THE ENDOGENEITY OF MONEY DURING THE GERMAN HYPERINFLATION: A REAPPRAISAL

by

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Introduction

One of the main points that emerges from the early research on the German hyperinflation, notably in Frank Graham (1930) and Bresciani-Turroni (1937), is that the government depended on the creation of money for most of its revenue. Recent studies of the German hyperinflation by Sargent and Wallace (1973), Sargent (1975), and Frenkel (1976) have reported statistical evidence supporting the hypothesis that the supply of money was endogenous during this episode. The postulated chain of events is that the higher the nominal price of goods and services the government demands, the more money the government needs to create, which in turn leads to further increases in prices. Accordingly, it seems reasonable to postulate some sort of stable government reaction function to prices making the money supply endogenous.

The primary evidence for the endogeneity of money given by Frenkel and by Sargent and Wallace is that the price series appears to lead the money series. Frenkel uses cross-correlation estimates while Sargent and Wallace estimate two-sided regressions to arrive at this conclusion.

Their results are summarized in Figure 1 and 2 below. From his cross-correlation estimates Frenkel (1976) concludes, "Thus prices cause (or are exogenous to) money in the Granger sense." He finds that causality runs from prices to money when he uses the cost of living index and finds two-way feedback between money and the wholesale price index. Sargent and Wallace (1973) on the other hand reject the hypothesis that there is no feedback from current inflation to future rates of money creation at the 99 percent level and also reject the hypothesis that there is no feedback from the rate of money creation to subsequent rates of inflation at the 95 percent level. They conclude that the results imply that inflation strongly influences subsequent rates of money creation but that the influence of money creation on subsequent rates of inflation is harder to detect.

Using a semilog demand for money function where real income and the real rate of interest are assumed exogenous, the rational expectations solution for the price level given by Sargent and Wallace is

$$(1) \qquad p_t = \tilde{\varepsilon}_t + A + \sum_{j=0}^{\infty}(1-\theta)^j \theta m^e_{t+j}(t),$$

where p_t is the logarithm of the price level and $m^e_{t+j}(t)$ is the forecast of the logarithm of the money supply (m_{t+j}) j periods hence, made at time t. A is a constant, $\tilde{\varepsilon}_t$ is assumed to be a normally distributed error term, and $\theta = \frac{1}{1+\alpha}$ where α is the interest rate semielasticity of the demand for money.

The empirical finding that the money supply leads prices does not necessarily imply that prices cause money in the Granger sense, since the current price should depend on the expected future money supplies. The direction of causality has to be decided based on whether the unanticipated component of the money supply leads prices. The techniques proposed by Granger (1969) and Sims (1972) attempt to distinguish anticipated from unanticipated changes in the variables involved. What enables Sargent and Wallace and Frenkel to use the Granger and Sims tests is the assumption that the money and price series of the German hyperinflation are generated by linear autoregressive or moving average (ARIMA) processes. Neither of the papers investigated the empirical validity of this assumption.

This paper marshals evidence that the linearity assumption is not valid and presents revised exogeneity tests that do not assume linearity. The results no longer favor the hypothesis that prices are exogenous. An economic explanation of this finding is then provided.

Univariate Time Series Models and Prediction[1]

Figure 3 summarizes part of the sample autocorrelation structure of the price and money series of the German hyperinflation as successive observations are deleted starting with the last date of the sample (August 1923). The impact of removing even one observation is sufficiently large to cast doubt on the assumption that either of these series could have been generated by a single linear ARIMA model.

One way to deal with the problem is to construct some nonlinear process that could generate series such as the ones observed. The

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[1]The money supply data is taken from Rogers (1934). The data differ from those reported in "Zahlen Zur Geldentwertung" (1925) only for 1923 when Rogers includes authorized issues of money by non-banks. The data are identical for the rest of the period. The Cost of Living index and Government bonds are reported in "Zahlen Zur Geldentwertung" (1925). These are the same data as those used in all the papers dealing with the hyperinflation. Government expenditure data were collected from "Germany's Economy, Currency and Finance" (1924).

possibilities are numerous, however, and the properties of estimators of such nonlinear processes are not well known. Furthermore, since the economic intuition associated with a complex nonlinear function that appears to trace the sample path is unclear, there is no theoretical guidance as to what type of nonlinear form to choose.

It is generally true that for sufficiently small distances away from a particular point and with sufficient smoothness conditions, a nonlinear system can be approximated by a linear one. Invoking that analogy, assume that economic actors will form expectations based on past values of the variable as if, for short forecasting horizons, a linear rule approximates the nonlinear underlying process sufficiently well. In practice, this amounts to reevaluating the linear ARIMA process each period and using the updated version to forecast the future path. As one tries to forecast further into the future, the efficiency of the forecasts will decline, since the linearity assumption will be increasingly less appropriate.[2]

The procedure of continuously updating the linear approximation model is carried out for the cost of living index (COL) and the money supply (MON). Figure 4 contains the set of ARIMA models that seem to describe the sample autocorrelation structure most adequately. There is no single formalized and generally accepted method of estimating ARIMA models. It is possible that different researchers confronted

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[2]A problem not dealt with here, is how many observations one should include in re-evaluating the ARIMA process. If the process is highly nonlinear, then the best linear approximation may involve using only recent observations. I have chosen to include all past data.

with the same time series will end up with different models, at least in appearance. It is much less likely, however, that these models will yield significantly different predictions.

The method used for the estimation is as follows:

(1) Starting with December 1920 (2012) the sample autocorrelation structure for the different time periods is computed and examined. In all cases all available data up to the end of the interval are used in computing both the sample autocorrelation and the coefficient estimates.

