INFLATION AND MILLER'S MODEL OF CAPITAL STRUCTURE

by

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Working Paper No. 10-83

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Preliminary
Not for quotation

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*This paper has benefitted from the comments of M. Flannery, G. Gerber, R. Papetti, and A. Santomero.

I. Introduction

Academic studies concerning inflation have grown in tandem with the price level. A number of researchers have considered the effect of the rate of inflation on the interest rate. The simplest models show that, without taxes, a one-percent rise in the rate of anticipated inflation increases nominal interest rates by one-percent. Recently, the effect of taxation on the relationship between interest rates and inflation has been investigated. For example, Darby [1975] has pointed out that a one-percent increase in the inflation rate should increase the after-tax interest rate by one percent, implying a greater movement in the pre-tax interest rate. This result has been confirmed and extended by Gandolfi [1976], Feldstein [1976], and others.

However, empirical evidence has not supported the above theoretical work. Beginning with Irving Fisher, many researchers have found that a one-percent increase in the inflation rate yields less than a one percent increase in the pre-tax interest rate. The results of Feldstein-Eckstein [1970] and Fama [1975] suggest a one-to-one relationship. To our knowledge, there is no convincing evidence of a greater than one-to-one relationship.

In addition to the lack of empirical validation, there is at least one conceptual shortcoming in past studies of inflation and interest rates. The discipline of corporate finance has not produced a theory of corporate capital structure suitable to macromodels. For many years the basic paradigms here have been the Modigliani-Miller theorems in [1958] and [1963]. These papers yield the unrealistic conclusion that, in the presence of corporate taxes, a firm should issue only debt. An interior optimum to the debt to equity ratio can occur when bankruptcy costs, agency costs, and signalling behavior are considered. However, due to their qualitative nature, the received results are not easily applied to macromodels.

Miller [1977] has recently provided an elegant theory of capital structure in the presence of both personal and corporate taxes. Our paper shows that this framework can be readily integrated with a macromodel. While Miller's work has been criticized,[1] its general concepts are of such central importance to corporate finance that its ramifications, such as the effect of inflation on interest rates, should be investigated.

Our paper yields at least three developments. First, we show a set of circumstances in which the theoretical relationship between interest rates and inflation is near that observed empirically. We do not wish to overplay this result since another set of circumstances in our model produces a responsiveness of the rate of interest to inflation above that found by Darby. Also, we are by no means the first to develop a theory to explain the empirical results concerning the Fisher effect. For example, Mundell [1963] argues that a cash balance effect can lower the responsiveness of interest rates to inflation rate changes. Gandolfi [1982] obtains similar results by considering depreciation changes and capital gains taxes.

More importantly, we posit a macromodel that is consistent with both the theory and practice of corporate finance. Many previous works posit only debt financing, an assumption not consistent with real world practice. Others, such as Feldstein, Green and Sheshinski [1978], assume mixed financing but do not ground their treatment in corporate financial theory.

Thirdly, the effect of inflation on income distribution is considered. Some previous works have posited that any change in taxes due to inflation is immediately restored to the public in such a way that the income of each individual, his saving behavior, and/or his utility curves are unaltered.

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[1] For example, see DeAngelo and Masulis [1980] and Taggart [1980].

This assumption seems acceptable in these models since the results are essentially obtained in a world of identical individuals. However, individuals are clearly not identical in Miller's system of progressive taxation. Results concerning income redistribution are, we feel, useful additions to our paper.

Our purpose is to incorporate Miller's insight into the received macro-models. We use the simplest assumptions whenever possible in order to compare most clearly our results with those of the literature. Therefore, we mention only briefly such refinements as depreciation, capital gains taxes and real balance effects.

Our paper is organized as follows. A review of the Miller model is presented in section II. In section III, we analyze the impact of changes in the rate of inflation on interest rates in the basic Miller model. A generalization of our results to a world with capital gains taxes appears in section IV. Concluding remarks are presented in section V.

II. Review of the Miller Model

Miller [1977] posits a model with a flat corporate tax rate of tc and a system of graduated personal taxes. Firms desire to minimize the cost of capital after corporate taxes. Since interest costs are tax-deductible and payments to equity holders are not deductible, firms are indifferent between issuing equity and debt when:

$$i_D(1 - tc) = i_E \text{ ,}^{1} \qquad (1)$$

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[1]We assume perfect certainty in order to make our discussion in this section more compatible with the macro-models to be presented later.

where i_D is the pre-tax cost of debt

i_E is the required return on equity.

Under the assumption that individuals are taxed on interest income but do not pay taxes on equity returns,[1] the individual prefers an investment in equity when:

$$i_D(1 - tpi) < i_E \text{ ,} \tag{2}$$

where tpi is the marginal personal tax rate for individual i. An individual prefers debt when the inequality runs in the other direction. Inequality (2) holds for individuals in relatively high tax brackets and the reverse inequality holds for individuals in relatively low tax brackets.

Using the above model, Miller determines equilibrium from a demand and a supply schedule.[2] For given i_E, the demand for bonds is a positive function of i_D.[3] Only zero tax-bracket investors desire bonds when $i_D = i_E$. As i_D rises, more investors choose bonds. For a given i_E, the supply of bonds outstanding is infinitely elastic at the value of i_D such that equation (1) holds.

