes that no rebalancing occurs during the year--a buy-and-hold strategy.[14] The second assumes that the investments in each security are rebalanced at the closing prices each day to the initial equal proportions--a rebalanced strategy. In the buy-and-hold series, each dividend is assumed reinvested in the issue paying the dividend, whereas in the rebalanced series each dividend is reinvested equally over all issues.[15] Any stock that is dropped or delisted from the CRSP file during the year is assumed sold at the last available price. In the buy-and-hold series, the proceeds are reinvested in the stocks remaining in the portfolio in proportion to the then current portfolio weights, whereas in the rebalanced series, the proceeds are reinvested equally in the remaining stocks. There is no adjustment in either series for transaction costs.

3.2 Single Period Returns

Previous studies, beginning with Reinganum (1981a), use rebalanced portfolios and compute differences between the average daily returns on the portfolio with the smallest firms and the portfolio with the largest firms. These differences--the so-called size effect--average about 0.1 percent per day. Due to the bid-ask bias, the average computed daily return overestimates the expected true daily return. If the magnitude of the bias differs across firm size, then the difference between returns for two portfolios is also biased. Average returns for buy-and-hold strategies contain less bid-ask bias, due to the diversification effect discussed above.

The empirical results are consistent with the existence of a bid-ask bias. In virtually every instance, the average daily rebalanced return exceeds the average daily buy-and-hold return, but the numerical difference is much greater for the portfolios of smaller firms (table 1). For the large-firm (tenth) portfolio, the average difference for the overall period is only 0.001 percent per day, whereas the average difference for the small-firm (first) portfolio is 0.056 percent—over fifty times as great. The differences between the two strategies decline monotonically with increases in firm size.

Using rebalanced portfolios, the average size effect for the overall period is 0.105 percent per day, which is close to the similarly calculated estimates in earlier studies. However, for buy-and-hold portfolios, the size effect is only 0.051 percent—less than half of the rebalanced size effect.

The summary statistics for three six-year subperiods presented in table 1 suggest that the size effect is nonstationary across subperiods, a result previously noted by Blume and Friend (1974) and by Brown, Kleidon, and Marsh (1982). The nonstationarity is particularly evident in the buy-and-hold results. In the first and third subperiods, average buy-and-hold portfolio returns decline as one moves from smallest to largest, but that pattern is reversed in the second subperiod. In fact, the largest firms' returns in the second subperiod exceed those of the smallest firms by an average of 0.045 percent with a t statistic of 2.22. Although the size effect in the second subperiod with the rebalanced portfolios is also much smaller, there is no reversal as there is with the buy-and-hold strategy.

In an analysis of seasonality, Keim (1982) reports that approximately half of the average size effect can be attributed to the month of January. The average daily returns for rebalanced portfolios shown in table 2 are consistent with Keim's analysis. In contrast, inferences about the size effect based upon buy-and-hold returns differ markedly from those of Keim. On average over the 1963-80 period, there is a strong January seasonal but no pronounced size effect over the eleven months from February through December. The average return on the smallest firms minus the average return on the largest firms is -0.005 percent per day with a t statistic of -0.47. In fact, of the ten portfolios, the smallest-firm portfolio actually has the lowest average daily return from February through December. Moreover, for each of the twelve months, the average size effect for the rebalanced strategy exceeds that for the buy-and-hold strategy by about 0.05 percent per day. This relatively constant difference is to be expected if the bid-ask phenomenon is stationary over time.

A closer examination of the month-by-month buy-and-hold returns from February through December reveals some size effects that, depending on the month, go in either direction. For example, average returns in February decline monotonically moving from smallest to largest, and the February size effect is 0.136 percent per day with a t statistic of 3.39. In contrast, October and November each exhibit opposite size effects of about the same magnitude: October has a size effect of -0.118 percent with a t statistic of -2.84, and November's size effect is -0.100 percent with a t statistic of -2.63. Average portfolio returns for each of those months increase almost monotonically moving from smallest to largest firms. Whether these month-by-month averages truly reflect a size-related phenomenon is unclear. Even the largest numbers in absolute value for these eleven months are small in comparison to those of January--February's value is about one-fifth of January's.

