The Real Exchange Rate, The Current Account And The Speed Of Adjustment

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1. Introduction

A stylized recent model of exchange rate determination would include the following features: high capital mobility, rational expectations and continuous clearing of asset markets. Such a model would exhibit saddle-path stability and, in order to solve it, one would typically first determine its long run (steady-state) equilibrium following a disturbance, and then identify the unique path along which convergence would be obtained.[1] In this paper we present and discuss a class of models for which the steady state equilibrium seems to admit a priori an infinity of solutions so that there would appear to exist an infinity of convergence paths.[2] It will be shown that this indeterminacy in only apparent: the long run equilibrium, and the path that leads to it, are uniquely determined by the dynamic characteristics of the model. In other words, the parameters which set the speed of adjustment of the model have a permanent effect on the evolution of the economy.

This interesting property is obtained in two-country models with infinite intertemporal optimization where agents typically consume their permanent income, which, in the stationary state, coincides with their actual income. Consequently, under assumptions to be specified later, the requirement that the current account be in equilibrium vanishes, opening up the possibility of an indeterminacy of the real exchange rate. Models of real exchange rate determination with inter-temporal optimization have recently received considerable attention, expecially in Svensson and Razin (1981), Obstfeld (1981a, 1981b), Sachs (1982), and Dornbusch (1981). There are actually at least two reasons why such models are interesting. First, Dornbusch and Fischer (1980), Rodriguez (1980), and Mussa (1980) have emphasized the role of present and future current account imbalances in driving the exchange rate, following the earlier contribution by Kouri (1976). An important implication of the re-emergence of the current account is a renewed interest in the intertemporal allocation of resources and spending among countries which is implied by such surpluses or deficits, and, therefore, the need to model carefully this process. Another reason is related to the widespread use of the rational expectations assumption. As pointed out by Muth (1961) in his original contribution, if one models optimizing agents, one has to assume also that they use all available information in forming their expectations. But then, if they incorporate future anticipated events in their rational expectations, it seems natural to replace static by dynamic optimization.

It has become a standard property of exchange rate models under perfect foresight, that the impact effect of an exogenous disturbance is the function of the speed at which some slow moving variables are able to adjust, as exemplified in Dornbusch (1976). But, this effect does not concern the stationary state to which the model converges which, typically, remains uniquely defined and easily characterized. The class of models discussed in the present paper opens up new interesting possibilities. For example, it is shown that the degree of flexibility of wages, or the rate at which capital is accumulated, have permanent effects on such variables as the real exchange rate and a country's external indebtedness.

The point made here will seem intuitively clear and is but a special case of the general treatment of linear models under perfect foresight by Blanchard and Kahn (1980) and Buiter (1981). Still, it does not seem to have been directly addressed in the exchange rate literature, although Obstfeld (1981a) and Sachs (1982) have signaled its existence. In a completely different set-up, Drazen (1980) obtains the same property and argues, as we do, that it presents attractive economic implications.

The problem at hand is illustrated through an example in the next section. The analytical solution is presented in section 3, and put to work in section 4, where another example shows the role of the labor market in determining the stationary state value of the real exchange rate through its effect on cumulated current account imbalances. Section 5 offers some concluding remarks.

2. The Nature of the Problem: A Simple Example

The model presented in table 1 assumes perfect foresight and intertemporal optimization. It describes two countries which trade goods and securities with each other. Each country produces one good, using capital as the sole factor of production. A symmetrical model with labor instead of capital is presented in section 4 below. A model with both labor and capital is too large to be solved analytically, and has been simulated in Sachs (1982) and Giavazzi, Odekon and Wyplosz (1982). The production technology is identical in both countries and exhibits decreasing return to scale (equation (1)). Each good is used for private consumption at home and abroad, and for domestic capital formation. The two goods are imperfect substitutes in consumption and the demand equations (2) and (3) are derived in appendix 1 from the intertemporal optimization of an instantaneous Cobb-Douglas utility function.[3] The variable $\lambda = eP^*/P$ is the real exchange rate, with e the nominal exchange rate and P and P^*, respectively, the prices of domestic and foreign goods. With this specification, total real consumption, $C + \lambda C_m$, in each period, is a constant share of real wealth A, the constant being the rate of time preference δ. The assumption that δ is constant and identical across countries is crucial and will be discussed later. Consumption of each good is, by virtue of the Cobb-Douglas assumption, a constant share of total

Table 1

The asterisk denotes foreign country's variables.

	Home	Foreign
(1)	$y = y_0 K^{\alpha}$	$y^* = y_0^* K^{*\alpha}, \quad 0 < \alpha < 1$
(2)	$C = a\delta A \ , \quad a > 1/2$	$C^* = (1 - a^*)\delta A^* \ , \quad a^* < 1/2$
(3)	$\lambda C_m = (1 - a)\delta A$	$\lambda^{-1} C_m^* = a^* \delta A^*$
(4)	$A = q(K - Z)$	$A^* = q^* K^* + \lambda^{-1} q Z$
(5)	$\dot{q} = rq - D/K$	$\dot{q}^* = r^* q^* - D^*/K^*$
(6)	$D = y - I + q\dot{K}$	$D^* = y^* - I^* + q^* \dot{K}^*$
(7)	$I = \dot{K}\left(1 + \frac{\phi}{2}\frac{\dot{K}}{K}\right)$	$I^* = \dot{K}^*\left(1 + \frac{\phi^*}{2}\frac{\dot{K}^*}{K^*}\right)$
(8)	$\dot{K} = K(q - 1)/\phi$	$\dot{K}^* = K^*(q^* - 1)/\phi^*$
(9)	$r = r^* + \dot{\lambda}/\lambda$	
(10)	$q\dot{Z} = \lambda C_m - C_m^* + DZ/K$	
(11)	$y = C + C_m^* + I$	$y^* = C^* + C_m + I^*$

consumption and, in each country, a larger share of consumption falls on the locally-produced good ((2) and (3)).

Equity claims on the domestic and foreign capital stocks are the only assets and are taken as perfect substitutes. Consequently, the assumption, implicit in the definition of wealth (4), that only domestic claims are traded, is inocuous and $Z \gtrless 0$ represents the volume of domestic equities held abroad. The variable q in (4) is the market value of installed capital, i.e. Tobin's q. It is given in differential form in (5), where the dividends D are defined in (6). The definition of dividends assumes that all capital outlays are financed through issues of equities, so that dividends include the proceeds of the issue of new stocks less spending on investment, I. The investment function (7), in turn, follows the cost of investment literature,[4] in assuming that total investment expenditures exceed the value of actually installed capital $\dot{K}$, this cost being here a simple linear function of K. The optimal rate of investment (8) is derived in appendix 1,[5] and shows the role of the cost of investment, ϕ. Equation (5) is the arbitrage condition which follows from the assumption of perfect asset substitutability so that expected real returns, adjusted for expected real exchange rate changes, are equalized. With perfect foresight there is no distinction between expected and actual variables. Finally, in (10), current account deficits at home, the sum of the trade deficit and of dividend payments, are matched by changes in the foreign ownership of domestic stocks, as we assume that these are the only traded assets. The model is closed with the conditions (11) that both goods markets are in equilibrium.

