COMMENT ON INFLATION AND THE STOCK MARKET

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In a recent article in this review, Martin Feldstein attributes a crucial share of the failure of share prices to rise during a decade of substantial inflation to basic features of the current U.S. tax laws, particularly historic cost depreciation and the taxation of nominal capital gains.[1] This comment will indicate that Feldstein's model rests on improperly specified asset demand functions. As a result, the implications of his model are different from those indicated by a more theoretically defensible model based on expected utility maximization. Perhaps most important we show that the implied impact of inflation upon equity share prices is so dependent on the values assumed for certain critical parameters, notably the effective capital gains tax rate, that even the direction to say nothing of the magnitude of the implied impact is unclear.[2]

The Feldstein Model

To elucidate the deficiencies in the basic model used by Feldstein, which he considers a market equilibrium model of share valuation in the absence of inflation, we shall for the sake of simplification initially assume there are no taxes. Thus, Feldstein would write the portfolio equilibrium of households in a two asset world as

$$(1) \quad \frac{\rho}{q} = r_o + \delta_{ho} \quad \text{(See his Eq. 7)}$$

where ρ can be considered the pretax earnings per unit of capital or per share of corporations as a whole (which are assumed to be unlevered and

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[1] Martin Feldstein, "Inflation and the Stock Market," American Economic Review, December, 1980.

[2] A detailed examination of the relationship between inflation and stock prices by the present authors appears in "Effect Of Inflation On The Profitability and Valuation of U.S. Corporations", Working Paper No. 13-80, Rodney L. White Center for Financial Research, Wharton School, University of Pennsylvania.

pay out all earnings as dividends), q is the share price, r_o is the rate of interest per dollar invested in government bonds, and δ_{ho} is the relevant risk premium. Since δ_{ho} is assumed by Feldstein to be proportional to the standard deviation of the dollar rate of return on the household's portfolio, he writes

$$(2) \qquad \delta_{ho} = \delta_h \, s_{ho} \, \sigma_\rho$$

where s_{ho} is the number of shares the household holds and δ_h is a constant. In the absence of market equilibrium, δ_{ho} need not be a constant over households though it is a constant for a specific household. Similarly, δ_h and s_{ho} may vary over households.

It should be stressed that the model implied by Eqs. (1) and (2) is based on ad hoc assumptions by Feldstein and not on expected utility optimization. We shall indicate below that the customary expected utility maximization leads to a substantially different model with quite different implications for tax effects. However, before introducing the more customary and more theoretically defensible model, we should like to point out that Feldstein's model seems questionable even on an ad hoc basis.

In the final equilibrium, assuming homogeneous expectations which are implicit in Feldstein's model, $\delta_{ho} = \delta_h s_{ho}$ as well as q must be the same for all households. We would take no issue with this implication if it were solely a consequence of a representative household aggregation convenience. Unfortunately, this implication results from more fundamental deficiencies in Feldstein's asset demand function. If δ_h representing a not clearly defined measure of risk aversion is the same for two households, so is s_{ho} which implies that regardless of differences in their wealth, the two households hold the same dollar amount of risky assets. Conversely, if the two households hold the same amount of risky assets, risk aversion as measured by δ_h would also be the same regardless of differences in the level of their wealth and holdings of risk-free assets. These implications of the method do not seem plausible. To avoid these difficulties, δ_h would have to be considered a function of the level of wealth and holdings of the risk-free asset, an interpretation which leads to other difficulties. Were δ_h to be considered a function of wealth, it would implicitly be an endogenous variable, since wealth is surely a function of share price q, variations in which constitute a prime focus of the model.

