Private Discrimination and Social Intervention in Competitive Labor Markets

by

Richard Startz
and
Shelly J. Lundberg

Working Paper No.19-81

RODNEY L. WHITE CENTER
FOR FINANCIAL RESEARCH

The Wharton School
University of Pennsylvania
Philadelphia, PA 19104

Private Discrimination and Social Intervention in Competitive Labor Markets

Richard Startz
The Wharton School
University of Pennsylvania

and

Shelly J. Lundberg
University of Pennsylvania
and NBER

November 1981

Do laws forbidding discrimination reduce allocative efficiency? A common thread in economists' discussions of equal opportunity laws has been a presumption that equal pay and/or quota constraints placed on firms act as transfer mechanisms which, as a rule, cause efficiency losses. The implicit model which commentators employ, however, is based on competitive markets with perfect information, so that it is unclear how discrimination could have arisen in the first place.[1] Our purpose in this paper is to consider the efficiency effects of equal opportunity type intervention in the context of a conventional model of discrimination; that is, one which does not depend upon differences in innate ability between groups of workers to produce differences in wages.

The economic analysis of labor market discrimination has produced two general types of models: "taste" discrimination and informational or "statistical" discrimination. Taste models, such as Gary Becker's prototype, produce wage differentials based on the preferences of majority employers, employees, and customers, but of a type which should not persist in competitive markets. Statistical models, on the other hand, demonstrate that treating two groups of workers differently may be the rational response of firms to uncertainty about an individual's productivity. In this case, persistent wage differentials may arise between workers with the same productivity who belong to different, identifiable groups, even in competitive markets.

We present in this paper a simple model of statistical discrimination and examine the effects of prohibiting group-specific treatment of workers on both

---

[1]For example, Finis Welch, in the course of an illuminating discussion of affirmative action enforcement, constructs a simple model in which equal pay for workers who differ in ability does indeed distort occupational choices and cause efficiency losses.

net social product and the distribution of income. The agents are competitive firms who pay wages equal to the expected value of a worker's marginal product, conditional upon all information available to them, and income-maximizing workers who decide on the size of their human capital investments based on known wage schedules. Each worker is characterized by a level of innate ability and by affiliation with one of two groups. Firms are able to assess the marginal product of members of one group more reliably than for the second group's members, and so offer different wage schedules. The main result is that the allocation achieved by rational agents in this labor market can be improved by prohibiting discrimination based on group membership.

I. Imperfect Information and Discrimination

Consider two groups of workers, defined according to race, sex, or some other easily-observable, exogenous, characteristic. Each group contains individuals with varying levels of ability or skill which determine their marginal products in any employment. Risk neutral firms, though they know the density functions which describe the distribution of ability for each type of worker, cannot observe directly the marginal product of an individual i, MP_i. They do, however, observe a test score T_i which is some function of the worker's marginal product and group membership, I_i.

The group index I_i will enter this relationship if the testing procedure differs across groups; i.e. is biased or less reliable for one or the other. Since all firms are equally effective in assessing MP_i via the test, a competitive equilibrium will involve paying each individual a wage equal to the expected value of marginal product conditional on the test score and group membership.

$$w_i = E(MP_i | T_i, I_i)$$

The wage schedule $w(T_i)$ will generally be different for the two groups, though for each group the average wage will equal its average marginal product.[2]

Aigner and Cain present a simple model of statistical discrimination which illustrates the general characteristics of this approach. They assume that the exogenously given (normal) distributions of productivity in each of two groups of workers (black and white) are identical. The test scores which firms observe, however, are more reliable indicators of ability for whites than for blacks. Thus,

$$T_i^B = MP_i + \varepsilon_i^B$$

$$T_j^W = MP_j + \varepsilon_j^W \qquad \text{where } \sigma^2_{\varepsilon^B} > \sigma^2_{\varepsilon^W}.$$

It is straightforward to show that the equilibrium wage is a weighted average of mean productivity and the individual's test score, where the test score of a black worker is weighted more lightly than the test score of a white worker. The wage schedule $w^B(T_i^B)$ will have a smaller slope than the schedule $w^W(T_i^W)$, though mean wages will equal mean productivities, which are identical for the two groups. High scoring blacks will thus be paid less than whites with the same test score; the reverse will be true for workers with low scores.[3]

---

[2]We assume that firms are able to assess average group productivity accurately, but not the productivity of individual workers. Wage differentials between equally skilled workers, however, may be allowed to erode slowly as each firm acquires information about individual productivities without changing the essential character of the model.

