Are Forward Exchange Rates Really Useful Predictors of Future Spot Rates? Some Evidence from Daily Dollar-DM Data

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ABSTRACT

The Speculative Efficiency Hypothesis is the proposition that a transform T of the current τ-period forward exchange rate, $T(F_t(\tau))$, is an unbiased predictor of a transform of the spot exchange rate at time $t+\tau$, $T(S_{t+\tau})$. This paper argues that one should test Speculative Efficiency using daily data and the transform $T(x) = \log x$, compare the usefulness of the forward rate to alternative predictors, and make allowances for heavy tails in the data--in particular considering the possibility that percentage, or log, changes in spot and forward rates have a non-normal stable distribution. Two tests--sample kurtosis and maximum-likelihood estimation of the characteristic exponent--show that if percentage, or log, changes in the daily spot and one-month forward dollar prices of DM over the period July 1973 to June 1979 follow a stable distribution, then this distribution is non-normal stable. This conclusion is not altered when allowance is made for time-varying scale or when tested in a model that allows for other types of distributions. The Speculative Efficiency Hypothesis is then tested using a regression procedure that is valid for non-normal stable distributions. The results show unambiguously that the log forward rate is an unbiased estimator of the future log spot rate. The log forward rate is not, however, useful: the current log spot rate is an equally good predictor, and the discount or premium on forward exchange has no relationship to future appreciation or depreciation of the spot rate. This empirical finding raises questions about the relevance of exchange rate models which assign a privileged role to the forward rate as an expectations variable.

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I. Introduction.

The Speculative Efficiency Hypothesis[1] is the proposition that a transform T of the current t-period forward exchange rate, $T(F_t(\tau))$, is an unbiased predictor of a transform of the spot exchange rate at time $t+\tau$, $T(S_{t+\tau})$. The hypothesis is tested in the form

$$T(S_{t+\tau}) = a + bT(F_t(\tau)) \qquad (1)$$

where the coefficients (a,b) are hypothesized to be (0,1). Sometimes the identity transform $T(X) = X$ is used, but more often, for reasons explained below, T is taken as $T(X) = \log X$.

The Speculative Efficiency Hypothesis is usually assumed to hold in rational expectation models of the foreign exchange market. If E_t represents the expectations operator at time t, and $T(X) = X$, then a typical theoretical assumption is

$$F_t(\tau) = E_t(S_{t+\tau}) .$$

One then invokes the Interest Parity Theorem, which relates $F_t(\tau)$ to S_t in terms of relative interest rates, and thus a relationship between S_t and $E_t(S_{t+\tau})$ is obtained.

Despite its widespread use as a theoretical proposition, the Speculative Efficiency Hypothesis has surprisingly little empirical support. While many researchers have concluded that they "cannot reject the hypothesis" of

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speculative efficiency, inspection of their results shows that neither could they have rejected neighboring alternative hypotheses, such as $(a,b) = (0,.9)$ or $(a,b) = (0,1.1)$. Standard errors are too large for "confirmation" of the hypothesis to be convincing. Others have rejected the hypothesis empirically, and then proceeded as though it were a valid theoretical assumption.

A number of comments on the hypothesis, its tests, and the literature are in order, which will also serve to motivate the present paper.

1. The Speculative Efficiency Hypothesis is of interest because it implies the forward rate $F_t(\tau)$ conveys more information about the future spot rate, $S_{t+\tau}$, than does the current spot rate, S_t. The divergence of the forward rate from the current spot rate, $F_t(\tau) - S_t$, is then a useful indicator of the direction and magnitude of the future movement of the spot rate, $S_{t+\tau} - S_t$. Suppose, however, that we also discovered that we could not reject the hypothesis that the current spot rate S_t, or its transform $T(S_t)$, is an unbiased predictor of the future spot rate $S_{t+\tau}$, or its transform $T(S_{t+\tau})$. Then the Speculative Efficiency Hypothesis, even if valid, would lose much of its appeal. Corporate treasurers would know that the forward rate would contain no additional useful information about future movements in the spot rate not already contained in the current spot rate, and model-builders would have to give up the privileged role of the forward rate as an expectation variable. For many of the models in vogue the consequence would be serious: one could no longer use the Interest Parity Theorem to tie the present spot rate to the future spot rate in terms of interest rates or interest rate determinates like money supplies, income, prices, and so on.

Bilson (1981) reports evidence that the current spot rate is not only as good, but better, than the forward rate as a predictor of future spot rates.

2. Unbiasedness is not the same as usefulness. If the winning number in a state lottery is chosen from a uniform distribution over the natural numbers from 1 to 9999, then the number 5000 is an unbiased predictor of the winning number. But one would make no more money betting continually on 5000, than one would betting that tomorrow's winning number will be the same as today's (which is also an unbiased predictor). When undergraduate finance texts report that the forward rate is an unbiased predictor of the future spot rate, the student is led to believe he has acquired useful information, though this is not necessarily so.

Let Z_t, ε_t, η_t, ε_s, η_s be mutually independent random variables for all $t \neq s$, where Z_t is a Martingale

$$E_t(Z_{t+\tau} | Z_t, Z_{t-1}, \ldots) = Z_t,$$

and $E(\varepsilon_t) = E(\eta_t) = 0$ for all t. Suppose further that the forward rate F_t and the spot rate S_t are generated according to

$$S_t = Z_t + \varepsilon_t$$
$$F_t = Z_t + \eta_t \ .$$

Then the forward rate would be an unbiased predictor of the future spot rate, the spot rate would be an unbiased predictor of the future forward rate, and each would be an unbiased predictor of its own future value:

$$E(S_{t+\tau}) = E[E(S_{t+\tau} | S_t)] = E(Z_t) = E(S_t) = E(F_t),$$

and similarly for $E(F_{t+\tau})$. While this example may seem extreme, the evidence presented below shows that over the six-year period July 1973 to June 1979 the dollar price of DM did in fact behave precisely in this fashion once the transform $T(x) = \log x$ was used. With respect to information about the future, the log spot and log forward rates were interchangeable variables.