The Optimization Model

Turning next to a more acceptable model of the expected return on a risky asset based on expected utility optimization, it has been shown that a continuous time model plus the plausible assumption of constant proportional risk aversion at the micro level leads to a simple macro equilibrium relationship between the relative demand for risky asset (α) and the market price of risk $\left(\frac{E(r_m) - r_f}{\sigma_m^2}\right)$, i.e.,

$$(3) \quad E(r_m) - r_f = C\,\alpha\,\sigma_m^2,$$

if (like Feldstein) we abstract from human wealth (or other non-marketable assets).[1] $E(r_m)$ and σ_m^2 are the expected value and variance of the rate of return on the market portfolio of risky assets, r_f is the risk-free rate of return, C is the harmonic mean of individual investor's Pratt-Arrow measures of proportional risk aversion, and α is the ratio of the value of risky assets to the value of all marketable assets.[2] Thus, capital

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[1] Irwin Friend and Marshall Blume, "The Demand for Risky Assets", American Economic Review, December, 1975. Using Feldstein's symbols for $E(r_m)$ and r_f, Eq. 5 becomes $(\rho/q) = r_o + C\alpha \mathrm{Var}(\rho/q)$.

[2] To introduce supply considerations into this one-period model, we can write

$$\alpha = \frac{E(R_m)}{1+E(r_m)} \div \left\{ \frac{E(R_m)}{1+E(r_m)} + \frac{F}{(1+r_f)} \right\}$$

where $E(R_m)$ represents the expected dollar return over the period on the available supply of risky assets at the beginning of the period, and F represents the corresponding known dollar return on riskless assets. See Friend and Blume, op. cit.

asset pricing theory would imply that contrary to Eqs. (1) and (2) which assume that the risk differential in Eq. (1) is proportional to the standard deviation of return, it is proportional to the variance of return (either at the macro level or at the micro level substituting C_k and α_k for C and α). The significance of this difference will become apparent when the asset demand models are applied in situations with varying share price. Feldstein's ad hoc model leads to an expression for the risk premium in which the share price q does not appear: the risk premium is a function of σ_ρ, the standard deviation of the economic return to a unit of capital. This fortuitous elimination of q improperly simplifies the subsequent analysis. In contrast, the model presented here admits to no such simplification: the variance of the market rate of return depends inversely on the square of the share price q and in consequence the risk premium will be an endogenous function of q.[1] Thus even if the riskiness of economic return as measured by either the standard deviation or variance of economic return is held constant, Feldstein's model will yield misleading results unless q also is held constant, which is contrary to the aim of the analysis.

Now introducing corporate and personal income taxes, where corporate taxes are assumed to be proportional to before-corporate-tax income (with proportional tax credits associated with losses) and personal income taxes are assumed to be a function only of the level of before-personal-tax

[1]This point may be illustrated using Feldstein's framework as follows. Eq. (2) in his model is a consequence of taking the risk premium as a linear function of the product of market value of risky-asset holdings and the standard deviation of market return:

$$\delta_{ho} = \delta_h \cdot (s_{ho} \cdot q) \cdot (\sigma_\rho / q)$$

If the variance of market return is used here, eq. (2) will become $\delta_{ho} = \delta_h s_{ho} \sigma_\rho^2 / q$, which implies that the risk premium is dependent on q.

income and not its composition (again with proportional credits associated with losses), it is easily shown that Eq. 3 can be written as

$$(4) \qquad (1-\theta)(1-\tau)E(r_m) - (1-\theta)r_f = C\alpha(1-\theta)^2(1-\tau)^2\sigma_m^2$$

either at the micro or macro level, if at the macro level $(1-\theta)$ is interpreted as the weighted harmonic mean of the complement of individual household's personal tax rates and $(1-\tau)$ is the complement of corporate tax rates.[1] $E(r_m)$ and σ_m^2 now refer to before-corporate-income tax returns. Comparing our Eq. (4) and Feldstein's Eqs. (7) and (8), it is obvious our $C\alpha$ corresponds to his $\delta_h s_{ho}$ and our σ_m^2 replaces his σ_ρ.

