BOND INDEXATION AND EXHAUSTIBLE RESOURCES DEPLETION

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Abstract

In this paper it is shown that an exhaustible resource owner's supply response to the offer of an indexed bond depends on the correlation between the relative price of the resource and the price to which the bond contract is indexed. The indexed bond encourages current extraction only if this correlation is weak; a strong (positive or negative) correlation implies that the availability of the indexed bond discourages current production. This result is then explained in terms of two distinct and sometimes competing insurance effects generated by the indexed bond offer.

I. Introduction

In the past decade, production in the industrialized economies has had to adjust to pronounced variations in the supply of imported natural resources. The magnitude of these swings in prices and quantities has caused an extensive reinvestigation of the economic theory of optimal extraction of exhaustible resources.[1] To date, however, few attempts have been made to evaluate the responsiveness of the time path of resource extraction to changes in the menu of financial assets in which resource owners may invest excess revenue.[2]

Several observers have expressed the concern that resource suppliers will reduce current sales if available financial assets become sufficiently unattractive. Discussing OPEC's production decisions, Robert Dunn has written:

> ...if ever the surplus countries decide that oil in the ground is a more attractive investment than financial claims on the industrialized countries, the result will be ... [that] the OPEC countries ... will have to force their current accounts into equilibrium ... The industrialized countries would be well-advised to see to it that the OPEC surplus countries continue to find it attractive to accumulate financial claims on the rest of the world.[3]

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[1]The book by Dasgupta and Heal (1979) offers an excellent collection of results in the economic theory of exhaustible resources. For a brief and lucid presentation of the fundamental analytics of exhaustible resource extraction, see Solow (1974). Hotelling (1931) pioneered formal analysis of the problem.

[2]A high proportion of those studies available understandably focus on the Organization of Petroleum Exporting Countries (OPEC). Agmon, Lessard and Paddock (1979) informally analyze OPEC's oil supply decision in terms of a portfolio problem. Calvo and Findlay (1978) characterize the way in which OPEC should simultaneously supply oil and invest in real capital assets in the oil-consuming economies, taking into account the fact that changes in the supply of oil may alter the return on these assets. The dependence of the cartel's extraction policy on the oil producers' domestic development objectives is analyzed by Schmalensee (1976).

[3]Dunn (1979), page 11.

This reasoning has provided the basis for several proposals for the design of securities more to the liking of resource suppliers. For example, one suggestion is that the Western governments offer the OPEC nations an indexed bond,[4] that is, a bond with a nominal return linked to the price level in the West so that the real return is guaranteed. The potential advantages of this proposal for the oil exporters are obvious. An indexed bond would protect the future purchasing power of surplus oil revenue without forcing OPEC to hold a diversified and illiquid portfolio of equity investments in the West.

Formal economic analysis of the impact on the supply of oil to the West of offering OPEC an indexed bond is limited to that provided by Levy and Sarnat (1975). Using a mean-variance portfolio model, these authors conclude only that offering OPEC an indexed rather than a nominal bond would have an ambiguous effect on the quantity of oil currently supplied.[5] No attempt is made to isolate factors which would resolve this ambiguity.

The purpose of this paper is to present and interpret conditions under which offering the owners of an exhaustible resource an indexed instead of a nominal bond would lead these agents to increase the amount of the resource currently supplied. The problem is analyzed in a two-period framework. The resource owner begins the first period with a known quantity of the resource, and decides then how much to extract immediately. I assume that all remaining reserves are sold in the second period. In both periods, the resource owner consumes a single composite commodity, which is produced by the resource

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[4]See, for example, Farmanfarmaian, et al. (1975), page 217.

[5]See pages 367-368. The authors do draw the natural conclusion that, if the real interest rate guaranteed on the indexed bond were high enough, OPEC would increase its current rate of extraction.

consuming nations. The price of this good is known in the first period, but is stochastic in the second period. Finally, the resource owner is assumed to be a price-taker in the market for the good it consumes, but may exercise monopoly power in the market for the commodity it supplies. In this setting, I analyze the ways in which the availability of an indexed bond would affect the welfare and, thereby, the supply decision of the resource owner.[6]

I begin in the next section by describing the model and stating the main result, which relates the response of current resource supply to the extent to which resource reserves provide a hedge against unexpected price level variation. It is shown that current resource supply increases under the indexed bond regime only if the correlation between the relative price of the resource and the nominal price of the good which the resource owner consumes is relatively weak. If this correlation is strongly positive or negative, the indexed bond discourages current extraction. In Section III, I present an interpretation of these findings in terms of two distinct insurance effects generated by the availability of the indexed bond. Results are then summarized in the concluding section.

II. Bond Indexation and the Resource Extraction Decision

The owner of the exhaustible resource is assumed to maximize the expected utility derived from current (c_1) and future consumption (c_2):

$$W(c_1, c_2) = U_1(c_1) + E[U_2(c_2)] \tag{1}$$

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[6] It should be noted that the analysis which follows only describes the impact of the indexed bond offer on a resource owner's supply decision. No attempt is made to determine how the resulting response affects the welfare of the resource-consuming economies. In particular, I offer no judgment regarding the assumption implicit in Dunn and Farmanfarmaian, et al., cited above that the West would benefit if OPEC pushed up its oil supply timetable.

where E is the expectation operator. I also assume (a) first period consumption is the same (and equal to $\bar{c}_1$) whether an indexed or a nominal bond is available,[7] and (b) the resource supplier consumes only a composite commodity produced in the West.[8] Given the two-period framework, all of second period income is consumed. One source of second period income is revenue from the sale of resource deposits untapped in the first period, all of which are extracted in the second period.[9] In addition, resource revenue earned in the first period in excess of consumption is lent to the West. In the nominal bond (NB) regime, these loans are repaid in the second period with a fixed nominal return, which is subject to fluctuation in real value due to variation in the price of the Western good. In the indexed bond (IB) regime, these loans are repaid with a fixed real return, independent of the realization of the Western price level. Choosing the Western good as numeraire, second period consumption by the resource owner in the respective regimes is, therefore:

$$\text{(2NB)} \qquad \tilde{c}_2 = (Q_1 y_1 - \bar{c}_1)\left(\frac{1+r}{\tilde{P}_2}\right) + \tilde{Q}_2(D-y_1)$$

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[7] This assumption is restrictive as long as the owner's second period resource revenue is stochastic. This is true even in the special case in which second period utility can be written as $U_2(c_2) = \log c_2$, which under other circumstances makes first period consumption independent of second period uncertainty (see Mirman (1971)). In the OPEC context, this assumption may be justified if the oil producers consume beyond optimal levels at the behest of oil importing countries trying to minimize current account deficits.