On average over these eighteen years, there seems to be little evidence of any consistent size effect in the last eleven months of the year. Indeed, the full-year average size effect of 0.051 percent is roughly 1/12 of the

January value of 0.649 percent.[16] Nonetheless, an examination of the data by subperiods discloses some non-stationary size effects in the last eleven months of the year. For the last eleven months, the size effect is 0.096 percent for 1963-68, -0.113 percent for 1969-74, and 0.0041 percent for 1975-80 with respective t-values of 6.45, -6.13, and 0.20. For comparison, the January effect varies only slightly across these three subperiods relative to its magnitude—from 0.429 percent for 1963-68 to 0.815 percent for 1975-80 with a minimum t-value of 6.40.

The portfolio returns discussed above are not adjusted for risk because there is no longer a generally agreed upon method for risk adjustment and previous studies, such as Reinganum (1981b, 1982a), find that various methods of risk adjustment do little to change inferences about the size effect. Nevertheless, some analysis of risk-adjusted returns is certainly warranted.

A common criterion for adjusting for risk is to define excess returns as those that violate the implications of the Sharpe-Lintner version of the two-parameter model. To implement this criterion, define R_{St} as the return on the small-firm portfolio, R_{Lt} as the large-firm return, R_{Mt} as the return on the market portfolio, and R_{Ft} as the riskless interest rate. Consider the regression equation

$$(R_{St} - R_{Lt}) = \alpha + \beta(R_{Mt} - R_{Ft}) + \varepsilon_t \, , \qquad (15)$$

where ε_t is an independent disturbance with zero expectation. The Sharpe-Lintner model implies that $\alpha = 0$. A nonzero value of α is interpreted as the excess return of small firms relative to large firms or, alternatively, the risk-adjusted size effect.

Roll (1981) suggests that infrequent trading is more often associated with small firms, and, if so, ordinary least-squares estimates of betas for these firms using daily data may be downward biased. The aggregated coefficient method of Dimson (1979) can be used to adjust for this effect by estimating the regression

$$(R_{St} - R_{Lt}) = \alpha + \sum_{k=-15}^{5} \beta_k (R_{M,t+k} - R_{F,t+k}) + \varepsilon_t , \qquad (16)$$

where R_M is the daily return on the S & P 500 index and R_F is the daily return on a one-month T-bill (held constant within a given calendar month). The estimates of α and their t statistics are reported as "Sharpe-Lintner excess returns" in table 2. The magnitude of the size effect is reduced slightly, but the changes are too small to alter any of the previous discussion.

3.3 Compounded Returns

Following the practice of much prior research, the last section reports estimates of daily expected returns, but also of interest are expected returns over longer periods of time. As shown in table 3, the compounded returns for the daily rebalanced portfolios exceed those of the buy-and-hold portfolios of the same firms, and the difference is greatest for the portfolios of the smaller firms.[17] The average yearly holding period return decreases monotonically with firm size for the rebalanced portfolios and, with one exception, for the buy-and-hold portfolios. The average yearly size effect for the rebalanced portfolios is 41.33 percent, and for the buy-and-hold portfolio 21.76 percent with t-values of 3.46 and 2.07.

Table 3 also reports the average holding period returns for both January and February through December. The average size effect using the buy-and-hold portfolios is 15.53 percent in January but only 3.91 percent in the last eleven months. The t-values of these two numbers are 5.96 and 0.45 respectively, suggesting that the size effect is only significant on average in January—the same conclusion as reached with the analysis of the daily returns themselves. In contrast, the rebalanced compounded returns indicate a significant size effect on average in both January and the remaining eleven months.

4. A FURTHER INVESTIGATION OF THE BIAS

Demsetz (1968) and more recently Branch and Freed (1977) postulate and find that the bid-ask spread of an individual stock as a percentage of its price is negatively and strongly related to the price of the stock itself in models that hold other possible explanatory variables constant. Since the bias due to the bid-ask effect is related to the variance of the percentage bid-ask spread, it seems natural to examine the relation between price and this bias. As a rough attempt to hold other variables constant, the subsequent analysis of price will control for differences in market value.

For each year, the stocks in each market decile are partitioned into subgroups according to the closing price of the prior year. The price classifications are \$2 or less, \$2 to less than \$5, \$5 to less than \$10, and \$15 to less than \$20. Stocks with closing prices of \$20 or more are dropped since the two studies cited above suggest that the strong relation of percentage bid-ask spread to price is due primarily to lower priced stocks.