[3]Aigner and Cain also describe a model which results in different mean wages for blacks and whites, though ability distributions are identical. This model relies upon employer risk aversion and the authors express some reservations about its empirical relevance.

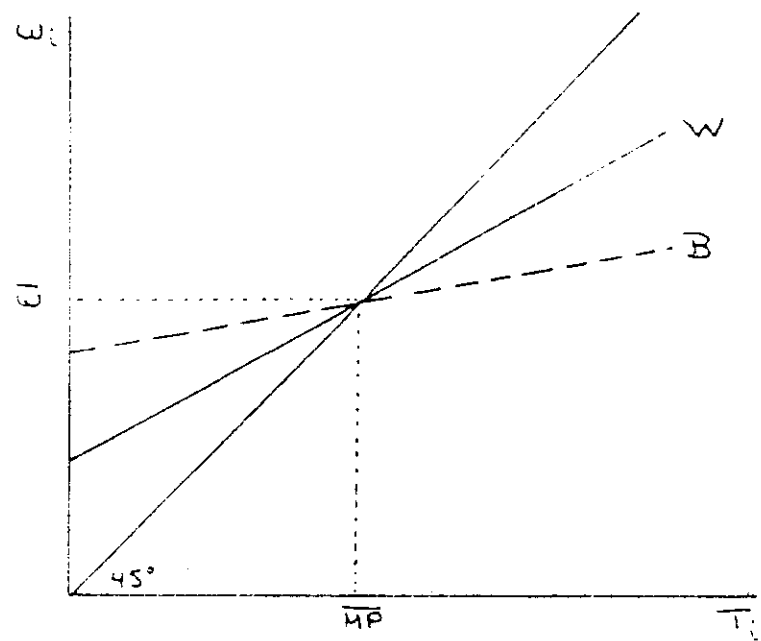

Figure 1

What implications do such informational models of discrimination have for the efficiency of discriminatory equilibria? Since the wage schedules have no real effects on resource allocation--i.e. on labor supply or on the sorting of workers among jobs in which their productivities differ--the sole effect of discrimination is a redistribution of income among workers with the same level of ability. A crucial assumption is stated by Aigner and Cain,

> "Our focus is on labor market discrimination, which means we will generally assume that the worker's pre-labor market investments and endowments are given."

In this paper we recognize that human capital investment decisions will be affected by the presence of labor market discrimination and model this dependence explicitly. If wages are based on the results of an imperfect test, the returns to an investment in skills which cannot be directly observed are reduced. The result is a suboptimal level of human capital which will vary over groups if the quality of testing varies. We show in the remainder of the paper that equilibrium allocations under this type of imperfect information can be improved by such simple forms of labor market intervention as enforcing group-blind compensation rules.

II. Informationally Efficient Discriminatory Equilibria

We assume that workers have certain characteristics, both innate and acquired, which determine their productive abilities. These characteristics are distributed randomly in the population. Each worker knows his own characteristics exactly and invests in acquiring human capital to the point where the marginal cost of further investment just balances the increment to wages produced by the increased investment.[4] Employers know the density function describing the distribution of characteristics through the population and observe a "test score" for each worker that provides information about the worker's marginal product.[5] The employer then offers a wage equal to the conditional expectation of the worker's marginal product. We set out below a simple stochastic model of worker characteristics and calculate the unique linear rational expectations equilibrium.

