THE AGGREGATION OF CONSUMER PREFERENCES OVER FIRM INVESTMENT AND FINANCIAL DECISIONS IN IMPERFECT MARKETS

by

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ABSTRACT

This paper considers a two-period, complete markets model in which consumers' income from initial portfolio endowments is taxed at rates which depend both on the consumer being taxed and on whether the income is ordinary income or capital gains. Such differential taxation causes consumers to disagree about the objectives of the firm, both as these relate to the firm's investment decision and the firm's financial policy. While consumers will not unanimously favor any specific firm policy (in particular, most consumers will not favor firm value maximization), policies exist which are supported by a majority vote of the firm's shareholders.

Keywords: consumer preferences, imperfect markets, voting equilibria.

Introduction

The problem of aggregating stockholder preferences regarding firm investment and financial policies did not arise in neoclassical economic models, where firms were assumed to be owned by a sole decision maker who operated under conditions of complete certainty. Fisher [1930] was the first to pose the unanimity question: Under what conditions can the management of a firm adopt a single production plan which will be accepted unanimously by all shareholders? Fisher (and later Hirschleifer [1958]) concluded that where markets are competitive and perfect[2] all shareholders prefer the production plan that maximizes the value of the firm. As shown by Arrow [1963-64] the introduction of uncertainty does not alter this result, provided that markets -- in addition to the above assumptions -- are assumed to be complete.[3]

Although not posed as a question of shareholder unanimity, the question of the optimal financing of a firm's production which arises in the financial economics literature bears many of the hallmarks of the Fisherian unanimity question. Thus, for example, the well-known result of Modigliani and Miller [1958] may be rephrased as follows: When markets are perfect (in the sense of footnote 2) and complete,[4] shareholders will unanimously approve *any* financing scheme proposed by management. Whereas, however, unanimity in the investment case follows from the desires of shareholders to see their current wealth from the firm maximized, unanimity in the financing case results from shareholder indifference to any and all financing schemes, all of which are shown to leave shareholder wealth unchanged.

In an effort to extend the above-cited results, a number of recent papers in the investment area have studied the sufficient conditions for stockholder unanimity when market completeness does not hold, but under the assumption that markets are perfect. When markets are incomplete, one of two types of conditions guarantees stockholder unanimity: 1. The returns of any new production proposal are spanned by the returns of existing production plans; this is the approach adopted by Ekern and Wilson [1974], Leland [1974], Radner [1974], Nielsen [1976], and Grossman and Stiglitz [1977]. 2. All investors' utility functions belong to the same family of hyperbolic absolute risk-aversion utility functions; this condition was shown by Rubinstein [1974] also to be sufficient for unanimity with respect to the firm's financing decision.[5]

Extensions of the Modigliani and Miller results, on the other hand, have taken a somewhat different path: Discussions in the financial economics literature have *assumed* that consumers unanimously desire the maximization of the firm's value, and have proceeded to investigate the impact of market imperfections (most notably taxation) on optimal firm policies. As early as 1963 Modigliani and Miller introduced corporate taxation as a source of market imperfection and concluded that all investors will prefer full debt financing of the firm. In an effort to explain why, nevertheless, corporations are generally seen to have less than maximal leverage, Miller [1977] introduced individual taxation as well as corporate taxation into a financial markets model. Miller (and later DeAngelo and Masulis [1980] -- in a formalization of Miller's model) investigated the optimal financing decision of the firm when personal tax rates vary among individuals and among different types of securities. As in the earlier paper by Miller and Modigliani, the discussion in these recent papers assumes, however, that value maximization is the desired shareholder objective of the firm, and the question of whether this is in fact so (and we shall see in this paper that this is not so) is not examined.[6]

To sum up the research in this area: On the one hand, the production literature has investigated sufficient conditions for unanimity in perfect, incomplete markets, but has not examined the impact of market imperfection on this unanimity. On the other hand, the finance literature has concentrated on the impact of market imperfections (most notably taxation of corporate and individual incomes), but has taken for granted that consumers will unanimously desire market value maximization.

Neither of the two approaches cited above has dealt with the following question: Will consumers be unanimous in their preferences over firm financial and production plans in markets with taxation? In this paper we examine this question. We construct a model of an economy with complete markets in which there exists a sole imperfection: Tax rates vary across individuals in the economy.[7] We show that the effect of differential taxation on shareholder preferences in such a model is to cause shareholders to *disagree* about the objectives of the firm, both as these relate to the firm's investment decision (choice of inputs) and the firm's financial policy (debt/equity ratio). Having established a lack of shareholder unanimity, we then proceed to ask whether the preferences of shareholders may be aggregated in some manner. We show that an aggregation procedure indeed exists; decisions about both the firm's investment policies and its financial policies may be made by a majority vote of the firm's shareholders.

The majority voting procedure is shown to be well-defined and non-manipulable.

The structure of the paper is as follows: In Section I we model a single-good, two-period, stock-market economy in the spirit of the standard Arrow-Debreu model. Three kinds of taxes are levied in the model: 1. Corporate incomes, suitably defined, are subject to a corporate income tax. 2. Individual dividends and interest income are subject to an "ordinary" income tax. 3. Capital gains by shareholders from the sale of their portfolios in the first period are subject to a capital gains tax. Both the ordinary and the capital gains taxes are assumed to be proportional, consumer-dependent, taxes.

In Section I of the paper we study the effect of this differential taxation of consumer incomes on consumer preferences about firm input choices. Consumer preferences are shown to depend on their tax rates; by our assumption of complete markets, we show that each consumer wishes to see his first-period, after-tax income from his initial portfolio holdings maximized, and this income is shown to depend on the individual tax rates. There is thus disagreement about preferred firm input choice, even though the effect of marginal changes in firm input choices on market prices and consumer incomes is accurately predicted (this follows from the complete market assumptions). We conclude Section I by showing that the differing consumer preferences may be aggregated in this case by a majority voting procedure.

