TESTING THE CAPM WITH BROADER MARKET INDEXES: A PROBLEM OF MEAN-DEFICIENCY

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1. INTRODUCTION

A recent study by Stambaugh (1981a) investigates the sensitivity of inferences about the Capital Asset Pricing Model (CAPM) to altering the composition of the market index. Tests employing broader market indexes which include bonds, real estate, and consumer durables yield inferences about the CAPM virtually identical to those produced by using a stocks-only index. However, these broader indexes do not capture the total return on a feasible investment portfolio--a portion of the expected return is missing. This misstatement of the mean stems from the nature of the time series used as proxies for returns on real estate and durables. Such a deficiency in the index requires caution in constructing a test of the CAPM. In essence, such a test cannot address mean-variance efficiency of the actual index, but rather it must test the efficiency of a feasible portfolio whose returns are perfectly correlated with those of the index.

The basis for most tests of the CAPM is the linear relation

$$\mu_i = \gamma_1 + \gamma_2\beta_i \quad , \qquad \text{for all } i, \tag{1}$$

where $\beta_i = \text{cov}(r_i, r_m)/\sigma^2(r_m)$, r_i is the return on asset i, r_m is the return on the market portfolio, and $\mu_i = E(r_i)$. Also necessary (and sufficient) for mean-variance efficiency of the market portfolio is that γ_2 in (1) exceed zero and

$$\gamma_2 = E(r_m) - \gamma_1 \quad . \tag{2}$$

Imposing this additional restriction eliminates γ_2 as an additional free parameter, and such a step has been taken in order to estimate γ_1 [eg., Black, Jensen, and Scholes (1972)] and to test the restriction in (1) [eg., Gibbons (1981)].

Any test of (1) and (2) must replace the unobservable r_m with the return on some index (proxy) portfolio, r_p. Thus, the mean-variance efficiency of the index portfolio is tested. Drawing inferences about the CAPM requires that one make assumptions about (i) the degree to which the index approximates returns on the market portfolio and (ii) the extent to which inferences are sensitive to alternative specifications of the index. Use of a broad market index often creates a problem aside from those inherent in making these assumptions.[1] If the index excludes a portion of the return on a feasible proxy portfolio, then imposing (2) is no longer appropriate. However, if the excluded portion is constant, then (1) is still testable as a restriction implied by mean-variance efficiency of the proxy protfolio. In addition, if (1) is assumed to hold for the proxy portfolio, the restriction in (2) may be used to estimate the portion of that portfolio's return excluded from the index.

The paper is organized as follows. Section 2 describes the broader indexes used here and explains why a portion of the actual portfolio return is excluded. Section 3 conducts tests (and estimates parameters) of (1) alone as well as (1) combined with (2). Rejections of the latter restriction produced by the broader indexes are consistent with exclusion of a portion of the portfolio's expected return. This excluded return is estimated in section 4, and the resulting quantity is interpreted as an estimate of the net rental return on residential real estate and consumer durables.

2. THE BROADER MARKET INDEXES

2.1 Index Components[2]

Construction of a market index requires estimates of both returns and relative market values for each of the index components.[3] Data for both items (albeit, at times, imperfect) was obtained for seven major asset classes during the period February 1953 through December 1976: (1) NYSE common stocks, (2) corporate bonds, (3) government bonds, (4) treasury bills, (5) residential real estate (structures), (6) housefurnishings, and (7) automobiles. Weights for an index combining these seven assets are displayed in table 1.[4] NYSE common stocks range from 15% of the index's value in 1953 to 34% in 1968 and, on average, account for about one-fourth of the index. Real estate and consumer durables together make up approximately half of the index.