The expected difference between the calculated average daily rebalanced and buy-and-hold return is a function of the number of securities in the portfolio and increases as the number of securities increases. With 10 or more securities, an examination of (13) shows that the expected differences in the calculated returns do not change rapidly with the number of securities. For example, the expected difference in calculated returns for a portfolio of 100 securities, all with the same statistical properties, is only 10% greater than for a portfolio of 10 of these securities. Thus, to avoid having to consider explicitly the number of securities in each portfolio, the following analysis uses only portfolios of 10 or more securities.

The average daily differences between the rebalanced and buy-and-hold returns, shown in table 4, exhibit a strong negative relation to price, when market value is held constant, but little, if any, relation to market value, once price is held constant. For instance, for the second smallest market value group, the average difference between the calculated daily percent returns for the rebalanced and buy-and-hold portfolios is 0.136 for the 0 to $2 range but only 0.0008 for the $15 to $20 price range. Similar negative relations appear in every market value category for which data are available.

To test the significance of the relations suggested by table 4, the yearly differences between the average daily returns are regressed upon a constant, four dummy variables for price classes, and nine dummy variables for market value classes. In addition, to allow for possible contemporaneous correlation in the residuals, the regression includes seventeen dummy variables for time periods. The adjusted R-squared is 0.84. The F-statistic that tests whether the coefficients on the price dummy variables are jointly zero is 394.84 with 4 and 397 degrees of freedom, which is significant at any usual level. The corresponding F-statistic for the coefficients on the dummies for market value is 0.3417 with 9 and 397 degrees of freedom, which is not significant at any usual level.[18]

In sum, the differences between rebalanced and buy-and-hold average daily returns are significantly related to price and weakly, if at all, to market value. The strong negative relation to price is consistent with the "bid-ask" phenomenon, but it does not preclude other explanations.

5. CONCLUSIONS

Individual stock returns computed with closing prices are upward biased, primarily due to a "bid-ask" effect. The computed return on a rebalanced portfolio is also upward biased, since such a return is simply an arithmetic average of returns on individual stocks. The computed return on a buy-and-hold portfolio largely avoids the bid-ask bias due to a "diversification" effect. The size of the bias in daily returns on stocks of small firms is sufficient to alter substantially conclusions about the size effect. Based on buy-and-hold daily returns, the full-year size effect is half as large as previously reported using rebalanced returns, and all of the size effect is due to the month of January.

The implications of this study reach beyond consideration of the size effect. Any study that forms equally-weighted portfolios should be alert to the potential biases introduced by the use of quoted closing prices in calculating returns. Evidence presented here indicates that these biases can sometimes be substantial. These biases can be greatly reduced by using returns implicit in a buy-and-hold strategy.

FOOTNOTES

[1]Banz (1981) finds a significant size effect using monthly returns for New York Stock Exchange (NYSE) stocks during the 1936-75 period. Evidence of a size effect also appears in earlier literature. Friend, Blume, and Crockett (1970, pp. 52-59) find that, for the 1964-68 period, risk-adjusted returns on equally-weighted portfolios of NYSE stocks significantly exceed returns on value-weighted portfolios. Blume and Friend (1974) find that, for the 1928-68 period, risk adjusted returns on small stocks exceed those of large stocks, with the exception of the 1948-58 subperiod when the reverse occurs. These results are roughly consistent with those of Banz.

[2]See also Reinganum (1981a, 1981b).

[3]If there are no trades in a day, CRSP uses as the quoted closing price the average of the bid and ask prices. To the extent that the bid and ask prices are kept up to date, this practice of CRSP could help reduce the differences between true and quoted prices.

[4]Niederhoffer and Osborne (1966) explain how the bid-ask effect leads to "reversals," or negative autocorrelation in price changes. The infrequent or "nonsynchronous" trading effect, first discussed by Fisher (1966), is shown by Scholes and Williams (1977) to imply negative autocorrelation in individual security returns and positive autocorrelation in index (portfolio) returns.

[5]Our work is not without precedent, however. Although he does not consider the bid-ask effect, Fisher (1966) discusses how deviations of closing prices from "true" prices can bias computed returns.

[6]Although this example is intended primarily for illustration, it is at least roughly consistent with empirical evidence. For the 1963-1980 period, the average price per share for firms in the lowest market-value decile is about \$5. We randomly selected one day, December 13, 1973, and found an average closing bid-ask spread of \$0.19 for stocks on the NYSE with bid prices between \$4 and \$6. (Spreads were obtained from Stock Quotations on the New York Stock Exchange, a publication of Francis Emory Fitch.)