3. If the dollar price of DM is not to have different characteristics as a biased or unbiased predictor from the DM price of dollars, then the

transform $T(X) = \log X$ is implied. Suppose one believes the forward exchange rate is an unbiased predictor of next period's spot rate:

$$F_t(\tau) = E_t(S_{t+\tau}) .$$

Then in terms of the foreign currency, the forward rate is necessarily a biased predictor of the future spot rate, since by Jensen's inequality

$$E_t(1/S_{t+\tau}) > 1/E_t(S_{t+\tau}) = 1/F_t(\tau) .$$

However, making the alternative assumption that

$$\log F_t(\tau) = E_t(\log S_{t+\tau})$$

removes the paradox induced by choice of currency-unit since in terms of the other currency

$$E_t(\log(1/S_{t+\tau})) = E_t(-\log S_{t+\tau}) = -\log F_t(\tau) = \log (1/F_t(\tau)) .$$

The feeling that the currency of denomination ought not affect the biasedness or unbiasedness of the forward rate as a predictor rests on an appeal to symmetry or aesthetics. In the empirical results reported here I use the transform $T(X) = \log X$, but profess to no revelation that the world necessarily conforms to my sense of symmetry.

Besides inducing independence of currency unit, the transform $T(X) = \log X$ has another important implication. Sometimes it is argued, with reference to the possibility of risk premia in forward exchange rates

> ...it is difficult to see how such an argument could possibly hold symmetrically for both sides of the foreign exchange market--for if a French speculator in the franc/sterling forward market is <u>rewarded</u> for taking risk, it follows that a British speculator holding the opposite side of the market is <u>penalized</u> for taking risk![2]

In fact, however, if $F_t(\tau)$ and S_t are lognormally distributed, and stationary, and the relationship $\log F_t(\tau) = E(\log S_{t+\tau})$ holds, then the forward rate is a downwardly biased predictor of the future spot rate in terms of both currencies. If

$$\log F_t(\tau) = E(\log S_{t+\tau}) = m$$

then

$$E(S_{t+\tau}) = \exp(m + \tfrac{1}{2}\sigma^2) = F_t(\tau)\exp[\tfrac{1}{2}\sigma^2] > F_t(\tau)$$

where σ^2 = Variance ($\log S_t$). The same holds in terms of the other currency

$$E(1/S_{t+\tau}) = \exp[-m + \tfrac{1}{2}\sigma^2] = \frac{\exp[\tfrac{1}{2}\sigma^2]}{F_t(\tau)} > \frac{1}{F_t(\tau)} .$$

Thus, given that the log forward rate is an unbiased predictor of the future log spot rate, there is room for a risk premium from the viewpoint of either currency, since the level of the forward rate is a downwardly biased predictor of the level of the future spot rate in terms of both currencies.

4. One obvious way to improve the precision of the estimates in equation (1) is to increase the size of the data set. Now properly speaking the Speculative Efficiency Hypothesis only applies to daily data. The 30-day forward rate (or its transform) is hypothesized to be an unbiased predictor of the spot rate (or its transform) 30 days from now.[3] If the hypothesis is true, then it should be true more often than once a week or once a month.

A drawback in the use of daily data, however, is that daily data involves overlapping prediction intervals. Since there are about 20 trading days to a month, the OLS regression model of the spot rate on the 30-day forward rate would take the form

$$\log S_{t+20} = a + b \log F_t + \varepsilon_t$$

This sets up a 20th order moving-average process, so that the correlation coefficient of ε_t and ε_{t-1} would be about 95% (19/20) and hence Durbin-Watson statistics would have a value on the order of D.W. $\cong 2(1-.95) = .10$. But even with serial correlation, the point estimates of (a,b) are not biased.

5. The employment of OLS or other estimators based on a least-squares metric may, however, seriously bias the estimates of (a,b) for an entirely different reason. Westerfield (1977), in comparing the variability of exchange rates under "fixed" and floating regimes, found that log spot and log forward changes ($\log S_t - \log S_{t-1}$, $\log F_t - \log F_{t-1}$) can be better described as if they were generated from the non-normal members of the class of stable distributions, associated with the name of Paul Lévy. These distributions will be briefly described.

Stable distributions are the only possible distributions that exist as limit distributions of suitably transformed sums of independent and identically distributed (i.i.d.) random variable. That is, central limit arguments lead us to consider stable distributions in cases where an observed variable can be thought of as the outcome of very many i.i.d. random influences. In addition, stable distributions are closed under convolutions. The sum of two stable distributions is also stable, though possibly with different location and scale parameters.

Stable distributions $S_{\alpha,\beta}(s;x,m)$ have the characteristic function

$$\zeta(z) = \int_{-\infty}^{\infty} \exp(ixz)dS_{\alpha,\beta}(x;c,m) = \exp(imz-|cz|^{\alpha}(1-i\beta(z/|z|)\cdot w),$$

where $-\infty < m < \infty$, $c > 0$, $|\beta| \leq 1$, $0 < \alpha \leq 2$, and $w = \tan(\pi\alpha/2)$ if $\alpha \neq 1$, or $w = 2/\pi \log|z|$ if $\alpha = 1$. When $\alpha = 2$, this becomes the characteristic function of a normal distribution. The parameter β is an index of skewedness, c is a scale parameter (corresponding to the S.D./$\sqrt{2}$ for a normal distribution), and m is a location parameter. The variable $t = c^{\alpha}$, which corresponds to one-half the variance when $\alpha = 2$, will be referred to as the levy.