The weights in table 1 are used to combine monthly return estimates on these asset classes into a broader market index.[5] Actually, two broader indexes are constructed--they differ only in that the CRSP value-weighted NYSE stock index is used in the first index whereas the equally-weighted NYSE stock index is used in the other. Return series for corporate bonds, government bonds, and treasury bills are taken from Ibbotson and Sinquefield (1979). The monthly percent change in the home purchase component of the U.S. Consumer Price Index (CPI) is used as an estimated return on residential real estate. This series is also employed by Fama and Schwert (1977) who conclude that it "seems to be the best available quality adjusted index of transaction prices for real estate." Rates of return on housefurnishings and automobiles are similarly estimated as the monthly percent changes in the housefurnishings and used automobiles components of the CPI. All of the return series employed in the study are "real" returns in that they are deflated according to changes in the total CPI.[6] Table 2 displays the sample means and standard deviations of these time series for four subperiods.[7] Perhaps the most striking feature of these statistics is the magnitude of the standard deviation of common stock returns compared to the standard deviations of the other asset returns. As a result, common stocks tend to play a dominant role in determining variations in returns on the broader index. For example, the sample correlations between the value-weighted stock index and the corresponding broad index are 0.88, 0.97, 0.97, and 0.95 for the four subperiods.

2.2 The "Mean-Deficiency"

Use of the CPI series for real estate and consumer durables (housefurnishings and automobiles) in estimating rates of return requires some simplifying assumptions about the true underlying returns. Percentage changes in these price indexes reflect only the capital gains on the assets. The portion of the total return due to the flow of rental services (net of depreciation and other costs) provided by the assets is excluded. It is assumed that these net rental services constitute the same fraction of the asset's value each period, i.e., the net rental return ("dividend yield") is constant. Fama and Schwert (1977) point out the need for this assumption when they use the home purchase index. The assumption is entertained here as plausible for most durables and real estate. Of course, the more precise notion is that the rental return rate is assumed to be [in a phrase penned by Merton (1980)] "a slowly-varying function of time relative to the time scale of market price changes."[8]

If the above assumption is true, then any index which includes these CPI series to estimate returns does not capture the total return on an actual portfolio of these assets--the rental return is missing. However, if this is the only imperfection in the index, then the variance of the actual portfolio's return and its covariance with other returns is unaffected by exclusion of the constant rental return rate in computing the index. As a result, (1) remains a testable restriction--the β_i's are unaffected. However, (2) becomes an inappropriate constraint. The next section demonstrates the importance of allowing for this mean-deficiency in the index. Then estimates of the excluded rental return are constructed in section 4.

3. TESTING THE CAPM

3.1. Methodology

The statistical framework in which (1) is analyzed consists of a set of familiar market-model equations

$$\underline{r}_i = \underline{\iota}_T \alpha_i + \underline{r}_I \beta_i + \underline{\varepsilon}_i \quad , \qquad i = 2, \ldots, K, \tag{3}$$

where $\underline{r}_i' = (r_{i1}, r_{i2}, \ldots, r_{iT})$ denotes a T-vector of returns on the i^{th} asset, $\underline{r}_I$ is a vector of returns on the market index, $\underline{\iota}_T$ is a unit T-vector, and $\underline{\varepsilon}_i$ is a vector of regression disturbances. It is also assumed that the $\underline{\varepsilon}_i$ obey a multivariate normal distribution,

$$E\ \underline{\varepsilon}_i \underline{\varepsilon}_j' = \sigma_{ij} I \quad , \tag{4}$$

and that α_i and β_i in (3) are stationary over the T time periods.

K-2 over-identifying restrictions are placed on α_i and β_i across equations by the linear pricing relation in (1). To see this, first note that, even in the absence of any pricing relation, a direct implication of (3) is

$$\alpha_i = \mu_i - \beta_i \mu_I \quad , \qquad i = 1, \ldots, K \quad . \tag{5}$$

Substitution of the right-hand side of the pricing relation in (1) for μ_i in (5) yields

$$\alpha_i = \gamma_1 + (\gamma_2 - \mu_I)\beta_i \quad , \qquad i = 1, \ldots, K \quad .$$

or, in vector form,

$$\underline{\alpha} = \gamma_1 \underline{\iota}_K + (\gamma_2 - \mu_I)\underline{\beta} \quad . \tag{6}$$

Written in terms of the parameters actually identified in (6), the above expression is

$$\underline{\alpha} = \gamma_1 \underline{\iota}_K + \gamma_2^* \underline{\beta} \tag{7}$$

where $\gamma_2^* = \gamma_2 - \mu_I$. Of the K restrictions (equations) contained in (7), two equations are needed to identify γ_1 and γ_2^*. Thus, K-2 over-identifying restrictions on $\underline{\alpha}$ and $\underline{\beta}$ are implied by the linearity of (1).