[8] The "West" should be thought of as the industrialized resource-importing countries. It produces the good the resource owner consumes, and dictates the set of financial assets in which the resource producers can invest first period excess revenues.

[9] Obviously, reserves will be totally depleted in the two periods only if the marginal net revenue derived in the second period from the last unit of the remaining deposit is positive. I assume that the level of initial reserves and the first period supply decision are such that this condition is met.

(2IB) $$\tilde{\tilde{c}}_2 = (\hat{Q}_1\hat{y}_1 - \bar{c}_1)(1+\theta) + \tilde{\tilde{Q}}_2(D-\hat{y}_1) ,$$

where a ^ designates a variable in the IB regime, a ~ denotes a random variable, and

- Q_1, Q_2 ≡ the relative price per unit of the resource in the first and second periods, respectively.[10]
- P_2 ≡ the price of the Western good in the second period.[11]
- y_1 ≡ the number of units of the resource supplied in the first period.
- r ≡ the nominal rate of interest on the nominal bond, exogenously set by the Western bond-issuing authority.
- θ ≡ the real rate of interest on the indexed bond, exogenously set by the Western bond-issuing authority.
- D ≡ the owner's total resource reserves, in units.[12]

Obviously, the resource owner would supply more in the first period under the IB regime if θ, the real interest rate on the indexed bond, were set high enough relative to r, the nominal rate on the nominal bond. To isolate the effect of offering the resource owner a stable real interest rate, the real return on the indexed bond is assumed to be equal to the expected real return on the nominal bond:[13]

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[10]For expositional convenience I assume that the marginal cost of resource extraction is zero. All results can be generalized to the non-zero marginal cost case by reinterpreting Q_1 and Q_2 as net revenue, in units of the Western good, per unit of the resource in the first and second periods, respectively.

[11]The distribution of the second period price of the Western good is assumed to be independent of the type of bond offered. This specification is consistent with the characterization of the resource owner as a price taker in the market for the good it consumes.

[12]The quantity of resource reserves is assumed known. Pindyck (1980) describes the resource owner's production and exploration behavior when future reserve levels are uncertain.

[13]While Fischer (1975), Liviatan and Levhari (1977), and Siegel and (continued)

(3) $$1 + \theta = E\tilde{x} \equiv \bar{x}$$

where

(4) $$\tilde{x} \equiv \frac{1+r}{\tilde{P}_2} .$$

Employing (3), equations (2) can be rewritten as:

(2NB') $$\tilde{c}_2 = (Q_1 y_1 - \bar{c}_1)\tilde{x} + \tilde{Q}_2(D - y_1)$$

(2IB') $$\tilde{\tilde{c}}_2 = (\hat{Q}_1 \hat{y}_1 - \bar{c}_1)\bar{x} + \tilde{\tilde{Q}}_2(D - \hat{y}_1) .$$

In the NB regime, both sources of second period income (consumption) are likely to be stochastic. The ex post real return on nominal loans to the West varies with the Western price level. Also, in the absence of futures markets and long term commodity contracts, it is unlikely that the resource owner is able to forecast with certainty the relative price at which it can sell its remaining reserves. In the IB regime, this relative price uncertainty represents the only source of variability in the resource owner's second period earnings.

In what follows, I assume for simplicity that a single underlying stochastic process generates both sources of uncertainty in the resource owner's real income stream: the Western price level and the relative price at which resource deposits held over to the second period can be sold. The correlation between the Western price level and the demand for the resource measures the success with which unextracted reserves hedge the owner against inflation risk. This is obviously a critical factor in determining the impact of the indexed bond policy on current resource supply.

A simple representation of this potential correlation between the relative price of the resource and the Western price level is:

$$(5) \qquad \tilde{Q}_2 = \alpha + \beta(D - y_1) + \frac{\gamma'}{\tilde{P}_2} \qquad \alpha > 0,\ \beta < 0,\ \gamma' \gtrless 0 .$$

Restricting β to negative values implies a second period demand curve for the resource which is downward sloping for every realization of $\tilde{P}_2$. If $\gamma' < 0$, the relative price the West is willing to pay for a given amount of the resource rises with the Western price level, so that reserves held over from the first period tend to hedge the resource owner against unexpected inflation in the West. If $\gamma' > 0$, Western inflation erodes the real value of unextracted resource deposits.[14]

Using equations (4) and (5), $\tilde{Q}_2$ can be written as a linear function of $\tilde{x}$:

$$(5') \qquad \tilde{Q}_2 = \alpha + \beta(D - y_1) + \gamma\tilde{x}$$

where $\gamma \equiv \frac{\gamma'}{1 + r} \gtrless 0$ as $\gamma' \gtrless 0$. (5') implies a value for second period resource earnings:

$$(6) \qquad \tilde{R}_2 = \tilde{Q}_2(D - y_1) = (\alpha + \gamma\tilde{x})(D - y_1) + \beta(D - y_1)^2,$$

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[14] One set of economic circumstances which would generate the relationship hypothesized in (5) is the following. Suppose that the resource is an input to the production process for the Western good. Then when Western output increases, Western firms are willing to pay a higher relative price for any given amount of resource supplied. If, in the second period, unexpected Western price level and output fluctuations are caused by "demand-side" shocks (such as unexpected monetary or fiscal policy responses), changes in the Western price level and output, and therefore in $\tilde{P}_2$ and $\tilde{Q}_2$, will be positively correlated ($\gamma' < 0$). Alternatively, if the unexpected changes in the Western price level are generated by "supply-side" shocks, $\tilde{P}_2$ and $\tilde{Q}_2$ will be negatively correlated ($\gamma' > 0$).

which, in turn, allows us to rewrite the expressions for the resource owner's second period consumption:

$$(2NB") \qquad \tilde{c}_2 = (Q_1 y_1 - \bar{c}_1)\tilde{x} + (\alpha + \gamma\tilde{x})(D - y_1) + \beta(D - y_1)^2$$

$$(2IB") \qquad \tilde{\tilde{c}}_2 = (\hat{Q}_1 \hat{y}_1 - \bar{c}_1)\bar{x} + (\alpha + \gamma\tilde{x})(D - \hat{y}_1) + \beta(D - \hat{y}_1)^2 .$$