In the model of Section I firms made only choices over inputs. In Section II we redefine the model so that firms decide not only upon inputs but also upon a debt-equity structure. The addition of a financial policy variable to the model increases the model's complexity considerably, since we must now add a bond market to the model and consider the possibility of firm bankruptcy. Having done this, we reconsider consumer preferences over both financial policy and firm investment polciy given the tax environment of Section 1. As in Section 1, we find that consumers disagree about firm policies, and that this disagreement is a function of consumer tax rates. We consider each of the two firm decisions variables separately. All consumers are shown to prefer only one of two possible firm financial policies: Either a consumer prefers that the firm have no debt at all, or he prefers that the firm choose the maximal debt level consistent with the receipt of the tax shield from the debt.[8] The consumer preference over firm debt policy is shown, furthermore, to be independent of the firm's investment policy, and we again show that consumer preferences may be aggregated by a majority choice procedure.

I. *Consumer Preferences over Firm Production Decisions When Incomes Are Taxed*

Ia. *The Model*

Consider a two-period model with one physical good used for both consumption and production. We assume that the price of the physical good is unity both today ("period 0") and in every state of the world tomorrow ("period 1"). There are assumed to by N such states; while we shall not require that agents in the model have homogeneous expectations (i.e., consumers and firms may have different subjective probabilities of the occurrence of states), we shall assume that all consumers believe every state of the world to have a positive probability of occurrence and that all agents agree on the number and description of states of the world.

There will be J firms and I consumers in our model. Each of the firms is assumed to purchase inputs today for production tomorrow; similarly, each firm's production today is determined by the inputs purchased by the firm before the start of the model. Let $\bar{y}_{jo}$ denote firm j's income from production today, T_c denote the (uniform) corporate tax rate, and z_j the quantity of the physical good purchased by firm j today for production tomorrow. Then the after-corporate tax income available to firm j's stockholders today will be[9]

(1) $$r_{jo} = \bar{y}_{jo}(1 - T_c) - z_j.$$

Given purchases z_j of inputs by firm j, denote the production tomorrow of the physical good in state n by $y_{jn}(z_j)$, where the functions y_{jn}, n=1,...,N, will be assumed to be continuously differentiable and weakly concave, with y_{jn} strictly concave for at least one n. In state n of the world tomorrow, firm j has production income $y_{jn}(z_j)$; its taxable income is $y_{jn}(z_j) - z_j$, and hence the after-corporate tax income of firm j in state n will be

(2) $$r_{jn}(z_j) = (1 - T_c)y_{jn}(z_j) + T_c z_j.$$

In our setting consumers (i=1, . . ., I) come into the market in period 0 with initial proportional ownership shares $\bar{s}_{ij}$ for each firm j, j=1, . . ., J, in the economy. These initial holdings (which are assumed to satisfy $\sum_i \bar{s}_{ij} = 1$ and $\bar{s}_{ij} \geq 0$) entitle their holders to a share in the dividends distributed in period 0 by each firm, r_{jo}. Furthermore our model assumes that investors liquidate their initial holdings in period 0 and then purchase new share portfolios. If the market price of firm j's equity is given by p_j, investor i will realize the amount $\bar{s}_{ij}p_j$ (before taxes) from this sale of his initial equity in the firm. Consumer i's before-tax proceeds from dividends and the liquidation of his initial portfolio are thus

(3) $\bar{s}_{ij}\{[\bar{y}_{jo}(1 - T_c) - z_j] + p_j\}$.

From this amount consumer i now has to pay regular income tax at a rate T_d^i on the dividends received and capital gains tax at a rate of T_g^i on the excess of the selling price p_j over the original purchase price, which -- with no loss in generality -- we shall assume to have been zero.[10] The after-tax cash flow to consumer i from his initial holding in firm j is therefore

(4) $\bar{s}_{ij}\{(1 - T_d^i)[y_{jo}(1 - T_c) - z_j] + (1 - T_g^i)p_j\}$.

In period 0 individual i not only collects income from his holdings of shares in the firms, but he also purchases a share portfolio which will provide him with income (and consumption) in period one. Denote this portfolio by $s_i = (s_{i1}, \ldots, s_{iJ})$, where -- as before -- s_{ij} denotes the proportion of firm j's equity purchased by consumer i. We shall assume that s_{ij} may be either positive or negative; if $s_{ij} < 0$, this will represent a short sale by individual i of firm j's equity; such a short sale will require individual i to pay out in period 1 the gross payment paid out by firm j. Given individual i's marginal tax rate on ordinary income T_d^i, the net payout of firm j in state n to individual i will be

(5) $r_{jn}^i(z_j) = [(1 - T_d^i)r_{jn}(z_j) + T_d^i p_j]s_{ij}$.[11]

Given individual i's portfolio s_i, his total net income in state n from portfolio s_i purchased in period 0 will be

(6) $\sum_j r_{jn}^i(z_j)$.

We may now define the maximization problem faced by individual i. Given firm decisions (z_j) and market prices (p_j), individual i will be assumed to maximize a utility function of today's consumption x_{io} and tomorrow's (state-dependent) consumption (x_{in}):

$$\max_{s_i} U_i(x_{io}, x_{i1}, \ldots, x_{iN})$$

such that

$$x_{io} = \sum_j \bar{s}_{ij}\{(1 - T_d^i)[(1 - T_c)\bar{y}_{jo} - z_j] + (1 - T_g^i)p_j\} - \sum_j s_{ij}p_j \quad .$$

(7)

$$x_{in} = \sum_j r_{jn}^i \ .$$

We shall assume that u_i is concave, increasing in each x_{in}, n=0, ..., N, continuous, and has the additional following property

$$\frac{\partial U_i}{\partial x_{in}} \to +\infty \qquad \text{as } x_{in} \to 0,\ n=0, 1, \ldots, N. \tag{8}$$

This last property guarantees that no individual will risk default in any state of the world.