Each worker produces a marginal product MP_i that depends on innate ability a_i and acquired ability X_i. The desired level of X_i can be purchased by each worker at increasing marginal cost, reflecting diminishing returns to time and money spent on training activities and increasing disutility of

---

[4]Though we refer to acquired abilities rather indiscriminately as "human capital" or "training", we are following the interpretation of Arrow:

> "Hence, the investments are not the usual types of education or experience, which are observable, but more subtle types of personal deprivation and deferment of gratification which lead to the habits of action and thought that favor good performance in skilled jobs..." (p. 27).

[5]This test score need not result from a literal "test", but is simply a summary measure of all information the employer is able to acquire during the hiring process and on the job.

foregone leisure. Formulae for marginal product and the cost of acquired training are

(1) $MP_i = a_i + bX_i$

(2) $C(X_i) = \frac{1}{2} cX_i^2$, $c'(X_i) = cX_i$

In a full-information equilibrium every worker purchases b/c units of education at a cost $\frac{1}{2} b^2/c$. The per capita net social product of education is

(3) $MP(X_i) - MP(0) - C(X_i) = \frac{1}{2} b^2/c$

Employers do not observe true productivities, but rather a "test score," T_i, for each worker. The test measures the worker's marginal product with a random error.

(4) $T_i = MP_i + \varepsilon_i$

The worker characteristics a_i and ε_i are drawn from a bivariate normal distribution with known parameters $\bar{a}$, $\bar{\varepsilon}$, σ_a^2, σ_ε^2. We assume that a_i and ε_i are uncorrelated, though this assumption is unimportant. Workers know their own individual characteristics and maximize wages net of education costs. Employers are competitive and maximize profits by setting wages equal to the expected value of marginal product, conditioning the expectation on all available information. The parameters of the joint density function, as well as b and c, are public knowledge.

There exists a unique linear rational expectations equilibrium for wages and human capital investments. In determining this equilibrium, workers look to the wage offer schedule to decide on the optimal level of human capital investment and firms look to the joint distribution of test scores and marginal product to decide on the wage offer schedule. As a solution technique, we initially write optimal human capital investment as a linear function of worker characteristics with undetermined coefficients, as in (5). Properties of the equilibrium solution below allow us to fix unique values for the coefficients and thus completely characterize the equilibrium.

$$X_i = \rho_o + \rho_a a_i + \rho_\varepsilon \varepsilon_i \tag{5}$$

The firm's problem is to establish a wage offer schedule as a function of test scores.

$$w_i = E(MP_i|T_i) = E(T_i - \varepsilon_i|T_i) = T_i - E(\varepsilon_i|T_i) \tag{6}$$

Since the test score is a linear function of normal random variables, the test score itself is normally distributed with mean $\bar{T} = b\rho_o + (1+b\rho_a)\bar{a} + (1+b\rho_\varepsilon)\bar{\varepsilon}$ and variance $\sigma_T^2 = (1+b\rho_a)^2\sigma_a^2 + (1+b\rho_\varepsilon)^2\sigma_\varepsilon^2$. T_i and ε_i have a bivariate normal distribution with correlation coefficient $(1+b\rho_\varepsilon)\sigma_\varepsilon/\sigma_T$. The expectation of ε_i conditional on T_i follows immediately.

$$E(\varepsilon_i|T_i) = \bar{\varepsilon} + (1+b\rho_\varepsilon)\sigma_\varepsilon^2/\sigma_T^2\,[T_i - \bar{T}] \tag{7}$$

For convenience, we write the coefficient of the test score in (7) as (1-β). Substituting (7) into (6), we write the wage schedule offered by employers as

$$w_i = \overline{MP} + \beta(T_i - \bar{T}) \tag{8}$$

Note that if $\bar{\varepsilon} = 0$, the individual wage is a simple weighted average of the group mean and individual test scores.

