DO FORECAST ERRORS OR TERM PREMIA REALLY MAKE THE DIFFERENCE BETWEEN LONG AND SHORT RATES?

by

Richard Startz

Working Paper No. 8-81

RODNEY L. WHITE CENTER
FOR FINANCIAL RESEARCH
University of Pennsylvania
The Wharton School
Philadelphia, PA 19104

Do Forecast Errors or Term Premia Really Make the Difference Between Long and Short Rates?

Richard Startz

The Wharton School

University of Pennsylvania

July 1981

It is fairly well established that long-term interest rates contain time-varying term premia as well as forecasts of future short-term rates. Is the variation in term premium important? The natural way to pose such a question is from the vantage of an investor, speculator, or capital budgeteer trying to extract a prediction of future short rates from the term structure. Suppose one estimates the future short-term rate by the difference between current long and short rates, perhaps adjusting further by a constant term premium. The difference between this estimate and the short rate actually realized in the future contains two sources of error: the market's error in forecasting the future short rate and the premium embedded by the market in the long rate. In this paper we ask whether the variation in the premium is substantial as compared to variation in the market's forecast error. (The answer is yes.)

There are really two distinct methods for looking at forecasting errors and term premia. The first method is to explicitly model expectations formation and/or the components of the premium. The second method uses the statistical properties of "rational expectations". Procedures in this second class, the one below being a case in point, have both a strength and weakness in that structural relationships aren't identified. The disadvantage is self-evident. The advantage is that the validity of the empirical results isn't dependent on one particular theoretical model. The evidence below can't tell us whether variation in term premia is due to changes in risk preference, or to institutional changes, or to any of a number of other sources. The evidence does prove that this variation, taken from all sources, is substantial.

We can interpret the term structure as a set of spot rates and implied forward rates.[1] The spot rate realized next period, s, is the market's expectation formed today, s^e, plus a forecast error, ε. The implied forward rate, f, is the expected spot rate plus a premium that the market chooses today, P. Define the market's information set today, a set that obviously includes s^e and P, as Φ. We assume that the market forms s^e rationally, in the usual sense of s^e being a mathematical expectation. Restating all this in algebraic form:

(1) $s \equiv s^e + \varepsilon$

(2) $f \equiv s^e + P$

(3) $s^e = E(s|\Phi)$

Rational expectations implies that ε is uncorrelated with elements of the information set. The following second moments are immediate.

(4) $\text{var}(s) = \text{var}(s^e) + \text{var}(\varepsilon)$

(5) $\text{var}(s-f) = \text{var}(P) + \text{var}(\varepsilon)$

(6) $\text{cov}(s, f) = \text{var}(s^e) + \text{cov}(s^e, P)$

Sample statistics provide unbiased and consistent estimators of the true population statistics for (4), (5), and (6).[2] Shiller [3] observed that a test for the pure expectations hypothesis, that P is zero or at least

---

[1]Suppose we want to examine the forward rate on a τ period bond beginning $n-\tau$ periods in the future, based on the current n-period long rate. Under certainty we have $(1 + {}_tR_n)^n = (1 + {}_{t+n-\tau}R_t)^\tau (1 + {}_tR_{n-\tau})^{n-\tau}$ We define the realized spot rate as $s = \log(1 + {}_{t+n-\tau}R_\tau)^\tau$ and the forward rate as $f = \log(1 + {}_tR_n)^n - \log(1 + {}_tR_{n-\tau})^{n-\tau}$

[2]We need the minor assumption that the distributions involved possess second moments and the less minor assumption that we are able to repeatedly sample from a fixed distribution. We need not assume that the draws are independent.

constant through time, is to check whether var(s) > var(f), as it must be in the absence of a time-varying risk premium.[3] Fama [2] looked at the question of market efficiency by making use of the fact that Φ might be useful in predicting s^e and P, but not ε. Both Shiller and Fama find ample evidence that a premium is present in forward rates, though neither looks for a measure of the size of the premium.

Four underlying population statistics, $var(s^e)$, var(P), $var(\varepsilon)$, and $cov(s^e, P)$, generate the three sample statistics in (4) through (6). If we can only discover any one of the four, the other three are immediately identified. Rational expectations can not identify any of the basic time-series *per se*. However, a simple statistical filtering device identifies useful bounds on the second moment statistics.

If we regress s on any subset of Φ, all the explained variation is due to s^e and none to ε since ε is uncorrelated with Φ. The standard error of the regression is an *upper bound* on the standard deviation of ε.[4] The following estimators of the underlying population statistics follow immediately.