Ib. *First-Order Conditions and Market Completeness*

We shall henceforth assume that the returns $(r_{jn}(z_j))$ span an N-dimensional Euclidean space for all choices of firm inputs (z_j). This assumption, which is termed *market completeness*, is sufficient to guarantee that for any set of market prices (p_j) for firms' equity, there exists a unique set of *state prices* $\beta = (\beta_1, \ldots, \beta_N)$ which price the firms' gross returns (r_{jn}). We prove this fact in the following theorem:

Theorem 1. Let markets be complete. Then given any set of market prices (p_j) and firm inputs (z_j) there exists a vector of state prices $\beta(p_j)$ such that

$$(9) \qquad p_j = \sum_n \beta_n r_{jn}, \; j = 1, \ldots, J.$$

Proof of Theorem 1. Denote by s_i^* individual i's utility maximizing portfolio given prices (p_j) and inputs (z_j) and let x_i^* denote the corresponding consumption. Then it follows from setting

$$\frac{\partial U_i}{\partial s_{ij}} = 0 \quad \text{for every} \quad j,$$

that

$$p_j = \sum_n \beta_n^i r_{jn},$$

where

$$(10) \qquad \beta_n^i = \frac{q_n^i(1-T_d^i)}{1 - \sum_n q_n^i T_d^i}$$

and

$$(11) \qquad q_n^i = \frac{\partial U_i/\partial x_{in}}{\partial U_i/\partial x_{io}} \quad \text{evaluated at} \quad (x_i^*) .$$

By the assumption of market completeness it follows that there exists a vector $\beta = (\beta_1, \ldots, \beta_N)$ such that

$$\beta_n = \beta_n^i \quad \text{for every} \quad i=1, \ldots, I.$$

Q.E.D.

The theorem shows that in complete markets all consumers have the same implicit prices for consumption in any state of period 2. The complete market assumption guarantees that in any given equilibrium, the state-returns of firms will be uniformly priced by all consumers. In addition to the assumption of completeness, we shall assume that all consumers believe that no action of a single firm can affect either the state prices of the market or the actions and prices of other firms. In such a market (see Svennson [1977]) consumers can thus use the state prices to predict the effect on the value of any individual firm of a change in firm policy.

Ic. *Consumer Preferences Over Firm Inputs in Complete Markets with Taxation*

In no-taxation models of consumer choice of firm inputs, the assumption of complete markets is sufficient to guarantee unanimity of consumer choice over firm input purchases. As we shall see, the introduction of a tax system with varying taxes among individuals implies that there will not, in general, be such unanimity. To see this, we first differentiate an individual consumer's utility function with respect to z_j, assuming that state prices, portfolio choices, and input choices of firms other than j are constant. Setting this derivative equal to zero, we find that

$$(12) \qquad \frac{\partial U_i}{\partial z_j} = \frac{\partial U_i}{\partial x_{io}} \frac{\partial x_{io}}{\partial z_j} + \sum_n \frac{\partial U_i}{\partial x_{in}} \frac{\partial x_{in}}{\partial z_j} = 0,$$

since

$$(13) \qquad \frac{\partial x_{io}}{\partial z_j} = \bar{s}_{ij} \{- (1 - T_d^i) + (1 - T_g^i) \frac{\partial p_j}{\partial z_j}\} - s_{ij} \frac{\partial p_j}{\partial z_j},$$

and

$$(14) \qquad \frac{\partial x_{in}}{\partial z_j} = s_{ij}\{(1 - T_d^i)[(1 - T_c)\frac{\partial y_{jn}}{\partial z_j} + T_c] + T_d^i \frac{\partial p_j}{\partial z_j}\} .$$

It follows from Theorem 2 that under our assumptions (completeness and competitivity), consumers desire firm actions to be taken so as to maximize their current wealth from their initial portfolios (the first expression on the left-hand side of (15)); the *consumption effect* -- all those terms which involve the choice of new portfolios (s_{ij}) made in period 0 -- plays no part in the choice by the consumer of the utility-maximizing z_j. Writing $\partial p_j/\partial z_j$ as in (17), Theorem 2 shows that consumer i wants firm j to choose z_j^{i*} so that

$$(20) \qquad \sum_n \beta_n \frac{\partial r_{jn}}{\partial z_j} = \sum_n \beta_n \{(1 - T_c) \frac{dy_{jn}}{dz_j} + T_c\} = \frac{1 - T_d^i}{1 - T_g^i}$$

The utility-maximizing action desired by any consumer of a firm in which he is an initial shareholder is thus a function of that consumer's ordinary and capital-gains tax rates, and is *not* necessarily the action which maximizes the firm's net-present value. Net-present-value maximization will be desired by any single consumer only if his capital gains tax rate is equal to his ordinary income tax rate. Equation (20) indicates that if consumer i's ordinary tax rate is higher than his capital gains rate, i.e., $T_d^i > T_g^i$, then the wealth-maximizing investment decision desired of any firm j by consumer i is larger than the net-present-value-maximizing investment decision.

Since the maximizing z_j^{i*} is a function of individual tax rates, there will, in general, be no unanimous choice of firm inputs. We may, however, establish that in our model there will be a choice of inputs which will be the preferred choice of consumers having a majority of the initial shareholdings in firm j.

