Why Do Companies Pay Dividends?: Comment

by

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In a recent article in this review, Martin Feldstein and Jerry Green provide a novel theoretical explanation of why corporations pay dividends in spite of unfavorable taxation. The payment of dividends, according to these authors, reflects appropriate management response to the desire for a diversified investment portfolio by risk-averse investors who regard each firm's return as "both unique and uncertain." This explanation, if it were correct, would represent no mean accomplishment since, as Feldstein and Green (F-G) point out, "the nearly universal policy of paying substantial dividends is the primary puzzle in the economics of corporate finance." Unfortunately, the flaws in their analysis are so serious that it does not seem to us to cast any light on the dividend policy of corporations.

The basic but by no means the only important deficiency in the model on which their analysis and conclusions are based lies in its treatment of risk-free assets. In the F-G model, any net supply of risk-free assets held by investors can be provided only by corporate dividend payments. Such a supply theory for risk-free assets is highly contrived, but once this assumption is made the model ensures that sufficiently risk-averse investors (at the extreme those who will hold only risk-free assets) will want to receive dividends. However, the F-G paper never takes the obvious next step in their analysis, i.e. to determine whether the market's risk aversion (as measured for example by the Pratt-Arrow measure of relative risk aversion) is sufficiently high to justify dividend payments even under the questionable assumptions of their model. We attempt to fill in this serious gap and find that, to the extent risk aversion in their model can be measured from available data, no dividends can be justified.

A second questionable assumption in the F-G model is that while corporations provide the only funds available to acquire net risk-free assets, they cannot hold them in their own portfolios. We shall show that relaxation of this restriction will lead to total retention, and that this result is invariant over all levels of risk aversion. F-G do state at the end of their paper that one worthwhile extension of their model "would be to recognize...that corporations as well as investors can earn the risk-free return." However, they surmise that the link between dividends and real corporate investment which is implicit in their present model, though weakened, would persist. We show that this surmise is incorrect.

Still another serious deficiency in the F-G model is their mis-specification of the quadratic utility function, which they say they are using. While we hold no special brief for the quadratic utility function, we shall demonstrate that not only do F-G mis-specify it, but more importantly, the utility function they use does not seem to conform to any reasonable decision rule.

Finally, we shall discuss briefly other significant problems in the F-G analysis which they do not recognize, at least explicitly. One troublesome problem which they do mention at the end of their paper is the dependence of their result on the number of firms in the economy. The model they use assumes two firms. In the obviously more realistic multiple-firm economy, dividend payments are much more difficult to justify for diversification purposes.

F-G Model With Corporate Investment in Risk-Free Assets

To demonstrate that the F-G model cannot justify substantial dividend payments once corporations are allowed to hold risk-free assets, we shall for expository purposes greatly simplify their model but still retain all the essential features for investigating the net balance of the positive and negative effects of diversification and differential taxation respectively in dividend payout. We shall retain the F-G assumptions of two firms, each with one share outstanding and earnings of one dollar, but now assume only one investor, no initial net supply of risk-free assets, and with both firms equal in expected after-tax return (i.e., $r_1^e = r_2^e$ in the F-G terminology), in total risk ($\sigma_1 = \sigma_2$) and with a zero correlation between the returns of the two firms ($\sigma_{12} = 0$). It should be noted that this last assumption ($\sigma_{12} = 0$) maximizes the potential of diversification for risk-reduction, thus making the best possible case for the F-G position. Under their assumptions, obviously the optimal dividend payment d will be the same for the two firms and this will also be true of stock price p, though we shall not introduce stock price directly into the present analysis.

With these simplifying assumptions, the F-G one-period model would imply

$$(1) \qquad W = 2dR(1 - \Theta) + 2(1 - d)[fR + (1 - f)r] \quad ,$$

where W is terminal wealth, R is the risk-free rate before personal income taxes, Θ is the effective personal tax rate, and f is the proportion (or amount) of earnings which the corporations invest in risk-free assets. For the moment, we shall assume that R is the rate of return received by both individual and corporate investors on assets which are both tax-free and risk-free, but we shall show that our conclusion does not depend on this simplification. Under these assumptions, it is obvious that the variance of W becomes

$$(2) \qquad \text{Var } W = 4(1 - d)^2(1 - f)^2\sigma^2 \quad .$$

Then, holding risk constant at any level, say $(1-d)(1-f) = k$, we can write

$$(3) \qquad E[W] = 2dR(1 - \Theta) + 2(1 - d)R - 2kR + 2kr^{e} \quad .$$

To obtain the effect of dividend payments on wealth for given risk, we compute

$$(4) \qquad \frac{dE[U(W)]}{dd} = \frac{dE[W]}{dd} = 2[R(1 - \Theta) - R] = -2\Theta R < 0 \quad ,$$

so that zero dividends will be paid out. Similarly, under the assumptions leading to Equation (4), we obtain

$$(5) \qquad \frac{dE[W]}{dd} = \frac{2kR\Theta}{(1 - f)^2} \geq 0 \quad ,$$

which again suggests earnings will be retained and invested in risk-free assets in preference to distribution of earnings to taxable investors.