Each worker faces the wage schedule (8) with certainty and invests in human capital to the point where the marginal cost of acquiring more training equals the marginal increase in wages. An additional unit of X increases the worker's marginal product and test score by b, so that wages rise by βb. The equilibrium level of acquired human capital is

(9) $X_i = \beta b/c$

for all workers.

Since the marginal cost of and marginal returns to each unit of X are identical across workers, X is nonstochastic in equilibrium -- ρ_a and ρ_ε in (5) are identically zero. The equilibrium value of β is

(10) $\beta = \sigma_a^2/\sigma_T^2$

so that β, which is also the ratio of marginal private to marginal social returns to acquired human capital, is between zero and one and depends only upon the relative sizes of the variances of innate ability and testing error.

Since $MP_i = a_i + \beta b^2/c$ and $w_i = \overline{MP} + \beta[(a_i - \bar{a}) + (\varepsilon_i - \bar{\varepsilon})]$ it is easy to show that $\bar{w} = \overline{MP}$ and that $\sigma_w^2 = \sigma_{MP}^2 = \sigma_a^2$.

The net social product of education is

(11) $MP(X_i) - MP(0) - C(X_i) = \beta(b^2/c)(1-\tfrac{1}{2}\beta)$

So social welfare increases monotonically with β. Private markets result in an underinvestment in education.[6]

We now consider discriminatory equilibria. Suppose that workers are drawn from two subpopulations, the star group (*) and the dagger group (†). We assume the groups have identical mean innate characteristics $\bar{a}$ and $\bar{\varepsilon}$ and the same test variance σ_T^2. The only innate difference between the two groups is that the star group has a relatively heterogenous innate ability and

---

[6]Note that there are two first-best policies the government could use to achieve social efficiency. They could order employers to use wage offer schedules with a one as the coefficient on test scores or the government could subsidize human capital investment to lower the marginal cost to $c\beta X_i$.

relatively homogeneous testing ability as compared to the dagger group.[7] Algebraically, $\sigma_a^2(*) > \sigma_a^2(\dagger)$ and $\sigma_\varepsilon^2(*) < \sigma_\varepsilon^2(\dagger)$. Using (10) we have $\beta^* > \beta^\dagger$.

Employers rationally discriminate between the star group and the dagger groups by offering separate wage schedules. Workers in each group respond to their available opportunities and separate equilibria are calculated as above. The star group, whose test scores are more reliable indicators of productivity, becomes the high wage/high training group. Every member of the star group acquires $(\beta^*-\beta^\dagger)b/c$ more training than every member of the dagger group, since the marginal return to each unit of X is higher. The average wage for each group, however, is equal to the group average marginal product.

Suppose the star group makes up α percent of the population and the dagger group the remaining $1-\alpha$ percent. Total training is $(\alpha\beta^* + (1-\alpha)\beta^\dagger)b/c$ and the net social product of training is

$$[\alpha\beta^*(1-\tfrac{1}{2}\beta^*) + (1-\alpha)\beta^\dagger(1-\tfrac{1}{2}\beta^\dagger)]b^2/c. \tag{12}$$

Wage schedules for the star and dagger groups are reproduced in Figure 2. Would the usual sort of test indicate dagger workers on the whole are "discriminated against"? The average star worker does receive higher wages, and higher wages net of training costs, than the average dagger worker. "Regression" tests of wages against test scores would reveal that the two groups are paid according to different schedules with star workers receiving larger raises for increased test scores. As Figure 2 shows, a dagger worker with an average dagger test score would receive a raise if paid according to the star schedule. A dagger worker who achieves the average star test score remains underpaid in comparison to the average star worker. Another way of

---

[7]Suppose, for example, that personnel managers are all members of the star group, and are more effective at assessing workers who are members of their own group.