Although Feldstein did not attempt to investigate the impact of changes in tax rates upon equity valuation, such an analysis may serve to indicate additional deficiencies in his model. Here, as above, the implications of the Feldstein model will be shown to differ significantly from those of the expected utility maximization model. Now Eq. (4) is an equilibrium demand relation. Introducing supply conditions we can write $\alpha = \frac{V_m}{V_m + V_f}$ so that the final equilibrium relation becomes

$$(5) \quad E(R_m) = \frac{r_f}{(1-\tau)} V_m + (1-\theta)(1-\tau)\ \sigma^2(R_m)\ \frac{C}{V_m + V_f}$$

where V_m is the market value of risky assets (and of the real capital stock), V_f is the market value of the risk-free assets (one-period government bonds), and R_m is the stochastic real before-corporate-tax dollar return on

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[1] See Friend and Blume, P. 905, and M. Blume, J. Crockett and I. Friend, Financial Effects of Capital Tax Reforms, Monograph Series in Finance and Economics, Monograph, 1978-9, New York University, p. 84. Though not relevant to the discussion here, the right hand side of Eq. (11) should have a negative sign in front of it.

the risky assets (and stochastic output of the capital stock). The assumption of fixed supply of physical capital is made by Feldstein in his inflation-impact analysis. The value of the risk-free asset plays no role in his model, and for convenience, we shall assume that it too is fixed. From Eq. (5) it is easy to derive[1]

$$(6) \qquad \frac{dV_m}{d\tau} = \frac{-\left[\frac{r_f}{(1-\tau)} - (1-\tau)(1-\theta)\sigma_m^2 C\alpha\right]\left[\frac{\alpha(V_m+V_f)}{(1-\tau)}\right]}{\frac{r_f}{(1-\tau)} - (1-\tau)(1-\theta)\sigma_m^2 C\alpha^2}$$

and using Feldstein's symbols $\frac{\rho}{q} = r_m = \frac{E(R_m)}{V_m}$

$$(7) \qquad \frac{dq}{d\tau} = \frac{\rho}{E(R_m)}\,\frac{dV_m}{d\tau}$$

The sign of $\frac{dq}{d\tau}$ will be positive only if

$$(1-\tau)^2(1-\theta)C\alpha^2\sigma_m^2 < r_f < (1-\tau)^2(1-\theta)C\alpha\sigma_m^2$$

Assuming C=2, $\alpha = .9$, $\tau = .4$, $\theta = .3$ and $\sigma_m^2 = .047$,[2] this condition becomes

$$.019 < r_f < .021$$

If, as Feldstein suggests, $r_f = .03$, this condition is violated and thus $\frac{dq}{d\tau}$ is negative, so that stock prices would decrease when the corporate tax is increased and the before-corporate-tax cost of capital $\frac{\rho}{q}$ increases. However,

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[1] This is equivalent to Eq. (6) in Blume, Crockett and Friend, _op. cit._, p. 85.

[2] See Friend and Blume (1975) for values of parameters. The value chosen for σ_m^2 is a post-war market return variance estimate divided by $.36 = (1-\tau)^2$. Higher variance estimates were obtained in the pre-war period.

the rate of return required by investors after corporate taxes $\frac{\rho}{q}(1-\tau)$ is decreased.

Similarly, it can be shown that if r_f is unaffected by a rise in the personal income tax rate, which Feldstein seems to assume, the effect of a change in personal rather than corporate income taxes is obtained from

$$(8) \qquad \frac{dV_m}{d\theta} = \frac{(1-\tau)\sigma_m^2 \alpha V_m}{[\frac{r_f}{1-\tau} - (1-\tau)(1-\theta)\sigma_m^2 C\alpha^2]}$$

and

$$(9) \qquad \frac{dq}{d\theta} = \frac{\rho}{E(R_m)} \frac{dV_m}{d\theta}$$

which are the personal tax counterparts of equations (6) and (7). In this case the numerator of the expression in equation (8) is always positive and the condition for the denominator to be positive is

$$r_f > (1-\tau)^2 (1-\theta) C\alpha^2 \sigma_m^2$$

Using the parameter values mentioned above, this implies $r_f > .019$, a condition met if Feldstein's attribution of .03 to the risk-free rate is correct. From (9), this would further imply $\frac{dq}{d\theta} > 0$.