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[1]We employ this assumption initially because Miller uses it in the major example illustrating his model. He points out [p. 270] that "As a practical matter, however, the assumption that the effective [tax] rate [on equities], at the margin is close to zero may not be so wide of the mark. Keep in mind that a 'clientele effect' is also at work in the market for shares. The high dividend paying stocks will be preferred by tax exempt organizations and low income investors; those stocks yielding more of their return in the form of capital gains will gravitate to the taxpayers in the upper brackets. The tax rate on such gains is certainly greater than zero, in principle. But holders need pay no taxes on their gains until realized and only a small fraction of accumulated gains are, in fact, realized and taxed in any year." However, we relax the assumption in a later section of the paper.

[2]See Miller [1977], Figure 1, p. 269. The notation in our paper differs slightly from the notation in Miller's work.

[3]The demand curve is a positive function of i_D and therefore a negative function of the price of a consul, $\frac{1}{i_D}$. See Miller [1977], p. 268.

Since equation (1) must be true in equilibrium, we can use (2) to classify investors into three categories:

(A) Any high tax bracket investor (tpi > tc) invests only in stocks.

(B) Any low tax bracket investor (tpi < tc) invests only in bonds.

(C) Any marginal investor (tpi = tc) is indifferent between stocks and bonds.

Since (1) implies that the after-tax cost of debt is equal to the after-tax cost of equity, the value of an individual firm is unaffected by its debt to equity ratio. Thus, the amount of debt issued by a given firm is indeterminant. However, total debt in the economy must be equal to the aggregate holdings in financial assets by all individuals with tpi < tc.[1]

This section of the paper is intended merely to acquaint the reader with the basic Miller model rather than to extend the model or to analyze it rigorously. Previously, extension and analysis has taken two tacts. First, researchers such as DeAngelo and Masulis [1980] and Taggart [1980] have examined the properties of the model under a more complicated tax system and under uncertainty. Second, others such as Hamada [1982], have analyzed the conditions where arbitrage can not take place.[2]

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[1]Miller's model ignores holdings of money.

[2]For example, without rigorous specifications in the model, high tax bracket individuals might borrow on an individual level, using the proceeds to lend through a corporation.

III. The Effect of Inflation on Interest Rates

We posit a model where the interest rate is determined by the supply and demand of loanable funds.[1] We specify the saving and investment functions below. The two functions are equal at equilibrium. Inflation impacts the equilibrium since the after-tax rates of interest applicable to saving and investment are affected by the rate of change in prices.

A. The Investment Function

In a world with neither taxes nor inflation, the firm invests until:

$$MP_K = i \ ,$$

where MP_K is the marginal physical product of capital and i is the interest rate. At this point we need no distinction between the cost of equity and the cost of debt.

With both inflation and corporate taxes, the firm invests until:

$$MP_K(1 - tc) + \pi = b \cdot i_D(1 - tc) + (1 - b)i_E \ , \qquad (3)$$

where π is the rate of inflation and b is the proportion of the firm financed with bonds. Both i_D and i_E are nominal quantities.

Under inflation, the two returns to capital are (1) the marginal physical product of capital and (2) the price increase in the asset. The first return is taxed at the corporate rate. The second return escapes taxation under our assumption of no capital gains tax. This latter return is equal to π if all assets increase at the same rate and there is no depreciation. Equation (3) reflects the fact that firms can deduct interest payments but cannot deduct

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[1] This type of model has been used frequently in the analysis of interest rates, inflation, and taxes. For example, see Cross [1980] and Gandolfi [1976], [1982].

payments to equity holders.

Using (1), we can rewrite (3) as:

$$MP_K(1 - tc) + \pi = i_E = i_D(1 - tc) \ . \tag{4}$$

In accordance with Miller's theory, b is indeterminant in (3) and therefore, does not appear in (4). We can now solve for MP_K as:

$$MP_K = \frac{i_E - \pi}{1 - tc} = i_D - \frac{\pi}{1 - tc} \ . \tag{5}$$

As mentioned above, investment is carried out until (3) is reached. In terms of (5), the rate of investment can be written as:

$$f\left(i_D - \frac{\pi}{1 - tc}\right) = f\left(\frac{i_E - \pi}{1 - tc}\right) \ . \tag{6}$$

We employ the traditional assumption that $f' < 0$.

B. The Saving Function

1. The After-Tax Rate of Return

We next treat the saving function. For simplicity, we assume that there are two groups of individuals.[1] Each individual in group 1 has personal tax bracket, $tp1 < tc$, and each member in group 2 has tax bracket, $tp2 > tc$. In keeping with the basic Miller model, we assume that interest payments are taxed at the individual's personal tax rate while returns on equity are completely untaxed. Thus, given (1), all individuals in group 1 prefer bonds while all individuals in group 2 prefer stocks. Each member of group 1 has identical income and possesses the same saving function. Similarly, each

[1]The equilibrium condition of (1) still holds when there are only two groups. Our results could be derived for any arbitrarily large number of groups, though the mathematics would be more involved. Since little understanding would be added by this complex approach, we choose the assumption of two groups.

member of group 2 has an identical income (which is presumably higher than the income of those in group 1) and possesses the same saving function (which need not be the same as the saving function of each member of group 1).

The saving of each individual is a function of his real after-tax rate of return and his real income. This real after-tax return is $i_D(1 - tpl) - \pi$ for each member of group 1. From (1), we can express this return in terms of i_E as $i_E(\frac{1 - tpl}{1 - tc}) - \pi$.[1] The real after-tax return is $i_E - \pi$ for each member in group 2.