2.2 The Stationary State

Assuming away growth, technological changes and depreciation of capital, stationarity requires that all variables become constant. With $\dot{\lambda} = 0$, real interest rates are equalized. With $\dot{K} = \dot{K}^* = 0$ we need to have $\bar{q} = \bar{q}^* = 1$. Then with $\dot{g} = \dot{g}^* = 0$ and $I = I^* = 0$, (5), (6) and (7) imply that $\bar{y} = \overline{rK}$ and $\bar{y}^* = \bar{r}^* \cdot \bar{K}^*$, which, together with (1), define uniquely $\bar{K}$ and $\bar{K}^*$ as functions of $\bar{r} = \bar{r}^*$. Next, we consider the two goods market equilibrium conditions (11). One of them can be replaced by the requirement that world spending equals world income:

$$y + \lambda y^* = (C + \lambda C_m) + (\lambda C^* + C_m^*) = \delta(A + \lambda A^*)$$

which, given the above stationary state conditions, implies:

$$(12) \qquad \bar{r}(\bar{K} + \overline{\lambda K}^*) = \delta(\bar{K} + \overline{\lambda K}^*)$$

Clearly then, the two interest rates must equal the rate of time preference. Otherwise, we would have permanent world net saving (when $r > \delta$) or dissaving (when $r < \delta$).

We then consider the current account condition (10). With goods markets in equilibrium, the current account in each country is the excess of income over spending, so that $\dot{Z} = 0$ implies:[6]

$$(13) \qquad \overline{yE} - \overline{rZ} = (\bar{r} - \delta)(\bar{K} - \bar{Z}) = 0$$

This is where the indeterminacy appears: with $\bar{r} = \delta$, the current account balance condition is always satisfied, so that it is not an active condition. As a consequence, we lose one equation to find the stationary state values of the two variables yet to be determined, λ and Z. The only remaining available condition is one of the two goods market equilibrium (11), any one of which gives:

$$(14) \qquad \bar{\lambda} = \frac{(1-a)\bar{K} + (a-a^*)\bar{Z}}{a^*\bar{K}^*}$$

so that any pair of values $(\bar{\lambda},\bar{Z})$ which satisfy (14) is *a priori* compatible with the stationary state requirement: the distribution of wealth $\bar{Z}$, and the real exchange rate $\bar{\lambda}$ can take an infinity of values.[7]

The economic reason for this apparent indeterminacy can be made intuitive by considering a transfer of wealth from domestic to foreign residents (an increase in $\bar{Z}$), starting from a stationary state situation. Such a transfer, given perfect assets substitutability, does not affect investment/saving decisions and does not upset world equilibrium as seen in (12). Its only effect is to shift world demand toward foreign goods (when $a > a^*$) and only requires a real depreciation to restore equilibrium in *both* goods markets.[8]

This indeterminacy of the stationary state is only apparent: following a disturbance, the model will converge to a unique stable equilibrium, but the resulting values of $\bar{Z}$ and $\bar{\lambda}$ will be a function of its dynamic characteristics. Unfortunately, these values cannot be found without first spelling out the complete dynamic solution.[9] Although we do not present such a solution for this model, it appears that the parameters describing the cost of investment, ϕ and ϕ^* in (7), will influence, not only the adjustment path, but also the ultimate values of Z and λ, and therefore the distribution of spending between the two countries. A higher cost of investment at home will slow down the accumulation (or decumulation) of K toward its optimal value, thus hampering the adjustment of domestic output and, usually, worsening, *ceteris paribus*, the current account, and its total cumulated value as measured by Z. This, in turn, will require a corresponding real exchange depreciation.

2.3. How General Is the Problem?

The property shown in the previous example follows from the fact that, with intertemporal optimization zero savings are an implication of the stationary state, achieved when the interest rate equals the rate of time preference. We now address the question whether this property is truly general or whether it follows from some special assumptions introduced in the model. The answer is that, indeed, there are several ways of eliminating this property. We now discuss some of them and argue that the assumptions that they entail are not obviously superior to those of the above model.

A first possibility is to do away with the perfect assets substitutability hypothesis, which is equivalent to assuming different rates of time preference in each country, since in the stationary state we will still need: $\bar{r} = \delta$, $\bar{r}^* = \delta^*$ and we now want $r \neq r^*$. To understand why the indeterminacy is removed, consider again a transfer of wealth ΔZ to the foreign country. Foreign spending increases by $\delta^* \cdot \Delta Z$ while domestic spending falls by $\delta \Delta Z$: the world equilibrium is disturbed, interest rates will have to adjust and the process will generate current account disturbances leading back to the initial distribution of wealth: the non-uniqueness property is removed. But this solution has some unattractive features. It implies either a corner solution where one country has continuously dissaved to the point of selling away all its wealth so that the other country owns the whole world and consumes all output, or else it implies no holding of foreign assets in the stationary state, since such holdings would have spending out of these assets proportional to the holding country rate of time preference, while earnings would be proportional to the issuing country's rate.

Another possibility is to allow for each country to have variable and endogenous rates of time preference. Obstfeld (1981b) has introduced such a rate, function of utility. In the stationary state, with perfect asset substitutability, we will still have identical rates of time preference in both countries and consumption is still proportional to wealth, δ being the coefficient of proportionality. But, the equalization of the rates of time preference effectively imposes a further condition which eliminates the non-uniqueness property. The reason is that a transfer of wealth, for example from the domestic to the foreign economy, would reduce wealth and therefore consumption at home, with the opposite effect abroad. This, then, would lower domestic utility, increase foreign utility and result in different rates of time preference, prompting current account imbalances until the initial situation is restored. In this case, there is a unique distribution of wealth, and a unique real exchange rate, compatible with the stationary state. But, the solution of the problem has a cost, as such endogenous rates of time preference are hard to justify: should the rate of time preference be an increasing or a decreasing function of utility?[10]

A third possibility would be to introduce wealth in the utility function, so that transfer would alter spending , generating a Metzler-type behavior, and prompting current account adjustments until the unique stationary state distribution of wealth is reached. The question, of course, is whether wealth belongs to the utility function.

The model discussed in the previous section does not include labor as a factor of production. In the following section labor is introduced and it will be seen that the indeterminacy remains. But could it be removed if leisure were an argument of the utility function?[11] In this case, the stationary state requires that real wages be equal to both the marginal productivity of labor and the marginal utility of leisure. If the utility function is not additive in leisure and consumption but assumes substitutability, a transfer of wealth abroad will reduce domestic consumption and increase the marginal utility of leisure, resulting in a reduction of labor supply. In the corresponding stationary state, the capital stock would be lower at home, higher abroad. Yet, it still is the case that equality between the interest rates in each country and the rate of time preference will guarantee balanced current accounts, so that the non-uniqueness property is preserved. But with labor and capital now depending upon wealth, the non-uniqueness spreads as it also affects these variables, as well as output levels.

Summing up, two-country models with inter-temporal optimization are quite likely to exhibit the property that the stationary state is not uniquely determined or, more precisely, that it will be related to some of their dynamic characteristics. The assumptions required to eliminate the property are not necessarily superior, while the indeterminacy may prove to yield interesting and intuitive results. Of course, once we leave the general optimizing framework, the property disappears. It is, of course, the case of ad hoc Keynesian consumption functions and models where consumers are facing quantity or liquidity constraints. It should also be the case of models where optimization is carried over a finite period of time, or of models with overlapping generations, unless bequests exist and enter the utility function, although this point is but a conjecture at this time.