(2) Given the autocorrelation structure, a tentative model is postulated and estimated. The characteristics of the residuals provide clues for further refinements of, additions to, and deletions from the original model. The Box-Pierce statistic, the autocorrelation structure of the error terms, and the skewness and kurtosis statistics of the residuals are used to decide on the appropriate model.

(3) ARIMA models that contain the maintained model are estimated in each case to see whether more coefficients should be added to the maintained model.

(4) The out-of-sample predictive ability is <u>not</u> taken into account in selecting the appropriate model for each time period.

Some observations about the models in Figure 4 are in order. In all cases, the form of the models seems somewhat more stable than the coefficient estimates. In the money supply series for instance, the form of the autoregressive part of the model remains the same from 46 to 51 observations, although the parameter estimates seem to vary widely and not in any systematic fashion. The models identified for 48 observations and 51 observations are almost identical, while the intervening ones are not. Somewhat surprisingly, 12-month seasonal parameters for the money supply series persist through all the estimated models. For the early part of the data, it is necessary to allow for both autoregressive and moving average seasonal parameters. No other combination of MA or AR parameters seems to vitiate the need for both seasonals.

The residual autocorrelation statistics are uniformly low and the t-statistics of the retained coefficients are high; all the coefficients retained in these models significantly contribute towards their explanatory power. The skewness and kurtosis statistics, on the other hand, suggest that there may be departures from the normality assumption for the residuals. It is typically the case that the last one or two residuals appear out of line with the rest of the sample, suggesting again that no one linear model can describe these time series.

Figure 5 compares the explanatory power of forecasts obtained in this manner with the simple assumption that the underlying time series processes are linear. The comparison is made for three different time periods. For each time period, row (a) is a regression of the dependent variable--the money supply (MON) in the first panel, the cost of living index (COL) in the second--on past and present values of the corresponding independent variable. Row (b) includes the forecasts of the independent variable (obtained through the above procedure) on the right-hand side. A significant F-value indicates that the forecasts of the independent variable contribute significantly to the explanatory power of the regression and implies that the linear approximation method captures significantly more information from the past history of the series than the linear hypothesis. In the case of COL on MON, the F-tests indicate that for the 2001-2303 and 2001-2307 periods the null hypothesis that the nonlinear model does not capture additional information must be rejected at the 1-percent level of significance; the hypothesis is not rejected for the 2001-2208 period. In the case of MON on COL, the results are similar; the hypothesis that the nonlinear model does not capture additional information for the 2001-2307 period is rejected at the 1-percent level of significance. The hypothesis is not rejected for the 2001-2208 and the 2001-2303 period.

These results show that restricting the time paths of money and prices to linear ARIMA processes is not in accordance with the data; the local linear approximation method captures significantly more information from the past history of each variable.

Two-Sided Regressions and Causality

The way in which Sims (1972), Sargent and Wallace (1973), and others have implemented Granger's definition of causal ordering is to regress Y on past and current values of X and also on past, current, and future values of X. If the second regression explains Y significantly better than the first, the conclusion is that the causal chain runs from Y to X. The test must be repeated with X as the left-hand variable to determine whether the causal chain runs from X to Y as well.

The two-sided regression tests reported in Sargent and Wallace and reproduced in Figure 2 have been duplicated with the present data and statistical packages used in this paper.[3] The results, reported in Figure 6, broadly confirm the Sargent and Wallace conclusions. The case for money creation's influencing prices is weaker than the case for prices' influencing money creation for the whole period. The hypothesis that past money does not influence current prices is just rejected at the 1-percent level of significance while the hypothesis that past prices do not influence current money is easily rejected at the 1-percent level of significance. For the 2001-2208 period the hypothesis that past money does not influence current prices is not rejected at the 5-percent level of significance, while the hypothesis that past prices do not influence current money is rejected at the 5-percent level of significance but not rejected at the 1 percent level. These tests seem to uphold the hypothesis that the causal ordering runs from prices to money for the sample period 1902-2307. For the 1902-2206 period, however, the same hypothesis fails two out of three times at the 95-percent confidence level.

The Sargent and Wallace tests assume that the data are generated by linear time series models. The evidence in section 2 suggests that this is not an appropriate assumption, casting doubt on the validity of the tests. The Sims test, however, can be modified to preserve its intent and to incorporate the underlying nonlinearity of the time series processes. In this test, the past and current values

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[3]Econometric Software Package, by Prof. P. Nelson.

of X in the regression of Y on X summarize the best forecast of the future values of X from its own past history. Since the underlying time series process is not linear, the expected values of X (X^e_{t+j}), calculated by the linear approximation method, should replace the lagged values of X in the regressions. The appropriate test of the direction of the causal chain is whether future values of X significantly increase the explanatory power of the regression.

The results of these tests are reported in Figure 7.[4] For the 2001-2208 period, the hypothesis that past prices do not affect current money cannot be rejected at the 5-percent level while the hypothesis that past money does not affect current prices is rejected at the 1-percent level. For the 2001-2307 period, the hypothesis that past prices do not affect current money is just rejected at the 1-percent level when the regression is run in levels, but is not rejected at the 1-percent level when the regression is run in first differences. For the same period, the hypothesis that past money does not affect current prices is easily rejected at the 1 percent level of significance. Furthermore, a regression of COL on expected money (excluding the current money stock term) when compared with the 2001-2307 regression in row (b) yields an F-statistic of 3.20, which leads to accepting the hypothesis that past prices do not influence current money at the 1-percent level of significance.[5]

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[4]The D. W. coefficients reported for these regressions are low. Allowing second order autocorrelation reduces the value of the F-statistic.