[7]If expected returns differ across stocks, then $r_{BH}(t)$ is not independent of the vector of prices at (t-1), $P'(t-1) \equiv [P_1(t-1), \ldots, P_N(t-1)]$. In that case, the expectations in (12) should first be taken conditional on $P(t-1)$. Then taking unconditional expectations shows that $\sigma^2\{\frac{\overline{\varepsilon_i(t-1)}}{P_i(t-1)}\}$ is replaced by $E\{\sigma^2[\frac{\overline{\varepsilon_i(t-1)}}{P_i(t-1)} \mid P(t-1)]\}$, where the expectation is taken over the distribution of $P(t-1)$. This step is omitted because it would make the exposition more cumbersome without materially affecting any of the conclusions.

[8]The true buy-and-hold return can be written, $r_{BH} = \Sigma\, w_i r_i$, where the

weights (w_i's) depend on past prices. If true returns for a given security are i.i.d., then

$$E\{r_{BH}\} = \Sigma \; E\{w_i\}E\{r_i\} \; .$$

If expected returns differ across securities, then the highest expected returns are multiplied by the highest expected weights, and $E\{r_{BH}\} > E\{r_{RB}\}$. If expected returns are identical for all securities, then $E\{w_i\} = 1/N$ for each i, and $E\{r_{BH}\} = E\{r_{RB}\}$. See also Cheng and Deets (1971) for a discussion of buy-and-hold versus rebalanced portfolios.

[9]The framework used here to analyze infrequent trading is the same as that of Scholes and Williams (1977). They show that continuously compounded returns are unbiased. That is, $E\{\ln[1 + \hat{r}_i(t)]\} = E\{\ln[1 + r_i(t)]\}$. It is easily seen that this is also true in the presence of the bid-ask effect, since $E\{\ln[1 + \delta_i(t)/1 + \delta_i(t - 1)]\} = 1$ if it is further assumed that the $\delta_i(t)$'s are identically distributed over time.

[10]A paper by Roll (1983), which came to our attention after earlier versions of this paper were written, also analyzes compounded returns in this context. He obtains results similar to those reported here (table 3).

[11]As shown by Cohen, Maier, Schwartz and Whitcomb (1979, p. 158) for the bid-ask effect, $\mathrm{cov}\{\hat{R}_i(t-1), \hat{R}_i(t)\} \approx -\sigma^2\{\delta_i(t)\}$. That is, if $\hat{P}_i = P_i \pm \varepsilon_i$, with either occurrence equally likely, then $\mathrm{cov}\{\hat{R}_i(t-1), \hat{R}_i(t)\} \approx -[\varepsilon_i/P_i]^2/4$.

[12]The precise method of firm selection varies across studies. A sample of 566 firms is used by Reinganum (1981a) and, subsequently, by Brown, Kleidon, and Marsh (1982). Reinganum (1981b) and Keim (1982) use all NYSE and AMEX firms, but they require that the security remains on the file for the entire year. All these methods, however, yield similar size effects.

[13]Some firms are excluded because data for number of shares outstanding are missing even though price data are available. This situation occurs more often for smaller firms.

[14]The analysis in section 2 assumes an equal investment at time 0 in each security, based on <u>true</u> prices. To implement this assumption empirically, the quoted price series for each stock is deflated by the initial <u>quoted</u> price rather than the true price. In other words, the implicit numbers of shares depend on initial quoted rather than true prices. The buy-and-hold return for the first day of the year is thereby biased by the same magnitude as the rebalanced return. (The first-day returns for both strategies are indeed identical.) Thereafter, however, the buy-and-hold returns reduce the bias through the diversification effect embodied in (12), because the implicit number of shares purchased is independent of subsequent returns (both true and computed). The bias in the first-day buy-and-hold return could be reduced in the same manner as subsequent days by instead deflating each price series by

the penultimate price of the prior year—a kind of instrumental variables approach. Such a refinement would, however, have a negligible effect upon the values reported in the tables.

[15]Following the procedures of numerous earlier studies, the dividends are assumed reinvested on ex-dividend dates despite the fact that the dividends would not be available until payment date. Since dividends are actually paid to stockholders at a later date, this may introduce a bias, but the magnitude should be small. Moreover, there is little reason to believe that this bias differs as between the buy-and-hold and the rebalanced strategies.