Thus, unlike the results implied by Feldstein's model, expected utility maximization implies that with the U.S. Government giving credits for losses, higher personal income tax rates would be associated with a rise in stock prices and a reduction in the before-corporate-tax cost of capital. Clearly these results depend on the assumed symmetry of tax effects, but to obtain any alternative theoretically defensible model would seem to require the application of expected utility maximization to an alternative specification of tax effects.

The correspondence between our Eq. (4) and Feldstein's Eqs. (7) and (8) may be made more explicit by setting $E(r_m) = \rho/q$. If in addition our model is expanded to permit a constant inflation at rate π, (4) becomes

$$\text{(10)} \qquad \frac{(1-\theta)[(1-\tau)\rho - \lambda\pi]}{q} - c\pi - [r_o(1-\theta) - \theta\pi] = C\alpha(1-\theta)^2(1-\tau)^2\sigma_m^2$$

where in addition to the parameters already defined, λ is the underdepreciation tax penalty and c is the effective rate of capital gains taxation.[1] Now since σ_m^2 is a market return variance (divided by $(1-\tau)^2$) and α is a market value ratio, both of these parameters will depend on q. However, if σ_m^2 and α may be assumed constant for the sake of simplification, (10) may be rearranged to isolate q.

$$\text{(11)} \qquad q = \frac{(1-\theta)[(1-\tau)\rho - \lambda\pi]}{(1-\theta)r_o - (\theta-c)\pi + C\alpha(1-\theta)^2(1-\tau)^2\sigma_m^2}$$

This equation is analogous to Feldstein's (20).

Comparison between this analysis and Feldstein's may be facilitated by restricting attention to households. In the absence of inflation, Feldstein assumes capital markets to be in equilibrium, implying that q, which represents the market price of a dollar's worth of capital at replacement prices, is equal to unity. Using Feldstein's estimates for the values of all parameters, his equation (20) implies that an 8% inflation

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[1] Equation (12) summarizes capital market equilibrium when the inflation is perfectly anticipated. Corresponding analyses in which inflation is uncertain are described in Friend, Landskroner and Losq, "The Demand for Risky Asset Under Uncertain Inflation," Journal of Finance, December, 1976; and Friend and Hasbrouck, op. cit.

will result in q = .86, a 14% drop in equity values.[1] By comparison, equation (11) suggests q = 1.29 in the absence of inflation and q = 1.23 with an 8% inflation, a much smaller decline. Our computations assumed Feldstein's values for his parameters and also assumed as before C = 2, $\alpha = .9$ and $\sigma_m^2 = .047$.

The Effect of Capital Gains Taxation

Although this difference in results reflects mainly the differences in the underlying theoretical approaches, it must be noted that both models are extremely sensitive to parameter choice. Among Feldstein's assumptions in this regard, the one which we believe most dubious is his estimate of .15 for c, the effective capital gains taxation rate. Although a precise figure is difficult to obtain, some insight into capital gains taxation may be obtained from special IRS studies conducted for the 1959 and 1962 tax years. In these years, net long-term capital gains reported to the IRS by individuals on their corporate stock holdings were $5 billion and $3.6 billion, respectively. By contrast, the average annual capital gains on stock held by households over the ten year periods preceding 1959 and 1962 were $33 billion and $32 billion respectively.[2] Assuming that the actual capital gains taxes paid in each of these years were 20% of the reported gains, the effective capital gains tax rates may be computed as .03 (=.2x5/33) for 1959 and .02 for 1962.

Several considerations may qualify the validity of these estimates. First, the reported capital gains figures used are solely long-term values; short-term capital gains are omitted. For 1962 the net short-term capital

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1 This is fairly close to the q=.812 obtained by Feldstein when an institutional sector is included in his model.