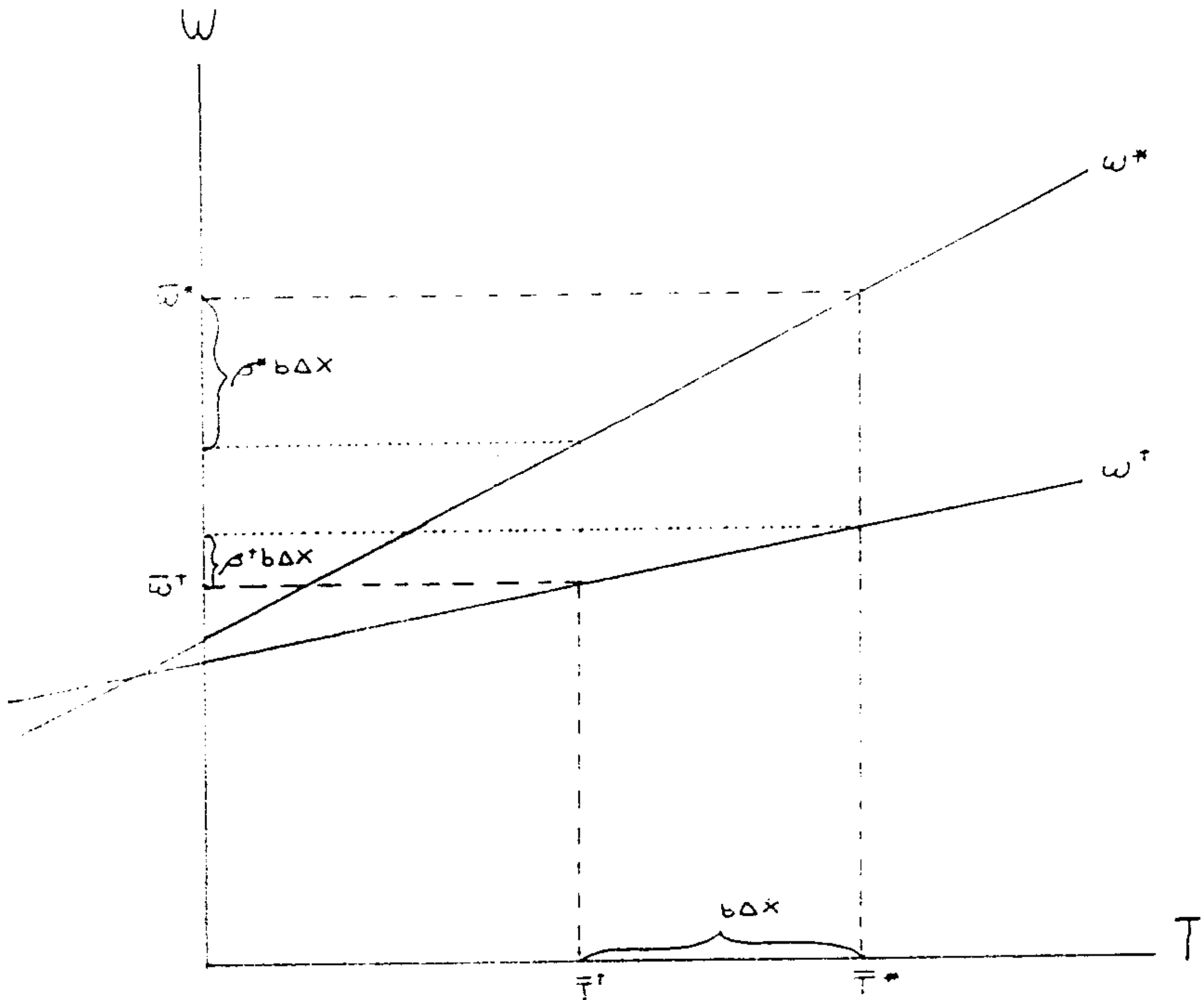

Figure 2

stating these comparisons is that dagger workers are underpaid even after accounting for group differences. The difference in productivity levels is, by construction, due only to the training differential but the average wage differential is more than can be explained by the difference in training using either the star or dagger schedule to calculate the wages due to increased training. In fact, for two workers with identical test scores, the star worker will generally receive the higher wage, the situation reversing only at very low test scores.