3. Analysis and Solution for Linear Models

In this section we present briefly the results derived in Giavazzi and Wyplosz (1982). We deal with the general case of a system of linear difference equations, characterize the mathematical aspects of the problem described in the previous section and sketch its solution. The reader uninterested in these technical aspects can proceed directly to section 4 without loss of continuity.

3.1. Formulation of the Problem

The general form of a system of linear differential equations is:

(15) $$\dot{x} = Ax - z$$

where x is an n-vector of endogenous variables, z an n-vector of (or combination of) exogenous variables.[12] If the n × n matrix A is non-singular, then there is a unique stationary state:

(16) $$\bar{x} = A^{-1}z$$

and (15) can be rewritten as:

(17) $$\dot{x} = A(x - \bar{x}) \ .$$

The solution of (15), under perfect foresight, is given in Blanchard and Kahn (1980) and Buiter (1981). Stability of the system requires that A admit as many positive eigenvalues as there exist non-predetermined variables in x. The problem under discussion corresponds to the case where the transition matrix A is singular: then we do not have unicity of the stationarity state. In the example of section 2, matrix A is of rank n − 1, so that it admits one zero eigenvalue. Yet some of the variables in x may still have well determined stationary state values, known functions of z. This is the case solved below.

3.2. Separation of the Endogenous Variables

We discuss the case where matrix A is of rank n − 1, yet k of the n endogenous variables, collected in the vector x_1, admit a well-known unique stationary state. We reorder the variables in x and matrix A accordingly, so that the stationary state condition $\dot{x} = 0$ implies:

$$(18)\qquad \begin{bmatrix} A_{11} & A_{12} \\ A_{22} & A_{22} \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

By assumption (18) admits a unique solution for $\bar{x}_1$, not for $\bar{x}_2$.

It is assumed that A can be diagonalized into Λ and we call V the matrix of right-eigenvectors of A, partitioned conformably as follows:

$V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}$ where $\begin{bmatrix} V_{11} \\ V_{21} \end{bmatrix}$ is the eigenvector associated with the zero eigenvalue.[13] Then, in a system like (15), the k first endogenous variables have a well defined stationary state, while the (n-k) last ones share one degree of freedom, when the k first elements of the eigenvector associated with the zero eigenvalue are null ($V_{11} = 0$), i.e. when the last (n-k) columns of the transition matrix A are linearly dependent, while the k first ones are independent.

3.3. Solution of the System

We first rewrite (17) so as to clarify the distinction made among the endogenous variables. The general form can be shown to be:

$$(19)\qquad \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 - \bar{x}_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} A_{12}u \\ A_{22}u \end{bmatrix}$$

where u is a vector of (n - k) exogenous variables. This allows us to interpret the terms in A_{11} and A_{21} as speeds of adjustment, and establishes $\bar{x}_1$ as a set of exogenous variables. For the system (19) to admit a stable solution, it must satisfy the conditions spelled out in Blanchard and Kahn (1980): if there are (n - p) non-predetermined variables in x_1, matrix A must possess (n - p) strictly positive eigenvalues. Then Λ and V can be reordered so that:

$$\Lambda = \begin{bmatrix} 0 & & \\ & \Lambda_1^p & \\ & & \Lambda_1^n \end{bmatrix}$$

where Λ_1^n is an $(n - p) \times (n - p)$ diagonal matrix with positive diagonal elements and Λ_1^p is a $(p - 1) \times (p - 1)$ diagonal matrix with negative diagonal elements. The solution to (19) is:

$$\text{(20)} \qquad \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 0 & V_{12}e^{\Lambda_1^p t} \\ V_{21} & V_{22}e^{\Lambda_1^p t} \end{bmatrix} \begin{bmatrix} (V_1^p)^{-1} \\ 0 \end{bmatrix} \left(x^p(0) - V^p \cdot \begin{bmatrix} 0 \\ \overline{y}_1 \end{bmatrix}\right) + V \begin{bmatrix} 0 \\ \overline{y}_1 \end{bmatrix}$$

where x^p collects the p predetermined variables in x, extracted from both x_1 and x_2, V^p is the corresponding $(p \times n)$ matrix extracted from V, V_1^p is the $(p \times p)$ matrix consisting of the first p columns of V^p and with the $(n - 1)$ vector $\overline{y}_1 = \overline{V}_{21}\overline{x}_1 + \overline{V}_{22}u$, where $\overline{V}_{21}$ and $\overline{V}_{22}$ are elements of V^{-1}:

$$V^{-1} = \begin{bmatrix} \overline{V}_{11} & \overline{V}_{12} \\ \overline{V}_{21} & \overline{V}_{22} \end{bmatrix}$$

To obtain the stationary state value $\overline{x}$, we take the limit of (20) as t goes to infinity. The k first rows of the right hand side reduce to $\overline{x}_1$ as required. Of interest are the last $(n - k)$ rows:

$$\text{(21)} \qquad \overline{x}_2 = [V_{21} \;\; 0](V_1^p)^{-1}\left(x^p(0) - V^p\begin{bmatrix} 0 \\ \overline{y}_1 \end{bmatrix}\right) + V_{22}\overline{y}_1$$

This is the central result of this section. It appears that the a priori indeterminacy of the stationary state has been eliminated through the solution of the system. Still, while $\overline{x}_2$ is unique, it exhibits some properties that do not usually appear:

- $\overline{x}_2$ depends upon the initial position of the system at time $t = 0$, as described by the values of the predetermined variables $x^p(0)$.

- The term in front of $x^p(0)$, $[V_{21}\ 0](V_1^p)^{-1}$ is an $(n - k) \times p$ matrix that does not seem to be amenable to further simplification. It collects elements of the eigenvectors associated with the eigenvalues which are negative or null. In general, this expression will not be independent of the particular values of these eigenvalues collected in Λ_1^p, and which play the role of speeds of adjustment for the dynamic system, as is clear from (20). For the same reason, V_{22} may usually include parameters function of the speeds of adjustment.

It is customary in perfect foresight (or rational expectations) models to have the initial values of the non-predetermined variables depend upon the speed of adjustment of the predetermined variables, as exemplified by the overshooting result of Dornbusch (1976). In the present case, the final value of a subset of the variables, $\overline{x}_2$, both predetermined and non-predetermined, will also be a function of such speed of adjustment parameters, for a given disturbance in the exogenous variables u or in the final values $\overline{x}_1$ of the other endogenous variables. This point is illustrated in the following section.