2. The Individual's Real Income

While the above calculation of the real after-tax interest rate is straightforward, the determination of an individual's income is more involved. We begin our discussion here with a world of no inflation, where each stockholder earns an after-tax rate of return of i_E^0. Each bondholder earns an after-tax rate of:

$$i_D^0(1 - tpl) = i_E^0(\frac{1 - tpl}{1 - tc}) . \tag{7}$$

With an inflation rate of π, the bondholder now earns a real after-tax rate of return of:

$$i_D(1 - tpl) - \pi = i_E(\frac{1 - tpl}{1 - tc}) - \pi , \tag{8}$$

implying that the change in the real after-tax return to the bondholder due to inflation is:

$$(i_E - i_E^0)(\frac{1 - tpl}{1 - tc}) - \pi . \tag{9}$$

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[1]While it is more intuitive to express a bondholder's return as a function of i_D, we shall frequently express this term as a function of i_E so that both the return to a stockholder and the return to a bondholder can be written as a function of the same parameter.

With no inflation, total taxes in the economy are:

$$C + J + D(i_D^0 \cdot tpl) - D(i_D^0 \cdot tc) = C + J + Di_D^0(tpl - tc) = C + J + Di_E^0\left(\frac{tpl - tc}{1 - tc}\right) , \quad (10)$$

where:

C is the corporate tax on net income before interest, i.e., the corporate tax if interest were not deductible,

J is the tax on labor income,

D is the debt outstanding in economy.

The third term on the left hand side (LHS) of (10) is the total tax paid by bondholders. The fourth term on the LHS is the total deduction taken by all corporations on interest paid to bondholders.

Under inflation, total taxes are:[1]

$$C + J + D\left(i_E\left(\frac{tpl - tc}{1 - tc}\right)\right) ,$$

implying from (10) that the change in taxes due to inflation is:

$$D\left((i_E - i_E^0)\left(\frac{tpl - tc}{1 - tc}\right)\right) . \quad (11)$$

This expression is negative since $tpl < tc$.

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[1]We assume that C, J, and D are invariant to the rate of inflation over a small interval of time. The constancy of C and J follows from the traditional assumption that investment over that interval is insignificant relative to the initial capital stock. Thus, the pre-tax incomes to capital and labor, both of which are functions of capital, can be treated as constants.

We assume that all debt has infinitesimal maturity so that capital gains and losses do not accrue to bondholders when the interest rate changes. In addition, no new debt is issued since the capital stock is a constant. Furthermore, if marginal tax brackets are unaffected by inflation, no existing equity (debt) is converted to debt (equity). Thus, debt is a constant.

With a constant level of real government spending and a balanced budget, a reduction in taxes in one sector of the economy must be offset by an increase in taxes in another sector. We assume that a head tax is used to keep the budget balanced, implying that marginal tax brackets are unaffected. For simplicity, we posit that the fraction K of the tax shortfall represented by (11) is paid by bondholders, and the fraction 1 - K of (11) is paid by stockholders.[1] From (9) and (11), we see that the income of bondholders rises (falls) during inflation if:

$$D\left[(i_E - i_E^0)\left(\frac{1 - tpl}{1 - tc}\right) - \pi - K(i_E - i_E^0)\left(\frac{tc - tpl}{1 - tc}\right)\right] \gtrless 0 \ . \qquad (12)$$

Since income is a constant in the economy, any increase (decrease) in the income of bondholders is offset by a decrease (increase) in the income of stockholders.

We write the aggregate saving function of the low tax bracket individuals during inflation as:

$$L(r^L_{A.T.},\ Y_L), \qquad (13A)$$

where $r^L_{A.T.}$ is the real after-tax interest rate and Y_L is the aggregate real income of these individuals. The arguments of the function L in (13A) are:[2]

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[1]We achieve similar results if the extra tax collected from each group is a more complex function of (11).

[2]For simplicity, we ignore the effect of real cash balances on the saving function. While a full treatment of this effect is algebraically complex, it is relatively straightforward conceptually. Following the work of Santomero [1974], we can include the cash balance effect by writing income as:

$$Y_L^* = Y_L - \ell^D\left(Y_L,\ i_E\left(\frac{1 - tpl}{1 - tc}\right),\ \pi\right)\pi \ , \qquad (13C')$$

where money demanded, ℓ^D, is a function of income, the nominal after-tax interest rate and the inflation rate. (Using Cagan [1956], we treat the interest rate and the inflation rate as separate arguments in (13C').) Given that money held is ℓ^D, the loss of income due to inflation is $\ell^D \cdot \pi$.

$$r^{L}_{A.T.} = i_E\left(\frac{1 - tpl}{1 - tc}\right) - \pi \tag{13B}$$

$$Y_L = \bar{Y}_L + D\left[(i_E - i^0_E)\left(\frac{1 - tpl}{1 - tc}\right) - \pi\right] - KD\left[(i_E - i^0_E)\left(\frac{tc - tpl}{1 - tc}\right)\right] . \tag{13C}$$

Some explanation of equation (13C) is needed. First, $\bar{Y}_L$ is the total income from both labor and capital accruing to low tax bracket individuals in a no-inflation world. The remainder of the expression is the left hand side of (12), the change in the bondholder's income from inflation.

The aggregate saving function of the high tax bracket individuals is:

$$H(r^{H}_{A.T.}, Y_H) , \tag{14A}$$

where $r^{H}_{A.T.}$ is the real after-tax interest rate for the high tax bracket individuals and Y_H is the aggregate real income of these individuals.