Three tests of the restriction in (7) are analyzed in Stambaugh (1981b): the likelihood ratio test, the Lagrangian multiplier test, and the Wald test. Each of these tests produces a statistic which is asymptoticlly distributed $\chi^2(K-2)$ under the null hypothesis of linearity.[9] However, Monte Carlo experiments indicate that only the Lagrangian multiplier test statistic is adequately approximated by $\chi^2(K-2)$ in finite samples with characteristics resembling those of samples actually employed in tests of the CAPM.[10] It is also possible to obtain maximum likelihood (ML) estimators of γ_1 and γ_2, and these estimators are found to exhibit Monte Carlo behavior very close to their asymptotic (theoretical) distributions.[11] (μ_I can be identified separately from (7), allowing γ_2 to be identified as $\gamma_2 = \gamma_2^* + \mu_I$.)

The tests and estimators are discussed above in the context of the linearity hypothesis in (1) and its implied parameter restrictions in (7). It is also possible to apply the same techniques after imposing the additional restriction in (2) which, when stated in terms of the market proxy portfolio, p, used in the test, becomes

$$\gamma_2 = \mu_p - \gamma_1 \tag{8}$$

where $\mu_p = E(r_p)$. Note that μ_p in (8) must be the total expected return on the portfolio. As observed earlier, μ_p is not necessarily equal to μ_I, the expected return on the index used in (3). However, if it is assumed, as in section 2, that the excluded return is constant, i.e.,

$$r_{pt} = r_{It} + c \quad , \qquad t = 1, \ldots, T,$$

then the form of (1) [and therefore (6)] is unaffected. Substituting the additional restriction (8) into (6) yields

$$\begin{aligned} \underline{\alpha} &= \gamma_1(\underline{\iota}_K - \underline{\beta}) + (\mu_p - \mu_I)\underline{\beta} \\ &= \gamma_1(\underline{\iota}_K - \underline{\beta}) + c\underline{\beta} \end{aligned} \tag{9}$$

which simplifies to a set of K-1 over-identifying restrictions,

$$\underline{\alpha} = \gamma_1(\underline{\iota}_K - \underline{\beta}) \quad , \tag{10}$$

if $\mu_p = \mu_I$ $(c = 0)$. Black, Jensen, and Scholes (1972) use (10) to construct a time series of γ_1's (the zero-beta factor), and more recently the set of restrictions is subjected to a likelihood ratio test by Gibbons (1981). Both studies employ an equally-weighted NYSE common stock portfolio as the market index so, of course, $\mu_p = \mu_I$ in that case. However, the use of a broader index as described earlier necessitates greater caution in constructing a correct test. If part of the mean return is excluded $(c \neq 0)$, then (10) fails even if the actual portfolio is mean-variance efficient. An additional parameter, γ_2^*, must be preserved as in (7) to allow for this mean-deficiency. If instead, the test is based on (10), type I error is likely. Evidence of just such a possibility appears in the results presented below.

3.2 Test Results

The first set of results to be discussed here pertains to the linearity restriction in (7) and the parameters γ_1 and γ_2. The two stocks-only indexes as well as the broader indexes described in section 2 are used in all tests.

Before proceeding to the tests, though, it is necessary to choose a particular set of K individual assets to test the restriction and estimate the γ's. The set chosen here consists of 28 assets which include (i) 19 common stock portfolios grouped by industrial classification, (ii) 4 government bond portfolios and 1 corporate bond portfolio, and (iii) 4 noncallable preferred stocks.[12] This is the most inclusive set of assets used by Stambaugh (1981a), who finds that inferences are often sensitive to the particular set of K assets chosen.