[5]Critical value at 1 percent is 3.83.

The results differ very substantially from the ones in Figure 6. Although there is some evidence that prices may be influencing money, the case for money's influencing prices is much stronger. Such a comparison suggests that a model that treats the money supply as an econometrically exogenous variable is not inconsistent with the data. This finding may be somewhat surprising, given the unanimity of earlier work. But more careful examination of the institutional setting and the actions of the central bank as undertaken in the next section, provides an explanation of this finding.

Government Revenue and Credit Creation

In previous hyperinflation studies, it has been customary to equate the government revenue from inflation with the central bank seignorage from money creation. As can be seen from Figure 8, during the entire hyperinflation the government continued to issue debt to cover its budget deficits. The central bank monetized some of the debt by purchasing it directly from the Treasury or in the open market.[6] This has the potential of becoming a superfluous step so that all the debt would be purchased by the Central bank as it is issued. The data in Figure 9 reveal that the correlation between the real government revenue from debt issues and the central bank seignorage ranges between .63 and .24 on a monthly basis, depending on the deflator used. In 20 out of 31 months over the whole sample

[6]The government was not prohibited from selling its debt directly to the central bank.

and 9 out of 19 months since 1922, the government revenue exceeds bank seignorage. From February to August 1922, the nominal debt held by the private sector declines, implying that the bank was issuing money in excess of new government debt. After that period, the nominal value of government debt held privately increases without interruption. The real value of privately held debt declines continuously from April 1921 to March 1923 with one interruption. Surprisingly, it increases between March and July of 1923 in spite of the extremely high inflation rates. It is interesting to note that the autocorrelation at lag 1 of all the revenue series is low. The average autocorrelation for government revenue is 0.22 and for bank seignorage 0.44. Furthermore, if one assumes that the government or the bank tried to extract a constant revenue each month, as postulated by Sargent and Wallace (1973), the mean revenue is not statistically distinguishable from zero in most of the measures. The value of privately held debt follows closely the value of total debt. In some instances bank seignorage exceeds total revenue from debt even though private holdings of nominal debt rise. The reason is that the central bank regularly bought private debt instruments and issued currency against them. This was one way in which cheap credit was rationed to corporations.[7]

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[7]See Bresciani-Turroni (1953). The following quotations are from Graham (1930), p. 62, and Bresciani-Turroni (1953), p. 78, respectively.

> But Reichsbank note issues were not dependent on Treasury borrowing only. . . . The use of commercial bills had almost entirely ceased during the war. . . . the resulting scarcity of credit, which grew marked as inflation advanced, impelled the Reichsbank in the summer of 1922, to urge the readoption of the commercial bill as a credit instrument. A low discount rate was applied and, under the existing condition of continuous currency depreciation, the rediscounting of commercial bills became an obvious recourse for the private banks. The Reichsbank's portfolio of commercial bills consequently grew rapidly in spite of a gradual diminution in the bank's enthusiasm in the matter and of subsequent large but always inadequate increases in the rate.
>
> . . . .
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> . . . . . only towards the middle of August 1923 did the president of the Reichsbank announce the adoption of the principle of loans at a "constant value".

The picture that emerges from these calculations is that there is considerable month-to-month deviation between the actions of the central bank and the needs of the government. Undoubtedly, the bank tried to limit interest rate rises and maintain market conditions favorable to the sale of the debt issues. But it was subjected to many other pressures and demands for cheap credit which may have caused continuing and unpredictable changes in its behavior pattern. It is not then surprising that feedback from prices to money is unpredictable and hard to detect. Taking the money supply as the econometrically exogenous variable does no more violence to the data than the opposite assumption.

Conclusion

The first section contains evidence that the covariance structure of the price and money supply series appears to be unstable. This could be explained by any number of reasons including the possibility that the series were generated by a stable but nonlinear process. There is no generally accepted methodology for identifying or estimating such processes.

Using the analogy of local linearization of nonlinear behavior, a procedure of a continuously updated linear approximation is proposed. It is shown that this procedure captures significantly more information than the best linear model. It is shown further that the conclusions about the direction of causality from the Sims test change considerably when the new procedure is used. In particular, assuming that the money supply is econometrically exogenous to prices is not inconsistent with the data.

The last section presents additional evidence that supports this finding. It is shown that the central bank appears to have operated independently from the revenue needs of the government in the short run. This provides an economic explanation of the apparent exogeneity of money during the German hyperinflation.

FIGURE 1

FRENKEL RESULTS ON EXOGENEITY

CROSS-CORRELATION OF PRICE(T) AND MONEY(T + K)

		Lags									
Variable		-4	-3	-2	-1	0	+1	+2	+3	+4	s.e.
COL	-.13	-.28	.27	-.04	.49*	.43*	-.34	.03	-.12	.18	
WPI	-.27	-.17	.07	.40*	.13	.50*	-.02	-.24	.18	.18	

*Statistically significant cross-correlations.
PRICE is the Cost of Living Index (COL) and the Wholesale Price Index (WPI) respectively. Both series have been pre-whitened by the same filter.