The tail of a stable distribution is Pareto

$$1 - S_{\alpha,\beta}(x) \cong Cx^{-\alpha}, \quad x \to \infty, \quad C \text{ a constant.}$$

For $\alpha < 2$, moments up to, but not including, α exist, while higher moments are infinite. Hence only the normal distribution has a finite variance.

In terms of its relative frequency graph, a non-normal symmetric stable distribution will have a greater concentration of data around the central part of the distribution than would a normal distribution, but at the same time more extreme observations. By comparison to the normal, the intermediate ranges of the probability density of a stable distribution with $\alpha < 2$ are robbed of weight, and this weight is redistributed to the center and tails.

As a consequence, measures of variability such as the sample variance, or techniques such as least squares regression, which weighs outlying observations more heavily, may be inappropriate. Cornell and Dietrich (1978) have similarly found evidence for stable distributions in exchange rates. Bilson (1981) does not consider stable distributions, but notices the frequent occurrence of outliers in the data. Bilson's results are particularly interesting in this context. Bilson begins with the same definition of Speculative Efficiency as used in this paper, but then implicitly switches to a second definition--called it "Speculative Efficiency II"--that says the discount or premium on forward exchange is an unbiased predictor of the actual change in the spot rate. He was able to reject Speculative Efficiency II for most currencies traded in active markets against the dollar. The sample evidence is not inconsistent with the hypothesis that the forward premium is unrelated to the actual rate of spot depreciation. When extreme observations are dropped, however, then Speculative Efficiency II has more support. One is left with many possible explanations for this result: (1) Speculative Efficiency II holds only in "normal" times; (2) Speculative Efficiency II holds all the time but Bilson's rejection of the hypothesis stems from inappropriate use of least squares on heavy-tailed data; (3) Speculative Efficiency II fails, but the existence of outliers makes least squares regression suspect as a way of demonstrating this. Bilson himself shows that, by a least squares criterion, the current spot rate is a better predictor of the future spot rate than is the current forward rate. He suggests that spot rates evolve as a random walk without discernible drift, and one should bet against the forward rate as a predictor of the future, especially in the case of a large premium or discount in the forward market.

Using an estimation procedure that allows for heavy-tailed data, I recapture Bilson's results regarding Speculative Efficiency II, but do not find that the current spot rate is a <u>better</u> predictor of the future spot rate than the forward rate, only that it is <u>equally as good</u>.

In summary, I have argued that one should test Speculative Efficiency using daily data and the transform $T(X) = \log X$, compare the usefulness of the forward rate to alternative predictors, and make allowances for heavy tails in the data—in particular considering the possibility log changes in spot and forward rates have a non-normal stable distribution.

II. Preliminary Testing of the Data

A. Tests for Non-normality.

The data used in this paper to investigate the issues raised in the introduction are exchange rate observations for the dollar price of DM for the floating rate period July 1973 to June 1979. There are 1472 observations of the bid prices for spot and 1-month forward rates, recorded daily to four or five decimal places at 1:00 pm New York Time. The observations are published in the annual reports of the International Money Market of the Chicago Mercantile Exchange. Three series were created: daily proportional changes in the spot rate, $\Delta s = (S_t - S_{t-1})/S_{t-1}$; daily proportional changes in the forward rate, $\Delta f = (F_t - F_{t-1})/F_{t-1}$; and the daily discount or premium on forward exchange, $p = (F_t - S_t)/S_t$.

If Δs, Δf, p are the outcome of very many i.i.d. random influences, than an appeal to the generalized central limit theorem suggests these variables should have a stable distribution. However, a normal distribution (a stable distribution with $\alpha = 2$) will result only if the maximal term in the set of these i.i.d. random influences is negligible compared to their sum.[4] Thus if the class of stable distributions is taken as a working hypothesis for Δs, Δf, p, the next step is to establish whether a normal or non-normal member of this class is involved.

Saniga and Miles (1979) have examined by Monte Carlo simulation the power of six goodness-of-fit tests for normality when the alternative is stable ($\alpha < 2$). The tests included sample kurtosis, sample skewedness, a joint kurtosis-skewedness test, the Shapiro-Wilk W, the studentized range test, and the D'Agostino D. Sample kurtosis is the most powerful test if the number of observations $N > 50$ and the skewedness parameter $|\beta| < .75$.

The expected value of the kurtosis for a normal distribution is 3 and the S.D. is $(24/N)^{1/2}$.

Applying this test to Δs, Δf, p shows the data is highly leptokurtic:

	Sample kurtosis	S.D. above expected value for normal
Δs	11.88	69.5
Δf	9.64	52.0
p	34.18	244.0

In Table A, next page, the observations have been converted into t-values and then grouped into interval classes. Thus +4 indicates observations four sample S.D. above the mean. There are a large number of outlying observations, yet at the same time a heavy concentration around the mean. Leptokurtosis was also found by Westerfield (1977) who looked at log changes in weekly data.

B. Maximum Likelihood Estimation of Stable Parameters.

This preliminary test indicates an infinite-variance stable distribution ($\alpha < 2$) is involved. As the next step, we would like to obtain more information about the exact value of α, as well as the other parameters β, c, m. These may be obtained by maximum-likelihood (ML) estimation.[5] DuMouchel (1973) has shown that the ML estimators of θ, $\theta = (\alpha,\beta,c,m)$, have approximately a $\mathcal{N}(\theta, I^{-1}/N)$ distribution, where I is the Fisher information matrix and N is the sample size. In addition, (DuMouchel, 1975), using a multinomial approximation to the likelihood function, he has tabulated the asymptotic S.D.'s and correlations of the ML estimators.