Part A of table 3 contains the results of a Lagrangian multiplier test of the linearity restriction in (7) for all four indexes. None of the results would reject linearity at any conventional significance level, and the statistics and p-values are remarkably similar across indexes--p-values for the overall statistic fall between 69.5% and 76.3%. ML estimates of γ_1 and γ_2 are shown in table 4. The last column of part A reports an overall test of the Sharpe-Lintner hypothesis, $\gamma_1 = r_F$, where r_F is the average real return on one-month T-bills. P-values are less than 0.01% for all indexes, thereby rejecting the hypothesis. An overall-period test of $\gamma_2 = 0$ is reported in the last column of part B. P-values are 0.32% or less for all indexes, so $\gamma_2 = 0$ is rejected in favor of $\gamma_2 > 0$ at any conventional significance level. The overall inference implied by these results is that the index portfolios are on the positively-sloped portion of the mean-variance frontier, but none of them is the Sharpe-Lintner tangency portfolio.

A different picture is presented by part B of table 3, which reports the results of a Lagrangian multiplier test of (10), i.e., linearity <u>and</u> $\gamma_2 = \mu_1 - \gamma_1$. The overall p-values for the stock indexes--33.6% and 69.0%--are well above any conventional significance level and would not reject the restriction. This is not surprising given that linearity is not rejected in part A and, since the stock indexes are feasible portfolios, $\mu_I = \mu_p$.[13] In contrast, the overall p-values for the broader indexes are less than 4%, which could easily lead one to reject the restriction in those cases. However, this rejection is precisely what would be predicted if $\mu_I \neq \mu_p$, i.e., if the index excludes part of the expected return on a feasible portfolio. The next section actually estimates this excluded return, and it appears to be both economically and statistically significant.

4. ESTIMATING THE EXCLUDED RETURN

If it is assumed that the actual portfolio represented by the index is mean-variance efficient, the (7) and (8) can be used to estimate the excluded return. First note that $\mu_I = \mu_p - c$, where c is the excluded return (a constant). Thus,

$$\gamma_2^* = \gamma_2 - (\mu_p - c) \quad .$$

Substituting (8) for γ_2 yields

$$\gamma_2^* = \mu_p - \gamma_1 - (\mu_p - c)$$

or

$$c = \gamma_1 + \gamma_2^* \quad . \tag{11}$$

A maximum likelihood estimator for c is, therefore, obtained by summing the estimates of γ_1 and γ_2^* in (7). An asymptotic standard error is obtained as the square root of

$$\mathrm{var}(\hat{c}) = \mathrm{var}(\hat{\gamma}_1) + (\hat{\gamma}_2^*) + 2\ \mathrm{cov}(\hat{\gamma}_1, \hat{\gamma}_2^*) \quad ,$$

and the covariance term is negative.[14] In a similar fashion, $\hat{\mu}_p$ can be constructed as $\hat{\mu}_I + \hat{c}$, and $\mathrm{var}(\hat{\mu}_p)$ is simply $\mathrm{var}(\hat{\mu}_I) + \mathrm{var}(\hat{c})$.[15]

Parts A and B of table 5 present ML estimates of c and μ_p along with estimates of their asymptotic standard errors. The estimates of c, the excluded (real) return, range form 0.08% to 2.3% per month, and the estimates are all large (by at least twice) relative to their standard errors. Somewhat striking is the difference in precision of the estimates of c and those of μ_p, the total return. Based on ratios of coefficients to their standard errors, it appears that c can be estimated about three to five times more precisely than μ_p. There is also less dispersion across the four subperiods in the estimates of c than in the estimates of μ_p. These observations are at least roughly consistent with the initial assumption that the excluded return is constant (or slowly-varying compared to prices). Another interesting result is the non-trivial fraction of total portfolio expected return accounted for by c--from somewhat less than half in the first two subperiods to virtually all of the total in the third and fourth subperiods. Given excluded returns of these magnitudes, it is not surprising that the tests of linearity and $\gamma_2 = \mu_I - \gamma_1$ failed for the broader indexes (table 3, part B).