This gives (dividing (12) through by $\partial U_i/\partial x_{io}$) $\partial U_i/\partial z_j = 0$ if and only if

$$(15) \qquad \bar{s}_{ij}\{-(1 - T_d^i) + (1 - T_g^i)\frac{\partial p_j}{\partial z_j}\} - s_{ij}\frac{\partial p_j}{\partial z_j} + $$
$$+ s_{ij}\sum_n q_n^i\{(1 - T_d^i)[(1 - T_c)\frac{\partial y_{jn}}{\partial z_j} + T_c] + T_d^i\frac{\partial p_j}{\partial z_j}\} = 0,$$

where $$q_n^i = \frac{\partial U_i/\partial x_{in}}{\partial U_i/\partial x_{io}} .$$

The last two terms on the left-hand side of the above equation equal zero; to see this, rearrange these terms to get

$$(16) \qquad s_{ij}\{\frac{\partial p_j}{\partial z_j}[-1 + \sum_n q_n^i T_d^i] + \sum_n q_n^i (1 - T_d^i)[(1 - T_c)\frac{\partial y_{jn}}{\partial z_j} + T_c]\} .$$

Since by the first-order conditions (9)

$$(17) \qquad \frac{\partial p_j}{\partial z_j} = \sum_n \beta_n \frac{\partial r_{jn}}{\partial z_j}$$

it follows by (10) that (16) is zero. Thus,

$$(18) \qquad \frac{\partial U_i}{\partial z_j} = 0 \iff -(1 - T_d^i) + (1 - T_g^i)\frac{\partial p_j}{\partial z_j} = 0.$$

We have thus established

Theorem 2. Given prices $(p_1, \ldots, p_J)$ and firm inputs $(z_1, \ldots, z_J)$ consumer i for whom $\bar{s}_{ij} > 0$ prefers that firm j choose z_j^* so as to set

$$(19) \qquad \frac{\partial p_j}{\partial z_j} = \frac{(1 - T_d^i)}{(1 - T_g^i)}$$

Theorem 3. Given prices $(p_1, \ldots, p_J)$ and firm inputs $(z_1, \ldots, z_J)$ there exists an input choice z_j^* for each firm j such that:

3.1. There exists no z_j^{i*} which is preferred by any coalition of consumers having a majority of initial shareholdings in firm j to z_j^*.

3.2. The choice of z_j^* is non-manipulable: A misrepresentation by consumer i of his true preference can only result in an input choice z_j less preferred by i than z_j^*.

Proof of Theorem 3. Since z_j^{i*} maximizes a concave function, it is unique for each individual. Arrange the z_j^{i*} in ascending order; without loss in generality, assume that

$$z_j^{1*} \leq z_j^{2*} \leq \ldots \leq z_j^{I*} .$$

Let $\hat{z}_j^{i*}$ be the minimum $\hat{z}_j^i$ having the property[12]

$$\sum_{i \leq \hat{i}} \bar{s}_{ij} \geq \tfrac{1}{2} . \tag{21}$$

It now follows by the concavity of consumer preferences over firm j's inputs that all consumers $i \leq \hat{i}$ will oppose the choice of any z_j^{i*} with $i > \hat{i}$. Similarly, the choice of any z_j^{i*} such that $i < i$ will be opposed by all $i \geq \hat{i}$ and favored only by $i < \hat{i}$. Thus no alternative proposal will be able to garner a majority of the votes of initial shareholders. This proves 3.1.

To establish 3.2, consider any shareholder i with $i < \hat{i}(i > \hat{i})$. The only way i can change the choice of z_j^* is to announce that he prefers some $\hat{z} > z_j^*$ $(\hat{z} < z_j^*)$. But this will change the choice of firm inputs in a direction not preferred by consumer i (i.e., the choice of inputs will be larger (smaller), whereas i would prefer smaller (larger) inputs).

Q.E.D.

Theorem 3 resolves the problem of aggregating the (non-unanimous) preferences of initial shareholders which were proved in Theorem 2. Even though initial shareholders will not, in general, agree about the desired level of firm inputs, a simple voting procedure may be used to decide upon the level of firm inputs desired by a majority of the initial shareholders. Note that this voting procedure is -- in addition to being non-manipulable -- in principle independent of the announced preferences of initial shareholders, since by Theorem 2 shareholder preferences depend only on individual tax rates, a datum which may be objectively confirmed.

II. *Consumer Preferences over Firm Financing Decisions*

Having examined the impact of taxation on the aggregation of consumer preferences over firm investment decisions, we now examine the problem of aggregating preferences over firm financial decisions -- in particular the problem of the optimal capital structure of the firm. We first redefine our model to account for the existence of corporate debt and the consequent possibility of corporate bankruptcy.

Consider a firm j which has chosen to invest z_j in inputs today for future production, and suppose that the firm has promised the holders of its debt a payment of B_j -- this payment to be made if the firm's after-tax cash flow will suffice. Given production $y_{jn}(z_j)$, the taxable income of the firm in state n will be

$$(22) \qquad y_{jn}(z_j) - (B_j - p_j^b) - z_j,$$

where p_j^b is the value today of the firm's debt and $(B_j - p_j^b)$ is the interest payable on the debt. We shall assume that if B_j is of reasonable size with respect to z_j, say $B_j \leq \bar{B}_j(z_j)$, that the interest on the debt is tax deductable (this is in keeping with the spirit of current Internal Revenue Service practice[13]). Thus for $B_j \leq \bar{B}_j(z_j)$, the after-corporate-tax cash flow available to the firm's equity holders will be

$$(23) \qquad r_{jn}^e(z_j, B_j) = \begin{cases} y_{jn}(z_j) - T_c[y_{jn}(z_j) - (B_j - p_j^b) - z_j] - B_j & \text{if this is} \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

If $B_j \leq \bar{B}_j(z_j)$, the corresponding cash flow available to the firm's debt-holders will be

$$(24) \qquad r_{jn}^b(z_j, B_j) = \begin{cases} B_j & \text{if } r_{jn}^e(z_j, B_j) \geq 0 \\ y_{jn}(z_j) - T_c[y_{jn}(z_j) - (B_j - p_j^b) - z_j] & \text{otherwise} \end{cases}$$

The second line of r_{jn}^b assumes that the firm can sell its tax losses if the benefit of these are not available to it.