For a data sample $(x_1,\ldots,x_n)$, the likelihood function is defined by

$$L(\theta) = \Pi_{j=1}^{n} s_{\alpha,\beta} \frac{(x_j - m)/c}{c} , \tag{6}$$

where $s_{\alpha,\beta}$ is the density function of a stable distribution having c=1 and m=0. Maximizing L by selection of θ requires computation of the stable density $s_{\alpha,\beta}$. No closed form representation of $s_{\alpha,\beta}$ exists except for the normal, $(\alpha,\beta) = (2,0)$, the Cauchy, $(\alpha,\beta) = (1,0)$, and the positive and negative forms of the Pearson V, $(\alpha,\beta) = (\frac{1}{2}, \pm 1)$. This problem may be circumvented by an asymptotic expansion of the stable density in terms of gamma functions (Bergström, 1952), or by writing the stable density as the inverse Fourier transform of its characteristic function

$$s_{\alpha,\beta}(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp(-ixz)\zeta(z)dz .$$

Table A

Class Interval t-Values for the Dollar Price of DM
Daily Data, July 1973 to June 1979

	From	To	Counts
Δs (spot rate)	-8.77654	-7.79161	1.
	-7.79161	-6.80668	0.
	-6.80668	-5.82175	1.
	-5.82175	-4.83682	1.
	-4.83682	-3.8519	3.
	-3.8519	-2.86697	10.
	-2.86697	-1.88204	25.
	-1.88204	-0.897109	129.
	-0.897190	0.08782	662.
	0.08782	1.07255	493.
	1.07275	2.05768	107.
	2.05768	3.04261	28.
	3.04261	4.02754	5.
	4.02754	5.01246	5.
	5.01246	5.99739	1.
Δf (forward rate)	-6.76761	-5.90629	1.
	-5.90629	-5.04497	2.
	-5.04497	-4.18365	3.
	-4.18365	-3.32233	5.
	-3.2233	-2.46101	8.
	-2.46101	-1.59969	39.
	-1.59969	-0.738374	165.
	-0.738374	0.122947	655.
	0.122947	0.984266	441.
	0.984559	1.84559	103.
	1.84559	2.70691	26.
	2.70691	3.56823	14.
	3.56823	4.42955	7.
	4.42955	5.29087	0.
	5.29087	6.15219	2.
p (forward premium)	-9.427698	-8.16021	3.
	-8.16021	-6.89365	2.
	-6.89365	-5.62708	3.
	-5.62708	-4.36051	3.
	-4.36051	-3.09395	3.
	-3.09395	-1.82738	18.
	-1.82738	-0.560811	213.
	-0.560811	0.705756	980.
	0.705756	1.97232	240.
	1.97232	3.23889	4.
	3.23889	4.50546	1.
	4.50546	5.77203	0.
	5.77203	7.03858	0.
	7.03858	8.30515	0.
	8.30515	9.57172	2.

The Fast Fourier Transform (FFT) is very accurate in the neighborhood of $x=0$, so that FFT combined with the asymptotic expansion above gives a practical and accurate method of numerically calculating the stable density for values both near and distant from zero.

Below are ML estimates of θ for the daily dollar/DM exchange rate variables Δs, Δf, p:

	$\hat{\alpha}$	$\hat{\beta}$	$\hat{c}$	$\hat{m}$
Δs (x100)	1.62 ± .04	−.023 ± .024	.298 ± .009	.021 ± .013
Δf (x100)	1.57 ± .04	0 ± .05	.301 ± .007	.018 ± .007
p x(100)	1.53 ± .01	−.080 ± .06	.117 ± .005	.174 ± .003

The β estimates show the distributions are fairly symmetric. The α estimates indicate an infinite variance model is appropriate.

It is interesting to compare the location-scale $(\hat{c},\hat{m})$ estimates here with those that would result from an assumption the distributions were Gaussian:

	S.D./ $\sqrt{2}$	mean
Δs (x100)	.438	.020
Δf (x100)	.432	.020
p (x1000)	.219	.175

The estimates of the scale parameter are too high by 47, 44, and 87 percent, respectively, when Δs, Δf, and p are assumed normal. Estimates of the mean, however, are indistinguishable under the two assumptions. These results are to be expected give $\alpha > 1$, so that the mean exists, but not the S.D. Also $\hat{\beta} \cong 0$. For $\beta = 0$, ML estimates of α and c are asymptotically independent of those for m and β.[6] Assuming normality imposes the constraint $\alpha = 2$, which biases the estimate of c, though not that of m.

The distributions of Δs and Δf for the floating period July 1973 to June 1979 are practically indistinguishable. The dollar/DM evidence does not support theoretical conclusions which argue that the forward rate is either more, or less, volatile than the spot rate. This follows from the fact that the $\hat{c}$ estimates are both about .003, so that Δs, Δf have equal variation.

C. Are the Distributions Really Stable?

In sections A and B two tests were used to distinguish non-normal stable distributions from normal stable distriutions: namely, sample kurtosis and the maximum likelihood estimate of the α parameter. These tests are powerful given the stable assumption, but a second possibility is a complete lack of stability--normal or otherwise. If we drop the assumption of stability, are there other explanations that account for the data? In this section the problem is investigated two ways. First, we look at the possibility of time-varying scale parameters. Second, we fit a general model to the right-hand tail of the distribution Δs--a model which allows for several distributional possibilities. Appendix A adds some relevant material concerning the student t-distribution.

1. Time-Varying Scale Parameters

Suppose the time series $\{Y_t\}$ is nonstationary, and may be written as $\{b(t)X_t\}$ where $\{X_t\}$ is a stationary and stable ($\alpha < 2$) series. then the scale parameter of Y_τ may be written $c_Y^\alpha = b^\alpha(\tau)c_X^\alpha$. Thus $\{Y_t\}$ may be a mixture of stable distributions with different scale parameters, and hence may not be stable. Moreover, a time-varying scale could generate outliers and bias our estimates of α.