The arguments of the function H in (14A) are:

$$r^{H}_{A.T.} = i_E - \pi \tag{14B}$$

$$Y_H = \bar{Y}_H - D\left[(i_E - i^0_E)\left(\frac{1 - tpl}{1 - tc}\right) - \pi\right] + KD\left[(i_E - i^0_E)\left(\frac{tc - tpl}{1 - tc}\right)\right] . \tag{14C}$$

As with (13C), some explanation of equation (14C) is needed. First, $\bar{Y}_H$ is the total income from both labor and capital accruing to high tax bracket individuals in a no-inflation world. The second and third terms, which also appear in (13C), are the gains to the bondholders from inflation. Since we are in a zero-sum situation, any increase in the wealth of bondholders is a decrease in the wealth of stockholders.[1] We assume that the partial

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[1]One can derive (14C) in an alternative manner. The income to equity holders without inflation is:

$$(Y_c - Di^0_D)(1 - tc) = Y_c(1 - tc) - D \cdot i^0_E , \tag{F1}$$

where Y_c is corporate profit before interest and taxes. The real income to (continued) derivatives, L_1, L_2, H_1, and H_2, are all positive in (13) and (14).

C. Equilibrium

Equilibrium occurs when saving equals investment, i.e., when (6) equals (13A) plus (14A). The full derivative of this equality with respect to i_E and π is:

$$\frac{f'}{1-tc}(di_E - d\pi) = H_1 \cdot (di_E - d\pi) - H_2 \cdot D\left(\left(\frac{1-tpl}{1-tc}\right) - K\left(\frac{tc-tpl}{1-tc}\right)\right)di_E$$
$$+ H_2 \cdot Dd\pi + L_1 \cdot \left(\frac{1-tpl}{1-tc}\right)di_E - L_1 d\pi \qquad (15)$$
$$+ L_2 \cdot D\left(\frac{1-tpl}{1-tc} - K\left(\frac{tc-tpl}{1-tc}\right)\right)di_E - L_2 \cdot D \cdot d\pi .$$

From (15) we derive:

$$\frac{di_E}{d\pi} = \frac{\frac{f'}{1-tc} - H_1 - L_1 + (H_2 - L_2)D}{\frac{f'}{1-tc} - H_1 - L_1 \cdot \left(\frac{1-tpl}{1-tc}\right) + (H_2 - L_2)D \cdot \left(\frac{1-tpl}{1-tc} - K\left(\frac{tc-tpl}{1-tc}\right)\right)} . \qquad (16)$$

1. Interest Rate Effect

An increase in the rate of inflation has two basic effects. First, the change in the real after-tax rate of interest for the high tax bracket individuals differs from this change for the low tax bracket individuals. We

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equity holders with an inflation rate of π is:

$$Y_c(1 - tc) - D\left(i_D(1 - tc) - \pi\right) = Y_c(1 - tc) - D(i_E - \pi) . \qquad (F2)$$

In (F2), the real after-tax corporate income, $Y_c(1 - tc)$, is assumed to be invariant to the rate of inflation. The change in income to equity holders from inflation is equal to (F2) minus (F1), yielding:

$$-D(i_E - i_E^0 - \pi) . \qquad (F3)$$

The extra amount of taxes to be paid by equity holders is (11) multiplied by $-(1 - K)$ or

$$-D(1 - K)(i_E - i_E^0)\left(\frac{tc - tpl}{1 - tc}\right) . \qquad (F4)$$

The sum of $\bar{Y}_H$, (F3) and (F4) is equal to (14C).

call this the interest rate effect. Second, income is redistributed across the two groups. Equation (16) can be interpreted intuitively by examining these two effects. The terms f', H_1 and L_1 are derivatives of investment and saving with respect to particular interest rates. The terms H_2 and L_2 are derivatives with respect to income. The interest rate effect is highlighted by focusing on f', H_1 and L_1, while the income redistribution effect is illuminated by following H_2 and L_2.

In order to focus on the interest rate effect we initially posit that $H_2 = L_2$,[1] causing the two terms in (16) involving $H_2 - L_2$ to be equal to zero. Here, any redistribution of income does not affect our results since the marginal propensity to save for the group gaining income is equal to this propensity for the group losing income. Alternatively, one could focus on the interest rate effect by setting $K = 1$. This assumption implies that the head tax offsetting lost taxes caused by inflation is borne totally by the bondholders. Here, since the same term, $(H_2 - L_2)D$, appears in both the numerator and the denominator of (16), the interest effect to be described below is at most mitigated.[2]

Given that either $H_2 = L_2$ or $K = 1$, equation (16) equals 1 if $tpl = tc$. However, Miller's work implies that the tax bracket of each bondholder is below the corporate tax rate ($tpl < tc$ in our model). Thus, the absolute

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[1]This assumption will be relaxed later in the paper.

[2]As a third possibility we could have rewritten (13C) and (14C) so that additional taxes paid by group 1 would be $D \cdot k\left((i_E - i_E^0)\left(\frac{1 - tpl}{1 - tc}\right) - \pi\right)$. That is, additional taxes would be a proportion of the increase in bondholders' income from inflation and not a proportion of the tax shortfall. Here, all income transfer effects would be eliminated if $k = 1$, so that the interest rate effect would be easily observed.