4. Second Example: Model with Labor Only

4.1. Presentation and the Stationary State

In this section, we present a model very similar in spirit to that discussed in section 2 but which turns out to reduce to a smaller dimension and allows for an easier analytical solution.[14] This model is presented in table 2 below. The difference is that production is now carried out with labor as the only factor of production, instead of capital (1'). The crucial speed of adjustment will be that of the labor market which functions as follows. Labor supply is infinitely elastic at the going real wage rate w, so

Table 2

(1')	$y = y_0 L^{\alpha}$	$y^* = y_0^* L^{*\alpha}$
(2)	$C = a\delta A$	$C^* = (1 - a^*)\delta A^*$
(3)	$\lambda C_m = (1 - a)\delta A$	$\lambda^{-1} C_m^* = a^* \delta A^*$
(4')	$A = X - Z$	$A^* = X^* + \lambda^{-1} Z$
(5')	$\dot{X} = rX - y$	$\dot{X}^* = r^* X^* - y^*$
(6')	$wL = \alpha y$	$w^* L^* = \alpha y^*$
(7')	$\dot{w} = \gamma(L - \bar{L})$	$\dot{w}^* = \gamma^*(L^* - \bar{L}^*)$
(9)	$r = r^* + \dot{\lambda}/\lambda$	
(10')	$\dot{Z} = \lambda C_m - C_m^* + rZ$	
(11')	$y = C + C_m^*$	$y^* = C^* + C_m$

that actual employment L can differ from the "natural" level $\bar{L}$. Excess demand for labor (resp. excess supply), in turn brings about an increase (resp. decrease) in the real wage: the speed at which this adjustment proceeds to reestablish full employment is captured in (7') by the parameter γ. Demand for labor follows from the firm optimizing choice, so that in (6') the real wage rate is equal to the marginal productivity of labor. Total domestic wealth A is defined in (4') as the present value of domestic output:

$$X(t) = \int_t^\infty e^{-\int_t^s r(V)dV} y(s)ds, \text{ or } \dot{X} = rX - y \text{ as in (5')},$$

less domestic indebtedness Z, where Z can be positive or negative. Trade in assets takes the form of indexed bonds, i.e. claims to units of output of the issuing country, and (9) ensures that the yields of such bonds are the same, irrespective of which country issues them. Equation (10') describes the current account and (11') represents the two goods markets' equilibrium conditions.

As in section 2, the stationary state implies:

$$\bar{r} = \bar{r}^* = \delta$$

and the two goods markets equilibrium then reduces to:

$$(22) \qquad \bar{\lambda} = \frac{(1-a)\bar{X} + (a-a^*)\bar{Z}}{a^*\bar{X}^*}$$

As $\bar{X} = \bar{y}/\delta = y_0\bar{L}^\alpha/\delta$ and $\bar{X}^* = y_0^* L^{*\alpha}/\delta$, $\bar{X}$ and $\bar{X}^*$ are clearly defined and we have, again, a relationship linking $\bar{\lambda}$ and $\bar{Z}$, leaving these two variables <u>a priori</u> undetermined.

We will consider a change in domestic productivity $\hat{y}_0 = dy_0/y_0$ which occurs unexpectedly in period $t = 0$. We know that in the new stationary state:

$\overline{w} = (y_0 + dy_0)\overline{L}^{-(1-\alpha)}$ and $\overline{w}^*$ is unchanged, so that domestic wealth will change proportionately to the productivity gain with no long-run effect on foreign human wealth.

4.2. Solution

We note that the interest rate variables are merely definitional and can be eliminated through (5'), (6') and (9) so as to obtain:

(9') $\quad \dot{\mu}/\mu = y/X - y^*/X^*$

where $\mu = \lambda X^*/X$ is the relative value of foreign and domestic gross wealths. The model is then driven by the four equations (7'), (9') and (10'), together with the goods market equilibrium conditions, which allows us to eliminate X and X*. The relative value of wealths μ, is a non-predetermined variable, while w, w* and Z are predetermined.

For the purpose of this example, computations can be greatly reduced by a careful choice of parameters and initial values. Specifically we assume:[15]

For $t < 0$, $X = X^* = 1$, $r = r^* = \delta$, $w = w^* = 1$, $y = y^* = \delta$, $\lambda = \mu = 1$, $Z = 0$.

The system is linearized and solved around this initial position in appendix 2. The resulting laws of motions of the four driving variables are:

$$w(t) = 1 + \hat{y}_0\left(1 - e^{-\gamma_1 t}\right)$$

$$w^*(t) = 1$$

(23) $$Z(t) = \frac{1 - a - a^*}{a + a^*} \cdot \frac{\alpha}{1 - \alpha} \cdot \frac{\delta}{\gamma_1+\delta}\,\hat{y}_0\left(1 - e^{-\gamma_1 t}\right)$$

(24) $$\mu(t)-1 = \frac{1-a-a^*}{a+a^*}\,\hat{y}_0\left(1+ \frac{\alpha}{1-\alpha}\,\frac{\delta}{\gamma_1+\delta}\right)+2\,\frac{1-a-a^*}{a+a^*}\,\frac{\alpha}{1-\alpha}\,\frac{\delta}{\gamma_1+\delta}\,\hat{y}_0\left(e^{-\gamma_1 t} - 1\right)$$

where $\gamma_1 = \gamma\overline{L}/(1 - \alpha)$ is a measure of the speed of adjustment in the domestic labor market.

From these formulae, it is easy to obtain the stationary state values for Z and μ:

$$\bar{Z} = \frac{1 - a - a^*}{a + a^*} \frac{\alpha}{1 - \alpha} \cdot \frac{\delta}{\gamma_1 + \delta} \hat{y}_0$$

$$\bar{\mu} - 1 = \frac{1 - a - a^*}{a + a^*} \hat{y}_0 \left(\frac{a + a^*}{a^*} + \frac{a - a^*}{a^*} \frac{\alpha}{1 - \alpha} \frac{\delta}{\gamma_1 + \delta}\right)$$

It appears that the sign of $(1 - a - a^*)$ plays an important role in the evolution of the system: in the following, we discuss the case where the home country captures less additional sales than the foreign country when world wealth increases, i.e. $1 - a - a^* > 0$. We also assume $a > a^*$, a "preferred habitat" in consumption.

We discuss the solution with the help of figure 1. In the long run, we know that $\mu = \lambda X^*/X$ and Z are linked only by the condition :

$$a^*\overline{\mu X} = (1-a)\bar{X} + (a-a^*)\bar{Z}$$

which is represented by the line LR. An increase in y_0 leads to a proportional increase in the new stationary state value of X, which shifts the line to LR'. The a priori indeterminacy of $\bar{\mu}$ and $\bar{X}$ means that any position along LR' is feasible. At time $t = 0$ when y_0 unexpectedly changes, Z cannot instantaneously move, but μ is non-predetermined and will jump to a point like or B. From there, the economy will follow the stable convergence paths going through or B and depicted on figure 1.

In order to interpret the solution described by (23) and (24), we turn to figure 1. The line LR represents the indeterminacy problem: a priori, in the stationary state, μ and Z can be anywhere along this line which is derived from (22):

$$a^*\mu\overline{X} = (1-a)\overline{X} + (a-a^*)\overline{Z}$$

We have assumed that, prior to the disturbance, the economy was at point A, with Z = 0 and μ = 1. The slope of the line LR increases with $\hat{y}_0$, the disturbance. On impact, Z cannot change instantaneously, but μ is free to jump. As in other models with perfect foresight, the magnitude of the jump is a function of the speed of adjustment of the economy: the slower the labor market reacts to a disequilibrium, i.e. the smaller γ, the larger the impact increase in μ. What is novel here, is that wherever μ jumps to, there will be a convergence path leading to a stationary state position along LR, as shown on figure 1 by the two impact positions B and C, and the corresponding long run points B' and C'.