To compare first period resource supply under the two regimes, the welfare function (1) is differentiated with respect to y_1 and $\hat{y}_1$, respectively, to find the first-order conditions:[15]

$$(7NB) \qquad V(y_1) = EU_2'(\tilde{c}_2)[\tilde{x}MR_1 - \tilde{MR}_2] = 0$$

$$(7IB) \qquad \hat{V}(\hat{y}_1) = EU_2'(\tilde{\tilde{c}}_2)[\bar{x}\hat{MR}_1 - \tilde{\tilde{MR}}_2] = 0 .$$

$MR_1 \equiv \dfrac{d(Q_1 y_1)}{dy_1}$ is assumed positive and is non-stochastic, and

$\tilde{MR}_2 \equiv \dfrac{d[\tilde{Q}_2(D - y_1)]}{d(D - y_1)}$ is found by differentiating (6) with respect to $(D - y_1)$:

$$(8) \qquad \tilde{MR}_2 = \alpha + 2\beta(D - y_1) + \gamma\tilde{x} .$$

The output levels y_1^* and $\hat{y}_1^*$ implied by (7NB) and (7IB), respectively, are optimal and uniquely determined if the second-order conditions for expected utility maximization:

$$(9NB) \qquad V'(y_1) = E\{U_2''(\tilde{c}_2)[\tilde{x}MR_1 - \tilde{MR}_2]^2 + U_2'(\tilde{c}_2)[\tilde{x}\frac{dMR_1}{dy_1} - \frac{d\tilde{MR}_2}{dy_1}]\} < 0$$

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[15]The resource owner is assumed to be a monopolist or one of a group of perfect competitors. The following analysis abstracts from the game theoretic considerations associated with an oligopolistic market structure. Analyses of the basic resource extraction problem under alternative market structures are presented in Dasgupta and Heal (1979), Chapter 11, and Salant (1976).

(9IB) $\hat{V}'(\hat{y}_1) = E\{U''_2(\tilde{\hat{c}}_2)[\bar{x}\hat{MR}_1 - \tilde{\hat{MR}}_2]^2 + U'_2(\tilde{\hat{c}}_2)[\bar{x}\frac{d\hat{MR}_1}{dy_1} - \frac{d\tilde{\hat{MR}}_2}{dy_1}]\} < 0$

are satisfied. It is assumed that the resource owner is risk averse ($U''_2 < 0$), and that MR_1 and MR_2 are decreasing in y_1 and $D - y_1$, respectively.[16]

The second-order conditions suggest a means of comparing optimal first period resource supply under the two bond regimes. Let y_1^* and $\hat{y}_1^*$ represent the levels of first period resource supply which are optimal under the nominal and indexed bond regimes, respectively. Then evaluate both (7NB) and (7IB) at $y_1 = \hat{y}_1 = y_1^*$. By definition, $V(y_1^*) = 0$. If $\hat{V}(y_1^*) > 0$, then by (9IB), $\hat{y}_1^*$ must exceed y_1^* in order to satisfy (7IB). Similarly, $\hat{y}_1^* < y_1^*$ if and only if $\hat{V}(y_1^*) < 0 = V(y_1^*)$. The comparison criterion can therefore be summarized as:

(10) $\hat{y}_1^* \gtreqless y_1^*$ as $\hat{V}(y_1^*) \gtreqless V(y_1^*) = 0$.

Evaluating all relevant terms at $y_1 = \hat{y}_1 = y_1^*$, the resource owner's second period consumption can be expressed as:

(11NB) $\tilde{c}_2 = k_0 + k_1\tilde{x} + k_2\tilde{x}$

(11IB) $\tilde{\hat{c}}_2 = k_0 + k_1\bar{x} + k_2\tilde{x}$,

where

(12) $k_0 \equiv \alpha(D - y_1^*) + \beta(D - y_1^*)^2$

(13) $k_1 \equiv R_1^* - \bar{c}_1 > 0$

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[16]The specific form of (8) satisfies this restriction since $\frac{d\tilde{MR}_2}{d(D-y_1)} = 2\beta < 0.$

(14) $k_2 \equiv \gamma(D - y_1^*) \gtrless 0 \text{ as } \gamma \gtrless 0 \qquad (\text{or as } \frac{\partial Q_2}{\partial P_2} \lessgtr 0)$,

and R_1^* and MR_1^* are the previously defined variables evaluated at y_1^*. The comparison criterion (10) can then be rewritten as:

(10') $\hat{y}_1^* \gtrless y_1^*$ as

$$E\{U_2'(k_0 + k_1\bar{x} + k_2\tilde{x})[\bar{x}MR_1^* - \delta - \gamma\tilde{x}]\} \gtrless$$

$$E\{U_2'(k_0 + k_1\tilde{x} + k_2\tilde{x})[\tilde{x}MR_1^* - \delta - \gamma\tilde{x}]\} = 0 .$$ [17]

The inequality (10') leads to the following proposition on the resource owner's response to the offer of an indexed bond.

<u>Proposition I</u>: Suppose (a) the resource owner's second period utility function is logarithmic in second period consumption, $U_2(c_2) = \log c_2$, and (b) the real return on the nominal bond, $\tilde{x}$, is uniformly distributed on the interval $(1-x', 1+x')$. Then if x' is small,

$$\hat{y}_1^* \left\{\begin{matrix} > \\ = \\ < \end{matrix}\right\} y_1^* \quad \text{as} \quad \left\{\begin{matrix} \frac{-k_1}{(D - y_1^*)} < \gamma < MR_1^* \\ \gamma = \frac{-k_1}{(D - y_1^*)} , \ \gamma = MR_1^* \\ \gamma < \frac{-k_1}{(D - y_1^*)} \ ; \ \gamma > MR_1^* \end{matrix}\right\}.$$ [18]

This proposition indicates that if the price of the Western good ($\tilde{P}_2$) and the relative price of the resource ($\tilde{Q}_2$) are weakly correlated, the resource

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[17] δ is defined as $\alpha + 2\beta(D - y_1^*)$.