Dropping the simplifying assumption that investors and corporations both invest in tax-free risk-free assets and assuming instead that they invest in taxable risk-free assets, with Θ_p the personal tax rate and Θ_c the somewhat higher corporate tax rate, Equation (1) is transformed into

$$(6) \qquad E[W] = 2dR(1 - \Theta_p) + 2d(R - 1)(1 - \Theta_p) + 2(1 - d)[f + f(R - 1)(1 - \Theta_c) + (1 - f)r^{e}] \quad ,$$

where R now represents a taxable risk-free return and r^{e} remains the after-tax expected return, but (2) is not affected so that (4) becomes

$$(7) \qquad \frac{dE[W]}{dd} = (1 - R)(\Theta_p - \Theta_c) + [R(1 - \Theta_p) - 1] \quad ,$$

which is clearly negative for any reasonable value of the parameters. (For illustrative purposes, F-G use 1.1 for R and 0.5 for Θ_p. Even using 0.3 for Θ_p and 0.5 for Θ_c, Equation (7) would still be highly negative.)

<u>The F-G Utility Function</u>

Individual behavior in the F-G economy is governed by expected utility maximization. They purport to assume a utility function quadratic in wealth, and claim that it leads to an expected utility of the form $E[U(W)] = E[W] - (\gamma/2)Var(W)$. As the utility function implied by this expected utility relationship constitutes a key component of their model, it is necessary to examine its economic validity and reasonableness.

In the first place, a quadratic utility function (say $U(W) = W-(b/2)W^2$ restricted to the domain $W < 1/b$) will generally lead to an expected utility of the form $E[U(W)] = E[W] - (b/2)(Var(W)+E(W)^2)$, a form not consistent with that claimed. Lest this point be dismissed as a technicality, we raise the more basic question: even though a quadratic utility function will not generally lead to an expected utility of the F-G form, is there some (possibly indeterminate) utility function that does? For suitable restrictions on the types of outcomes considered, it may be possible to construct such a utility function (as when final outcomes are certainties). The nature of these restrictions is far from obvious, however. The special case of normally distributed outcomes is of particular importance. In this case, Chipman (1973) has shown that when the expected utility $E[U(W)]$ is written as a function of mean and variance (μ and σ^2), existence of an underlying utility function is contingent on (among other things) a boundary condition of the form $(1/\sigma)dE[U]/d\sigma^2 = d^2E[U]/d\mu^2$. This condition is obviously not met in the F-G case, and in consequence it is clear that when outcomes are normally distributed, no utility function exists that will lead to the F-G expected utility function.

Nonexistence of any clear underlying utility function is certainly a fundamental defect. Nevertheless, granting this failing of the F-G expected utility function, let us propose a still more general question: does their function constitute a reasonable decision rule in selecting among uncertain prospects? The answer must be in the negative, for as shown below, their expected utility function is inconsistent with the principles of stochastic dominance.

The concept of stochastic dominance was developed to clarify those properties of random gambles which lead to unambiguous rankings under minimal assumptions about preferences.[1] One risky outcome is said to dominate another, in the first-order stochastic dominance (FSD) sense, if it offers enhanced probabilities of higher payoffs. Formally, the distribution function of dominating gambles' outcomes lies on or below that of the dominated gamble, which implies that the probability mass for the dominating gamble lies to the right of the dominated. Hadar and Russell show the equivalence of the following two propositions: (i) gamble A dominates (FSD) gamble B; and (ii) all expected utility maximizers with positive marginal utility of wealth will prefer A to B. As shown in the Appendix, gambles may be constructed in which the ranking obtained under stochastic dominance is reversed using the F-G expected utility rule. For a general utility function, this behavior is tantamount to marginal utility that is not everywhere positive, an extremely unattractive property. When employing quadratic utility, the aforementioned wealth restriction is used to exclude the region of negative marginal utility. It is unclear what similar restrictions if any might suffice to exclude such situations in the F-G utility function.

The F-G Risk Parameters

The preceding discussion has established the F-G expected utility function

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[1] The initial discussion of stochastic dominance is in Hadar and Russell (1969).

to be an aberrant formulation with few connections to more valid constructs. This being the case, we have some reservations about pursuing the analysis of their function in an attempt to assess its empirical validity. Nevertheless, their risk aversion parameter γ is in their model a key determinant of the optimal dividend payout level, and accordingly we shall attempt to shed whatever light possible on its value.