If $B_j > \bar{B}_j(z_j)$, interest is not tax-deductable and the cash flows to equity holders are given by

$$(23') \qquad r_{jn}^e(z_j, B_j) = \begin{cases} y_{jn}(z_j) - T_c[y_{jn}(z_j) - z_j] - B_j & \text{if this is} \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Cash flows to debtholders in this case are given by

$$(24') \qquad r^b_{jn}(z_j, B_j) = \begin{cases} B_j & \text{if } r^e_{jn}(z_j, B_j) \geq 0 \\ y_{jn}(z_j) - T_c[y_{jn}(z_j) - z_j] & \text{otherwise} \end{cases} .$$

Now consider a consumer i who in period 0 purchases a proportion s_{ij} of firm j's equity and a proportion b_{ij} of firm j's debt. If the consumer's ordinary income tax rate is T^i_d, and if--as in Section 2 all second period payments by firms to consumers are adjudged ordinary income for tax purposes--then the consumer's net of tax return from his shareholdings in firm j will be

$$(25) \qquad r^{ei}_{jn}(z_j, B_j) = s_{ij}[(1 - T^i_d)r^e_{jn} + T^i_d p^e_j],$$

where p^e_j is the market price of the firm's equity. Similarly, consumer i's net-of-tax return from his bondholding b_{ij} in the firm will be

$$(26) \qquad r^{bi}_{jn}(z_j, B_j) = b_{ij}[(1 - T^i_d)\, r^b_{jn} + T^i_d p^b_j],$$

where p^b_j is the market price of the firm's debt.

Assuming, as we did previously, that the firm produces in the current period from inputs purchased before the start of the model, and that this production is taxed at the corporate tax rate T_c, it follows that the current dividend of the firm (which accrues to the firm's initial shareholders) will be[14]

$$(27) \qquad (1 - T_c)\bar{y}_{jo} - z_j + p^b_j.$$

We shall assume that this dividend is taxed at each consumer's ordinary income tax rate, whereas the cpaital gains in period 0 from the sale of consumer i's initial portfolio of shareholdings are taxed at the capital gains tax rate T_g^i.[15] We may now write the consumer's maximization problem:

$$\max U_i(x_{io}, x_{i1}, \ldots, x_{iN})$$

subject to

$$(28)\quad \begin{aligned} x_{io} &= \sum_j \bar{s}_{ij}\{(1-T_d^i)[(1-T_c)\bar{y}_{jo} - z_j + p_j^b] + (1-T_g^i)p_j^e\} - \sum_j s_{ij}p_j^e - \sum_j b_{ij}p_j^b \\ x_{in} &= \sum_j (r_{jn}^{ei} + r_{jn}^{bi}) \end{aligned}$$

As in Section II,it follows from the first-order conditions that in a complete market there exist state prices $\beta = (\beta_1, \ldots, \beta_N)$ such that

$$(29)\qquad p_j^e = \sum_n \beta_n r_{jn}^e \quad \text{and} \quad p_j^b = \sum_n \beta_n r_{jn}^b,$$

where

$$(30)\qquad \beta_n = \frac{q_n^i(1 - T_d^i)}{1 - T_d^i \sum_n q_n^i}$$

To derive consumer i's preferences over firm j's debt level, we differentiate U_i with respect to B_j. In a manner similar to that used to establish Theorem 2, it may be shown that the complete merkets assumption implies that consumers wish to maximize

$$(31)\qquad W_j^i(B_j) = (1 - T_d^i)p_j^b + (1 - T_g^i)p_j^e.$$

Once again we see that in complete markets consumers are interested only in the maximization of their wealth from initial portfolio holdings, and that the "consumption effect" from portfolios purchased in period 0 is irrelevant.

We now turn to the main result of this section: In Theorem 3 we shall show that the maximization of $W_j^i(B_j)$ for any consumer i will always yield a corner solution; consumers will prefer only one of two possible debt levels for the firm--either no debt at all, or the maximal level of debt at which the tax authorities will still recognize interest payments on the debt as tax deductable. In order to establish the theorem, we first prove the following two lemmas.

Lemma 1. Given consumer portfolio choices, firm investment and financing decisions, and market prices for debt and equity, the derivative of the wealth function is given by

$$(32) \qquad \frac{\partial W_j^i}{\partial B_j} = \begin{cases} \dfrac{(\sum_A \beta_n \lambda_i + \alpha_i \sum_{\bar{A}} \beta_n T_c)(1 - T_g^i)}{1 + \sum_{\bar{A}} \beta_n T_c} & \text{if } B_j \leq B_j(z_j) \\[2ex] \sum_A \beta_n (T_g^i - T_d^i) & \text{if } B_j > B_j(z_j) \end{cases}$$

where

A = {states n | firm j is not bankrupt in n}

$\bar{A}$ = {states n | firm j is bankrupt in n}

$$\alpha_i = \frac{1 - T_d^i}{1 - T_g^i}$$

$$\gamma_i = \frac{T_g^i - T_d^i}{1 - T_g^i} + T_c(1 - \sum_n \beta_n)$$

Proof of Lemma 1. See Appendix.

An immediate corollary to Lemma 1 is the following

Corollary. Suppose that $T_g^i \leq T_d^i$ for every consumer i, i=1, . . ., I. Then no consumer will support an increase in the debt level of firm j beyond $B_j(z_j)$.

Proof of the Corollary. Suppose that $T_g^i \leq T_d^i$. Then as the second line of (32) indicates $\partial W_j^i / \partial B_j < 0$.

Q.E.D.

In the remainder of the paper, we shall assume that for every consumer i, $T_g^i \leq T_d^i$. There will be no need, therefore, in discussing consumer preferences about firm debt levels, to consider--for any firm j--debt beyond $B_j(z_j)$.