Finally, recall that the excluded return for these broader indexes is assumed to be the net rental return on residential real estate and consumer durables. The above estimates of c can be transformed into estimates of this rental return by making use of the weights received by these assets in the indexes. If w is the weight for real estate and durables as a whole, then the rental return for these assets (as a whole) is c/w. This calculation is reported in part C of table 5, and the rates of return are converted to annualized real returns. The average weight received by real estate and durables over each subperiod is used as w. Estimates of the rental return differ somewhat between indexes, which could reflect either sampling variability or a violation of one of the assumptions (probably some of both). However, the estimates all seem economically plausible, ranging from 1.7% to 5.4% per year. Separate estimates for real estate and durables could be constructed using estimated betas, but such an exercise must await future studies.

TABLE 1

MARKET INDEX WEIGHTS FOR SELECTED YEARS
(End of Year)

Assets	1952	1958	1964	1970	1976
NYSE Common Stocks	0.155	0.255	0.317	0.294	0.239
Corporate Bonds	0.078	0.060	0.055	0.050	0.052
Government Bonds	0.174	0.129	0.099	0.074	0.073
Treasury Bills	0.030	0.028	0.038	0.042	0.047
Residential Real Estate	0.385	0.361	0.340	0.362	0.399
Housefurnishings	0.121	0.106	0.096	0.116	0.130
Automobiles	0.058	0.062	0.054	0.063	0.059
Total Value ($Billions)	721.4	1080.9	1470.2	2068.3	3408.1

TABLE 2

MARKET INDEX COMPONENTS: MEANS AND STANDARD DEVIATIONS[a]

Assets	2/53-3/59	4/59-5/65	6/65-7/71	8/71-12/76
		Mean[b]		
NYSE Stocks (v.w.)	1.312	0.839	0.156	0.042
NYSE Stocks (e.w.)	1.366	0.810	0.585	0.236
Corporate Bonds	0.021	0.269	-0.251	0.175
Government Bonds	-0.036	0.217	-0.269	0.104
Treasury Bills	0.047	0.129	0.074	-0.079
Residential Real Estate	-0.037	-0.032	0.006	-0.065
Housefurnishings	-0.162	-0.131	-0.127	-0.105
Automobiles	-0.232	0.100	-0.150	0.173
		Standard Deviation[b]		
NYSE Stocks (v.w.)	3.311	3.433	4.114	5.133
NYSE Stocks (e.w.)	3.395	3.815	5.413	7.145
Corporate Bonds	1.449	0.823	2.199	2.356
Government Bonds	1.561	1.021	2.632	2.050
Treasury Bills	0.233	0.167	0.167	0.271
Residential Real Estate	0.366	0.303	0.355	0.433
Housefurnishings	0.522	0.274	0.224	0.400
Automobiles	1.655	1.988	2.070	2.543

[a]Monthly real returns.

[b]All values multiplied by 100.

TABLE 3

LAGRANGIAN MULTIPLIER TESTS OF PARAMETER RESTRICTIONS[a]

Market Index	2/53-3/59	4/59-5/65	6/65-7/71	8/71-12/76	Overall Statistic[d]
A. $\underline{\alpha} = \gamma_1 \underline{1} + \gamma_2^* \underline{\beta}$ (Linearity)[c]					
1. NYSE (value-weighted)	26.60 (43.1%)	27.38 (39.0%)	17.41 (89.6%)	23.01 (63.3%)	94.39 (74.0%)
2. NYSE (equally-weighted)	27.98 (36.0%)	27.40 (38.9%)	17.42 (89.6%)	23.37 (61.2%)	96.18 (69.5%)
3. 1 plus all other[b]	25.86 (47.1%)	26.79 (42.0%)	17.67 (88.8%)	23.07 (62.9%)	93.39 (76.3%)
4. 2 plus all other[b]	27.53 (38.2%)	26.93 (41.3%)	17.62 (88.9%)	23.33 (61.4%)	95.42 (71.4%)
B. $\underline{\alpha} = \gamma_1(\underline{1}-\underline{\beta})$ (Linearity and $\gamma_2 = \mu_I - \gamma_1$)[c]					
1. NYSE (value-weighted)	28.90 (36.6%)	27.60 (43.2%)	27.78 (42.3%)	29.38 (34.3%)	113.65 (33.6%)
2. NYSE (equally-weighted)	30.08 (31.1%)	27.94 (41.4%)	17.72 (91.2%)	24.50 (60.2%)	100.23 (69.0%)
3. 1 plus all other[b]	39.91 (5.2%)	33.47 (18.2%)	36.12 (11.3%)	26.24 (50.5%)	135.75 (3.7%)
4. 2 plus all other[b]	38.78 (6.6%)	44.68 (1.8%)	28.44 (38.9%)	25.08 (57.0%)	136.97 (3.1%)

