THE DETERMINANTS OF THE VARIABILITY OF STOCK MARKET PRICES*

by

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The most familiar interpretation for the large and unpredictable swings that characterize common stock price indices is that price changes represent the efficient discounting of "new information." It is remarkable given the popularity of this interpretation that it has never been established what this information is about. Recent work by Robert Shiller, and Stephen LeRoy and Richard Porter, has shown evidence that the variability of stock price indices cannot be accounted for by information regarding future dividends since dividends just do not seem to vary enough to justify the price movement. These studies assume a constant discount factor. In this paper, we consider whether the variability of stock prices can be attributed to information regarding discount factors (i.e. real interest rates), which are in turn related to current and future levels of economic activity.

The appropriate discount factor to be applied to dividends which are received k years from today is the marginal rate of substitution between consumption today and consumption k periods from today. We use historical data on per capita consumption from 1890-1979 to estimate the realized value of these marginal rates of substitution.

Robert Hall also studied these marginal rates of substitution and concluded that consumption is a random walk. We show that if current consumption and dividends are the best predictors of future consumption and dividends in Hall's sense, then the discount factor applied to stock prices would not vary. The variability of stock prices implies they do vary, so that we conclude that consumers must have a better method for forecasting future consumption than using only current consumption (e.g. consumers may know when the economy is in a recession).

I. Stock Returns and the Marginal Rate of Substitution

Consider a consumer who can freely buy or sell asset i and whose utility can be written as the present discounted value of utilities of consumption in future years $U_t = \sum_{k=0}^{\infty} \beta^k u(C_{t+k})$, where $\beta = 1/(1+r)$ and r is the subjective rate of time preference. A necessary condition for his holdings of the asset at t to be optimal, given that the consumer maximizes the expectation at time t, of this utility function is:

$$(1) \qquad u'(C_t)P_{it} = \beta E[u'(C_{t+1})(P_{it+1} + D_{it+1}) \mid I_t] \quad ,$$

where P_{it} is the real price (in terms of the single consumption good or "market basket" C_t) of asset i and D_{it+1} is the real dividend paid at $t+1$ to holders of record at t. E denotes mathematical expectation, here conditional on I_t which is all the information about the future which the agent possesses at time t. The left-hand side of (1) is the cost in terms of foregone current consumption of buying a unit of the asset, while the right-hand side of (1) gives the expected future consumption benefit derived from the dividend and capital value of the asset. This relation plays a central role in modern theoretical models of optimal dynamic consumption and portfolio decisions, such as those of Robert Lucas.

Since $u'(C_t)$ and P_{it} are known at time t (in contrast to P_{it+1}, D_{it+1} and C_{t+1} where are not), we can rewrite (1) as:

$$(2) \qquad 1 = E(R_{it}S_t \mid I_t) \quad ,$$

where $S_t = \beta u'(C_{t+1})/u'(C_t)$ is the marginal rate of substitution between present and future consumption (the reciprocal of the usual measure), and $R_{it} =$

$(P_{it+1} + D_{it+1})/P_{it}$ is the return (or rather one plus the rate of return as it is usually calculated). Note that the expectation in (2) conditional on information I_t is always 1. Hence it does not depend on I_t. Therefore, it equals the unconditional or simple expectation

$$(3) \qquad 1 = E(R_{it} S_t) \quad .$$

Thus, the proper stochastic interpretation of the familiar two-period diagram is that the expected product of the uncertain return and the uncertain marginal rate of substitution is one. This means that $E(R_{it})$ needn't equal the subjective rate of time preference nor need it be the same for all assets ("expected profit opportunities" may exist). Instead, (3) says that a weighted expectation of returns, with weights corresponding to marginal rates of substitution, is the same for all assets. Returns which come in periods of low marginal utility of consumption (i.e. when consumption is high) are given little weight, because they do little good in terms of utility. Returns which come in periods of high marginal utility are given a lot of weight. The same expression can also be written another way, using the fact that the expected product of two variables is the product of their means plus their covariance:

$$(4) \qquad E(R_{it}) = E(S_t)^{-1} \cdot (1 - \text{cov}(R_{it}, S_t)) \quad .$$

Equation (4) states that the expected return of an asset depends on the covariance of the asset's return with the marginal rate of substitution. An asset is very "risky" if its payoff has a high negative covariance with S. (Douglas Breeden has recently persuasively argued for the use of consumption correlatedness as the appropriate measure of risk.)