From Lemma 1 it follows that W_j^i will be a piecewise linear function of B_j. Each linear portion of W_j^i corresponds to a given partition of the states of nature between A--states in which the firm is not bankrupt--and $\bar{A}$--states in which the firm cannot fully satisfy the bondholders. If a change in the debt level B_j changes this partition, the following lemma shows the change in the derivative $\partial W_j^i / \partial B_j$:

Lemma 2: Let B_1 and B_2 be two debt levels for firm j such that

$$A(B_1) = \{1, \ldots, k\}$$

$$A(B_2) = \{1, \ldots, k-1\}$$

Then

$$(33) \quad \left.\frac{\partial W_j^i}{\partial B_j}\right|_{B_1} - \left.\frac{\partial W_j^i}{\partial B_j}\right|_{B_2} = \frac{(T_c \gamma_i \sum_n \beta_n + \gamma_i - \alpha_i T_c)\beta_k (1 - T_g^i)}{(1 + \sum_{k+1} \beta_n T_c)(1 + \sum_k \beta_n T_c)}$$

Proof of Lemma 2. See Appendix.

Theorem 4. Consumers will prefer only one of two possible levels of debt for any firm j: either no debt at all, or the maximal level of debt at which the tax authorities will still recognize interest payments on the debt as tax deductable.

Proof of Theorem 4. Consider first the case where $\gamma_i \geq 0$. In this case it follows from Lemma 1 that consumer i always prefers more debt, since $\partial W_j^i/\partial B_j > 0$.

Now consider the case where $\gamma_i < 0$. In this case it is clear from Lemma 1 that when $B_j = 0$, $\partial W_j^i/\partial B_j < 0$, so that initially consumer i will oppose any firm debt at all. It follows from Lemma 2, moreover, that as B_j increases, the derivative $\partial W_j^i/\partial B_j$ is non-decreasing (Lemma 2 shows that it is, in fact, increasing as the partition between A and $\bar{A}$ changes). Thus, for a consumer i with $\gamma_i < 0$, W_j^i is convex in B_j. Figure 1 shows two possibilities for this case. Both individuals i and h have negative γ's. However, individual i prefers that firm j have no debt at all (i.e., W_j^i is maximized for $B_j = 0$), while individual h prefers that firm j have the maximal allowable (by the tax authorities) debt level $B_j(z_j)$.

Q.E.D.

It follows from the above analysis that consumers will favor only one of two debt levels for the firm: Consumers i for whom $\gamma_i \geq 0$ will favor the maximal leverage over which interest tax deductability is allowed, while consumers for whom $\gamma_i < 0$ will have a convex preference function over the firm's debt level, preferring either a maximally leveraged or a no-debt firm.

Consumers will not, in general, be unanimous in their preferences over firm debt levels. The next theorem shows, however, that consumers' preferences may be aggregated and that for any firm there exists a debt level which is preferred by a majority of the firm's shareholders.

Theorem 5. Given prices $(p_1^e, \ldots, p_J^e; p_1^b, \ldots, p_J^b)$, firm inputs $(z_1, \ldots, z_J)$ and firm debt choices $(B_1, \ldots, B_J)$, there exists a debt level B_j^* for each firm j such that:

5.1. $B_j^* = 0$ or $B_j^* = B_j(z_j)$, where the latter is the maximal debt level at which the firm's bonds are recognized as a debt instrument by the tax authorities.

5.2. B_j^* is preferred by a coalition of consumers having at least 50% of the initial shareholdings in firm j.

5.3. The choice of B_j^* s non-manipulable: A misrepresentation by consumer i of his true preferences can only result in a choice of firm j's debt level less preferred by i than B_j^*.

Proof of Theorem 5. Let B_j^* be chosen by using a voting procedure in which consumers who are initial shareholders in the firm choose between the all-debt or no-debt option, and in which their votes are weighted by their shareholdings in the firm. It is easily established that this procedure will be non-manipulable, since a consumer wishing to change the majority choice debt decision will be able to do so only by indicating a preference for the debt level less favored by him, and in doing so will--if he affects the majority decision at all--affect it in a direction not desired by him. This voting procedure need not, furthermore, depend on announced preferences of the consumers at all, since by (32) above, the relative tax rates of each consumer--an objective factor--are the sole determinants of his preferences.

Q.E.D.

Concluding Remarks

We have shown that in a tax scenario similar to the one prevailing in the United States stockholders do not have unanimous preferences regarding the firm's investment and financing decisions. Management action rules which have been suggested until now (for example, net present value maximization or firm value maximization) are thus rendered inoperative. This result has widespread implications, especially in the finance literature, which has assumed that unanimity (through value maximization) holds in a world with taxation. Although in our world unanimity does not hold, it was shown in the paper that investors' preferences over the investment level and financial structure of the firm can be aggregated by means of a majority vote, so that each firm will choose an investment and financial policy which accords with the preferences of a majority of its shareholders. These preferences are dictated solely by the prices for contingent claims in the economy and the individuals' tax brackets. As far as the capital structure decision is concerned, our model implies that firms should either have no debt at all or should be financed by debt to the limit recognized by the IRS as debt. The model fails to explain why in the real world many firms do not comply with our results (but not always: there are corporations which do not issue any debt and there are corporations which finance up to the allowable limit). Other considerations, such as agency theory (Meckling and Jensen 1976) may perhaps be the cause for this behavior.