[16]We investigated whether the seasonality is sensitive to the time of the initial portfolio formation. The same year-end rankings were used to form buy-and-hold portfolios with equal weights at the beginning of the subsequent July, and the portfolios were then held through the following June. The full-year and month-by-month returns were virtually identical to those in table 2.

[17]See Blume and Friend (1974) for a comparison of five-year compounded returns of monthly rebalanced to buy-and-hold portfolios of NYSE stocks. They find substantial differences in the computed compounded returns between the two strategies in the 1928-48 period, but little difference in the 1948-68 period. This suggests that the bid-ask effect may be important for monthly returns of NYSE stock prior to 1948.

[18]The F-statistic for the coefficients on the dummies for time is 7.89 with 17 and 397 degrees of freedom, which is significant at the one percent level, confirming the possible presence of contemporaneous correlation. However, the coefficients on the time dummy variables bear no obvious relation to time; the correlation between time itself and the time dummy variable coefficients is -0.028.

Also, the significance of the price dummy variables does not hinge upon the inclusion of the time dummy variables, although the inclusion of the time variables probably improves the specification of the regression. In the regression excluding the time dummy variables, the calculated F-statistic for the price variables is 347.01 with 4 and 414 degrees of freedom, and the calculated F-statistic for the market value variables is 0.37 with 9 and 414 degrees of freedom.

REFERENCES

Banz, R. W., 1981, The relationship between return and market value of common stocks, Journal of Financial Economics 9, 3-18.

Blume, M. E. and I. Friend, 1974, Risk, investment strategy, and the long-run rates of return, Review of Economics and Statistics 56, 259-269.

Branch, B. and W. Freed, 1977, Bid-asked spreads on the AMEX and the Big Board, Journal of Finance 32, 159-163.

Brown, P., A. W. Kleidon, and T. A. Marsh, 1982, New evidence on the nature of size related anomalies in stock prices, forthcoming, Journal of Financial Economics.

Cheng, P. L. and M. K. Deets, 1971, Portfolio returns and the random walk theory, Journal of Finance 26, 11-30.

Cohen, K. J., S. F. Maier, R. A. Schwartz, and D. K. Whitcomb, 1979, On the existence of serial correlation in an efficient securities market, TIMS Studies in the Management Sciences 11, 151-168.

Demsetz, H., 1968, The cost of transacting, Quarterly Journal of Economics 82, 33-53.

Dimson, E, 1979, Risk measurement when shares are subject to infrequent trading, Journal of Financial Economics 7, 197-226.

Fisher, L., 1966, Some new stock market indices, Journal of Business 29, 191-225.

Friend, I., M. E. Blume, and J. Crockett, 1970, Mutual funds and other institutional investors (McGraw-Hill, New York).

Keim, D. B., 1982, Size related anomalies and stock return seasonality: Further empirical evidence, forthcoming, Journal of Financial Economics.

Niederhoffer, V. and M. F. M. Osborne, 1966, Market making and reversal on the Stock Exchange, Journal of the American Statistical Association 61, 897-916.

Reinganum, M. R., 1981a, Misspecification of capital asset pricing: Empirical anomalies based on earnings' yields and market values, Journal of Financial Economics 9, 19-46.

Reinganum, M. R., 1981b, The arbitrage pricing theory: Some empirical results, Journal of Finance 36, 313-321.

Reinganum, M. R., 1982a, A direct test of Roll's conjecture on the firm size effect, Journal of Finance 37, 27-35.

Reinganum, M. R., 1982b, The anomalous stock market behavior of small firms in January: Empirical tests for tax-loss selling effects, forthcoming, Journal of Financial Economics.

Roll, R., 1981, A possible explanation of the small firm effect, Journal of Finance 36, 879-888.

Roll, R., 1983, On computing mean returns and the small firm premium, manuscript, University of California, Los Angeles.

Scholes, M. and J. Williams, 1977, Estimating betas from nonsynchronous data, Journal of Financial Economics 5, 309-327.