FIGURE 2

SARGENT AND WALLACE RESULTS*

PRICE	-4	-3	-2	-1	0	+1	+2	+3	+4	R^2	F
INFLATION REGRESSED ON MONEY CREATION LAGS OF DMON (T + K)											
oefficients (T-Stats)	2.05 (4.71)	-.25 (.840)	.54 (1.90)	-2.21 (8.41)	1.26 (4.89)	1.53 (6.14)	-.39 (1.64)	-.46 (2.83)	.66 (7.03)	.977	38.47+
MONEY CREATION REGRESSED ON INFLATION LAGS OF DPRICE (T + K)											
oefficients (T-Stats)	.25 (4.16)	.11 (1.89)	.22 (4.08)	.21 (4.05)	.19 (5.58)	-.09 (2.93)	.04 (1.67)	-.04 (1.81)	.04 (1.75)	.879	4.77+

+Significant at 1-percent level.
*Sargent and Wallace use Cagan's data from 1921/04 to 1923/06. The regressions are all in first differences, denoted as DPRICE and DMON.

FIGURE 3

EFFECT OF DELETING OBSERVATIONS FROM THE END OF THE PERIOD:
COST OF LIVING INDEX

		Lagged Autocorrelations								
Date	OBS	1	2	3	4	5	6	Q(12)	Q(24)	Std. Dev.
2307	55	.07	-.05	-.03	-.33	.15	-.07	11.7	14.5	.14
2306	54	-.18	-.23	-.01	.00	.01	-.14	8.39	12.1	.14
2305	53	-.29	-.25	.30	-.13	-.01	-.05	16.6	21.3	.14
2304	52	-.30	-.10	.24	-.15	.05	-.09	12.7	17.0	.14
2303	51	-.33	-.08	.24	-.16	.06	-.10	13.9	18.4	.14
2302	50	-.16	-.11	.06	-.05	.05	.06	6.5	10.4	.14
2301	49	-.28	.18	-.13	-.02	-.11	-.01	10.3	14.3	.15
2212	48	-.24	.15	-.12	-.05	-.13	-.04	8.6	12.4	.15
2211	47	-.09	.14	.04	.05	.04	-.09	4.2	7.6	.15
2210	46	-.06	.00	-.04	-.11	.07	-.03	1.9	6.0	.15
2209	45	-.01	.03	.00	-.11	.05	-.03	1.6	5.6	.15
2208	44	-.11	-.14	.02	-.06	.06	-.01	5.8	12.3	.15

FIGURE 3 (Continued)

EFFECT OF DELETING OBSERVATIONS FROM THE END OF THE PERIOD: MONEY SUPPLY

		Lagged Autocorrelations[a]								
Date	OBS[b]	1	2	3	4	5	6	Q(12)[c]	Q(24)[c]	Std. Dev.[d]
2307	55	.23	-.28	-.10	-.02	-.13	.05	13.3	20.2	.14
2306	54	.03	-.36	.04	.04	-.21	.10	14.1	24.3	.14
2305	53	-.11	-.12	.32	-.15	-.20	.10	20.4	40.0	.14
2304	52	.00	-.07	.28	-.11	-.23	.07	19.0	37.5	.14
2303	51	-.27	.12	.26	-.08	-.20	.30	31.6	56.2	.14
2302	50	-.18	.04	.35	.00	-.13	.40	37.1	67.8	.14
2301	49	-.11	.00	.31	-.11	-.20	.35	37.8	67.1	.15
2212	48	-.07	.07	.42	-.07	-.13	.38	37.7	64.5	.15
2211	47	-.11	.02	.40	-.12	-.13	.38	36.2	62.2	.15
2210	46	-.20	-.02	.35	-.14	-.16	.40	35.9	63.2	.15
2209	45	-.29	-.14	.38	-.19	-.18	.47	56.8	99.6	.15
2208	44	-.36	-.15	.37	-.20	-.18	.46	57.5	101.0	.15

NOTES TO FIGURE 3

[a]The entries in each row are the lagged correlation coefficients of the second differences of the series starting with the observation of January 1919 (1901) and ending at the date indicated for the row.

[b]Number of observations in the sample.

[c]Box-Pierce statistic for 12 and 24 lags.

[d]Standard derivation of the correlation coefficients $\simeq 1/\sqrt{OBS}$

FIGURE 4

SEQUENTIAL ARIMA MODELS FOR THE COST OF LIVING INDEX

TIME[c] PERIOD	MODEL[a,d]	Residual Statistics[b] Q(12)	P(12)	Q(24)	P(24)	Skewness	Kurtosis
1901-2012 24 OBS	$(1-B)(1-\underset{(4.6)}{.6220}B)\,X_t = \underset{(1.8)}{.018} + \tilde{u}_t$	6.6	.88	16.1	.89	.37	6.6
2101-2112 36 OBS	$(1-B)(1-B^{12})(1-\underset{(6.2)}{.6757}B)\,X_t = (1-\underset{(5.2)}{.6217}B^{12})(1-\underset{(6.5)}{.6623}B^{10})\,\tilde{u}_t$	10.9	.21	19.5	.49	-.30	5.2
2201-2206 42 OBS	$(1-B)(1-\underset{(10.5)}{.8694}B)\,X_t = (1+\underset{(10.4)}{.8460}B^{12})(1-\underset{(13.2)}{.9002}B^{12})\,\tilde{u}_t$	5.1	.82	13.2	.90	.64	3.3
2207-2209 44 OBS	$(1-B)^2(1-\underset{(30.7)}{.9788}B^{12})\,X_t = (1-\underset{(9.8)}{.7123}B^{12})\,\tilde{u}_t$	7.4	.69	20.2	.57	.52	4.5
2209 45 OBS	$(1-B)^2(1+\underset{(3.7)}{.5407}B^{12})\,X_t = \underset{(1.7)}{.0104} + (1-\underset{(7.5)}{.7391}B^{12})(1-\underset{(2.9)}{.4235}B^{7})\,\tilde{u}_t$	5.9	.75	15.8	.78	.55	3.8
2210-2212 48 OBS	$(1-B)^2(1-\underset{(1.9)}{.3063}B^{12})\,X_t = (1+\underset{(10.9)}{.8498}B^{12})(1+\underset{(2.0)}{.2933}B^2-\underset{(1.9)}{.2166}B^3)\,\tilde{u}_t$	4.9	.76	8.7	.99	.21	3.1
2301-2303 51 OBS	$(1-B)^2\,X_t = (1-\underset{(4.2)}{.5670}B)(1+\underset{(5.2)}{.7419}B^{12})\,\tilde{u}_t$	7.1	.71	10.7	1.00	-1.9	14.3
2304-2305 53 OBS	$(1-B)^2(1+\underset{(2.5)}{.3502}B+\underset{(3.4)}{.5008}B^2)\,X_t = \underset{(2.3)}{.0157} + (1+\underset{(7.9)}{.8477}B^{12})(1-\underset{(16.3)}{.9043}B^{4})\,\tilde{u}_t$	4.9	.76	10.8	.95	.19	5.6
2306-2307 55 OBS	$(1-B)^2(1-\underset{(1.6)}{.2173}B+\underset{(2.0)}{.3554}B^2)\,X_t = \underset{(1.5)}{.0206} + (1+\underset{(4.3)}{.8055}B^{12})(1-\underset{(5.2)}{.7132}B^{4})\,\tilde{u}_t$	4.4	.82	6.6	1.00	-.25	8.1
	Simple Avg. Values	6.4	.71	13.5	.84	.19	6.1