In this third possibility, no redistribution of income occurs. Conversely, if $H_2 = L_2$, income can be redistributed, though any redistribution does not affect our results.

value of the term involving L_1 in the numerator is less than the absolute value of the term involving L_1 in the denominator. Since $f' < 0$, $H_1 > 0$, and $L_1 > 0$, it follows that $\frac{di_E}{d\pi} < 1$. From equation (1), we know that $\frac{di_D}{di_E} = \frac{1}{1 - tc}$, implying that:

$$\frac{di_D}{d\pi} < \frac{1}{1 - tc} . \qquad (17)$$

The inequality in (17) is interesting since other models, such as those of Darby [1975], Feldstein [1976], and Gandolfi [1982], yield the result that $\frac{di_D}{d\pi} = \frac{1}{1 - tc}$.[1] Thus, if either L_2 is nearly equal to H_2 or K is nearly equal to 1, the Miller model may help explain why the interest rate has not been found empirically to be as responsive to the inflation rate as previous theoretical models suggested.

We can achieve some additional understanding of the result by examining finite changes in (6), (13) and (14) in a world where saving is not a function of income. Let us imagine that $\Delta(1 - tc)i_D \equiv \Delta i_E = \Delta\pi > 0$. From (6), we see that f remains unchanged. Also (14) remains unchanged when the second argument of H is ignored. However, (13) rises when the second argument of L is ignored since, with $tpl < tc$, $\Delta i_E(\frac{1 - tpl}{1 - tc}) \equiv \Delta i_D(1 - tpl) > \Delta\pi$. Thus, an equilibrium cannot be maintained when $\frac{\Delta i_D}{\Delta\pi} = \frac{1}{1 - tc}$ or, equivalently, when $\frac{\Delta i_E}{\Delta\pi} = 1$. However, an equilibrium can be maintained if $\frac{\Delta i_E}{\Delta\pi} \equiv (1 - tc)\frac{\Delta i_D}{\Delta\pi} < 1$. Here, f rises and H falls, potentially offsetting the rise in L.

The above finding can be explained intuitively by focusing on the asymmetry in the Miller model. Low tax bracket individuals increase their saving if $\Delta i_E \equiv (1 - tc)\Delta i_D = \Delta\pi > 0$. High tax bracket individuals would <u>decrease</u> their saving if (1) $\Delta i_E \equiv (1 - tc)\Delta i_D = \Delta\pi > 0$ and (2) they were

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[1]Gandolfi finds that $\frac{di_D}{d\pi} = \frac{1}{1 - tc}$ in his simple model, though he reaches the inequality of (17) when depreciation and capital gains are considered.

required to invest in bonds. However, since these high tax bracket individuals are allowed to invest in equities, their saving function is unchanged. In summary, this asymmetry holds because of both (1) the differing tax treatment of the returns on the two financial instruments and (2) the graduated tax system.[1]

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[1]To show this more formally, we separately relax these two conditions. We first consider an equilibrium with a graduated tax system but where only bonds are employed. We assume that any income redistribution caused by inflation is offset precisely by a head tax so that the saving of the two groups can be written as a function of the real after-tax interest rate, yielding:

$$L\left(i_D(1 - tp1) - \pi\right) \text{ and} \tag{18}$$

$$H\left(i_D(1 - tp2) - \pi\right) . \tag{19}$$

Given (6), (18), and (19), equilibrium occurs when:

$$f\left(i_D - \frac{\pi}{1 - tc}\right) = L\left(i_D(1 - tp1) - \pi\right) + H\left(i_D(1 - tp2) - \pi\right) ,$$

implying the following condition:

$$\frac{di_D}{d\pi} = \frac{\frac{f'}{1 - tc} - H' - L'}{f' - H' \cdot (1 - tp2) - L' \cdot (1 - tp1)} . \tag{20}$$

If $tc = tp2 = tp1$, $\frac{di_D}{d\pi} = \frac{1}{1 - tc}$, which is the result achieved by Darby, Gandolfi, and Feldstein. However, let $tp2 > tc > tp1$ and, for symmetry, $tp2 - tc = tc - tp1$. Here, $\frac{di_D}{d\pi} \gtreqless \frac{1}{1 - tc}$ as $H' \gtreqless L'$. Thus, we have a more ambiguous relationship with only one financial instrument than we have with the Miller model. This result occurs since, if $(1 - tc)\Delta i_D = \Delta\pi > 0$, high tax bracket individuals reduce saving while low tax bracket individuals increase saving. One can compare the extent of the reduction to the extent of the increase by comparing H' and L'.

We next consider an equilibrium with both bonds and stocks but with no graduated tax system; we imagine that the personal tax rate is tc for all individuals. Here, $\frac{di_E}{d\pi} = 1$ in equation (16) under the assumption that either $L_2 = H_2$ or $K = 1$. (We do not consider the cases where either the personal tax rate is greater than tc for all individuals or the personal tax rate is less than tc for all individuals, since Miller's model does not hold in either of these two cases.)

Now that we have established and interpreted (17), we can focus on the extent of the inequality. If tpl = tc, $\frac{di_E}{d\pi} = 1$. This same relationship holds if $L_1 = 0$, i.e., the saving function of each low tax bracket individual is completely inelastic with respect to the real after-tax interest rate, and either K = 1 or $H_2 = L_2$. Alternatively, imagine that $H_2 = L_2$ and $f' = H_1 =$ tpl = 0 but $L_1 > 0$. Here $\frac{di_E}{d\pi} = 1 - tc$, implying that $\frac{di_D}{d\pi} = 1$. Thus, given that either K = 1 or $H_2 = L_2$, our model predicts that $1 < \frac{di_D}{d\pi} < \frac{1}{1 - tc}$ since we assume that $f' < 0$, $H_1 > 0$ and $L_1 > 0$.