TABLE 1

REBALANCED VERSUS BUY-AND-HOLD PORTFOLIOS
Average Daily Percent Returns[a]

Size Decile	Rebalanced Portfolio 1963-68	1969-74	1975-80	1963-80	Buy-and-Hold Portfolio 1963-68	1969-74	1975-80	1963-80	Rebalanced Minus Buy-and-Hold 1963-68	1969-74	1975-80	1963-80
	Average Return/(Standard Deviation) (in parentheses)[b]								Average Difference/(t Statistic)[c]			
Smallest	0.184 (0.658)	0.005 (1.01)	0.235 (0.913)	0.141 (0.881)	0.170 (0.731)	-0.054 (0.990)	0.142 (0.952)	0.085 (0.905)	0.015 (1.31)	0.059 (13.94)	0.093 (24.10)	0.056 (22.65)
2	0.147 (0.666)	-0.034 (0.996)	0.184 (0.854)	0.099 (0.856)	0.139 (0.724)	-0.067 (0.955)	0.140 (0.890)	0.070 (0.868)	0.008 (2.41)	0.032 (9.00)	0.045 (14.47)	0.028 (14.81)
3	0.129 (0.654)	-0.028 (1.01)	0.164 (0.854)	0.088 (0.857)	0.126 (0.697)	-0.048 (0.978)	0.137 (0.886)	0.072 (0.867)	0.003 (1.33)	0.019 (6.51)	0.026 (9.89)	0.016 (10.41)
4	0.116 (0.653)	-0.040 (1.01)	0.162 (0.860)	0.079 (0.858)	0.111 (0.687)	-0.052 (0.965)	0.146 (0.893)	0.068 (0.862)	0.005 (2.73)	0.013 (4.85)	0.016 (6.93)	0.011 (8.54)
5	0.098 (0.635)	-0.039 (0.959)	0.140 (0.869)	0.066 (0.837)	0.094 (0.661)	-0.047 (0.912)	0.128 (0.903)	0.058 (0.838)	0.004 (2.42)	0.008 (3.20)	0.012 (5.56)	0.008 (6.48)
6	0.090 (0.602)	-0.036 (0.950)	0.140 (0.859)	0.065 (0.822)	0.091 (0.618)	-0.039 (0.899)	0.133 (0.889)	0.062 (0.817)	-0.001 (-0.63)	0.003 (1.06)	0.007 (3.28)	0.003 (2.27)
7	0.085 (0.561)	-0.032 (0.926)	0.124 (0.790)	0.059 (0.778)	0.083 (0.576)	-0.035 (0.879)	0.118 (0.815)	0.055 (0.772)	0.002 (1.93)	0.003 (1.22)	0.006 (3.79)	0.004 (3.46)
8	0.070 (0.553)	-0.029 (0.951)	0.110 (0.777)	0.051 (0.781)	0.068 (0.565)	-0.031 (0.898)	0.105 (0.799)	0.047 (0.770)	0.002 (2.39)	0.002 (0.86)	0.005 (3.81)	0.003 (3.31)
9	0.065 (0.520)	-0.024 (0.908)	0.100 (0.744)	0.047 (0.744)	0.063 (0.533)	-0.025 (0.868)	0.096 (0.764)	0.045 (0.738)	0.002 (1.58)	0.001 (0.30)	0.003 (2.95)	0.002 (2.28)
Largest	0.046 (0.493)	-0.009 (0.895)	0.070 (0.765)	0.036 (0.739)	0.045 (0.502)	-0.010 (0.874)	0.069 (0.778)	0.035 (0.737)	0.001 (2.14)	0.001 (0.75)	0.001 (1.05)	0.001 (2.00)
	Average Difference/(t Statistic)[c]											
Smallest minus Largest	0.138 (10.28)	0.014 (0.65)	0.165 (7.83)	0.105 (9.67)	0.125 (8.50)	-0.045 (-2.22)	0.073 (3.42)	0.051 (4.60)				

[a]The reported statistics are derived from two series of daily returns for each year from 1963 through 1980 for each of ten portfolios formed by market value as of the end of the prior year. The first series are the daily returns for a specific year for a specific portfolio resulting from a daily rebalanced strategy, and the second series are the daily returns for a specific year for a specific portfolio resulting from a buy-and-hold strategy. Both strategies assume an equal amount invested in each security at the beginning of each year.

[b]The arithmetic averages of the daily returns for the years indicated for each size portfolio are shown. The numbers in parentheses are the standard deviations of the series.

[c]The indicated series of daily returns are differenced. The averages of these differences are given along with the t-values calculated on the basis of these differences, thus adjusting for any dependence between the original series.