The interquartile range is a robust measure of scale. Hence it can be used to adjust the data for time-varying scale. Observations on the exchange data Δs, Δf, and $\bar{p}$ were divided into yearly intervals and the interquartile range calculated for each year. Values obtained were:

Year	Δs	Δf	p
1	.00932	.00884	.00457
2	.00636	.00687	.00255
3	.00463	.00456	.00331
4	.00331	.00301	.00097
5	.00467	.00489	.00457
6	.00552	.00528	.00903

A non-constant scale could be indicated. Thus we need to see if this affects our estimated value for α. This possibility was checked for Δs. Below are class interval values for Δs, where each year's data for Δs has been divided by the respective interquartile range. (For convenience, the interquartile range was first multiplied by 100.) It is interesting to note that adjustment for time-varying scale makes the data more obviously symmetric.

	From	To	Counts
Δs (spot rate)	-9.81135	-8.78042	1.
divided by	-8.78042	-7.74949	0.
[interquartile	-7.74949	-6.71856	0.
range x 100]	-6.71856	-5.68763	1.
	-5.68763	-4.6567	1.
	-4.6567	-3.62577	4.
	-3.62577	-2.59484	10.
	-2.59484	-1.56391	51.
	-1.56391	-0.532979	264.
	-0.532979	0.497952	750.
	0.497952	1.52888	296.
	1.52888	2.55981	63.
	2.55981	3.59074	23.
	3.59074	4.62167	5.
	4.62167	5.6526	2.

Maximum likelihood estimates of paramters for this scale-adjusted distribution are

	$\hat{\alpha}$	$\hat{\beta}$	$\hat{c}$	$\hat{m}$
Δs	1.56±.04	-.06±.167	.514±.008	.041±.015

The estimate for α is about the same as before. The adjustment for time-varying scale does not affect our estimates or our previous conclusion that an infinite-variance model is appropriate.[7]

2. The Generalized Pareto Model of Tail Behavior

Because non-normal densities look quite different from the normal, the ML estimates of α are nonrobust to the assumption of stability. If the data is long-tailed and otherwise non-normal, but with finite variance, then the ML procedure will tend to generate an α-estimate indicating infinite variance. Thus, if the assumption of stability is in doubt, explicit investigation of the tail behavior is called for. In this context, the following model of the tail distribution, referred to as the generalized Pareto distribution P(x)[8] is useful:

$$1 - P(x) = \left(1 + \frac{AX}{B}\right)^{-1/A}, \quad -\infty < A < \infty,\ 0 < B < \infty,\ X > 0,\ AX > -B .$$

This distribution has the following properties:

(1) For A > 0, $\text{Prob}(X > X_0) \cong kX^{-1/A}$. Hence 1/A corresponds to the characteristic exponent α of the stable distribution, so the distribution may be stable or Pareto. A > .5 indicates infinite variance.

(2) A = 0. By definition, $\text{Prob}(X > X_0)$ is taken to be the exponential distribution.

(3) For A < 0, the generalized Pareto distribution has shorter tails than the exponential distribution.

(4) For A = -.5, the triangular distibution results.

(5) For A = -1, the uniform distribution over (0,B) results. The model therefore allows for a variety of tail behaviors. Further details on its use can be found in DuMouchel (1980).

The generalized Pareto model was fitted to the extreme 196 right-hand observations of the spot dollar/DM rate. Estimated A and B are

	$\hat{A}$	$\hat{B}$
Δs	$.606 \pm .018$	.078

The implied value for $\hat{\alpha}$ is $1.65 \pm .05$, so that an infinite variance model is still implied.

D. <u>A Cautionary Note of the Use of Logs</u>. I have defined, in the above preliminary tests of the data, Δs, Δf as

$$\Delta x = \frac{X_t - X_{t-1}}{X_{t-1}}$$

as opposed to

$$\Delta x' = \log X_t - \log X_{t-1} \ .$$

For small changes--in particular for daily exchange rate changes--Δx and $\Delta x'$ are indistinguishable. Hence, for practical purposes, either Δx or $\Delta x'$ will serve. However, in the context of stable distributions, the definition $\Delta x'$ can induce misunderstanding. If $\widetilde{\Delta x}'$ has a non-normal ($\alpha < 2$) stable distribution, then

$$E(\tilde{X}_t) = X_{t-1} E(\exp(\Delta\tilde{x}'))$$

is undefined ("equals infinity"). However, if $\Delta\tilde{x}$ has a non-normal ($\alpha < 2$) stable distribution, then

$$E(\tilde{X}_t) = [1 + E(\Delta\tilde{x})] X_{t-1}$$

is perfectly well-defined for any $\alpha > 1$.

Thus, in arguing for the use of the transform $T(X) = \log X$, I am only referring to <u>local</u> behavior ("in the small"). Log changes in exchange rates can be modeled "as if" they are non-normal stable. But there is no intention to imply that levels of exchange rates are undefined: that is an abuse of the model. Rather, "in the large" (which we really don't observe), the definition Δx will always be taken as implicit.