(7) $\hat{var}(\varepsilon)$ = standard error of the regression squared — *upper bound*

(8) $\hat{var}(s^e) = var(s) - \hat{var}(\varepsilon)$ — *lower bound*

(9) $\hat{var}(P) = var(s-f) - \hat{var}(\varepsilon)$ — *lower bound*

(10) $\hat{corr}(s^e, P) = [cov(s,f) - \hat{var}(s^e)]/\sqrt{\hat{var}(s^e) \times \hat{var}(P)}$ — *upper bound*

Looking at the forward rates contained in, say, a 12-month treasury bill, we might examine the projection for a one-month bill starting in 11 months, an 11-month bill starting next month, or combinations in between. The table

---

[3]Singleton [4] discusses hypothesis testing for these implied variance bounds.

[4]More precisely, the standard error is an unbiased and consistent estimator of the upper bound.

Table

τ+1 month ahead forecast of one month rate	percent of forward deviation due to premium	standard deviations at annual rates σ_p	σ_ε	σ_{s^e}	correlation coefficient s^e, P	number of observations
	lower bound		upper bound	lower bound	upper bound	
2	42.7	0.28	0.32	1.51	.31	223
3	32.0	0.31	0.45	1.48	.45	223
6	45.4	0.68	0.74	1.14	.28	150
9	54.2	0.92	0.84	0.67	.99	94
12	68.3	1.34	0.91	0.56	.91	94
1-month ahead forecast of τ+1 month rate						
2	42.7	0.28	0.32	1.51	.31	223
3	19.1	0.16	0.32	1.54	.48	223
6	10.0	0.10	0.30	1.36	.35	150
9	11.9	0.12	0.34	1.19	.88	94
12	11.3	0.12	0.35	1.16	.81	94

below presents results for long projections of one-month bills and one-month projections of long bills.[5] The first column gives the maturity of the bill, e.g., the first row reports results for the one-month forward projection on one-month rates. Column two reports variance of the premium as a percentage of the variance of the forward deviation s-f. The next three columns report bounds on standard deviations reported at simple annual interest rates, e.g., in row one the typical deviation of the premium is 28 basis points.

Is the premium important? The second column answers the question. If we use the forward rate to forecast one-month rates, one-third to two-thirds of our error is due to variations in the term premium, as a lower bound. For forecasting next month's long rate the result is less striking, though as a lower bound ten percent may be worth some thought.

The evidence tells us something of the value of economic analysis in the face of efficient financial markets. A planner interested in future short rates would be well advised not to take today's implied forward rate as an estimator. Even if we were to grant that economic analysis cannot reduce the forecast error, ε, the determinants of the premium, P, are potentially predictable. As the table shows, the variation in the premium is of the same

---

[5]The data is for U.S. treasury bills, which are pure discount notes, and was developed by Bildersee [1]. The sample period is the same as that used by Fama [2], monthly observations from 1/53 through 7/71 for the shortest bills. Longer bills have been generally available only more recently, the shorter periods reported are 2/59 through 7/71 and 10/63 through 7/71. (I reran the 2,3, and 6 month bill results over this last interval to ensure that reported differences are not due to use of different periods. The results are approximately the same as those reported. The percentage of forward deviation due to the premium is marginally higher in the shorter periods. Some of the correlation coefficients bounds are noticeably different.)

All the filtering regressions use as elements of Φ a constant, the forward rate, the lagged forward rate, the time t and time t-1 spot rates for the maturity of the bill. R^2's range from .96 to .31 for projections of one-month spot rates and from .96 to .92 for one month ahead projections.

order of magnitude as the variation in the forecast error. Concomitant with the large variation in the premium, we see that typical market forecasting errors are much smaller than had been previously thought. If we want to predict relatively long rates into the relatively near future, simple use of the term structure is fairly reasonable. For longer term predictions, attention to changes in the market premium is a must.

References

1. J. S. Bildersee, "Some New Bond Indexes," Journal of Business, October 1975, 48 pp. 506-525.

2. E. F. Fama, "Forward Rates as Predictors of Future Spot Rates," Journal of Financial Economics, October 1976, 3, pp. 361-367.

3. R. J. Shiller, "The Volatility of Long-Term Interest Rates and Expectations Models of the Term Structure," Journal of Political Economy, December 1979, 87, pp. 1190-1219.

4. K. J. Singleton, "Expectations Models of the Term Structure and Implied Variance Bounds", Journal of Political Economy, December 1980, 88, pp. 1159-1176.