TABLE 2

PORTFOLIO RETURNS BY MONTH, 1963-1980
Average Daily Percent Returns[a]

Size Decile	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	February through December
					Buy-and-Hold: Average Return/(Standard Deviation)[b]								
Smallest	0.731 (1.25)	0.128 (0.901)	0.055 (0.821)	0.019 (0.666)	-0.023 (0.935)	-0.012 (0.769)	0.047 (0.794)	0.086 (0.794)	0.080 (0.707)	-0.099 (0.955)	-0.027 (0.919)	0.037 (0.894)	0.026 (0.841)
2	0.518 (1.03)	0.064 (0.786)	0.068 (0.851)	0.039 (0.670)	-0.051 (0.965)	-0.016 (0.805)	0.070 (0.795)	0.054 (0.802)	0.065 (0.715)	-0.088 (0.952)	0.040 (0.962)	0.081 (0.845)	0.029 (0.839)
3	0.445 (0.992)	0.039 (0.769)	0.060 (0.866)	0.054 (0.687)	-0.032 (0.979)	0.005 (0.813)	0.061 (0.803)	0.047 (0.811)	0.077 (0.734)	-0.051 (0.956)	0.062 (0.953)	0.093 (0.858)	0.037 (0.846)
4	0.388 (0.940)	0.041 (0.738)	0.071 (0.868)	0.058 (0.710)	-0.037 (0.970)	0.004 (0.814)	0.057 (0.792)	0.055 (0.787)	0.064 (0.766)	-0.050 (0.947)	0.075 (1.02)	0.089 (0.839)	0.038 (0.848)
5	0.334 (0.906)	0.025 (0.715)	0.049 (0.872)	0.070 (0.701)	-0.046 (0.908)	0.005 (0.782)	0.049 (0.780)	0.042 (0.792)	0.057 (0.770)	-0.062 (0.934)	0.094 (0.949)	0.084 (0.819)	0.033 (0.827)
6	0.288 (0.858)	0.024 (0.673)	0.045 (0.858)	0.061 (0.720)	-0.027 (0.889)	-0.001 (0.769)	0.062 (0.776)	0.067 (0.767)	0.077 (0.750)	-0.043 (0.891)	0.094 (0.945)	0.094 (0.803)	0.041 (0.810)
7	0.242 (0.779)	0.020 (0.639)	0.039 (0.790)	0.065 (0.680)	-0.022 (0.815)	0.000 (0.718)	0.048 (0.735)	0.061 (0.761)	0.051 (0.728)	-0.030 (0.855)	0.100 (0.903)	0.090 (0.751)	0.038 (0.769)
8	0.182 (0.765)	0.001 (0.655)	0.050 (0.790)	0.061 (0.693)	-0.028 (0.841)	0.006 (0.719)	0.033 (0.749)	0.051 (0.741)	0.050 (0.752)	-0.015 (0.830)	0.097 (0.881)	0.082 (0.763)	0.035 (0.769)
9	0.149 (0.713)	-0.006 (0.609)	0.042 (0.709)	0.071 (0.668)	-0.029 (0.811)	0.014 (0.690)	0.040 (0.714)	0.043 (0.730)	0.046 (0.744)	-0.004 (0.793)	0.101 (0.871)	0.070 (0.745)	0.035 (0.739)
Largest	0.082 (0.683)	-0.008 (0.622)	0.034 (0.689)	0.069 (0.684)	-0.014 (0.793)	0.015 (0.688)	0.018 (0.721)	0.037 (0.751)	0.014 (0.762)	0.019 (0.840)	0.073 (0.839)	0.077 (0.724)	0.030 (0.742)
					Buy-and-Hold: Average Difference in Returns/(t-Statistic)								
Smallest minus Largest	0.649 (12.00)	0.136 (3.19)	0.021 (0.63)	-0.051 (-1.68)	-0.009 (-0.29)	-0.027 (-0.85)	0.029 (0.96)	0.049 (1.56)	0.066 (1.95)	-0.118 (-2.84)	-0.100 (-2.63)	-0.040 (-1.05)	-0.005 (-0.47)
					Buy-and-Hold: Sharpe-Lintner Excess Returns/(t-Statistic)								
Smallest minus Largest	0.544 (9.95)	0.120 (3.08)	0.024 (0.83)	-0.069 (-2.61)	0.013 (0.42)	-0.040 (-1.42)	0.027 (0.97)	0.066 (2.38)	0.034 (1.22)	-0.108 (-2.98)	-0.090 (-2.55)	-0.055 (-1.58)	-0.006 (-0.64)
					Rebalanced: Average Difference in Returns/(t-Statistic)								
Smallest minus Largest	0.704 (13.14)	0.204 (5.09)	0.076 (2.31)	0.010 (0.32)	0.055 (1.68)	0.021 (0.68)	0.080 (2.62)	0.084 (2.65)	0.120 (3.69)	-0.058 (-1.47)	-0.045 (-1.25)	0.013 (0.32)	0.050 (4.77)

[a]The reported statistics are derived from the same series of daily returns described in footnote a of Table 1.