[18] Proofs of the propositions appear in the appendix.

owner sells more of the total deposit in the first period under the IB regime. However, a strong (positive or negative) correlation between these variables leads the owner to reduce the amount of the resource currently supplied. Furthermore, the critical bounds for the correlation parameter γ depend on the amount of the resource optimally supplied in the first period under the NB regime. MR_1^* and $\frac{-k_1}{(D - y_1^*)}$ are both decreasing in y_1^*. Therefore, the higher is first period supply under the NB regime, the more likely it is that the indexed bond will inspire more (less) current resource supply if $\tilde{P}_2$ and $\tilde{Q}_2$ are positively (negatively) correlated.

I now provide an explanation for this finding in terms of the insurance effects generated by offering the resource owner an indexed asset.

III. An Insurance Interpretation of the Indexed Bond Policy

A more detailed analysis of the impact of the indexed bond offer on the welfare of the resource owner provides an intuitive explanation for the findings of Proposition I. Notice that both sides of inequality (10') can be written in the form:[19]

$$(15) \qquad \mu(\lambda) = E[\{U_2'(k_0+k_1[\lambda\bar{x} + (1-\lambda)\tilde{x}]+k_2\tilde{x})\} \{[\lambda\bar{x} + (1-\lambda)\tilde{x}]MR_1^* - \delta-\gamma\tilde{x}\}],$$

where the right-hand side of (10') corresponds to (15) evaluated at $\lambda = 0$, and the left-hand side to (10') at $\lambda = 1$. An increase in λ from zero to unity therefore represents movement from a nominal to an indexed bond regime. The resulting change in the function $\mu(\lambda)$ indicates the first period response of the resource owner to the availability of the indexed bond. Differentiating (15) with respect to λ,

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[19] I am indebted to Philip Dybvig for suggesting the following approach.

(16) $$\frac{d\mu(\lambda)}{d\lambda} = A + B ,$$

where

(17) $$A \equiv MR_1^* E[U_2'(k_0 + k_1\lambda\,\bar{x} + k'\tilde{x})\{\bar{x} - \tilde{x}\}]$$

(18) $$B \equiv k_1 E[U_2''(k_0 + k_1\lambda\,\bar{x} + k'\tilde{x})\{\bar{x} - \tilde{x}\}\{\phi(\tilde{x})\}]$$

(19) $$k' \equiv k_1(1 - \lambda) + k_2$$

(20) $$\phi(\tilde{x}) \equiv [\lambda\,\bar{x} + (1 - \lambda)\tilde{x}]MR_1^* - \delta - \gamma\tilde{x}.$$

Equation (16) indicates that the change from a nominal to an indexed bond regime causes the resource owner to increase (decrease) immediate extraction if both A and B are positive (negative) for all values of λ in the interval (0,1).[20] In this section I show that A and B represent insurance effects from offering a resource owner an indexed instead of a nominal bond. Term B, the "consumption insurance effect," measures the impact on the desirability of holding resource reserves, given the change in the distribution of the owner's second period consumption caused by the switch to the indexed bond regime. Bond indexation tends to insulate future consumption from unexpected price level variation, and therefore reduces the need to hold the resource as a hedge against inflation. Resource holdings provide a hedge when the relative price of the resource is positively correlated with the Western price level: indexation in this case tends to make a resource owner more willing to extract immediately. It is also true that indexation tends to reduce extraction in the first period if this correlation is negative.

[20] This condition is, of course, sufficient but not necessary.

Term A, the "bond insurance effect," determines whether the resource owner would prefer the indexed or the nominal bond, given the distribution of second period consumption. The indexed bond is more attractive, and therefore induces greater current extraction, if it pays more than the nominal bond in states in which the resource owner's second period consumption is low.

In the following subsections, I elaborate on these interpretations of A and B as bond and consumption insurance effects. It is shown that both effects unambiguously encourage current resource extraction only if the relative price of the resource and the Western price level are uncorrelated ($\gamma=0$). If these variables are correlated, four possibilities arise. If $\tilde{P}_2$ and $\tilde{Q}_2$ are strongly negatively correlated ($\gamma > MR_1^*$), the bond insurance effect is positive, but the consumption insurance effect is negative. If the negative correlation is weaker ($0 < \gamma < MR_1^*$), A remains positive but B becomes ambiguous. For $\gamma < \frac{-k_1}{(D - y_1^*)}$, the consumption insurance effect operates to boost current resource supply, while the bond insurance effect discourages it. But if $\frac{-k_1}{(D - y_1^*)} < \gamma < 0$, B remains positive while A becomes ambiguous. Thus, strong correlation between $\tilde{P}_2$ and $\tilde{Q}_2$ generates competing insurance effects and, by Proposition I, the effect which discourages current supply is dominant. If the absolute value of γ is small, however, a positive insurance effect overrides its ambiguous counterpart.

A. The Bond Insurance Effect

The relationship between A, the bond insurance effect given in equation (17), and the correlation parameter γ follows from Proposition II.

Proposition II: If the resource owner is risk averse, A is positive (negative) for all λ in (0,1) as k' (see equation (19)) is positive (negative) for all λ in (0,1).

The role that the correlation parameter γ plays in determining the sign of A then becomes clear if we expand the definition of k' (using (13) and (14) above):

$$(21) \qquad k' = (R_1^* - \bar{c}_1)(1-\lambda) + \gamma(D - y_1^*) \ .$$

Suppose first that γ is non-negative. Then k' is positive and the resource owner's second period consumption is increasing in x[21] (decreasing in the Western price level). This follows from the assumptions that $(R_1^* - \bar{c}_1)$, first period bond purchases, and $(D - y_1^*)$, reserves which are not extracted in the first period, are both positive. If γ is positive, the relative price of the resource and the Western price level are negatively correlated. Therefore, both real bond repayments and real resource earnings are higher than average when the Western price level is low. If $\gamma = 0$, real resource revenues are independent of the Western price level, so that c_2 is increasing in x because of the effect on real bond returns.

If γ is negative, low realizations of the Western price level imply smaller than average real second period resource earnings. However, for k' to be negative for all relevant values of λ, it must be the case that this impact on resource earnings dominates the effect on real bond returns; specificially, γ must satisfy $\gamma < \frac{-(R_1^* - \bar{c}_1)}{(D - y_1^*)} < 0$. If $\frac{-(R_1^* - \bar{c}_1)}{(D - y_1^*)} < \gamma < 0$, k' is ambiguous. It then follows from Proposition II that $A > 0$ if $\gamma \geq 0$, and $A < 0$ if $\gamma < \frac{-(R_1^* - \bar{c}_1)}{(D - y_1^*)}$.