A customary measure of risk aversion is the coefficient of relative risk aversion, defined for a utility function $U(W)$ as $C = -WU''/U'$. The empirical evidence, while not completely definitive, suggests that C appears to be constant across wealth levels. In addition, the numerical estimates of C are generally in excess of one, and usually in the range 2-6.[1] Given that under general assumptions about the probability nature of the outcomes, no underlying utility function corresponding to the F-G expected utility exists, one cannot soundly discuss its curvature as reflected in the coefficient of risk aversion. Such a coefficient can only be defined in an ad hoc fashion by hypothesizing a utility function conditional on knowledge of the expected outcome. Since $Var(W) = E[W^2] - E[W]^2$, the F-G expected utility function may be written as $E[U(W)] = E[W] - (\gamma/2)[E[W^2] - E[W]^2]$. The function which might logically be viewed as underlying this is $U(W) = W - (\gamma/2)[W^2 - E[W]^2]$, although since this depends on prior knowledge of the expected value of the outcome, it is not a legitimate utility function. Its coefficient of relative risk aversion may be computed as $C = \gamma W/(1-\gamma W)$.

In the F-G model, γ appears only as the product $\gamma\sigma^2$, for which a value of 1.87 is assumed in order to lead to a dividend payout of 0.8 and a stock price of 0.87. If each investor's initial endowment is assumed to consist of one-half

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[1] See Friend and Blume (1975), Friend and Hasbrouck (1982), and Grossman and Schiller (1981).

of each of the two firms' shares, the initial wealth is also 0.87. A realistic estimate for σ^2 follows from noting that the annual return variance for New York Stock Exchange stocks as a whole is approximately 0.04, implying a γ of about 47. Using these values for initial wealth and γ leads to a negative coefficient of relative risk aversion.[1] Additional insight on this difficulty is obtained by starting with the reasonable assumption that the coefficient of relative risk aversion is about 2 and computing (via numerical analysis) the implied values of γ and dividend payout. The results of this analysis suggest that no positive value of γ is compatible with $C = 2$ and a positive dividend payout.

Additional problems with the F-G model, although of lesser importance than those addressed above, stem from the competitive structure of the economy. One troublesome feature of the F-G numerical example is that the current share price seems to be greater than the expected value of the future wealth accruing to the share, at least from the viewpoint of the household investor. Using the F-G parameter values, the share price is 0.78, but the expected value of terminal wealth for the household investor is $dR(1-\Theta) + (1-d)r^e = .8(1.1)(1-.5) + .2(1.3) = .70$. This state of affairs could not conceivably constitute an optimum for a price-taking investor. The source of this valuation discrepancy is unclear, but appears to arise from the fact that the two firms are not taking investors' implicit required rate of return as exogenous. Merton and Subrahmanyam (1974) show that such a violation of the perfect competition assumption will lead to non-Pareto optimal allocations. They also note that monopoly access to projects (an F-G assumption) will generally lead to investment levels in excess of those socially optimal.

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[1] The average σ^2 for individual stocks would obviously be higher than the σ^2 for the market as a whole, but the use of any reasonable σ^2 would still yield a negative risk aversion coefficient.

The consequences of large numbers of firms have already been noted, but the behavior of the F-G model with large numbers of investors is also of some interest. The essential features of the problem may be demonstrated within the framework of the F-G model by assuming an investor clientele consisting of a number of identical households. In such a model, the optimal retention ratio may be shown to depend positively on the number of investors.[1]

Conclusion

The conclusion from our analysis of their model would appear to be the opposite of that drawn by F-G. There is no reason for believing that in spite of unfavorable taxation the desire for diversification by risk-averse investors helps to explain why corporations pay dividends.

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[1] If n is taken as the number of identical households, the F-G aggregate feasibility constraint (12) becomes $ns_H = 1$. Assuming for simplicity that $\sigma_1^2 = \sigma_2^2 = \sigma^2$ and $\sigma_{12} = 0$, F-G (14) becomes for each firm

$$p_i = \frac{1}{R}[R(1 - \Theta)d_i + (1 - d_i)r_i^e] - (\gamma/nR)(1 - d_i)^2\sigma^2 \quad ,$$

leading to

$$(1 - d_i^*) = \frac{n}{\gamma} \cdot \frac{1}{2\sigma^2} \cdot [R(1 - \Theta) - r_i^e] \quad .$$

Appendix

In this appendix, it is shown that the F-G expected utility rule is inconsistent with the principles of first order stochastic dominance. Consider gambles in which the wealth outcome is distributed uniformly on the interval (a,b). For fixed a, an increase in b will lead to a new gamble which dominates (FSD) the original (the new distribution function is shifted to the right). We now show that it is possible to construct for the F-G utility function uniform gambles for which an increase in the upper end point will lead to diminished expected utility.

For a random wealth uniformly distributed on (a,b), the mean and variance are $E(W) = (a+b)/2$ and $Var(W) = (b-a)^2/12$.[1] Substituting these expressions into the F-G utility function and differentiating with respect to the upper end point yields:

$$dE[U(W)]/db = 1/2 - \gamma(b - a)/12 \quad .$$

It is clearly possible to select a and b so that this expression is negative.

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[1] See Larson (1974).

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