[a]P-values in parentheses.

[b]The remaining six assets combined with the weights shown in table 1.

[c]Test statistics are asymptotically distributed χ^2 under the null with degress of freedom equal to 26 in part A and 27 in part B.

[d]The sum of the individual subperiod statistics, also distributed χ^2 under the null with 104 d.o.f. in part A and 108 d.o.f. in part B.

TABLE 4

MAXIMUM LIKELIHOOD ESTIMATES OF γ_1 and γ_2

$$\underline{\mu} = \gamma_1 \underline{\iota} + \gamma_2 \underline{\beta}$$

Market Index	2/53-3/59	4/59-5/65	6/65-7/71	8/71-12/76	Overall Statistic[c] and p-value
			A. Estimates of γ_1^a		
1. NYSE (value-weighted)	0.118 (0.026)	0.192 (0.022)	0.172 (0.036)	-0.007 (0.054)	4.82 (0.00%)
2. NYSE (equally-weighted)	0.119 (0.025)	0.193 (0.022)	0.172 (0.036)	-0.003 (0.054)	4.91 (0.00%)
3. 1 plus all other[b]	0.099 (0.027)	0.187 (0.023)	0.172 (0.036)	-0.011 (0.055)	4.23 (0.00%)
4. 2 plus all other[b]	0.103 (0.026)	0.188 (0.026)	0.172 (0.036)	-0.005 (0.054)	4.43 (0.00%)
			B. Estimates of γ_2^a		
1. NYSE (value-weighted)	1.384 (0.394)	0.586 (0.408)	0.323 (0.485)	0.533 (0.649)	3.22 (0.06%)
2. NYSE (equally-weighted)	1.230 (0.396)	0.625 (0.444)	0.407 (0.631)	0.259 (0.888)	2.73 (0.32%)
3. 1 plus all other[b]	0.330 (0.087)	0.206 (0.117)	0.053 (0.170)	0.136 (0.178)	3.31 (0.05%)
4. 2 plus all other[b]	0.282 (0.085)	0.212 (0.127)	0.084 (0.214)	0.009 (0.232)	2.88 (0.20%)

[a]Asymptotic standard errors in parentheses. Values multiplied by 100.

[b]The remaining six assets combined with weights from Table 1.

[c]A normal (0,1) statistic if $\gamma_1 = R_F$(ave. T-Bill return) in part A or $\gamma_2=0$ in part B. P-values in part A reflect a two-tailed test; those in part B reflect a postive one-tailed test.

TABLE 5

ML ESTIMATES OF NET RENTAL RETURN (EXCLUDED FROM INDEX) AND PORTFOLIO EXPECTED RETURN[a]

Market Index	2/53-3/59	4/59-5/65	6/65-7/71	8/71-12/76
A. Estimates of c: average monthly real return excluded from index[c]				
3. NYSE (value-weighted) plus all other[b]	0.200 (0.031)	0.133 (0.031)	0.228 (0.033)	0.134 (0.047)
4. NYSE (equally-weighted) plus all other[b]	0.136 (0.028)	0.146 (0.017)	0.117 (0.024)	0.081 (0.038)
B. Estimates of μ_p: total expected monthly real return on index portfolio[c]				
3. NYSE (value-weighted) plus all other[b]	0.429 (0.216)	0.393 (0.174)	0.226 (0.283)	0.124 (0.219)
4. NYSE (equally-weighted) plus all other[b]	0.385 (0.159)	0.400 (0.192)	0.256 (0.244)	0.084 (0.244)
C. Estimates of annualized real net rental return on Residual Real Estate and Consumer Durables				
3. NYSE (value-weighted) plus all other[b]	4.4% (0.7)	3.1% (0.7)	5.4% (0.8)	2.8% (1.0)
4. NYSE (equally-weighted) plus all other[b]	3.0% (0.6)	3.4% (0.4)	2.8% (0.6)	1.7% (0.8)