2 These estimates are based on Federal Reserve Board Flow-of-Funds data.

gains reported on the sale of corporate stock were $.5 billion, considerably less than the $3.6 billion of long-term gains. No short-term data are available for 1959. On the basis of the relative magnitude of the 1962 figure, however, and since investors would logically tend to time transactions so as to qualify for the preferential long-term rates, we believe the omission of short-term gains in the above computations to be of negligible importance.

A more puzzling aspect of the data concerns the small value of reported capital gains relative to the capital gains that apparently occurred. This may reflect the carrying over of these gains into estates, individual tax evasion or sampling error. If this discrepancy reflects capital gains which have merely been postponed in realization, the above computations will understate the true rate.

A final consideration involves the timing of the taxation. The $5 billion reported in 1959, for example, reflects capital gains which may have accrued in previous years. Taxes paid on these gains should properly be discounted back to the year in which the gains occurred. This line of reasoning would suggest that the computations overstate the effective tax rate. Since the tax liability is a nominal obligation, it should properly be discounted at a nominal interest rate. Nominal interest rates were relatively low during the period surrounding the two sample years, and in consequence, the degree of overstatement may be moderate. In recent years, however, with much higher nominal rates, the time dimension of the problem may be much more important.

The implications of a lower effective capital gains taxation rate in the Feldstein analysis may be seen by redoing the calculations taking $c = .05$, a figure somewhat higher than suggested by the above analysis. Under this assumption, with all other parameter values unchanged, the implied value of q with an 8% inflation is 1.12, a 12% increase. In other words, using Feldstein's model for households alone, the implied value of q associated with an 8% inflation rate varies from -14% to +12% depending on the tax rate assumptions.[1] We acknowledge that, despite its superiority on other counts, the optimization model is subject to similar problems of parameter sensitivity.

Almost as disconcerting as the extreme sensitivity of Feldstein's analysis to the capital gains tax assumption is the lack of dependence on the value chosen for the risk-free rate suggested by the implementation of his model. If q is constrained to unity in the absence of inflation, it may be shown by combining his Eqs. (14) and (20) that with inflation,

$$(12) \qquad q = \frac{(1-\theta)\ \{(1-\tau)\rho - \lambda\dot{\pi}\}}{-(\theta-c)\pi + (1-\tau)(1-\theta)\rho}$$

in which r_0 no longer appears. In contrast, our Eq. (11) which follows from expected utility maximization exhibits a negative dependence of q on r_0, and in numerical simulations this dependence is quite significant.[2] This is of considerable importance as Feldstein's attribution of .03 to

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[1] It has been brought to our attention that in later work, Feldstein accepts a .05 capital gains tax rate, and using a model extended beyond the one presently under discussion, claims that the basic result of an inflation-induced decline in equity values remains (Journal of Monetary Economics, July, 1980). We do not intend to comment on the entire corpus of his work in the area, but it is important to note that there exist tenable parameter assumptions under which his models may yield almost any result. We hold, however, that the extreme sensitivity of Feldstein's analysis vitiates whatever claim to validity such an *ad hoc* model may possess.

[2] See Friend and Hasbrouck, *op. cit.*

the real risk-free rate seems excessive.[1]

One last point should be made about the strong adverse effect on stock prices that Feldstein attributes to the joint effect of the capital gains tax and inflation. It has already been pointed out that his attribution of .15 to the present value of the capital gains tax rate that will be paid in the future upon sale of the stock seems greatly exaggerated and may be closer to .05. However, the problem of this effect of capital gains taxation on stock prices associated with changes in the steady-state rates of inflation is more complex than indicated in either the Feldstein or the simple expected utility maximization model presented above. Because of the way capital gains and losses are realized, dollar for dollar the tax laws provide a greater degree of insurance against unanticipated capital losses than penalty for unanticipated capital gains.

[1] A real risk-free rate in the neighborhood of .01 is suggested by the empirical analysis carried out by Fama, "Short-Term Interest Rates as Predictors of Inflation," <u>American Economic Review</u>, June, 1975.