FIGURE 4 (Continued)

SEQUENTIAL ARIMA MODELS FOR THE MONEY SUPPLY

TIME[c] PERIOD	MODEL [a,d]	Residuals[b] Q(12)	P(12)	Q(24)	P(24)	Skewness	Kurtosis
2002-2112 36 OBS	$(1-B)(1-B^{12})(1-\underset{(6.1)}{.7633}B)X_t=(1-\underset{(5.2)}{.6134}B^{12})(1-\underset{(2.8)}{.3776}B^8)\tilde{u}_t$	9.3	.32	17.1	.64	.42	2.7
2201-2206 42 OBS	$(1-B)(1-\underset{(1.9)}{.2301}B^6-\underset{(6.3)}{.7577}B^{12})(1-\underset{(7.8)}{.8100}B)X_t=\tilde{u}_t$	8.2	.51	16.1	.76	.55	3.4
2207-2208 44 OBS	$(1-B)^2(1-B^{12})X_t=(1-\underset{(10.4)}{.7116}B^{12})\tilde{u}_t$	6.8	.74	19.8	.60	.50	4.8
2209-2210 46 OBS	$(1-B)^2(1-B^{12})(1-\underset{(2.8)}{.4716}B^2)X_t=(1-\underset{(9.2)}{.7397}B^{12})\tilde{u}_t$	4.7	.86	15.8	.78	.60	4.3
2210-2212 48 OBS	$(1-B)^2(1-B^{12})(1-\underset{(2.6)}{.3818}B^2)X_t=(1-\underset{(10.0)}{.7022}B^{12})\tilde{u}_t$	5.4	.80	16.2	.76	.63	3.5
2301 49 OBS	$(1-B)^2(1-B^{12})(1-\underset{(1.3)}{.1888}B^2)X_t=(1-\underset{(3.0)}{.4357}B^{12})(1+\underset{(10.0)}{.7894}B^5)\tilde{u}_t$	6.6	.58	18.3	.57	.58	2.8
2302 50 OBS	$(1-B)^2(1-B^{12})(1-\underset{(1.4)}{.2150}B^2)X_t=(1-\underset{(1.6)}{.2782}B^{12})(1+\underset{(11.4)}{.8144}B^5)\tilde{u}_t$	7.6	.48	20.8	.41	.60	2.7
2303 51 OBS	$(1-B)^2(1-B^{12})(1-\underset{(2.2)}{.3587}B^2)X_t=(1-\underset{(7.9)}{.6877}B^{12})\tilde{u}_t$	7.9	.54	16.3	.75	-.63	6.8
2304 52 OBS	$(1-B)^2(1-\underset{(46.18)}{.9874}B^{12})(1-\underset{(2.2)}{.3660}B)X_t=(1-\underset{(10.8)}{.7506}B^8)\tilde{u}_t$	9.2	.42	16.2	.76	-1.2	7.3
2305 53 OBS	$(1-B)^2(1-\underset{(32.08)}{.9883}B^{12})X_t=(1-\underset{(12.3)}{.7604}B^8)\tilde{u}_t$	8.2	.61	15.3	.85	-.41	5.5
2306 54 OBS	$(1-B)^2(1-\underset{(13.32)}{.9739}B^{12})X_t=(1-\underset{(3.8)}{.3917}B^2+\underset{(8.0)}{.7196}B^8)\tilde{u}_t$	9.5	.39	18.1	.64	-.45	7.2
2307 55 OBS	$(1-B)^2(1-B^{12})(1-\underset{(3.3)}{.4643}B+\underset{(4.4)}{.6358}B^2+\underset{(2.1)}{.3287}B^3+\underset{(1.8)}{.3116}B^4)X_t=\tilde{u}_t$	8.7	.27	15.7	.68	-.31	6.4
	Simple Avg. Values	7.7	.54	17.1	.68	.07	4.8

NOTES TO FIGURE 4

[a]B stands for the backshift operator. $B^j X_t \equiv X_{t-j}$

Numbers in parentheses under the coefficients are the T-statistics.