[a] Asymptotic standard errors in parentheses.

[b] The remaining six assets combined with weights shown in Table 1.

[c] Values multiplied by 100.

FOOTNOTES

[1]The latter problems are well-discussed by Roll (1977), and his critique of CAPM tests served as the primary motivation for the sensitivity analysis of Stambaugh (1981a,b).

[2]The contents of this section are discussed in further detail in Stambaugh (1981a,b)

[3]Generally speaking, the lack of return data, particularly monthly, is the binding constraint. Estimates of market values of major components of national wealth have been available since the seminal work of Goldsmith (1952, 1955). Kendrick and Lee (1976) present a concise history of the wealth-estimation research.

[4]The estimated market values underlying these weights are obtained from the following sources:

NYSE Common Stocks: CRSP, University of Chicago.
Corporate Bonds: File constructed by Roger Ibbotson, University of Chicago.
U.S. Government Bonds and Treasury Bills: CRSP Government Securities File.
Residential Real Estate: Musgrave (1976) and Survey of Current Business.
Housefurnishings and Automobiles: Musgrave (1979).

[5]Monthly weights are obtained from year-end weights by linear interpolation.

[6]The method used is defined by Fama (1976, pp. 172-3).

[7]The subperiods are chosen to be roughly equal in length and to confine the period of wage-price controls (which began in August 1971) to the last subperiod.

[8]Merton refers to changes in variance in this manner.

[9]See Silvey (1975, chapter 7) for a discussion of these tests.

[10]In particular, the Wald test accepts the null hypothesis too often, whereas the likelihood ratio test rejects the null in too many cases when the number of equations (K) exceeds 10.

[11]See Stambaugh (1981b) for details.

[12]The common stock portfolios are formed according to the classification scheme of MacBeth (1975). The bond portfolios consist of the Ibbotson-Sinquefield (1979) corporate and government portfolios (20-year maturity) plus 3 government portfolios with maturities of 1 to 2 years, 2-5 years, and 5-10 years. The four preferred stocks are American Can, Ligget and Myers, Pacific Telephone and Telegraph, and Uniroyal.

[13] One might argue that this result is not necessarily to be expected, because linearity will hold in the absense of the additional restriction in (8) if each of the K individual assets is mean-variance efficient (even if the index is a single stock). However, this scenario is unlikely given that some of the assets here are individual preferred stocks.

[14] "^" denotes a ML estimator. As shown in Stambaugh (1981b)

$$\mathrm{var}\begin{vmatrix}\hat{\gamma}_1\\ \hat{\gamma}_2^*\end{vmatrix} = \frac{1}{T}\left[1 + \frac{(\gamma^*_2 + \mu_I)^2}{\sigma_I^2}\right]\begin{vmatrix}\underline{\iota}'\Sigma^{-1}\underline{\iota} & \underline{\iota}'\Sigma^{-1}\underline{\beta}\\ \underline{\iota}'\Sigma^{-1}\underline{\beta} & \underline{\beta}'\Sigma^{-1}\underline{\beta}\end{vmatrix}^{-1},$$

where Σ is the K by K matrix with (i,j) element σ_{ij}.

[15] The asymptotic covariance of $\hat{\mu}_I$ and $\hat{c}$ is zero, since the asymptotic covariance of $\hat{\mu}_I$ with both $\hat{\gamma}_1$ and $\hat{\gamma}_2^*$ is zero [also shown in Stambaugh (1981b)].

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