The legal system would almost certainly consider this equilibrium one of illegal discrimination, since dagger workers are generally paid less than star workers even after accounting for all observable individual characteristics. An economist, however, might well feel that the equilibrium does not exhibit invidious discrimination, since all parties are acting rationally and wages equal expected marginal products.

III. Socially Preferable Nondiscriminatory Equilibrium

We demonstrate that a policy forbidding separate wage schedules for star people and dagger people results in an increase in allocative efficiency. Consider the consequences of the following policy restriction: employers may offer wages equal to the expectation of a worker's marginal product conditioned on his test score, but may not consider group membership.

We now derive the linear rational expectations equilibrium by arguments analogous to those in section II. The key step is the derivation of $E(\varepsilon_i|T_i)$. A few intermediate calculations are required because the joint density function of ε_i and T_i is no longer bivariate normal. Let $f(\cdot)$ represent the density function of the mixture and $f^*(\cdot)$ and $f^\dagger(\cdot)$ represent the densities for the star and dagger groups. The density function of the mixture is

$$(13) \qquad f(\varepsilon_i, T_i) = \alpha f^*(\varepsilon_i, T_i) + (1-\alpha)f^\dagger(\varepsilon_i, T_i)$$

Suppose, as we shall demonstrate, that there exists a linear rational expectations equilibrium. Let β be the, yet to be determined, coefficient of the test score in the wage offer equation. Every worker, star and dagger, will choose $\beta b/c$ units of education, so that X_i is nonstochastic. The density functions $f^*(\cdot)$ and $f^\dagger(\cdot)$ are therefore bivariate normal as in section II, except that star and dagger functions share a common mean test score which may differ from the mean test score of either group in the discriminatory equilibrium.

To find the conditional expectation we need to find the conditional density function $f(\varepsilon_i|T_i) = f(\varepsilon_i, T_i)/f_T(T_i)$, where $f_T(T_i)$ is the marginal density function with respect to T. This marginal is a weighted sum of the star and dagger marginal densities, which are identical by construction, so

$$(14) \qquad f_T(T_i) = \int f(\varepsilon_i, T_i)d\varepsilon_i = \alpha\int f^*(\varepsilon_i, T_i)d\varepsilon_i + (1-\alpha)\int f^\dagger(\varepsilon_i, T_i)d\varepsilon_i$$
$$= \alpha f_T^*(T_i) + (1-\alpha)f_T^\dagger(T_i) = f_T^*(T_i) = f_T^\dagger(T_i)$$

The marginal distributions with respect to T are $N(\bar{T}, \sigma_T^2)$ for both the star and dagger group and therefore for the mixture as well. The conditional density for the mixture is a weighted sum of the individual conditionals, given (13) and (14).

$$(15) \qquad f(\varepsilon_i|T_i) = \alpha f^*(\varepsilon_i|T_i) + (1-\alpha)f^\dagger(\varepsilon_i|T_i)$$

Since the expected value is a linear operator,

$$(16) \qquad E(\varepsilon_i|T_i) = \alpha E^*(\varepsilon_i|T_i) + (1-\alpha)E^\dagger(\varepsilon_i|T_i)$$

Using the properties of bivariate normal distributions once again we have

$$(17) \qquad E(\varepsilon_i|T_i) = \alpha[\bar{\varepsilon} + (1-\beta^*)[T_i-\bar{T}]] + (1-\alpha)[\bar{\varepsilon} + (1-\beta^\dagger)[T_i-\bar{T}]]$$

Notice that the conditional expectation of marginal product will be linear in the test score, even though the conditional distribution is nonnormal.