III. A Test of the Speculative Efficiency Hypothesis.

Evidence presented in part II suggests that percentage changes or (for small changes) log changes in the spot and one-month forward dollar price of DM over the period July 1973 to June 1979 may be modelled as if they are drawn from a symmetric stable distribution with characteristic exponent $\alpha \cong 1.6$. The results are (written in logs for convenience):

$$\log S_{t+1} = \log S_t + \varepsilon_{t+1}$$
$$\log F_{t+1} = \log F_t + \eta_{t+1}$$

where ε_{t+1}, η_{t+1} have the distribution $S_{\alpha,\beta}(c,m) = S_{1.6,0}(.003,.0002)$. For an interval of size 20, corresponding approximately to one month, we have

$$\log S_{t+20} = \log S_t + \sum_{i=0}^{19} \varepsilon_{t+20-i} = \log S_t + \bar{\varepsilon} .$$

The error term $\bar{\varepsilon}$, being a sum of stable-distributed variables, is also stable, and is a moving average process of order 20, the sum of 20 i.i.d. random variables.

Leaving these results aside for the moment, let us turn to a direct test of the Speculative Efficiency Hypothesis. Since we are using the one-month forward rate, which corresponds to about twenty trading days, the Speculative Efficiency Hypothesis for our sample may be written

$$\log S_{t+20} = \log F_t + \gamma_{t+20}$$

where F_t is the one-month forward rate. We will assume that γ_t is the sum of 20 i.i.d. random variables drawn from a symmetric stable distribution with characteristic exponent $\alpha = 1.6$.[10] The regression equation is

$$\log S_{t+20} = a + b \log F_t + \gamma_{t+20} \qquad (2)$$

where (a,b) is hypothesized to be (0,1), and $\gamma_{t+20} = \gamma_{t+19} + \varepsilon_{t+20} - \varepsilon_t$. Or, rewritten, $\gamma_{t+20} = \sum_{=0}^{19} \varepsilon_{t+20-i}$. Our estimation procedures must take into account the non-normal stability of γ_{t+20}, as well as serial correlation.

Consider the linear regression model $V = gU + \xi$. Let $(v_1, v_2, \ldots, v_n)$ be a vector of n successive observations on V and $(u_1, u_2, \ldots, u_n)$ on U. The n disturbances $\xi = (\xi_1, \xi_2, \ldots, \xi_n)$ are assumed to be i.i.d. symmetric stable random variables with characteristic exponent α. Then the unbiased linear estimator of g that is best in the sense that it has minimum mean absolute error (Kadiyala, 1972), is given by

$$\hat{g} = \frac{\sum_i v_i(\text{sign}(u_i))|u_i|^{\frac{1}{\alpha-1}}}{\sum_i |u_i|^{\frac{\alpha}{\alpha-1}}} .$$

The estimator $\hat{g}$ has a symmetric stable distribution with mean g. Hence hypothesis-testing may be conducted in terms of $C_{\hat{g}}$, the scale parameter of the estimator.

Kadiyala'a estimator remains unbiased when the elements of the disturbance vector ε are not independent but rather come from a moving average process of the type hypothesized for γ in equation 2. Theoretically, however, there exists an unbiased estimator with lower scale parameters than those reported below. For more on this, see Appendix 2.

The intercept a in equation 2 was constrained to equal zero. Hence the test simply involves the confidence interval around $\hat{b}$. The estimated equation is

$$\log S_{t+20} = 1.000 \log F_t$$

$$C_{\hat{b}} = 6.31E - 05.$$

where $C_{\hat{b}}$ is the scale parameter of the estimate of b. Since the original data was quoted to four decimal places, I have followed the usual procedure of reporting point estimates rounded to three decimal places. The log of the one-month forward rate is clearly an unbiased predictor of the log of the future spot rate.

However, it is not necessarily useful. The current spot rate also predicts the future spot rate:

$$\log S_{t+20} = 1.000 \log S_t$$

$$C_{\hat{b}} = 6.28E - 05.$$

Similarly, to make the point, the current forward rate predicts the future forward rate

$$\log F_{t+20} = 1.000 \log F_t$$

$$C_{\hat{b}} = 6.33E - 0.5$$

and the current spot rate almost predicts the future forward rate

$$\log F_{t+20} = 1.001 \log S_t$$

$$C_{\hat{b}} = 6.33E - 05.$$

Only in the last equation can the hypothesis that b=1 be rejected (.001/6.33E − 05 = 15.8 has probability of less than .0025).

These results indicate that both the log spot and log forward rates evolve as Martingales. The result that the spot rate is an equally good predictor of the forward rate indicates that the discount or premium on forward exchange is not a useful guide to future appreciation or depreciation of the spot rate. We see this in the following regression. (Because of the longer time intervals, the switch is made back to percentage changes.)

$$\frac{S_{t+20} - S_t}{S_t} = -.014 \frac{F_t - S_t}{S_t}$$

$$C_{\hat{b}} = .093.$$

The coefficient −.014 is insignificantly different from zero, and we can reject the hypothesis that it is 1.0 at a high level of significance

(1.014/.093) = 10.9, which has probability of less than .005, for a symmetric stable distribution with $\alpha = 1.6$).

These results are not surprising, given the finding that log S_t and log F_t evolve essentially as martingales.[11] Hence, each is an unbiased predictor of its own future value. Given also that, for a sufficiently long sample period, log F_t and log S_t have the same mean, then each will also be an unbiased predictor of the future value of the other. The latter turns out to be the case

$$\log F_t = 1.000 \log S_t$$
$$C_{\hat{g}} = 2.28E - 06.$$

The error terms ε and η have a contemporaneous "correlation" of about 80%, as can be seen in the following regression of Δf on Δs:

$$\Delta f_t = .797\ \Delta s_t$$
$$C_{\hat{g}} = .00797.$$

As a final check on the temporal independence of ε_t, ε_s as well as η_t, η_s, autoregressions were run for Δs and Δf. Kanter and Steiger (1974) have shown that in the autoregression $X_n = a_1 X_{n-1} + \ldots + a_k X_{n-k} + Un$, the OLS estimates of a_i are consistent and apparently very efficient as long as the roots of the equation $1-a_1 Z - a_2 Z^2 - \ldots -a_k Z^k$ are outside the complex unit circle, X_n is independent of U_{n+j}, $j > 1$, and the U_n are independent and identically distributed and in the domain of attraction of a stable law. Hence the following equations were estimated by OLS. I report also some of the usual statistics produced by an OLS package, though the reader is warned these are here of uncertain interpretation

$$\Delta s = -0.048\Delta s_{-1} - .054\Delta s_{-2} + .04\Delta s_{-3} - .036\Delta s_{-4} - .008\Delta s_{-5}$$