[b]Q(j), P(j) are summary statistics designed to test the overall statistical significance of the first j autocorrelations taken together. A high value of Q(j) will result in a low value of P(j), its associated F-statistic. P(24) = .1 is to be interpreted as the probability that the first 24 sample autocorrelations come from random errors.

[c]This is the period during which the estimated model is valid and can be used for prediction. In each case the ARIMA model has been estimated from all available data starting in 1901.

[d]In all cases the seasonal parameters were initially estimated. In cases where the estimates were very close to 1.00 (> .99, < 1.01) and statistically significant, these parameters were set equal to one.

FIGURE 5

TESTS OF THE VARIABLE COEFFICIENTS MODEL

COST OF LIVING ON MONEY SUPPLY (T + k)[a]; First Differences

Time Period	Constant	Past Values -4	-3	-2	-1	0	Expected Values +1	+2	+3	+4	R^2	F-Stat.[b]	D.W.[c] Coeff.	F-Test[d] Compar.
2001-2208 (a)	-.03 (1.10)	.27 (.79)	.06 (.16)	.47 (1.56)	.57 (1.73)	.85 (3.06)					.7022	11.79	1.69 ρ = .31	
2001-2208 (b)	-.02 (.88)	.24 (.64)	.28 (.72)	[illegible] (.57)	.31 (.88)	.40 (.96)	.50 (1.46)	.08 (.22)	.01 (.04)	-.07 (.49)	.7717	7.89	1.77 ρ = .29	1.52 (4,20)
2001-2303 (a)	.04 (1.66)	-.33 (.61)	-.73 (1.43)	.52 (1.04)	-.34 (.76)	1.65 (4.59)					.7743	22.6	2.05	
2001-2303 (b)	.03 (1.64)	-.67 (1.40)	.10 (.20)	.11 (.27)	-.34 (.89)	.65 (1.57)	.70 (1.47)	.77 (1.46)	[illegible] (1.58)	.02 (.15)	.8719	21.9	1.85	5.14** (4,27)
2001-2307 (a)	.03 (1.23)	.13 (.43)	-1.20 (3.12)	.59 (1.57)	-.36 (1.10)	1.78 (8.72)					.8802	54.4	2.33	
2001-2307 (b)	.03 (1.48)	-.25 (.92)	-.17 (.42)	.31 (1.05)	-.45 (1.48)	.68 (1.92)	.62 (1.81)	.77 (1.58)	-.63 (1.90)	.07 (.53)	.9387	56.1	2.10	7.50** (4,31)

MONEY SUPPLY ON COST OF LIVING (T + k)[e]; First Difference

Time Period	Constant	Past Values -4	-3	-2	-1	0	Expected Values +1	+2	+3	+4	R^2	F-Stat.[b]	D.W.[c] Coeff.	F-Test[d] Compar.
2001-2208 (a)	.01 (1.56)	.16 (1.61)	-.10 (.88)	-.03 (.26)	.14 (1.30)	.28 (3.10)					.6641	9.89	2.10 ρ = .03	
2001-2208 (b)	.01 (1.40)	.13 (1.29)	-.05 (.43)	.02 (.19)	.18 (1.50)	-.03 (.17)	.15 (.64)	.39 (1.83)	-.15 (.66)	-.10 (.80)	.7340	6.44	2.01 ρ = .14	1.33 (4,20)
2001-2303 (a)	-.01 (.48)	.08 (1.04)	.12 (1.61)	.08 (1.13)	.33 (6.22)	.26 (5.74)					.9640	171.2	2.30 ρ = .48	
2001-2303 (b)	-.01 (1.32)	.13 (1.69)	.06 (.74)	.11 (1.64)	.30 (4.85)	.13 (1.32)	.00 (.00)	.34 (2.29)	-.06 (.54)	-.12 (1.56)	.9727	110.8	2.18 ρ = .51	2.15 (4,27)
2001-2307 (a)	-.01 (.57)	.31 (6.88)	-.05 (1.07)	.00 (.00)	.29 (6.54)	.40 (13.43)					.9676	214.8	2.18 ρ = .43	
2001-2307 (b)	.01 (.36)	.10 (1.48)	.08 (1.65)	.13 (2.19)	.27 (4.59)	.18 (3.07)	.00 (.00)	[illegible]	-.08 (.83)	-.12 (1.78)	.9814	187.6	2.09 ρ = .71	5.75** (4,31)

NOTES TO FIGURE 5

[a] COST OF LIVING (COL) is regressed on the Pasted, Current and Expected MONEY SUPPLY (MON). The expected values are derived from forward forecasts of the appropriate ARIMA models detailed in Table 4. Regression (a) is COL on current and past MON while regression (b) also includes expected MON.

[b] F-statistic of the regression.

[c] Durbin-Watson statistic testing first order autocorrelations. The critical values are given below for selected sample sizes and five independent variables.

n	dL	du	Level of Significance
31	0.90	1.60	1%
31	1.03	1.85	5%
38	1.02	1.58	1%
38	1.21	1.79	5%
42	1.07	1.58	1%
42	1.25	1.79	5%

[d] F-test comparing regressions (a) and (b) in each case. A high F-value implies that regression (b) has significantly higher explanatory power than regression (a). Critical values for the F-statistic are given below:

	F(4,31)	F(4,27)	F(4,20)
95%	2.68	2.73	2.87
97.5%	3.24	3.31	3.51
99%	4.00	4.11	4.43

*Shows at least a 5% level of significance. **Shows at least a 1% level of significance.

[e] MON is regressed on the Past, Current and Expected COL. The expected values are derived from forward forecasts of the appropriate ARIMA models detailed in Table 4. Regression (a) is MON on Past and Current COL, while regression (b) also includes Expected COL.