[21]Throughout the paper, particular ex post realizations of a random variable $\tilde{w}$ are denoted by w.

This result that the effect which term A represents encourages greater current resource extraction under the IB regime if k' is positive, and less if k' is negative, can be explained intuitively as follows. The function $\{\bar{x} - \tilde{x}\}$ in A is $\frac{d[\lambda \bar{x} + (1 - \lambda)\tilde{x}]}{d\lambda}$, the gain in real bond proceeds that an indexed bond holder experiences relative to a nominal bond holder. For example, when the Western price level is low, $x > \bar{x}$, and a nominal bond holder reaps an increase in real bond proceeds. By definition, an indexed bond holder foregoes these gains since the real return on that asset is fixed. Alternatively, indexed bond holders gain relative to nominal bond holders when $x < \bar{x}$ (P_2 is high).

If k' is positive, c_2 is high and $U_2'(c_2)$, the marginal utility of consumption, is low when $x > \bar{x}$. But, as just argued, this is exactly when an indexed bond holder is a relative loser in real bond proceeds. On the other hand, when x is low, $U_2'(c_2)$ is high and the indexed bond holder is a relative gainer. This argument, and a similar one for the case $k' < 0$, is summarized in Table 1.

These findings reveal why a resource owner willingly supplies more (less) to the West now in the IB regime if $k' > (<) 0$.[22] If $k' > 0$, a resource owner earns more by holding an indexed rather than a nominal bond in those states of nature in which the return advantage is worth more, i.e., when $U_2'(c_2)$ is higher. On the other hand, the nominal bond pays a higher return relative to the indexed bond in those states in which the return advantage to the nominal bond is less valuable. Therefore, if $k' > 0$, the smoothing of the real return which the indexed bond provides raises the resource owner's expected utility. This favorable "bond insurance effect" makes Western bonds a more attractive

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[22]Still concentrating only on the effect embodied in term A in (17).

Table 1

The Bond Insurance Effect

Case	$x - \overline{x}$	$U_2'(c_2)$	Indexed Bond Holder Relative Gainer (+) /Loser (-)
$k' > 0;\ \gamma > 0$	+	low	-
	-	high	+
$k' < 0;\ \gamma < \frac{-(R_1^* - \overline{c}_1)}{(D - y_1^*)}$	+	high	-
	-	low	+

investment, and therefore inspires the owner to sell more now, and to lend the additional earnings to the West. Of course, if k' is negative, a similar analysis reveals that an indexed bond provides unfavorable bond insurance, and we would therefore expect a curtailment of current extraction under the IB regime.

B. The Consumption Insurance Effect

The second insurance effect generated by the indexed bond offer is the consumption insurance effect represented by B (see (18)) in equation (16). Proposition III describes the dependence of this effect on the correlation parameter γ.

Proposition III: Suppose (a) the resource owner's second period utility function is of the form $U_2(c_2) = \log c_2$, and (b) the real return on the nominal bond, $\tilde{x}$, is uniformly distributed on the interval $(1-x', 1+x')$. Then, if x' is small, B is positive if $\gamma < 0$, negative if $\gamma > MR_1^*$, and ambiguous if $0 < \gamma < MR_1^*$.

This finding is a result of the correlation between the resource owner's second period consumption and the ex post relative performance of the two portfolio options: (1) first period resource extraction and bond purchase versus (2) resource conservation for second period depletion. To see this, observe first that, in B, the function $\{\bar{x} - \tilde{x}\}$ represents $\frac{dc_2}{d\lambda}$, the gain in second period consumption experienced by an indexed bond holder relative to a nominal bond holder for a given outcome of $\tilde{x}$. Low realizations of $\tilde{x}$ correspond to high P_2 and, therefore, low real proceeds from nominal loans to the West. Hence an indexed bond holder consumes relatively more in the second period in the states in which $x < \bar{x}$ and less in those in which $x > \bar{x}$.

The other critical element in B is the function $\phi(\tilde{x})$ (see (20)), which is linear in $\tilde{x}$ with slope:

(22) $$\phi'(\tilde{x}) = (1 - \lambda)MR_1^* - \gamma .$$

Since $MR_1^* > 0$, it follows that $\phi'(\tilde{x})$ takes on the same sign for all λ in (0,1) if either:

(23a) $\gamma < 0$, in which case $\phi'(\tilde{x}) > 0$,

or

(23b) $\gamma > MR_1^*$, in which case $\phi'(\tilde{x}) < 0$.

Consider the first of these possibilities. For low realizations of $\tilde{x}$, $\phi(x) = [\lambda \bar{x} + (1 - \lambda)x]MR_1^* - MR_2^* < 0$. This means that ex post, the strategy of selling off resource deposits in the first period and lending the excess proceeds to the West is outperformed at the margin by the strategy of holding reserves for extraction in the second period. That is, second period earnings on the overall portfolio of bonds and unextracted resource deposits would have been higher if more of the resource had been saved for later depletion.

Thus, if $\phi'(\tilde{x}) > 0$, when x is low, the following events occur simultaneously: (1) the bond investment fares worse, on the margin, than resource reserve holdings, and (2) the resource owner experiences higher second period consumption holding an indexed rather than a nominal bond. When x is high, $\phi(x)$ is positive, and the resource owner wishes, ex post, that more of the resource had been sold off in the first period. At the same time, high x implies that the resource owner is consuming less in the second period under an indexed rather than a nominal bond regime.

These findings yield the following interpretation of B as the "consumption insurance effect" of offering a resource owner an indexed bond.

As long as the resource owner is risk averse, marginal utility of consumption is declining in second period consumption. Therefore, when x is low, the resource owner consumes relatively more, and experiences lower marginal utility, as an indexed rather than a nominal bond holder. Since, when x is low, the resource owner's unextracted reserve holdings outperform its bond investments, offering the owner an indexed bond reduces the marginal utility of the relatively high return on held-over reserves. On the other hand, when x is high, the owner's marginal utility is higher under the indexed bond regime and bond investments fare better at the margin than resource deposits. Thus, an indexed bond holding increases the marginal utility of the relatively high return on excess first period resource earnings.

These observations are summarized in the lower half of Table 2. When $\phi'(\tilde{x}) > 0$ ($\gamma < 0$), the indexed bond alters the distribution of a resource owner's second period consumption in a way which raises marginal utility when bond holdings are the preferred investment <u>ex post</u>, and lowers marginal utility when resource reserve holdings have returned more. Thus, a resource owner increases its expected utility under the IB regime by extracting more of the resource in the first period to increase bond purchases.