APPENDIX

Proof of Lemma 1. Note that it follows from (31) that

$$\text{(A.1)} \qquad \frac{\partial W_j^i}{\partial B_j} = (1 - T_d^i)\frac{\partial p_j^b}{\partial B_j} + (1 - T_g^i)\frac{\partial p_j^e}{\partial B_j} \quad .$$

From (23) and (29) we obtain

$$\text{(A.2)} \qquad \frac{\partial p_j^e}{\partial B_j} = \sum_A \beta_n [T_c - T_c \frac{\partial p_j^b}{\partial B_j} - 1] \; .$$

Thus

$$\text{(A.3)} \qquad \frac{\partial W_j^i}{\partial B_j} = \frac{\partial p_j^b}{\partial B_j} \; [(1 - T_d^i) - T_c(1 - T_g^i) \sum_A \beta_n] - (1 - T_c)(1 - T_g^i) \sum_A \beta_n \quad .$$

Since (from (23) and (24))

$$\text{(A.4)} \qquad p_j^b = \frac{1}{1 + \sum_{\bar{A}} \beta_n T_c} [\sum_A \beta_n B_j + \sum_{\bar{A}} \beta_n \{y_{jn}(1 - T_c) + T_c z_j + T_c B_j\}] \quad ,$$

it follows that

$$\text{(A.5)} \qquad \frac{\partial p_j^b}{\partial B_j} = \frac{1}{1 + \Sigma_A \beta_n T_c} \left[\Sigma_A \beta_n + \Sigma_{\bar{A}} \beta_n T_c \right] \quad .$$

Write $(1 - T_d^i) = \alpha_i(1- T_g^i)$ and substitute (A.5) into (A.3) to get

$$\text{(A.6)} \qquad \frac{\partial W_j^i}{\partial B_j} = (1 - T_g^i) \left[\frac{(\sum_A \beta_n + \sum_{\bar{A}} \beta_n T_c)(\alpha_i - T_c \sum_A \beta_n) - (1 - T_c) \sum_A \beta_n (1 + T_c \sum_{\bar{A}} \beta_n)}{1 + \sum_{\bar{A}} \beta_n T_c} \right]$$

The numerator of the bracketed expression of (A.6) is

$$\text{(A.7)} \qquad \alpha_i \Sigma_A \beta_n + \alpha_i \Sigma_{\bar{A}} \beta_n T_c - T_c(\Sigma_A \beta_n)^2 - T_c^2(\Sigma_A \beta_n \Sigma_{\bar{A}} \beta_n)$$

$$- \Sigma_A \beta_n - T_c \Sigma_A \beta_n \Sigma_{\bar{A}} \beta_n + T_c \Sigma_A \beta_n + T_c^2(\Sigma_A \beta_n \Sigma_{\bar{A}} \beta_n).$$

Simplifying this expression gives the desired result.

Q.E.D.

Proof of Lemma 2. It follows from Lemma 1 that (temporarily ignoring the factor $(1 - T_g^i)$ in the numerator of (33))

$$\left.\frac{\partial W_j^i}{\partial B_j}\right|_{B_1} - \left.\frac{\partial W_j^i}{\partial B_j}\right|_{B_2} = \left[\frac{\sum^k \beta_n \gamma_i + \alpha_i \sum_{k+1} \beta_n T_c}{1 + \sum_{k+1} \beta_n T_c}\right] - \left[\frac{\sum^{k-1} \beta_n \gamma_i + \alpha_i \sum_k \beta_n T_c}{1 + \sum_k \beta_n T_c}\right]$$

The numerator of the second term above may be written

$$\sum^k \beta_n \gamma_i + \alpha_i \sum_{k+1} \beta_n T_c - \beta_k \gamma_i + \alpha_i \beta_k T_c$$

and therefore

$$\left.\frac{\partial W_j^i}{\partial B_j}\right|_{B_1} - \left.\frac{\partial W_j^i}{\partial B_j}\right|_{B_2} = [\sum^k \beta_n \gamma_i + \alpha_i \sum_{k+1} \beta_n T_c] \left[\frac{\beta_k T_c}{(1 + \sum_{k+1} \beta_n T_c)(1 + \sum_k \beta_n T_c)}\right]$$

$$+ \frac{\beta_k \gamma_i - \alpha_i \beta_k T_c}{1 + \sum_k \beta_n T_c}$$

Simplifying this expression gives the desired result.

QED

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FOOTNOTES

1. In writing this paper, we benefited from the helpful comments of Eitan Muller.

2. I.e.: " . . . All goods and assets are perfectly divisible; any information is costless and available to everybody; there are no transaction costs or taxes; all individuals pay the same price for any given commodity or asset; no individual is wealthy enough to affect the market price of any asset; and no firm is large enough to affect the opportunity set facing consumers." (Fama and Miller [1972, p. 272]).

3. By complete we shall mean a market in which the number of linearly independent securities is equal to the number of states of nature.

4. Modigliani and Miller did not address themselves to the completeness requirement. The necessity of completeness was spelled out by Grossman and Stiglitz [1977].

5. As shown by DeAngelo [1981], the unanimity results may be derived from the dominance of individual opportunity sets at a single firm decision. This will in general be so only in complete markets. In the incomplete markets case the lack of unanimity results from the presence of a consumption effect (as opposed solely to a wealth effect) from firm decisions. Hart [1979] has shown, however, that in large markets the consumption effect may be absent even though the market is incomplete. In an exception to the general stress on unanimity in this literature, Benninga and Muller [1979] have shown that under certain conditions consumer preferences in incomplete markets may be aggregated by a majority voting procedure, even though no consumer unanimity exists.

6. The unanimity assumption in financial markets was questioned by Schneller [1980] and Taggart [1980] . Neither of these authors, however, considered how consumer preferences might be aggregated if these were not unanimous.

7. Miller [1977] and DeAngelo and Masulis [1980] have considered models in which tax rates vary not only among individuals but are different depending upon whether the income under consideration is derived from debt or equity securities. As we have shown in Benninga and Schneller [1981] , such a framework (intended as a simplification of the two tax--dividend and capital gains--framework considered in this paper) depends critically on the restriction of short sales; once such an assumption is made consumers will be unanimous in their desire that all firms have a certain minimal level of debt.

8. We assume that for a given distribution of pre-tax, pre-debt period 1 earnings, no firm can enjoy a tax shield on debt if the promised debt service is "excessive" in comparison to the returns from real assets. In particular the maximal level of debt consistent with the reception of the debt tax shield must always imply some probability of non-bankruptcy. This assumption of the model is consistent with current American tax practice (see, for example, Federal Tax Course [1981, paragraph 1915]).