FIGURE 6

TESTS OF CAUSAL ORDERING WITH CONSTANT COEFFICIENTS

COST OF LIVING ON MONEY SUPPLY (T + k)[a]

		Past Values					Future Values						
Time Period	Constant	-4	-3	-2	-1	0	+1	+2	+3	+4	R^2	D.W.[c] Coeff.	F-Test[d]
2001-2208 (a)	-7.14 (5.90)	.38 (1.10)	.04 (.12)	.52 (1.73)	.22 (.62)	.98 (3.49)					.9933	1.51 ρ = .91	
2001-2208 (b)	-6.73 (4.23)	.43 (1.32)	.18 (.56)	.01 (.03)	.10 (.31)	.66 (2.06)	.97 (3.05)	-.12 (.38)	-.32 (1.01)	.14 (.56)	.9957	1.00 ρ = .92	2.93* (4,21)
2001-2307 (a)	-2.07 (6.56)	.06 (.23)	-1.21 (3.18)	.76 (2.06)	-.51 (1.53)	1.94 (9.81)					.9982	2.29 ρ = .82	
2001-2307 (b)	-.88 (.78)	-.02 (.04)	-.14 (.36)	-.43 (1.20)	-.63 (2.08)	1.69 (6.78)	.88 (5.44)	-.30 (1.97)	-.40 (3.33)	.25 (2.52)	.9992	1.87 ρ = .97	10.00** (4, 3[illegible])

MONEY SUPPLY ON COST OF LIVING (T + k)[e]

		Past Values					Future Values						
Time Period	Constant	-4	-3	-2	-1	0	+1	+2	+3	+4	R^2	D.W. Coeff.	F-Test
2001-2208 (a)	3.15 (9.94)	.17 (1.62)	-.10 (.82)	.05 (.40)	.12 (1.01)	.34 (3.64)					.9945	2.02 ρ = .95	
2001-2208 (b)	3.41 (13.01)	.10 (.99)	-.13 (1.19)	.02 (.22)	.12 (1.08)	.11 (.73)	.08 (.55)	.02 (.16)	.09 (1.00)	.02 (.36)	.9964	2.35 ρ = .89	2.77 (4,21)
2001-2307 (a)	1.90 (11.00)	.36 (7.22)	-.07 (1.19)	-.02 (.33)	.23 (4.04)	.43 (13.59)					.9994	1.42 ρ = .94	
2001-2307 (b)	1.93 (8.47)	.24 (3.06)	-.01 (.25)	.01 (.16)	.27 (4.97)	.32 (5.78)	.02 (.46)	.05 (1.12)	-.02 (.55)	.01 (.49)	.9996	1.20 ρ = .95	4.00** (4, 32)

NOTES TO FIGURE 6

[a]COST OF LIVING (COL) is regressed on the Past, Current and Future MONEY SUPPLY (MON). Regression (a) is COL on current and past MONEY while regression (b) also includes future MONEY supplies.

[b]F-statistic of the regression.

[c]Durbin-Watson statistic testing first order autocorrelations. The critical values are given below for selected sample sizes and five independent variables.

n	dL	du	Level of Significance
31	0.90	1.60	1%
31	1.03	1.85	5%
38	1.02	1.58	1%
38	1.21	1.79	5%
42	1.07	1.58	1%
42	1.25	1.79	5%

[d]F-test comparing regressions (a) and (b) in each case. A high F-value implies that regression (b) has significantly higher explanatory power than regression (a). Critical values for the F-statistic are given below:

	F(4,31)	F(4,27)	F(4,20)
95%	2.68	2.73	2.87
97.5%	3.24	3.31	3.51
99%	4.00	4.11	4.43

*Shows at least a 5% level of significance. **Shows at least a 1% level of significance.

[e]MON is regressed on the Past, Current and Future COL. Regression (a) is MON on current and past MON while regression (b) also includes future MON.

FIGURE 7

TESTS OF CAUSAL ORDERING WITH VARIABLE COEFFICIENTS.

COST OF LIVING ON MONEY SUPPLY (T + k)[a]

Time Period	Constant	Expected Money Supply +4	+3	+2	+1	0	Actual Money Supply +1	+2	+3	+4	R^2	F-Stat.[b]	D.W. Coeff.[c]	F-Test Compar.[d]
2001-2208 (a)	-4.67 (6.06)	.06 (.52)	-.29 (1.15)	.18 (.61)	.68 (2.30)	.98 (3.45)					.9938	795	1.49 $\hat{\rho}$ = .88	
2001-2208 (b)	-4.40 (3.96)	.01 (.06)	-.42 (1.22)	.98 (2.01)	-.76 (1.44)	.80 (2.48)	1.58 (2.95)	-.78 (1.56)	-.10 (.25)	.24 (.91)	.9958	552	0.92 $\hat{\rho}$ = .90	2.50 (4,21)
2001-2307 (a)	-2.43 (6.89)	-.03 (.26)	-.39 (1.53)	.53 (1.36)	.94 (3.19)	.11 (.56)					.9988	5,937	1.89 $\hat{\rho}$ = .93	
2001-2307 (b)	-2.64 (6.97)	.06 (.51)	-.19 (.72)	.58 (1.32)	.19 (.45)	.11 (.53)	.87 (3.72)	-.17 (1.39)	-.18 (1.83)	.04 (.91)	.9992	4,378	1.45 $\hat{\rho}$ = .93	4.00** (4, 32)
First Differences[e]														
2001-2307 (a)	.02 (1.02)	-.02 (.19)	-.46 (1.75)	.63 (1.60)	.93 (3.09)	.02 (.08)					.9260	[illegible]	1.88	
2001-2307 (b)	.02 (.99)	-.06 (.46)	-.25 (.90)	.69 (1.51)	.16 (.35)	.33 (.15)	.86 (3.49)	-.16 (1.25)	-.17 (1.62)	.04 (.72)	.9479	66.7	1.50	3.36* (4, 32)