One often hears from real world practitioners that there should be a near one-to-one relationship between Δi_D and $\Delta\pi$ since the bond market is almost completely dominated by pension funds and other tax-free investors. Miller's model shows that this analysis is not quite right. Given the premise that tpl = 0 for all bondholders, the one-to-one relationship would hold only if, in addition, the saving function of the high-tax bracket individuals and the investment function are both totally inelastic with respect to $i_E - \pi$.

2. Redistribution Effect

Previously, we considered the effect that the elasticity of saving with respect to the real after-tax interest rate has on the equilibrium relationship between the inflation rate and interest rates. We now wish to investigate the effect of inflation-caused income transfers on the relationship between inflation and interest rates.

To examine this, we posit the Friedman [1957] saving function:

$$S = g(r_{A.T.}) \cdot y \,, \tag{21}$$

where $r_{A.T.}$ is the real after-tax interest rate facing the individual, y is the permanent real income of the individual and g is a monotonically increasing function of $r_{A.T.}$.

Employing (6) and (21), we can now write the equilibrium condition as:

$$f\left(\frac{i_E - \pi}{1 - tc}\right) = g\left(i_E \cdot \frac{1 - tpl}{1 - tc} - \pi\right) \cdot Y_L + g(i_E - \pi) \cdot Y_H \,. \tag{22}$$

The first expression on the right hand side (RHS) of (21) is the saving function of group 1. Here, the argument of g is the real after-tax interest rate of bondholders. The second expression on the RHS is the saving function of group 2. The argument of g, here, is the real after-tax rate of return of equity holders.[1] Y_L and Y_H are expressed in more detail in (13C) and (14C), respectively.

Differentiating (22) with respect to i_E and π, we obtain:

$$\frac{di_E}{d\pi} = \frac{\frac{f'}{1 - tc} - Y_L \cdot g_L' - Y_H \cdot g_H' + (g_H - g_L) \cdot D}{\frac{f'}{1 - tc} - Y_L \cdot \left(\frac{1 - tpl}{1 - tc}\right) \cdot g_L' - Y_H \cdot g_H' + (g_H - g_L) \cdot D\left(\frac{1 - tpl}{1 - tc} - K\left(\frac{tc - tpl}{1 - tc}\right)\right)} \,. \tag{23}$$

In (23), g_L and g_H are defined to be $g(i_E(\frac{1 - tpl}{1 - tc}) - \pi)$ and $g(i_E - \pi)$, respectively. That is, g_L and g_H are not partial derivatives. Rather, they are saving functions evaluated at the real after-tax rate of return applicable to groups 1 and 2, respectively.

Expression (23) is merely (16) applied to the special case of the Friedman saving function. We previously focussed on the interest rate effect by setting $H_2 = L_2$ in (16). However, this equality is not possible in (23). The marginal propensity to save out of income for bondholders is $g(i_E(\frac{1 - tpl}{1 - tc}) - \pi) \equiv g_L$ and the marginal propensity to save out of income for stockholders is $g(i_E - \pi) \equiv g_H$. Since $i_E(\frac{1 - tpl}{1 - tc}) - \pi > i_E - \pi$ and g is monotonically increasing, the marginal propensity to save is greater for group 1.

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[1]Since saving is homogeneous of degree one in income, we can aggregate (21) across all the individuals in a group to get each term on the RHS of (22).

We can now investigate the possible values of (23). We first note that the relevant parameters are f', tpl, tc, g_H, g_L, g_L', g_H', Y_L, Y_H, and D. The boundaries on these parameters are $f' < 0$, $0 < tpl < tc < 1$, $0 < g_H < g_L$, $0 < k < 1$, $0 < g_L'$, $0 < g_H'$, $0 < Y_L$, $0 < Y_H$ and $0 < D$. Regardless of the values of the other parameters, $\frac{di_E}{d\pi} = 1$ $\left(\frac{di_E}{d\pi} < 1\right)$ when tpl = tc (tpl < tc). The derivative, $\frac{di_E}{d\pi}$, can equal 1 - tc only if $f' = g_H' = k = tpl = 0$ and $g_L' > 0$. However, the statement that $g_H' = 0$ violates the condition that Friedman's saving function is a monotonically increasing function of the interest rate. Thus, $1 - tc < \frac{di_E}{d\pi} < 1$.

Thus, the redistribution of income effect, given Friedman's consumption function, is in the same direction as the interest rate effect discussed earlier. The redistribution of income effect can be explained intuitively. In our model, inflation causes income to be redistributed from the equityholders to the bondholders. Since the bondholders have a greater marginal propensity to save, the saving schedule increases with inflation, putting downward pressure on the interest rate.

Of course, our results here are specific to the Friedman saving function. While there is much to recommend (21), it has the controversial property in our model that the marginal propensity to save is higher for the low tax bracket (and presumably low income) individuals of group 1. Alternatively, we can examine the redistribution of income effect more generally by focussing on (16).