$\bar{R}^2 = .005$ D.W. = 2.00 F(4/1461) = 2.73

$$\Delta f = -.046\Delta f_{-1} - .014\Delta f_{-2} - .008\Delta f_{-3} + .001\Delta f_{-4} - .005\Delta f_{-5}$$

$\bar{R}^2 = -.001$ D.W. = 2.00 F(4/1461) = .453

If the F-statistic has the usual interpretation, then the hypothesis that all coefficients are zero cannot be rejected for the forward rate. For the spot rate, the hypothesis can be rejected at the .05 level, though not at the .025 level. Given the lack of explanatory power of the equations, however, I would interpret both equations as evidence for independent increments.

IV. Conclusion

The conclusions of the paper can be briefly summarized:

(1) Proportional or (locally) log changes in the daily spot and one-month forward exchange rates for the dollar price of DM over the period July 1973 to June 1979 may be modelled as though these changes are drawn from a common symmetric stable distribution with characteristic exponent $\alpha \cong 1.6$.

(2) Both the log spot and one-month log forward rates evolve as Martingales. They have equal volatility as measured by the scale parameters of their daily incremental changes. The daily incremental changes of each series is independent of the past. There is, however, a same-day cross correlation of about 80% between incremental changes in the log spot and log forward rates.

(3) The current one-month log forward rate is an unbiased predictor of the future log spot rate. The current log spot rate, however, is also an unbiased predictor of the future log spot rate. In addition, the current log forward rate predicts its own value one-month later, and the current log spot rate predicts (with a slightly biased coefficient of 1.001) the future log forward rate. Thus, even though the Speculative Efficiency Hypothesis holds, the forward rate does not provide any additional information about the future once the current spot rate is known.

(4) The discount or premium on forward exchange is entirely unrelated to future appreciation or depreciation of the price of spot exchange.

(5) These conclusions would seem to call for a new generation of models of the foreign exchange market. Current models typically begin with the Interest Parity Theorem, which may be written (with continuously compounded domestic (i) and foreign (i*) interest rates)

$$\log F_t = \log S_t + i - i^*.$$

The variables i and i* are replaced by interest rate determining variables (such as those found in usual LM equations). But the crucial assumption is

$$\log F_t = E_t[\log S_{t+1}]$$

Thus, dropping the privileged role of the forward rate as an expectations variable would appear to undermine the structure of some of the more popular models of the foreign exchange market.

Appendix A:

A Comment on the Student t-Distribution

Stable and other distributions can be viewed as arising out of subordinated processes. This framework is useful for comparing stable distributions (arising out of the increments of a stable process) to the Student t-distribution.

Let X(v) be a stochastic process with stationary independent increments. Let T(t), $t > 0$, be a process wth nonnegative independent increments. Then Z(t) = X(T(t)) is referred to as a subordinated process. (T(t)) is called a directing process.) T(t) is viewed as a randomized operational time.

(a) If X(v) is a stable process with characteristic exponent α_1, and T(t) is a completely positive stable process ($\alpha_2 < 1$, $\beta = 1$), then X(T(t)) is a stable process with characteristic exponent $\alpha_3 = \alpha_1\alpha_2 < 2$. (Feller, 1971) Thus, for example, a stable process with finite mean may be generated by a completely positive stable process subordinated to Brownian motion.

(b) If 1/T(t) has a gamma-2-distribution, and X(v) is Browian motion, then Z(T) = X(T(t)) will have a Student (t) distribution. If d denotes the degree of freedom of a Student distribution, than all moments of order $r < d$ are finite. As $d \to \infty$, the Student distribution converges to the normal, for which moments of all orders exist. The standardized student variable $\frac{\tilde{X} - E(\tilde{X})}{(\mathrm{Var}(\tilde{X}))^{1/2}}$ has fatter tails than the standard normal, and is higher than the standard normal in the neighborhood of the mean. Blattberg and Gonedes (1974) find that leptokurtic daily stock returns have a t-distribution, so that monthly returns (sums of 20 i.i.d. variables) are close to a normal distribution (estimated d = 25). Thus the normal model would not be applicable to daily returns, but would be appropriate for returns defined over a longer period (1 month). They also show by Monte Carlo simulation that the Fama-Roll (1971) symmetric stable parameter estimators of α will increase with aggregation of the data, provided the data really follow a t-distribution.

The Fama-Roll estimator of α, although simple to compute and very useful in context, is only about 77 percent efficient as compared to the maximum likelihood estimator (DuMouchel, 1975). Moreover, it applies only to symmetric distributions. Blattberg and Gonedes find their data is approximately symmetric, but others have found significant skewedness in stock data (Fielitz and Smith, 1972; Fielitz, 1976; Simkowitz and Beedles, 1980). Thus one must be cautious in comparing goodness-of-fit when ML-estimated parameters of the t-distribution are used but only the simple Fama-Roll estimates are used for the stable.

For exchange rate observations, however, the data seems to be more obviously symmetric, so the T-distribution may serve as a plausible model.