This consumption insurance effect works in the opposite direction if $\phi'(\tilde{x}) < 0$ ($\gamma > MR_1^*$, see the top half of Table 2). In this case, marginal utility is higher under the IB regime in those states in which having held over reserves is the preferred investment <u>ex post</u>. Hence, if $\phi'(\tilde{x}) < 0$, the resource owner raises expected utility by extracting more of the deposit if offered an indexed bond.

In summary, the consumption insurance effect predicts that, if offered an indexed bond, a resource owner will supply more in the first period if demand for the resource is positively correlated with the Western price level

Table 2

The Consumption Insurance Effect

Case	$x - \bar{x}$	c_2 higher (+) or lower (−) under IB relative to NB	U_2' higher (+) or lower (−) under IB relative to NB	$\phi(x)$[a]
$\phi' < 0;\ \gamma > MR_1^*$	−	+	−	+
	+	−	+	−
$\phi' > 0;\ \gamma < 0$	−	+	−	−
	+	−	+	+

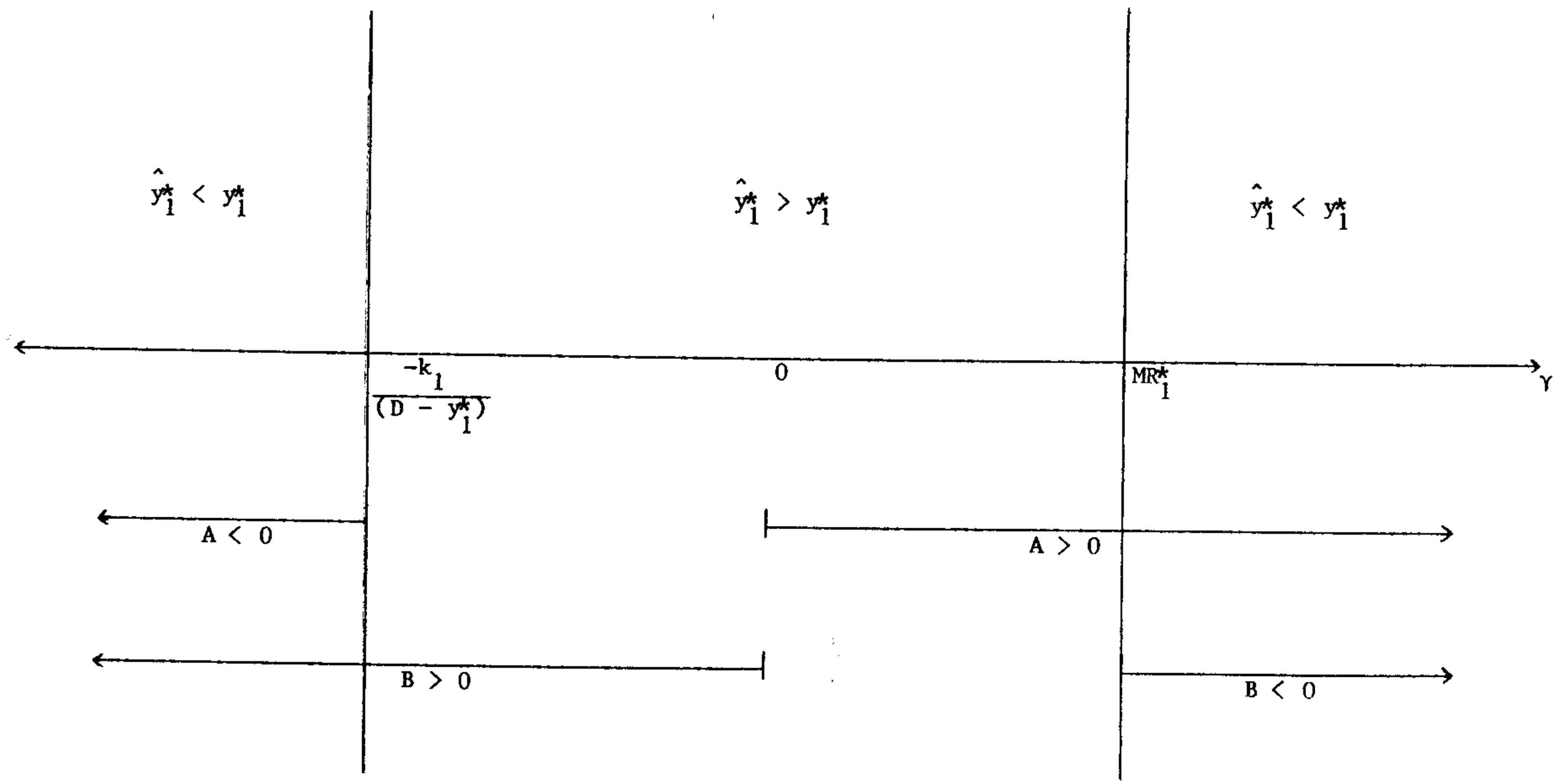
[a]If $\phi(x) > (<) 0$, the *ex post* marginal return on the resource extracted exceeds the marginal return on reserves held over and *sold* in the second period.

($\gamma < 0$). However, from Proposition II, the sufficient condition for the bond insurance effect to be positive cannot be satisfied in this case for all values of λ in (0,1). Furthermore, A is negative if the positive correlation between $\tilde{P}_2$ and $\tilde{Q}_2$ is strong enough ($\gamma < \frac{-k_1}{(D - y_1^*)}$). If, on the other hand, resource demand and the price level are negatively correlated ($\gamma > 0$), the bond insurance effect is positive, while the consumption insurance effect is negative ($\gamma > MR_1^*$) or ambiguous. Both insurance effects are unambiguously positive only if the relative price of the resource is independent of Western inflation experience ($\gamma = 0$).

These findings help explain the results of Proposition I (see Figure 1). Consider the case in which $\tilde{P}_2$ and $\tilde{Q}_2$ are positively correlated. If $\gamma < 0$, the resource provides its owner with an inflation hedge, and intuition suggests that the indexed bond offer would displace the need to hold resource deposits. Term B represents this portfolio effect which by itself indicates that current extraction increases ($B > 0$). However, while this positive consumption insurance stimulus is neither offset nor reinforced if $\frac{-k_1}{(D - y_1^*)} < \gamma < 0$, a stronger positive correlation makes the indexed bond less attractive to the resource owner than the nominal bond it replaces. In this case, the negative bond insurance dominates the favorable consumption insurance and immediate extraction declines under the indexed bond regime.