In order to understand how this happens, we consider first the long run effects of the disturbance. We note first that foreign output, employment and wage rate stay constant. In the stationary state, therefore, world wealth will have increased proportionately to domestic output, making for equal

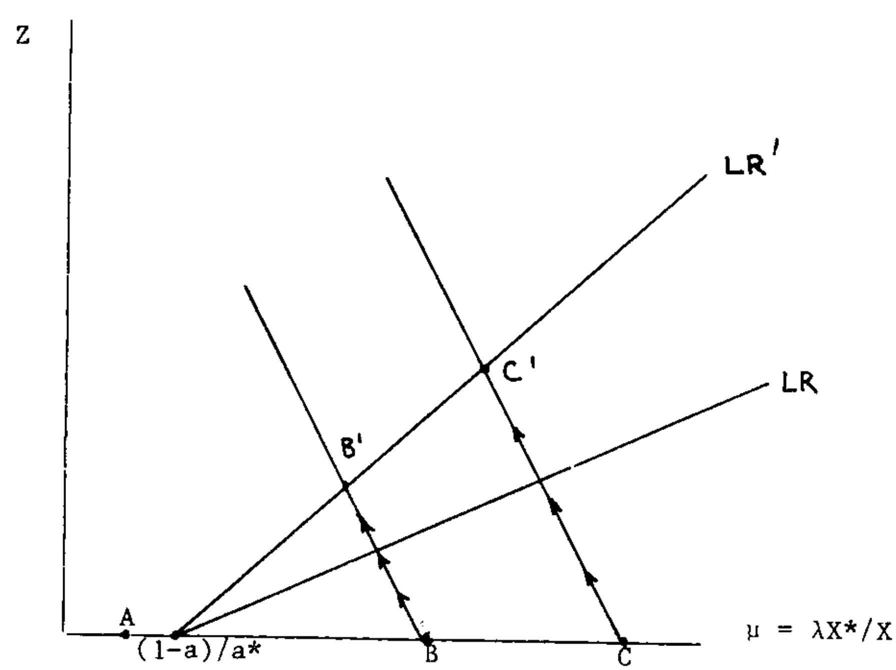

Figure 1

augmentations of world spending and domestic output. With spending directed to both domestic and foreign goods, a real exchange rate depreciation is needed for goods markets to be in equilibrium. Now consider $\mu = \lambda X^*/X$. If we had $a + a^* = 1$, the increase in world wealth per se would not affect the relative demand for domestic and foreign goods so that there would be no need for μ to change; with X^* constant, the increase in λ, proportional to the increase in X, would be enough to maintain both goods markets in equilibrium. If, however, $a + a^* < 1$, as world wealth increases, relative demand tilts toward foreign goods, which requires a further depreciation and an increase in μ; the relative value of foreign wealth, expressed in domestic goods units, must increase in order to eliminate the excess supply of domestic goods. This explains the stationary state value of μ in figure 1.

The impact effect of the increase in y_0 is, in many respects, similar to the long run case just described. Domestic output increases but attracts only a fraction $a + a^*$ of the increase in world wealth so that λ has to increase on impact, as well as μ, when $a + a^* < 1$.

Over time, the domestic labor market adjusts to the increased demand for labor generated by the productivity gain. As the real wage rate increases, demand for labor and domestic output decreases, which requires a real exchange appreciation in order to reduce demand for domestic goods. We thus obtain an overshooting for λ (and μ).[16] This appreciation being correctly anticipated, is accompanied, because of (9), by an interest rate differential so that for $t \geqslant 0$, $r \leqslant \delta$ and $r^* \geqslant \delta$. This interest rate effect is important since it leads to a drop in X^*, the present value of the constant flow of foreign output; as a consequence, the foreign current account turns into a surplus as foreign spending is reduced, and this is matched by a domestic deficit.

We can now discuss the role of γ, the speed of adjustment of the domestic labor market. With a high speed of adjustment, the current account imbalances are eliminated faster, thus making for a smaller cumulated debt of the home country and, therefore, requiring a smaller real exchange rate appreciation.[18] On figure 1, the adjustment path BB' describes the response of the economy for a higher γ than along CC'.

4.3. Welfare Implications

As the consumption behavior is derived from the optimization of Cobb-Douglas intertemporal utility functions, it is easy to draw implications concerning welfare in the new stationary state. This requires computing the values of total domestic and foreign wealth. As shown in appendix 2, foreign wealth $A^* = X^* + Z$ has to increase in the long run as X^* goes back to its initial value while Z is positive. However, A^* initially drops as X^* is reduced on impact, and Z increases only over time. Domestic wealth $A = X - Z$ increases in the stationary state if the loss in wealth Z through cumulated

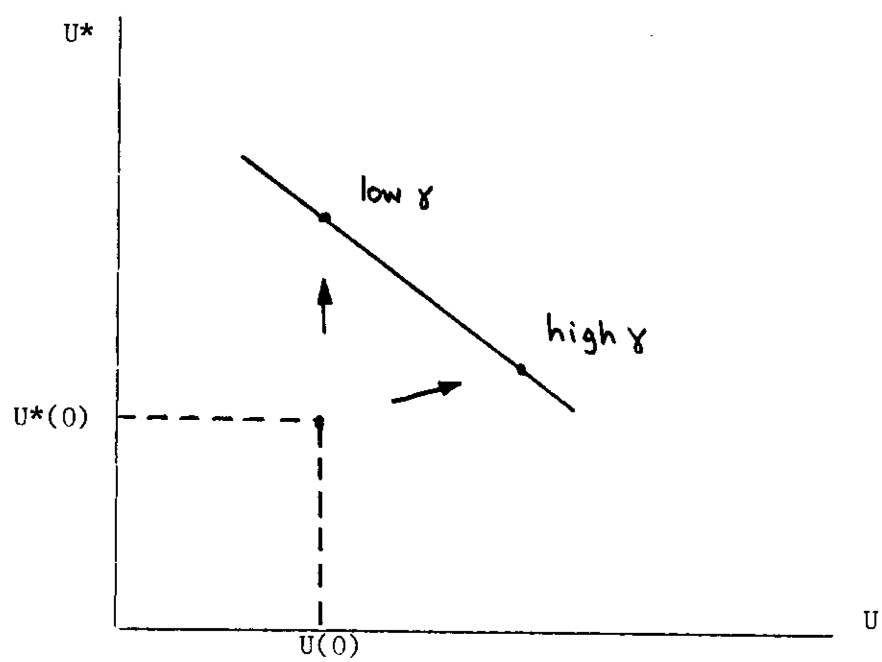

Figure 2

deficits does not offset the gain in X. The possibility that a productivity gain proves to be "immiserizing" augments when the speed of adjustment is small, as current account deficits are more prolonged. If U, and U*, are the domestic, and foreign, Cobb-Douglas welfare functions, then we have:

$$U = A\lambda^{-(1-a)} \qquad U^* = A^*\lambda^{a^*}$$

so that, while U* increases unambiguously through both its wealth and its terms of trade arguments, chances that U decreases grow as γ is lower since it not only reduces wealth gains, but it also worsens the domestic terms of trade.[19] The role of the speed of adjustment is illustrated in figure 2. The line LR shows all the possibilities for the stationary state values of U and U*. The exact position along LR is, as usual, *a priori* unknown. With a high speed of adjustment γ, both countries' welfare improves. With a low γ, the gain at home is lower and can even be negative, while the gain abroad is enhanced.