The theory of asset returns embodied in each of expressions (1) through (4) is very powerful because it can be applied so generally. It holds for any asset, or portfolio of assets. It holds for any individual consumer who has the option of investing in stocks (even if he chooses not to hold stocks) and thus it must hold for aggregate consumption so long as some peoples' consumption is well represented by the aggregate consumption. It holds even if the individual's choices regarding other assets are constrained (e.g. the individual cannot trade in his or her "human capital," is constrained by institutional factors in housing investment, or is unable to borrow money) so long as such constraints do not affect his ability to change his saving rate through stock purchases or sales. It incorporates all sorts of uncertainty that people consider in making investment decisions, since these factors are reflected in consumption. The model holds for any time interval, whether a month, a year, or a decade.

II. Perfect Foresight Stock Prices

By iterating (1), we find that price is the expected present value of dividends and a terminal price discounted by the marginal rates of substitution:

$$(5) \qquad P_{it} = E\left[\sum_{j=1}^{n} \beta^j \frac{u'(C_{t+j})}{u'(C_t)} D_{it+j} + \beta^n \frac{u'(C_{t+n})}{u'(C_t)} P_{it+n} \;\Big|\; I_t\right] .$$

It is useful to define the perfect foresight stock price P^*_{it}, which is the price at t given that the consumer knows the whole future time path of consumption, dividends and the terminal price P_{it+n}:

$$(6) \qquad P^*_{it} = \sum_{j=1}^{n} \beta^j \frac{u'(C_{t+j})}{u'(C_t)} D_{it+j} + \beta^n \frac{u'(C_{t+n})}{u'(C_t)} P_{it+n} .$$

Clearly (5) states that $P_{it} = E[P^*_{it} \mid I_t]$. Further, we assume the $u(C)$ is of the constant relative risk aversion form

$$(7) \qquad u(C) = \frac{1}{1-A} C^{1-A} , \quad 0 < A < \infty ,$$

where A is the "coefficient of relative risk aversion," which is a measure of the concavity of the utility function or the disutility of consumption fluctuations.

Figure 1 shows a plot of P_t from 1889-1979, where P_t is the annual average Standard and Poor Composite Stock Price Index divided by the consumption deflator. On the same figure, we plot the perfect foresight real price P_t^* for A=0 and A=4 using (6) and (7), where we use actual realized real annual dividends for the Standard and Poor series, the Kuznets-Kendrick-US NIA per capita real consumption on nondurables and services and the terminal condition $P_{1979}^* = P_{1979}$. For each A, we generate a value of β so that (3) holds, as estimated by the sample mean. The case A=0 is revealing; this is the case of risk neutrality, and of a constant discount factor. Notice that with a constant discount factor, P_t^* just grows with the trend in dividends; it shows virtually none of the short-term variation of actual stock prices. The larger A is, the bigger the variations of P_t^* and A=4 was shown here because, for this A, P and P* have movement of very similar magnitude. Irwin Friend and Marshall Blume estimated A to be about 2 under the assumption that the only stochastic component of wealth is stock returns. Irwin Friend and Joel Hasbrouck estimated A to be about 6 when stock returns and human capital are the stochastic components of wealth. We also computed a P* series using after tax returns. It did not look much different from the P* shown here in the first half of the sample when income taxes were generally unimportant, and did not seem to fit P any better in the second half.

The rough correspondence between P* and P (except for the recent data) shows that if we accept a coefficient of relative risk aversion of 4, we can to some extent reconcile the behavior of P with economic theory even under the assumption that future price movements are known with certainty. In a world of certainty, the marginal rate of substitution S_t would equal the inverse of one plus the real interest rate, r_t. Hence our equilibrium condition becomes $(P_{t+1} + D_{t+1}) \div P_t = 1+r_t$. Thus it can be shown that real stock prices as well as real prices of other assets whose dividend is stable in real terms will rise dramatically over periods when real interest rates are very high. Real interest rates will be high when C_{t+1} is high relative to C_t, e.g. in periods of depression when C_t is abnormally low.

Hence it is an equilibrium for P_t to be low (relative to P_{t+1}) because otherwise people will desire to dissave (e.g., by selling stock at t) in order to maintain their consumption level. Movements in real interest rates which are necessary to equilibrate desired savings to actual savings will lead to changes in stock prices even if dividends are unchanged. It is these movements which are brought out in the figure when P* with A=4 is compared with P* with A=0.