MONEY SUPPLY ON COST OF LIVING (T + k)[f]

Time Period	Constant	Expected COL +4	+3	+2	+1	0	Actual COL +1	+2	+3	+4	R^2	F-Stat.[b]	D.W.[c] Coeff.	F-Test[d] Compar.
2001-2208 (a)	3.24 (16.50)	-.12 (.89)	-.18 (.74)	.38 (1.61)	.21 (.86)	.22 (1.31)					.9948	948	1.74 $\hat{\rho}$ = .91	
2001-2208 (b)	3.42 (22.84)	-.13 (.93)	-.18 (.74)	.41 (2.00)	.21 (.79)	-.08 (.50)	.03 (.25)	.01 (.06)	.11 (.92)	.05 (.68)	.9973	855	2.22 $\hat{\rho}$ = .86	4.86** (4,21)
2001-2307 (a)	4.09 (4.38)	-.51 (2.58)	.63 (1.80)	-.34 (.90)	.42 (1.43)	.50 (3.82)					.9977	3,077	1.29 $\hat{\rho}$ = .98	
2001-2307 (b)	2.54 (19.53)	-.40 (2.98)	.32 (1.36)	.05 (.17)	-.14 (.60)	.86 (8.70)	.02 (.23)	.11 (1.68)	-.25 (5.48)	.18 (6.30)	.9992	4,543	1.62 $\hat{\rho}$ = .87	15.00** (4, 32)

NOTES TO FIGURE 7

[a]COST OF LIVING (COL) is regressed on the Expected and Current MONEY SUPPLY (MON). The Expected values are derived from forward forecasts of the appropriate ARIMA models detailed in Table 4. Regression (a) is COL on Current and Expected MON while regression (b) also includes future MON.

[b]F-statistic of the regression.

[c]Durbin-Watson statistic testing first order autocorrelations. The critical values are given below for selected sample sizes and five independent variables.

n	dL	du	Level of Significance
31	0.90	1.60	1%
31	1.03	1.85	5%
38	1.02	1.58	1%
38	1.21	1.79	5%
42	1.07	1.58	1%
42	1.25	1.79	5%

[d]F-test comparing regressions (a) and (b) in each case. A high F-value implies that regression (b) has significantly higher explanatory power than regression (a). Critical values for the F-statistic are given below:

	F(4,32)	F(4,28)	F(4,21)
95%	2.67	2.71	2.84
97.5%	3.23	3.29	3.48
99%	3.98	4.07	4.37

*Shows at least 5% level of significance. **Shows at least 1% level of significance.

[e]Similar to the regression above. However, the variables on both sides have been first differenced.

[f]MON is regressed on the Expected and Actual COL. The Expected values are derived from forward forecasts of the appropriate ARIMA models detailed in Table 4. Regression (a) is MON on Current and Expected COL, while regression (b) also includes Future COL.

FIGURE 8

INCOME AND GOVERNMENT SPENDING

	1920	1921	1922	1923
National Product (NP) (in billion Goldmarks)	31	38	34	27
Government Expenditures as % of NP	30%	26%	18%	42%
Taxes as % of NP	8%	12%	8%	5%
% of Expenditures Financed by Borrowing	72%	54%	54%	87%
Real Money Balances (in billion Goldmarks)	4.7	4.1	1.4	0.8
Velocity of Circulation	6.6	9.2	24.3	32.0

National Product figures were computed from Graham (1930). All the figures are yearly averages.

FIGURE 9

REAL GOVERNMENT REVENUE AND BANK SEIGNORAGE [a,b]

	MEAN	VARIANCE	T-STATISTIC [c]	AUTO CORRELATION [d]	CROSS CORRELATION [e]
BANK	0.410	0.081	1.44	0.555	0.627
GOV.MT	0.574	0.094	1.87	0.134	
BANK	0.208	0.017	1.60	0.388	0.296
GOV.MT	0.329	0.027	2.00	0.110	
BANK	0.271	0.037	1.41	0.482	0.435
GOV.MT	0.416	0.054	1.79	0.237	
BANK	0.143	0.012	1.31	0.328	0.302
GOV.MT	0.251	0.026	1.56	0.362	
BANK	0.311	0.043	1.50	0.541	0.426
GOV.MT	0.459	0.054	1.98	0.178	
BANK	0.162	0.012	1.48	0.356	0.241
GOV.MT	0.274	0.025	1.73	0.312	

NOTES TO FIGURE 9

[a]Real government revenue is computed by deflating the change in the outstanding government securities by a series of deflators. Real bank seignorage is computed by deflating the change in the money supply by the same series of deflators.

[b]The deflators in the order they are used are:

COL_t, WPI_t.

COL_{t+1}, WPI_{t+1}.

$\frac{1}{2}(COL_t + COL_{t+1})$, $\frac{1}{2}(WPI_t + WPI_{t+1})$.

where COL and WPI are the Cost of Living and the Wholesale Price Indices respectively.

[c]The statistic refers to the mean under the assumption that all the observations are drawn from the same distribution.

[d]Autocorrelation at lag 1 of each series.

[e]This is the cross-correlation at lag 0 between government revenue and bank seignorage for each deflator used.

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