IV. Conclusion

This paper is an attempt to evaluate the responsiveness of the timetable of exhaustible resource extraction to a particular change in the menu of financial assets in which resource owners may invest. I have examined the way in which an owner would react if offered an indexed bond, that is, a bond which pays a fixed real return independent of price level variation. The analysis suggests that eliminating the inflation risk in the bond return would



<u>**Figure 1: Resource Supply Response and the Insurance Effects**</u>

not necessarily inspire more current resource extraction, and could dampen current sales.

The main result is that the resource supplier's response to an indexed bond critically depends on the strength of the correlation between the relative price of the resource and the price of the good to which the bond contract is indexed. Current resource supply is higher under the indexed than under the nominal bond regime if the correlation between these variables is weak. However, a strong positive or negative correlation leads owners to reduce current resource supply. Furthermore, the critical boundaries for the correlation parameter depend on the rate of extraction which is optimal under the nominal bond regime. The higher is first period supply under the nominal bond regime, the more likely it is that the indexed bond will generate more (less) current extraction if the relative resource price and the general price level are positively (negatively) correlated.

An intuitively appealing approach is to view these results in terms of the insurance effects generated when a resource owner is offered an indexed bond. Replacing a nominal with an indexed bond creates incentives for the resource owner which can be decomposed into bond and consumption insurance effects. The consumption insurance effect, common in portfolio models, measures the impact on the desirability of holding resource reserves, given the change in the distribution in the owner's second period consumption induced by the shift to an indexed bond regime. For example, if the resource hedges its owner against inflation, the indexed bond tends to displace the need to maintain resource reserves. Current extraction is therefore encouraged if the relative resource price and the Western price level are positively correlated, and discouraged if this correlation is negative. The bond insurance effect measures whether the owner would prefer the indexed or the nominal bond, given the distribution of second period consumption. The indexed bond is more attractive, and therefore induces greater current extraction, if it pays more than the nominal bond in states in which the resource owner's second period consumption is low.

The findings of the paper show that both insurance effects unambiguously promote current resource extraction only if the relative resource price is independent of the general price level. Initial extraction is also higher under the indexed bond regime if these variables are weakly correlated, in which case one of the insurance effects is positive and strong enough to dominate its (ambiguous) counterpart. A stronger correlation, positive or negative, generates competing insurance effects, but in this case the sum of these effects is negative and the overall impact is for the indexed bond to discourage current extraction.

Appendix

Proof of Proposition I: From (10') in the text, $\hat{y}_1^* \gtrless y_1^*$ as

(A.1) $$m \equiv E\{U_2'(k_0 + k_1\bar{x} + k_2\tilde{x})[\bar{x}MR_1^* - \delta - \gamma\tilde{x}]\} \gtrless 0 \,.$$

Utilizing the assumption of Proposition I that

(A.2) $$U_2(c_2) = \log c_2$$

and that the probability density on $\tilde{x}$ is

(A.3) $$f(\tilde{x}) = \frac{1}{2x'} \quad ; \qquad 1-x' < \tilde{x} < 1+x' \quad , \quad 0 < x' < 1$$

(A.1) can be rewritten as:

(A.4) $$m = \frac{1}{2x'} \int_{1-x'}^{1+x'} \frac{z - \gamma x}{I + k_2 x}\, dx \gtrless 0 \,,$$

where

(A.5) $$z \equiv \bar{x}MR_1^* - \delta$$

and

(A.6) $$I \equiv k_0 + k_1\bar{x} \,.$$

Integrating the expression in (A.4) by parts, and using a second-order Taylor series approximation for $\ln \left[\frac{I + k_2(1+x')}{I + k_2(1-x')}\right]$ about $x'=0$,

(A.7) $$m \simeq \frac{z-\gamma}{I + k_2} \gtrless \quad \text{as} \quad z-\gamma \gtrless 0 \,.$$ [23]

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[23] $I+k_2$ is positive since it represents the expectation of second period consumption ($E(\tilde{c}_2) = I + k_2E(\tilde{x}) = I + k_2$ by (A.3)).

The proof is therefore complete if it can be shown that

$$\text{(A.8)}\qquad z-\gamma \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0 \quad \text{as} \quad \begin{matrix} \gamma < \dfrac{-k_1}{(D - y_1^*)} \;;\; \gamma > MR_1^* \\ \gamma = \dfrac{-k_1}{(D - y^*_1)} \;;\; \gamma = MR_1^* \\ \dfrac{-k_1}{(D - y_1^*)} < \gamma < MR_1^* \;. \end{matrix}$$

(A.8) follows directly from the portfolio first order condition under the nominal bond regime:

$$\text{(7NB)}\qquad E\{U_2'[k_0 + (k_1+k_2)\tilde{x}]\ \Psi(\tilde{x})\} = 0 \ ,$$

where $\Psi(\tilde{x}) \equiv \tilde{x}MR_1^* - \delta - \gamma\tilde{x}$. $U_2'[k_0 + (k_1+k_2)\tilde{x}]$ is positive and

$\begin{Bmatrix} \text{decreasing} \\ \text{constant} \\ \text{increasing} \end{Bmatrix}$ in $\tilde{x}$ as $(k_1+k_2) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0$, or as $\gamma \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \dfrac{-k_1}{(D - y_1^*)}$.

$\Psi'(\tilde{x}) = MR_1^* - \gamma \gtreqless 0$ as $\gamma \lesseqgtr MR_1^*$. The case where $\dfrac{-k_1}{(D - y_1^*)} < \gamma < MR_1^*$ is shown in Figure 2. In this case, U_2' decreases and Ψ increases in $\tilde{x}$. Since the probability density of $\tilde{x}$ is uniformly distributed about $\bar{x}$, the first order condition (7NB) can only be satisfied if

$$\text{(A.9)}\qquad \Psi(\bar{x}) = \bar{x}MR_1^* - \delta - \gamma \equiv z-\gamma > 0 \ .$$

This completes the proof for the case where $\dfrac{-k_1}{(D - y_1^*)} < \gamma < MR_1^*$. The other cases in (A.8) follow by similar arguments.