I took sums of size 20 for Δs. This leaves only 73 observations. The numerical routine I used has a downward bias in α estimates for small sample sizes. Hence, if the sample has a stable distribution, we would expect the ML estimate to drop slightly even in $\hat{\alpha}$ is constant for large sample sizes. But if the sample has a t-distribution, the $\hat{\alpha}$ estimate ought to increase toward $\alpha = 2$. The estimates parameters for sums of size 20 are

$\hat{\alpha}$	$\hat{\beta}$	$\hat{c}$	$\hat{m}$
1.40 ± .02	.093 ± .03	1.29 ± .02	.396 ± .041

The mean .396 is consistant with the previous estimate ($20 \times .021 = .42$), while the scale parameter is smaller than expected ($20^{1/1.6} \times .298 = 1.94$). The estimate $\hat{\alpha} = 1.40$ is smaller than before, as expected given the smaller sample size and the assumption $\hat{\alpha}$ is constant in large samples. The sample size is much too small for this to be conclusive but a t-distribution does not appear to be indicated. Westerfield (1977) also found that taking sums of daily data led to no increase in the α estimate.

Appendix B:

Regression with Stable Disturbances

Let the linear regression model be $V = gU + \gamma$. let $(v_1, v_2, \ldots, v_n)$ be a vector of n successive observations on V and $(u_1, u_2, \ldots, u_n)$ on U. The n disturbance terms $(\gamma_1, \gamma_2, \ldots, \gamma_n)$ are assumed to be generated according to the model

$$\gamma_t = \sum_{i=1}^{m} \varepsilon_{t-i+1} \tag{3}$$

where the ε_i are i.i.d. symmetric stable random variables with zero mean, scale parameter C_ε, and characteristic exponent α.

We want to find an unbiased estimator of g with the property that the loss function

$$L(g, \hat{g}) = E|g - \hat{g}|$$

is at a minimum for the estimator $\hat{g}$.

Let the variables a_i be such that

$$\sum_{i=1}^{m} a_i v_i$$

is a linear unbiased estimator of g. Since the estimator is unbiased, we have

$$E(\Sigma a_i v_i) = E(\Sigma a_i (gu_i + \gamma_i) = g\Sigma a_i u_i = g$$

which implies $\Sigma a_i u_i = 1$.

Thus the problem can be reformulated as

$$\min_{a_i} E|\Sigma \; a_i \gamma_i| \qquad \text{s.t. } \Sigma \, a_i u_i = 1 \; .$$

It can be shown that minimization of $E|\Sigma a_i \gamma_i|$ is obtained by minimizing the scale parameter of $\Sigma a_i \gamma_i$. The scale parameter of $\Sigma a_i \gamma_i$ is given the representation in equation (3),

$$C_{\hat{g}} = C_\varepsilon[|a_1|^\alpha + |a_1 + a_2|^\alpha + \ldots + |a_1 + a_2 + \ldots + a_m|^\alpha$$

$$+ |a_2 + a_3 + \ldots + a_{m+1}|^\alpha + \ldots + |a_{n-m+1} + \ldots + a_n|^\alpha \tag{4}$$

$$+ |a_{n-m+2} + \ldots + a_n|^\alpha + \ldots + |a_n|^\alpha]^{1/\alpha} \quad .$$

A check of the first order conditons shows this estimator is not the same as Kadiyala's. Neither, unfortunately, do the first order conditons appear to be solvable analytically for the variables a_i.

However, since γ_1, γ_{m+1}, γ_{2m+1}, ... are <u>independent</u> symmetric stable random varibles, the scale parameter of γ can be estimated by using every twentieth fitted residual $\hat{\gamma}$, $\hat{\gamma}_{m+1}$, $\hat{\gamma}_{2m+1}$, ... The estimated scale parameter for ε is then

$$\hat{C}_\varepsilon = m^{-1/\alpha} \hat{C}_{\hat{\gamma}}$$

The scale parameter of the estimator $C_{\hat{g}}$ is given by the above equation (4), where

$$a_i = \frac{(\text{sign } (u_i))|u_i|^{\frac{1}{\alpha-1}}}{\sum_i |u_i|^{\frac{\alpha}{\alpha-1}}} \, .$$

This estimator $\Sigma a_i v_i$ of g clearly remains unbiased, since $\Sigma a_i u_i = 1$ and $E(\Sigma a_i \gamma_i) = 0$.

FOOTNOTES

[1]The term is apparently due to John Bilson.

[2]Giddy (1976).

[3]There are additional factors to be considered, such as that the nearest market day to the 30th day is implied, and that the counting of the 30 days may not begin with the current trading day (settlement is usually two days later). I consider these problems trivial in comparison to the biases induced by monthly-averaged data.

[4]This proposition is proved in Lévy (1937).

[5]A number of stable-distribution parameter-estimation procedures, based on the sample characteristic function, have been proposed. See Koutrouvelis (1980), Paulson et. al. (1975), Press (1972). These estimators are not necessarily efficient, but may be justified by low computational cost. Fama-Roll (1971) give estimators based on sample fractiles. These estimators, however, are restricted to symmetric distributions.

[6]DuMouchel (1975).

[7]The average interquartile range is .005635. Throwing out two decimal places gives .5635. Our previous estimates of (c,m) were (.298,.021). Dividing these by .5635 yields (.529,.037), which is close to the current estimate of (c,m) as (.514,.041).

[8]DuMouchel (1980).

[9]The data analysis of part II could have been conducted in terms of $\gamma_t = \log S_{t+20} - \log F_t$. However, the distribution of γ would imply nothing about the distribution of log S or log F. The latter distributions are of independent interest. Hence the better procedure was to establish that log S and log F have stable distributions, which necessarily implies γ has a stable distribution.

[10]Actually they are submartingales, since the mean of ε and η is approximately .0002, and hence is larger than zero. However, this number is sufficiently small so that calling them martingales does not seem a great abuse of terminology.

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