[b]The arithmetic averages of the daily returns for the months indicated for each size portfolio for the 18 years are shown. The numbers in parentheses are the standard deviations of the relevant series.

[c]The indicated series are differenced. The averages of these differences are given along with the t-values calculated on the basis of these differences.

TABLE 3

AVERAGE COMPOUNDED RETURNS
1963-1980

Size Decile	January (1-month)	Feb.-Dec. (11-months)	Full Year (12 months)
	Rebalanced: Average Compounded Returns[a]		
Smallest	18.91%	25.32%	51.29%
2	12.78	17.33	33.44
3	10.84	15.14	28.75
4	9.29	14.07	25.63
5	7.91	11.41	20.99
6	6.76	11.85	20.11
7	5.62	11.10	17.93
8	4.24	9.96	15.22
9	3.44	9.40	13.65
Largest	1.86	7.72	9.96
	Buy-and-Hold: Average Compounded Returns[a]		
Smallest	17.40%	11.32%	31.42%
2	12.11	10.54	24.62
3	10.35	11.21	23.56
4	8.98	11.23	22.02
5	7.70	9.41	18.48
6	6.60	11.18	19.15
7	5.52	10.18	16.18
8	4.16	9.16	14.28
9	3.42	8.91	13.10
Largest	1.87	7.41	9.66
Rebalanced: Average Difference in Compounded Returns/(t-statistics)[b]			
Smallest minus Largest	17.05% (5.81)	17.60% (2.13)	41.33% (3.46)
Buy-and-Hold: Average Difference in Compounded Returns/(t-statistics)[b]			
Smallest minus Largest	15.53% (5.96)	3.91% (0.45)	21.76% (2.07)

[a]The reported statistics are derived from the same series used in table 1. For each year, the daily returns for the indicated portfolios are linked together to yield January, February through December, and one-year holding period returns. The averages of these holding period returns over eighteen years are reported. In view of the limited number of observations and the potential non-normality of yearly holding period returns, standard deviations are not presented.

[b]The individual series are differenced. The averages of these differences are given along with the t-values calculated on the basis of these differences.

TABLE 4

AVERAGE DAILY RETURNS FOR REBALANCED MINUS BUY-AND-HOLD PORTFOLIOS CROSS-CLASSIFIED BY MARKET VALUES AND STOCK PRICES, 1963-1980
(Number of Years of Available Data Shown in Parentheses)[a]

Price Range	Market Value Decile									
	1	2	3	4	5	6	7	8	9	10
$0 to less than $2	0.1412% (13)	0.1360% (8)	0.1425% (3)	0.1287% (1)	0.0987% (1)					
$2 to less than $5	0.0378 (17)	0.0454 (15)	0.0458 (9)	0.0416 (8)	0.0391 (6)	0.0373 (3)	0.0289 (2)	0.0444 (1)		
$5 to less than $10	0.0182 (15)	0.0195 (17)	0.0150 (16)	0.0149 (17)	0.0106 (11)	0.0059 (8)	0.0119 (8)	0.0170 (4)	0.0157 (2)	
$10 to less than $15	0.0075 (11)	0.0099 (15)	0.0064 (16)	0.0073 (17)	0.0075 (15)	0.0078 (15)	0.0048 (11)	0.0060 (8)	0.0078 (7)	0.0060 (4)
$15 to less than $20	0.0027 (3)	0.0008 (12)	0.0030 (15)	0.0043 (16)	0.0057 (16)	0.0027 (17)	0.0041 (15)	0.0056 (13)	0.0064 (10)	0.0038 (7)

[a]For each year, portfolios are formed by market value at the end of the prior year. Each of these portfolios is then partitioned according to the stock price at the end of the prior year; at this stage, any stock with a stock price of $20 or more is discarded for reasons discussed in the text. For each year for which there is a market value-price portfolio, the difference between the average daily rebalanced and buy-and-hold returns is calculated. The averages of these differences are reported in this table along with the number of years of available data.