As mentioned above, the Friedman saving function is a special case where $H_2 < L_2$. As long as this inequality holds, $1 - tc < \frac{di_E}{d\pi} < 1$. However, if $H_2 > L_2$, $\frac{di_E}{d\pi}$ can be greater than 1. To see this, imagine that $L_1 = 0$ and $K < 1$. Here, the coefficient of $(H_2 - L_2)D$ in the denominator is greater than one. Since $f' < 0$ and $H_1 > 0$, $\frac{di_E}{d\pi} > 1$ in this case.

D. A Note on Possible Extensions

As mentioned earlier, we are primarily interested in applying the Miller equilibrium condition to a basic macro-model. Therefore, in most cases, we do not extend Miller's insight on capital structure to other, more complex economies. However, there are two extensions worth discussing briefly in this section.

Without the imposition of a head tax, real tax revenues decline in our model. Conversely, real taxes may rise with the rate of inflation in practice, because, among other reasons, of tax penalties on tangible assets at the corporate level and tax bracket "creep" at the individual level.

We first examine penalties at the corporate level. Since replacement cost accounting is not allowed, costs of both inventories and depreciable assets are understated in an inflationary economy. A tax on capital gains is an excellent proxy for illusory inventory gains and might be at least a serviceable proxy for understated depreciation.[1] Gandolfi [1982] shows that, in the presence of this tax, the investment schedule is lowered as the rate of inflation rises, thereby reducing the responsiveness of the interest rate to the rate of inflation. Since this effect on the investment function applies directly to our model, no more will be said on it.

In addition, however, these corporate tax penalties are likely to redistribute income from stockholders to bondholders as inflation rises. The extra taxes reduce the value of equity. And, since this tax penalty reduces the tax shortfall caused by inflation (or even leads to a surplus), bondholders will pay a smaller head tax (or even receive a rebate). Thus, this corporate tax penalty accentuates the redistribution effect discussed previously.

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[1] The relationship of a capital gains tax and a tax on understated inventory costs was first suggested to me by M. Flannery.

Tax bracket "creep" also impacts both the interest rate effect and the redistribution effect. If a rise in the rate of inflation increases the marginal tax bracket of bondholders, their real after-tax rate of return is reduced for a given i_D and π. Thus, if redistribution effects are ignored, the values of both $\frac{di_E}{d\pi}$ and $\frac{di_D}{d\pi}$ are greater here than they were in the previous section.

The impact of bracket creep on the redistribution of income effect is indeterminate. Inflation will cause the labor income of both group 1 and group 2 to be taxed at a higher rate. Political pressures will determine which group suffers the most from bracket creep.

IV. Generalization to an Equity Tax

Because Miller [1977] derives his results for the case where there is no personal tax on the returns to equity holders, we used this assumption in the previous section. Though Miller's general conclusions also apply in the presence of a tax on equity, our results concerning inflation change in this case. We analyze this situation below.

One can model this additional personal tax in a variety of ways. We posit that the tax is $i_E \cdot a \cdot tpi$, where $a < 1$. This tax can arise for at least two reasons. First, $a \cdot tpi$ might be the capital gains tax. Second, if we employ the commonly used assertion that capital gains are not taxed, the dividend yield is $a \cdot i_E$. The reader can assume either of the two possibilities for much of the following discussion though previous authors have questioned voluntary dividend payments in Miller's model.[1]

For any tax bracket, tpi, the after-tax return to the stockholder is $i_E(1 - a \cdot tpi)$ while the return to the bondholder is $i_D(1 - tpi) =$

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[1]For example, see Robert Hamada [1982].

$i_E(\frac{1 - tp1}{1 - tc})$. By equating these two returns, we find that the individual is indifferent between bonds and stocks when:

$$tp^* = \frac{tc}{1 - a + a \cdot tc} . \tag{24}$$

Note that $tp^* > tc$ when $a > 0$.

We now posit that $tp1 < tp^* < tp2$, rather than $tp1 < tc < tp2$. To simplify matters, we assume that income is redistributed through taxation in such a manner that income for each of the two groups is unaffected by inflation.[1] Thus, we can express our saving functions in terms of the after-tax real rate only, yielding:

$$L\left(i_E\left(\frac{1 - tp1}{1 - tc}\right) - \pi\right) \text{ and}$$

$$H\left(i_E(1 - a \cdot tp2) - \pi\right) .$$

Our equilibrium condition is now:

$$f\left(\frac{i_E - \pi}{1 - tc}\right) = L\left(i_E\left(\frac{1 - tp1}{1 - tc}\right) - \pi\right) + H\left(i_E(1 - a \cdot tp2) - \pi\right) .$$

Taking the total derivative with respect to i_E, π, and a and then rearranging terms, we obtain:

$$\frac{di_E}{d\pi} = \frac{\frac{f'}{1 - tc} - L' - H' - H' \cdot \left(i_E \cdot tp2 \frac{da}{d\pi}\right)}{\frac{f'}{1 - tc} - L' \cdot \left(\frac{1 - tp1}{1 - tc}\right) - H' \cdot (1 - a \cdot tp2)} . \tag{25}$$

To interpret (25), first note that, if $a = \frac{da}{d\pi} = 0$, this equation is merely (16) when $H_2 = L_2$. As in (16), $\frac{di_E}{d\pi} < 1$.