5. Conclusion

We believe that the class of models in which the initial conditions and the speed of adjustment parameters have permanent influences on the path of the economy after a disturbance, is a large and important one. There are certainly several ways of making different assumptions which eliminate the *a priori* indeterminacy of the stationary state in these models. We have discussed some of them and argued that they do not necessarily seem more appealing than ours. We think that choosing these assumptions simply because they solve the problem discussed in the paper amounts to discarding what appears to be an intuitively interesting property, and is unnecessary since it turns out that the usual stationarity conditions remain sufficient to pin down a unique and stable long run equilibrium. The example which has been solved, shows results which seem to match what one would expect to find.

It is not clear how broad is the potential applicability of this approach. In this paper, the property hinges on the fact that we have two distinct groups of consumers who trade in goods and assets.[20] This is why it has natural applications in internalized macroeconomics for two-country models. It could as well be used in a Kaldorian economy with two classes of consumers who have different spending patterns.

But the same property might also obtain in a one small country model, provided its spending is, again, derived from infinite horizon intertemporal optimization, somehow leaving the rest of the world unspecified. The fact that Drazen (1980) reports a similar property arising in the production side is intriguing. His model has heterogeneous capital and labor, both susceptible of "investments," so that the indeterminacy stems from the possibility of adjusting labor to the existing structure of capital, or of adjusting capital to the existing structure of labor. The interesting aspect of this is that investments in capital and in labor (i.e. job training) are sluggish so that the final stationary state will be uniquely related to the speed of adjustment. There seems to be a scope for a generalization of the mechanisms brought up by Drazen and in the present paper.

APPENDIX 1: OPTIMIZATION

1. The Consumer's Problem

The consumer maximizes $\int_0^\infty e^{-\delta t}\, U(t)dt$ subject to the constraint that total spending $E = C + \lambda C_m$ exhausts, in present value, his wealth A, i.e. $A = \int_0^\infty e^{-\int_0^t r(s)ds} E(t)dt$, or, equivalently, $\dot{A} = rA - E$. We consider the special case where:

$$U(C,C_m) = \ln[U(C,C_m)]$$

and where $u(C,C_m)$ is a function homogeneous of degree 1. The first order conditions are:

$$\text{(A1)} \qquad \partial u/\partial C = \phi u \qquad\qquad \partial u/\partial C_m = \phi\lambda u$$

$$\text{(A2)} \qquad \dot{\phi} = (\delta - r)\phi$$

where ϕ is the Lagrange multiplier. Using the homogeneity of u through Euler equation, (A1) is reduced to: $E\phi = 1$. Differentiating this relationship logarithmically, we then eliminate r to obtain:

$$(\dot{A/E}) = \delta(A/E) - 1$$

which, when integrated forward, gives $E = \delta A$. If $u(C,C_m)$ is further specified as a Cobb-Douglas function, (A1) gives (2) and (3) in the text. Note that, in the stationary state, we have $\dot{\phi} = 0$ and $\dot{A} = 0$, so that, given the constraint and (A2), we must have:

$$r = \delta \qquad \text{and} \qquad E = \delta A,$$

irrespective of the functional form of the utility function $U(C,C_m)$. The reason why the simple formulation $E = \delta A$ also holds outside the stationary state is that the definition of $U(C,C_m)$ as $\ln[u(C,C_m)]$ renders this function Cobb-Douglas over time, thus yielding the usual constant share property.

2. The Firm's Problem

The firm maximizes its present value $\int_0^\infty [y - I]e^{-rt}dt$ given the cost of investment $I = \dot{K}[1 + \frac{\phi}{2}\frac{\dot{K}}{K}]$. Introducing the notation $\dot{K} = J$ the Hamiltonian is:

$$H = \left[y_0 K^\alpha - J\left(1 + (\phi/2)(J/K)\right) + q^m J\right]e^{-rt}$$

where q^m is the marginal cost of investment. The first conditions are:

(A3) $\quad \partial H/\partial J = 0$ so $\dot{K} = J = K(q^m - 1)/\phi$

(A4) $\quad -\partial H/\partial K = e^{-rt}\left(\dot{q}^m - rq^m\right)$, so $\dot{q} = rq^m - \left(\alpha y - I + q^m\dot{K}\right)/K$.

The average value of installed capital at time t, q^a is the present value of the firms earnings, the objective function in the previous optimization problem:

$q^a(t) \cdot K(t) = \int_t^\infty \left(y(S) - I(S)\right)e^{-r(S-t)}dS$, which after differentiation, and dropping the time parameter, gives:

(A5) $\quad \dot{q}^a = rq^a - \left(y - I + q^a\dot{K}\right)/K$.

Thus (5) and (6) in the text define q to be q^a as specified in (A5), while (8) is (A3) where q^m has been replaced by q^a. Comparison of (A4) and (A5) shows the nature of this approximation, discussed in footnote 4.

APPENDIX 2: SOLUTION OF THE MODEL

We first linearize the model around its initial position, characterized by $X = X^* = 1$, $\mu = \lambda = 1$, $y = y^* = \delta$, $Z = 0$ and $w = w^* = 1$. The wage adjustment equations (7'), after substitution of (6') into (1') and then plugging L and L* into (7') gives, for a given disturbance $\hat{y}_0 = \Delta y_0/y_0$ in home productivity:

(B1) $$\dot{w} = -\gamma_1(w - 1) + \gamma_1\hat{y}_0 \qquad \text{where } \gamma_1 = \gamma\overline{L}/(1 - \alpha)$$

(B2) $$\dot{w}^* = -\gamma_1^*(w^* - 1) \qquad \text{where } \gamma_1^* = \gamma^*\overline{L}^*/(1 - \alpha) \ .$$

Clearly, the only solution for (B2) which admits $w^*(0) = 1$ as assumed, is $w^* = \overline{w}^* = 1$, a constant. Thus, there will be no departure from full employment abroad. We use, in the following, the fact that w^* remains constant throughout.

From (B1), it is also clear that the stationary state value of w is <u>a priori</u> uniquely determined:

$$\overline{w} = 1 + \hat{y}_0 \ .$$

The goods markets' equilibrium conditions (11') are solved for X and X* after substitution of $\mu = \lambda X^*/X$. Actually, it is easier first to write that world income is equal to world spending:

$$y + \lambda y^* = \delta A + \delta\lambda A^*$$

which gives:

(B3) $$X + X^* = -\alpha/(1 - \alpha)[w - w^*] + \hat{y}_o/(1 - \alpha) \ .$$

Then the domestic goods market condition is solved for X:

(B4) $$(a+a^*)(X-1) = -[\alpha/(1-\alpha)](w-1) - a^*(\mu-1) + (a-a^*)Z + \hat{y}_0/(1-\alpha) \ .$$

The current account equation, when linearized and after substitution of (B3) and (B4), gives:

(B5) $(a+a^*)\dot{Z}/\delta = -(1-a-a^*)[\alpha/(1-\alpha)](w-1) - a^*(\mu-1) + (a-a^*)Z + (1-a-a^*)\hat{y}_0/(1-\alpha)$.