<u>Proof of Proposition II</u>: We wish to show that

$$\text{(17)}\qquad A \equiv MR_1^*\ E[U_2'(k_0 + k_1\lambda\bar{x} + k'\tilde{x})\{\bar{x} - \tilde{x}\}] \gtrless 0$$

for all λ in the interval (0,1) as k' (see equation (19)) $\gtrless 0$ for all λ in

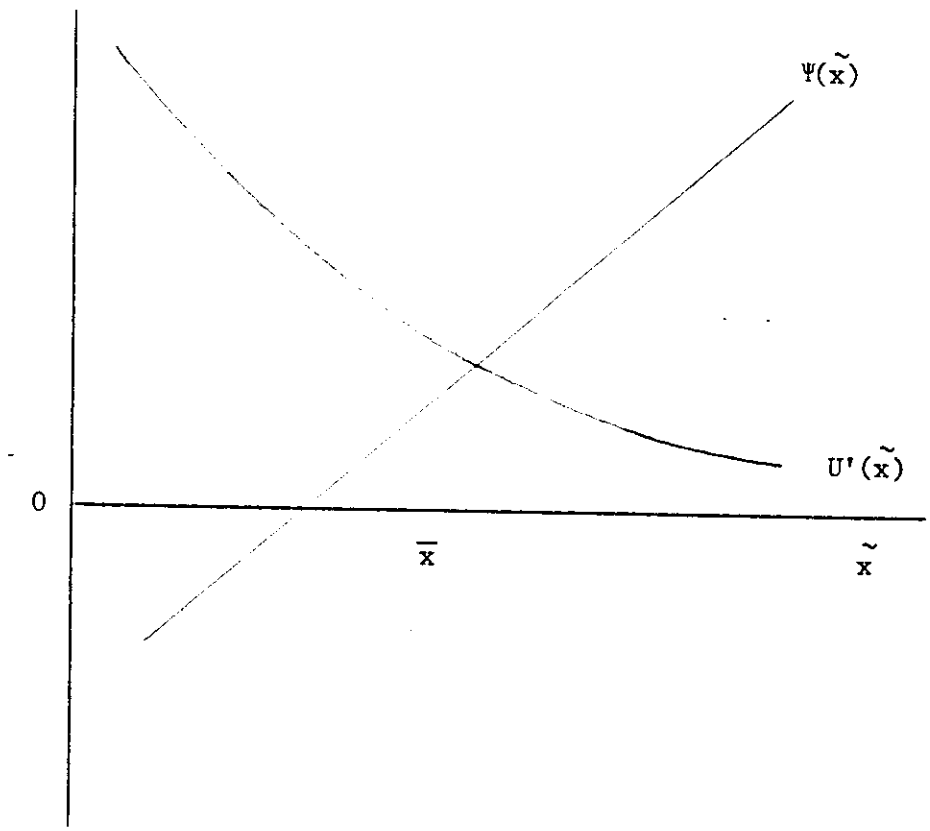

Figure 2: The Portfolio First Order Condition When $\frac{-k_I}{(D - y_I^*)} < \gamma < MR_I^*$.

(0,1). Since $MR_1^* > 0$, $A \gtrless 0$ as $E[U_2'(J + k'\tilde{x})\{\bar{x} - \tilde{x}\}] \gtrless 0$, where $J \equiv k_0 + k_1\lambda\bar{x}$. $U_2'(J + k'x)$ is positive and decreasing in x if $k' > 0$. Therefore, $U_2'(J + k'\bar{x}) \gtrless U_2'(J + k'x)$ as $x \gtrless \bar{x}$. Letting f(x) represent the probability density on x, it follows that

$$\int_0^{\bar{x}} U_2'(J + k'x)(\bar{x} - x)f(x)dx > U_2'(J + k'\bar{x}) \int_0^{\bar{x}} (\bar{x} - x)f(x)dx$$

and

$$\int_{\bar{x}}^{\infty} U_2'(J + k'x)(\bar{x} - x)f(x)dx > U_2'(J + k'\bar{x}) \int_{\bar{x}}^{\infty} (\bar{x} - x)f(x)dx \; .$$

Thus

$$\begin{aligned} E[U_2'(J + k'\tilde{x})\{\bar{x} - \tilde{x}\}] &= \int_0^{\infty} U_2'(J + k'x)(\bar{x} - x)f(x)dx \\ &> U_2'(J + k'\bar{x}) \int_0^{\infty} (\bar{x} - x)f(x)dx \\ &= 0 \; , \end{aligned}$$

since $\bar{x}$ is the mean of $\tilde{x}$. An analogous argument proves that $A < 0$ if $k' < 0$.

Proof of Proposition III: We wish to show that, if (A.2) and (A.3) hold,

(18) $$B \equiv k_1 E\left[U_2''\left(k_0 + k_1\lambda\bar{x} + k'\tilde{x}\right)\{\bar{x}-\tilde{x}\}\{\phi(\tilde{x})\}\right] \gtrless 0$$

for all λ in the interval (0,1) as $\gamma \lessgtr {0 \atop MR_1^*}$. First recall from (20) that $\phi(\tilde{x})$ can be written in the form:

(A.10) $$\phi(\tilde{x}) = s(\tilde{x}-\underline{x})$$

where

(A.11) $$s = (1-\lambda)MR_1^* - \gamma \; ,$$

and $\underline{x}$ is a constant independent of $\tilde{x}$. Using (A.2), (A.3) and (A.10), B can be rewritten as

$$\text{(A.12)} \qquad B = \frac{sk_1}{2x'} \int_{1-x'}^{1+x'} \frac{(x-\bar{x})(x-\underline{x})}{(J+k'x)^2} dx ,$$

where $J \equiv k_0 + k_1\lambda\bar{x}$. Carrying out this integration, and using a second-order Taylor series approximation for $\ln[\frac{J + k'(1+x')}{J + k'(1-x')}]$ about $x'=0$ yields

$$\text{(A.13)} \qquad B = \frac{sk_1(x')^2[J + k'\underline{x}]}{(J+k')[J + k'(1+x')][J + k'(1-x')]} .$$

All of the bracketed expressions in (A.13) involving J are positive since each represents second period consumption evaluated at possible realizations for $\tilde{x}$. Since $k_1 > 0$ (see (13)), it follows that the sign of s determines the sign of B. But by (A.11), $s \gtrless 0$ for all λ in (0,1) as $\gamma \lessgtr mR_1^*$ (upper case: $\gamma < 0$). This completes the proof of Proposition III.

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