9. We assume that inputs are deductable firm expenses in the period in which they are used for production. Thus, the tax credit from the purchase of z_j will accrue to the firm in period 1 (see equation (2)). In equation (1) we assume, with no loss in generality, that the firm receives no tax credit for the inputs (purchased before the start of the model) used to produce $\bar{y}_{jo}$.

10. Even though we shall use the assumption of constant marginal tax rates throughout the paper, all our results hold also for the case of a piecewise linear tax schedule.

11. We have assumed that in period 1 the tax authorities regard $s_{ij}[r_{jn}(z_j) - p_j]$ as taxable income derived by consumer i from his purchase of firm j's security, and that this income is taxed at rate T_d^i. This gives (5) as the consumer's net of tax return. This treatment is in reasonable accord with the current United States tax code.

12. If there are an odd number of shareholders having equal shares in firm j, our decision criterion corresponds to the median rule used by Black [1948] and Arrow [1951]. The criterion of inequality (21) is a generalization of this rule.

13. See footnote 8.

14. In defining the firm's period 0 dividend, we have assumed that it is impossible to differentiate between funds derived from production in period 0 and those derived from the sale of the firm's bonds *(pecunia non olet)*.

15. In order to simplify the model we have assumed initial holdings only of shares in the firm. The addition of initial bondholdings would not change the results of the model.

16. Under current United States tax schedules, this is most likely to be the case. To see this, consider first values of α_i: Since $T_g^i \leq .4T_d^i$, and since $0 \leq T_d^i \leq 0.6$, it follows that $0.53 \leq \alpha_i \leq 1$. Furthermore, assuming that the marginal corporate tax rate $T_c = 0.46$, and assuming that the one-period risk-free (real) interest rate is around 3 percent (thus implying $\sum_n \beta_n = 0.971$), we get that λ_i has a range of $-0.47 \leq \lambda_i \leq 0.013$.

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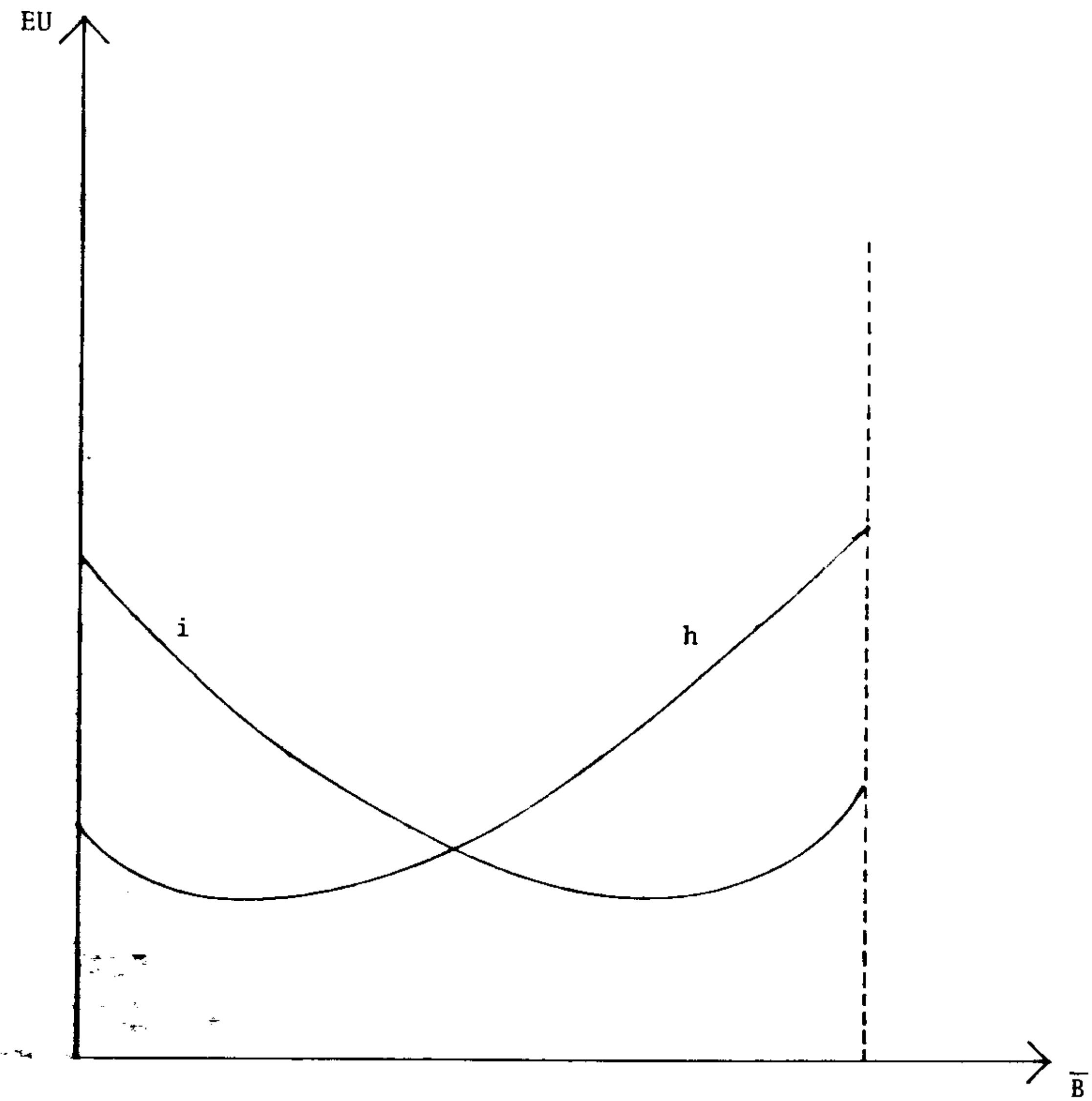

Figure I