The asset arbitrage condition (9'), similarly, yields:

(B6)
$$(a+a^*)\dot{\mu} = 2\delta(1-a-a^*)[\alpha/(1-\alpha)](w-1) + 2\delta a^*(\mu-1) - 2\delta(a-a^*)Z - 2\delta(1-a-a^*)\hat{y}_0/(1-\alpha) .$$

The system is reduced to the three equations (C1), (C5), (C6) and rewritten in matrix form as:

$$\begin{bmatrix} \dot{w} \\ \dot{Z} \\ \dot{\mu} \end{bmatrix} = \begin{bmatrix} -\gamma_1 & 0 & 0 \\ -\frac{1-a-a^*}{a+a^*}\frac{\delta\alpha}{1-\alpha} & \delta\frac{a-a^*}{a+a^*} & -\delta\frac{a^*}{a+a^*} \\ 2\frac{1-a-a^*}{a+a^*}\frac{\delta\alpha}{1-\alpha} & -2\delta\frac{a-a^*}{a+a^*} & 2\delta\frac{a^*}{a+a^*} \end{bmatrix} \begin{bmatrix} w \\ Z \\ \mu \end{bmatrix} + \begin{bmatrix} \gamma_1\bar{w} \\ u \\ 2u \end{bmatrix}$$

where $u = -\frac{1-a-a^*}{a+a^*}\frac{\delta\alpha}{1-\alpha} + \frac{\delta a^*}{a+a^*} + \frac{1-a-a^*}{a+a^*}\frac{\delta\hat{y}_0}{1-\alpha}$.

It can be immediately checked that the last two columns of the transition matrix are linearly dependent so that the matrix is singular and we cannot find a priori $\bar{Z}$ and $\bar{\mu}$. The eigenvalues are:

$$\lambda_1 = 0 \qquad \lambda_2 = -\gamma_1 \qquad \lambda_3 = \delta ,$$

so, with one positive root and one non-predetermined variable, the model is stable under perfect foresight. The corresponding matrix of eigenvectors is

$$V = \begin{bmatrix} 0 & \frac{a+a^*}{1-a-a^*} & 0 \\ \frac{a^*}{a-a^*} & \frac{\delta}{\gamma_1+\delta}\,\frac{\alpha}{1-\alpha} & -1 \\ 1 & -2\,\frac{\delta}{\gamma_1+\delta}\cdot\frac{\alpha}{1-\alpha} & 2 \end{bmatrix}$$

which contains terms with γ_1, the speed of adjustment.

We can solve for w, Z and μ:

(B7) $w(t) = \bar{w} + (1-\bar{w})e^{-\gamma_1 t}$ where, again, $\bar{w} = 1 + \hat{y}_0$

(B8) $Z(t) = \frac{1-a-a^*}{a+a^*}\,\frac{\alpha}{1-\alpha}\,\frac{\delta}{\gamma_1+\delta}\,\hat{y}_0\left(1 - e^{-\gamma_1 t}\right)$

(B9) $\mu(t) - 1 = \frac{1-a-a^*}{a^*}\left(1 + \frac{\alpha}{1-\alpha}\,\frac{\delta}{\gamma_1+\delta}\right)\hat{y}_0 + 2\,\frac{1-a-a^*}{a+a^*}\,\frac{\alpha}{1-\alpha}\,\frac{\delta}{\gamma_1+\delta}\,\hat{y}_0\left(e^{-\gamma_1 t} - 1\right)$.

The stationary state values for Z and μ immediately follow:

$$\bar{Z} = \frac{1-a-a^*}{a+a^*}\,\frac{\alpha}{1-\alpha}\,\frac{\delta}{\gamma_1+\delta}\,\hat{y}_0$$

$$\bar{\mu} - 1 = \frac{1-a-a^*}{a+a^*}\left(\frac{a+a^*}{a^*} + \frac{a-a^*}{a^*}\,\frac{\alpha}{1-\alpha}\,\frac{\delta}{\gamma_1+\delta}\right) .$$

Also, note from (B9) that the initial jump of μ at time zero will also be a function of γ_1.

Using (B3), (B4) and the linearized version of $\mu = \lambda X^*/X$, we can now compute:

$$X(t) - 1 = \hat{y}_0 + \frac{1}{a+a^*}\,\frac{\alpha}{1-\alpha}\,\hat{y}_0\left(1 - (1-a-a^*)\,\frac{\delta}{\gamma_1+\delta}\right)e^{-\gamma_1 t}$$

$$X^*(t) - 1 = -\frac{1-a-a^*}{a+a^*}\frac{\alpha}{1-\alpha}\frac{\gamma_1}{\gamma_1+\delta}\hat{y}_0 e^{-\gamma_1 t}$$

$$\lambda(t) - 1 = \frac{1-a}{a^*}\hat{y}_0 + \left[\frac{(1-a-a^*)(a-a^*)}{a^*(a+a^*)}\frac{\delta}{\gamma_1+\delta} + \frac{2-a-a^*}{a+a^*}e^{-\gamma_1 t}\right]\frac{\alpha}{1-\alpha}\hat{y}_0 .$$

Finally, if the domestic welfare function is $U = kC^a C_m^{1-a}$ with $C = a\delta A$, $\lambda C_m = (1-a)\delta A$, we obtain $U = A\lambda^{-(1-a)}$ when $k = \delta^{-1}a^{-a}(1-a)^{-(1-a)}$. Similarly, the foreign welfare function is $U^* = A^*\lambda^{a^*}$. Linearizing and computing the stationary state values gives:

$$\frac{\bar{U}-U(0)}{U(0)} = \left[1 - \frac{(1-a)^2}{a^*} - \frac{a}{a^*}\frac{(1-a-a^*)(1-a+a^*)}{a+a^*}\cdot\frac{\alpha}{1-\alpha}\cdot\frac{\delta}{\gamma_1+\delta}\right]\hat{y}_0$$

$$\frac{\bar{U}^*-U^*(0)}{U^*(0)} = (1-a)\hat{y}_0 + [a + (1-a^*)]\frac{1-a-a^*}{a+a^*}\frac{\alpha}{1-\alpha}\frac{\delta}{\gamma_1+\delta}\hat{y}_0 .$$

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Then the domestic goods market condition is solved for X:

(B4) $$(a+a^*)(X-1) = -\left[\alpha/(1-\alpha)\right](w-1) - a^*(\mu-1) + (a-a^*)Z + \hat{y}_0/(1-\alpha) \ .$$

The current account equation, when linearized and after substitution of (B3) and (B4), gives:

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The asset arbitrage condition (9'), similarly, yields:

$$\text{(B6)} \quad (a+a^*)\dot{\mu} = 2\delta(1-a-a^*)[\alpha/(1-\alpha)](w-1) + 2\delta a^*(\mu-1) - 2\delta(a-a^*)Z - 2\delta(1-a-a^*)\hat{y}_0/(1-\alpha) .$$

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where $u = -\frac{1-a-a^*}{a+a^*}\frac{\delta\alpha}{1-\alpha} + \frac{\delta a^*}{a+a^*} + \frac{1-a-a^*}{a+a^*}\frac{\delta\hat{y}_0}{1-\alpha}$.