Conveniently, we can define

$$(18) \qquad \beta = \alpha\beta^* + (1-\alpha)\beta^\dagger$$

and by setting the wage equal to expected marginal product we can reproduce the linear wage schedule

$$(19) \qquad w_i = \overline{MP} + \beta(T_i - \overline{T})$$

As a result of the policy restriction on wage schedules, all workers choose the same level of training. Average wages are the same for both groups and individuals with identical characteristics receive identical wages regardless of group membership. While the average marginal products of both groups are now equal, the variability in marginal product is greater for the star group. This implies that an individual employer faced with the equilibrium mixture still has an incentive to discriminate. The privately rational (but illegal) wage schedule based on both test scores and group membership pays each group the same on average but pays high wages to star workers with above average test scores than to dagger workers with equal test scores and higher wages to dagger than star workers with equal but below average test scores.

How does social welfare compare in the nondiscriminatory equilibrium with the discriminatory equilibrium? By substituting (18) into (11) and comparing it with (12), we can see that total training is the same in both cases, but net social product is <u>higher in the restricted, nondiscriminatory equilibrium</u>. The improvement in social efficiency occurs because some high cost units of training have been shifted from star workers to dagger workers, for whom marginal training costs are lower. The ratio of nondiscriminatory to discriminatory costs is

$$(20) \qquad \frac{\left(\alpha\beta^{*} + (1-\alpha)\beta^{\dagger}\right)^2}{\alpha\beta^{*2} + (1-\alpha)\beta^{\dagger 2}} < 1$$

As an example, consider the case of maximum private discrimination, where $\beta^*=1$, $\beta^\dagger=0$, and the ratio of nondiscriminatory to discriminatory costs is α.

IV. Summary and Conclusions

The model constructed above provides a spare representation of labor market discrimination. It produces differentials in average wages and rates of return to observable training between groups without appealing to differences in innate ability, risk aversion, or testing bias. We have shown that a competitive equilibrium under certain types of imperfect information can be improved by enforcing equal wage schedules for different groups of workers.

Our specific model can only give specific results. What more general lessons ought we draw about social policy toward discrimination?

At a general level, the results of our paper are an example of the theory of the second best. In a first-best world, economic agents would use all available information. In a second-best world, there is no reason to assume that approaching the first-best - using more information - is welfare-improving. Since the problem of incomplete information is endemic in situations of discrimination, considerations of the second best are a general concomitant to policy questions in this area.

Specifically, our results arise because social marginal conditions diverge from private marginal conditions. Comparing the discriminatory to the nondiscriminatory equilibrium, we see that one group (the star workers) had its private incentives pushed closer to socially correct incentives while the other group had its private incentives pushed even farther from the socially desirable level. The loss to society from the divergence between private and social incentives varies directly with the distance between the social and private incentives. The gain to society from discrimination, which reduces the small loss for the advantaged workers, is smaller than the loss to society from discrimination, which increases the large loss to the disadvantaged workers. While this result is not invariant with respect to specifications of cost and production functions, neither is it a peculiar case or the result of some special "trick".

Our arguments have been intentionally and openly one-sided. We recognize the omission of social costs of anti-discrimination policy which might arise from production losses due to mismatches of workers and jobs and from the costs to the government of maintaining any sort of social policy which must work against the private incentive structure. However, we believe we have demonstrated the need to be cautious in assessing the allocational consequences of equal opportunity action type policies.

The appropriate social response to rational discrimination must be determined by analysis of specific problems. It is not an appropriate response, even for those of us who generally believe in the efficacy of private markets, to dismiss discrimination as something "the market will handle".

References

D. J. Aigner and G.G. Cain, "Statistical Theories of Discrimination in Labor Markets," Industrial and Labor Relations Review, Jan. 1977, 175-87.

K. J. Arrow, "The Theory of Discrimination," in Orley Ashenfelter and Albert Rees, eds., Discrimination in Labor Markets, Princeton 1973.

G. S. Becker, The Economics of Discrimination, Chicago 1957.

E. S. Phelps, "The Statistical Theory of Racism and Sexism," American Economic Review, Sept. 1972, 659-61.

F. Welch, "Affirmative Action and Its Enforcement," American Economic Review, May 1981, 127-33.