Let us now consider the case where $a > 0$ and $\frac{da}{d\pi} = 0$. Here, the

[1]We employ this assumption purely for simplicity. The interested reader can add our earlier discussion on income redistribution to this section to get a more complete model.

relationship between L' and L' $\cdot$ $(\frac{1 - tp1}{1 - tc})$ is ambiguous. Since tp* > tc, tp1 could be greater than tc. This, of course, differs from the previous section where tp1 < tc. In addition, H' $\cdot$ (1 - a $\cdot$ tp2) < H'.

These two effects allow the possibility that $\frac{di_E}{d\pi} > 1$. To see this, imagine that tp1 > tc, implying that L' > L' $\cdot$ $(\frac{1 - tp1}{1 - tc})$. Given that $\frac{da}{d\pi} = 0$, f' < 0, H' > 0 and L' > 0, we find that $\frac{di_E}{d\pi} > 1$. This same inequality can be reached when tp1 < tc if L' is small and H' is large.

The difference between this section and the previous one can be explained intuitively. In the last section, the saving of equity holders was unaffected when $\Delta i_E = \Delta\pi > 0$. However, the saving of these individuals would be reduced in this section because of the new tax on equity. Furthermore, any bondholder whose tax bracket is above tc would also be hurt if $\Delta i_E \equiv \frac{\Delta i_D}{1 - tc} = \Delta\pi > 0$. In other words, all individuals with personal tax brackets above tc must reduce saving if $\Delta i_E = \Delta\pi$. This is in marked contrast to the previous section. There, all individuals whose tax rate exceeded tc were unaffected when $\Delta i_E = \Delta\pi$, since all of them invested in the then untaxed equity securities.

We can now consider (25) when a is endogenous. Since f' < 0, H' > 0, and L' > 0, the effect of π on i_E is reduced (increased) if $\frac{da}{d\pi} < 0$ $(\frac{da}{d\pi} > 0)$. Intuitively, while stockholders are hurt by the tax on equity, this tax is reduced as a is lowered.

We believe that a is likely to be a function of inflation. First, stocks are probably sold less frequently during inflationary periods since nominal returns are higher then, implying that $\frac{da}{d\pi} < 0$. Second, inflation may affect dividend policy.

Unfortunately, it is quite difficult to model this second relationship because, even with stable prices, there is no complete theory of dividend policy. On the one hand, many researchers state that a corporate strategy of no dividends is preferred since the tax rate on dividends is above the tax rate on capital gains. On the other hand, some have argued theoretically that dividends may be irrelevant even in a world of differential taxes.[1] Furthermore, though the empirical literature on the subject is impressive, the debate is far from settled on this front either.[2]

We lean toward the assumption that $\frac{da}{d\pi} < 0$, since a zero or positive relationship between dividends and price level changes is difficult to sustain, particularly at high rates of inflation. For example, imagine a no growth all-equity firm where initially $i_E = 8\%$, $\pi = 0$ and $a = 1/2$, implying that the dividend yield is 4%. Later, if $\pi = 10\%$, $i_E = 18\%$[3] and $a = 1/2$, the dividend yield is 9%. However, the ratio of cash flow to market value is 8% in a no-growth firm, implying that dividends exceed cash flow if $\Delta a = \Delta\pi$.

IV. Conclusions

Recently, Miller [1977] has analyzed capital structure in the presence of both corporate and personal taxes. Our work investigates the effect of inflation on interest rates when the Miller equilibrium condition is employed in a loanable funds model.

We view saving as a function of both the real after-tax interest rate and real income. To nullify any effects from the redistribution of income, we initially assume that the marginal propensity to save is unrelated to one's tax bracket. Here $(di_E/d\pi) < 1$, i.e., the change in the return on equity must be less than the change in the inflation rate, a result caused by the

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[1] See Black and Scholes [1974], and Miller and Scholes [1978].

[2] For example, see Litzenberger and Ramaswamy [1979] and Blume [1979].

[3] We are assuming that $\Delta i_E = \Delta\pi$ in this example. A similar conclusion would still follow if this equality did not hold.

asymmetry in the Miller model.

Next, we consider redistribution of income. The shares of national income held by bondholders and stockholders are affected by the rate of inflation. This income redistribution is not merely a function of after-tax rates of return on stocks and bonds but of the government's taxation policy. Because taxes are reduced with an increase in the rate of inflation in our model, the government must raise taxes by another means. Thus, government cannot easily escape being an agent of redistribution. If (1) income is redistributed from the stockholders to the bondholders when the rate of inflation increases, and (2) the Friedman consumption function holds, $1 - tc < \frac{di_E}{d\pi} < 1$. However, the Friedman consumption function implies in our model that the marginal propensity to save of low tax bracket individuals is above that of the marginal propensity to save of high tax bracket individuals. If this relationship is reversed, $\frac{di_E}{d\pi}$ can be greater than 1. In other words, $\frac{di_D}{d\pi}$ can be greater than $\frac{1}{1 - tc}$.

We find three additional effects when the Miller model is generalized to include taxation on equities. First, an individual whose personal tax bracket is higher than t_c may hold bonds. His real after-tax return is lowered if $(di_E/d\pi) \equiv \left(di_D(1 - t_c)\right)/d\pi = 1$. Second, the real after-tax return is lowered for all equity holders if $(di_E/d\pi) = 1$. Ignoring all redistribution effects, we find that these two factors increase the responsiveness of interest rates to changes in the rate of inflation. Third, inflation rate changes may affect a, that portion of equity returns subject to tax. The responsiveness of interest rates is reduced (increased) if $(da/d\pi) < 0$ $\left((da/d\pi) > 0\right)$.

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