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The stationary state values for Z and μ immediately follow:

$$\bar{Z} = \frac{1-a-a^*}{a+a^*}\frac{\alpha}{1-\alpha}\frac{\delta}{\gamma_1+\delta}\hat{y}_0$$

$$\bar{\mu} - 1 = \frac{1-a-a^*}{a+a^*}\left(\frac{a+a^*}{a^*} + \frac{a-a^*}{a^*}\frac{\alpha}{1-\alpha}\frac{\delta}{\gamma_1+\delta}\right) .$$

Also, note from (B9) that the initial jump of μ at time zero will also be a function of γ_1.

Using (B3), (B4) and the linearized version of $\mu = \lambda X^*/X$, we can now compute:

$$X(t) - 1 = \hat{y}_0 + \frac{1}{a+a^*}\frac{\alpha}{1-\alpha}\hat{y}_0\left(1 - (1-a-a^*)\frac{\delta}{\gamma_1+\delta}\right)e^{-\gamma_1 t}$$

$$X^*(t) - 1 = -\frac{1-a-a^*}{a+a^*}\frac{\alpha}{1-\alpha}\frac{\gamma_1}{\gamma_1+\delta}\hat{y}_0 e^{-\gamma_1 t}$$

$$\lambda(t) - 1 = \frac{1-a}{a^*}\hat{y}_0 + \left[\frac{(1-a-a^*)(a-a^*)}{a^*(a+a^*)}\frac{\delta}{\gamma_1+\delta} + \frac{2-a-a^*}{a+a^*}e^{-\gamma_1 t}\right]\frac{\alpha}{1-\alpha}\hat{y}_0 \ .$$

Finally, if the domestic welfare function is $U = kC^a C_m^{1-a}$ with $C = a\delta A$, $\lambda C_m = (1-a)\delta A$, we obtain $U = A\lambda^{-(1-a)}$ when $k = \delta^{-1}a^{-a}(1-a)^{-(1-a)}$. Similarly, the foreign welfare function is $U^* = A^*\lambda^{a^*}$. Linearizing and computing the stationary state values gives:

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$$\frac{\overline{U}^*-U^*(0)}{U^*(0)} = (1-a)\hat{y}_0 + [a + (1-a^*)]\frac{1-a-a^*}{a+a^*}\frac{\alpha}{1-\alpha}\frac{\delta}{\gamma_1+\delta}\hat{y}_0 \ .$$

FOOTNOTES

[1]For a representative sample of these studies see Dornbusch (1976), Dornbusch and Fischer (1980), Wilson (1979), Mussa (1980) and Kouri (1981).

[2]This indeterminacy should not be confused with the well known problem associated with saddle-path stability, according to which one needs additional conditions, such as ruling out explosive solutions, to identify a unique convergence path. On this problem, see for example Blanchard (1979).

[3]The essential result does not depend upon the specification of the utility function, because, in the stationary state, total spending $E = C + \lambda C_m$ is always equal to δA. In order to obtain $E = \delta A$ at any point in time, i.e. also outside the stationary state, we need the utility function to be the logarithm of a linear homogeneous function of C and C_m. See appendix 1 for proofs and a discussion.

[4]This investment function is described in Abel (1979) and used in Blanchard (1980).

[5]In the presence of decreasing returns, one should distinguish the shadow price of investment, Tobin's marginal q, from the present value of installed capital, Tobin's average q. Doing so would increase the order of the dynamic system, making it intractable, so that we approximate marginal q by its average (observable) value. The exact values of the two q's are given in appendix 1. For a discussion of the issue, see Hayashi (1981). For a simulated version of the model allowing for the two q's, see Giavazzi, Odekon and Wyplosz (1982). Also, note that the interest rate is not equal to the marginal productivity of capital. This is because, with only one factor of production and decreasing returns to scale, stockholders enjoy a rent which is implicitly redistributed as part of dividend payments so that all earnings are accounted for.

[6]Thus, at home, income is $y - rZ$, spending is $\delta A = \delta(K-Z)$ and $\dot{Z}$ measures the current account deficit.

[7]With $r = r^* = \delta$, (1), (5) and (6) imply: $\delta\bar{K} = y_0\bar{K}^{\alpha}$, so that $\bar{K}$ (and $\bar{K}^*$) are uniquely determined.

[8]When $a = a^*$, the transfer has no effect on relative demand for domestic and foreign goods; $\bar{\lambda}$ is determined but $\bar{Z}$ is irrelevant for any other variable: we actually have only one consumer. Also note that Branson (1979) has emphasized that a current account deficit will require a permanent real exchange rate depreciation in order to generate the trade surplus needed to pay for the increased foreign debt. Equation (14) seems to confirm this result when $a > a^*$, but for a totally different reason. The debt effect vanishes in (13) as domestic residents recognize that their wealth is reduced and lower their spending accordingly. Here the effect on the real exchange rate is entirely due to the shift in relative demand for domestic and foreign goods, as discussed in the transfer example, and with $a < a^*$ a current account deficit implies a long run real appreciation.

[9]Thus, the general analytical solution provided by Blanchard and Kahn (1980) remains valid in this case, and will provide the unique stationary state values. Yet, Blanchard and Kahn have not drawn the important consequences of the singularity of the transition matrix, as discussed in the next section.

[10]This issue has been recently revived by Lucas and Stokey (1982). Koopmans, Diamond and Williamson (1964) had derived a set of postulates conveying the concept of time impatience,and characterized the utility functions which satisfy these postulates. They came up with two examples, one with a constant rate of time preference, one with time preference an increasing function of utility. Lucas and Stokey build upon Koopmans et. al. to study the optimum equilibrium allocation in a many agents growth model. Very interestingly, they argue against a constant rate of time preference precisely because any distribution of utility is compatible with the stationary state, i.e., they reach the same indeterminacy property, but reject it.

[11]This case is treated in a simulation context by Lipton and Sachs (1980).

[12]We assume that z is a vector of constant terms. This allows us to simplify the presentation considerably without affecting the substance of the argument.

[13]If A cannot be diagonalized, the solution is possible by using the Jordan canonical transformation instead; see Blanchard and Kahn.

[14]We have been able, so far, to obtain analytical solutions for the model of section 2 only in cases where the dynamics is uninteresting and does not lead to current account imbalances, because of the simplifying assumptions which make it tractable.

[15]Yet, we do not assume that the model was resting in a stationary state since, with $Z = 0$ and $X = X^*$, (22) would imply $a + a^* = 1$. In this case we obtain a trivial solution where λ jumps to its new stationary state value, with $Z(t) = 0$, $\forall t$, and no dynamics at all. The reason will appear clearly in the following discussion where we show the role of the assumption $a + a^* \neq 1$.

[16]The overshooting in μ (and in λ) is now a familiar feature in exchange rate models, since Dornbusch (1976) and Black (1977). Here it follows from the stickiness of wages and the corresponding difference in speeds of adjustments on labor and assets markets.

[18]While the domestic current account is more quickly eliminated with a high speed of adjustment, its initial size is larger. With a high γ, the exchange rate appreciation, following the depreciation on impact, is faster, pushing r further down and thus leading to larger domestic wealth and spending. Yet the accumulated debt is unambiguously smaller as shown by (23).

[19]At this point, it is worth re-emphasizing that the foregoing discussion assumes $1 - a - a^* > 0$. Taking $1 - a - a^* < 0$ would reverse this result and put the burden of potentially decreasing wealth and welfare on the foreign economy.

[20]The production part of the models presented here is not required to obtain the result. We have assumed that firms optimize for the sake of coherence only.

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