The correlation between P* and P is perhaps not altogether surprising, given the correlation between the stock market and aggregate economic activity over the business cycle noted long ago by many people, e.g. Arthur Burns and Wesley Mitchell. However, P_t^* is not merely a proxy for aggregate economic activity or consumption at time t. If we assume, as an approximation, that dividends follow a growth path $D_t = D_o\delta^t$ and if we set $n=\infty$ in (6) to ignore the terminal price, then P_t^* is given by: $P_t^* = D_o\delta^t[C_t^A \sum_{k=0}^{\infty} (\beta\delta)^K C_{t+k}^{-A}]$. This says that P_t^* follows a growth path times the ratio of C_t^A to a weighted harmonic average of future C^A. The weights decline exponentially into the future. Thus, for example, P* declines gradually between 1907 and 1919 not because consumption declined, since real per capita consumption remained more or less level over this period, but because the gap between current consumption and the longer-run outlook widened. In other words, P* fell at this time because of perfect foresight individual, knowing his economic fortune would eventually improve following the war period, wished to try to smooth his consumption over this period. This kind of relationship between P and C would not have been visible by looking at raw stock price and economic activity index series alone, as the earlier scholars did. On the other hand, the short-run correspondence between P and P* around such episodes as the panics of 1893 or 1907 was in effect noted by these authors.

Our construction implies that P* (as well as P) is a leading indicator of future levels of economic activity, but it does not suggest the conventional notion of a fixed lead of a few months to a year between P and aggregate economic activity. However, such a fixed lead has never been quantitatively established (see C.W.J. Granger and M. Hatanaka).

Once we drop the assumption of perfect foresight, there need not be very much relationship between P_t and P_t^*. If consumers have no information about P_t^* then P_t will be a constant and P_t^* will vary. We can write $P_t^* = P_t + U_t$ where $U_t = P_t^* - E[P_t^* | I_t]$ is a forecast error. Since P_t is in the information set I_t, U_t must be uncorrelated with P_t, so that the variability of the stochastic process $\{P_t^*\}$ will be larger than that of the stochastic process $\{P_t\}$. Further, if we consider any subset of the information set at t, say I_t^s, then $\mathrm{Var}(P_t^* | I_t^s) \geq \mathrm{Var}(P_t | I_t^s)$. If we make the assumption that the variability of the stochastic processes $\{P_t\}$ and $\{P_t^*\}$ can be estimated from the sample variability of observed P_t and P_t^*, then the figure can give some evidence in favor of the hypothesis that A is at least 4. From the figure, it is clear that with A=0 the variance inequality is reversed: P_t^* varies less than P_t. This is evidence against the hypothesis that the discount factor does not vary. Once we raise A to say A=4, then the variability of the discount factor forces P_t^* to vary a lot. The larger A is, the larger is the variability induced in P_t^* by changes in the consumption path. Another way that the reader can see that discount factor variability is important is to apply the above variance inequality with $I_t^s = D_t$, yielding $\mathrm{Var}(P_t^* | D_t) \geq \mathrm{Var}(P_t | D_t)$. If the discount factor was constant, then this states that current dividends should be a better predictor of the current stock price than current dividends can predict weighted future dividends. Casual observation suggests this is false. Current dividends are a very good forecaster of future dividends, and a terrible forecaster of the current stock price. Once we permit the discount factor to vary, the inequality has a much greater chance of being true, since the current dividend is a poor forecaster of future discount factors.

If it is accepted that the variability of the discount factor is important, then we can use this to provide evidence against Hall's assertion that short-term movements in consumption are not forecastable by consumers. To see this, write the jth term in the summation in (5) as $E(\beta^j u'(C_{t+j})/u'(C_t) | I_t)E(D_{t+j} | I_t) + \mathrm{cov}(\beta^j u'(C_{t+j})/u'(C_t), D_{t+j} | I_t)$. If neither the expectation of $\beta^j u'(C_{t+j})/u'(C_t)$ nor its covariance with dividends is forecastable (depends on I_t), then this term varies only due to changes in the expectation of D_{t+j}, i.e. due to information about dividends. If, moreover, $E(\beta^j u'(C_{t+j})/u'(C_t) | I_t) = \gamma^j$ (as might be suggested by Hall's random walk hypothesis), then P_t equals $E(\hat{P}^*_t | I_t)$ where $\hat{P}^*_t = \sum \gamma^j D_{t+j}$ (plus a deterministic term due to the covariance). $\hat{P}^*_t$ has a constant discount factor and is proportional to P^* in Figure 1 with $A=0$. Because P^*_t with $A=0$ fails the variance test as mentioned previously, we tend to reject models with constant discount factors. Hence we conclude that consumption changes are forecastable. This implies that expected real interest rates vary (contrary to the claims of E. Fama and others).

This conclusion does not contradict Robert Hall's assertions that (i) to an econometrician who does not know as much as consumers, the marginal utility of consumption is a random walk and (ii) that income may be a proxy for lagged consumption in econometric models which have shown that consumption is very sensitive to income. The fact that stock prices vary so much with consumption suggests that consumers have more information about consumption than is contained in current consumption, and this leads expected real interest rates to vary with information.

III. Further Research

We have some preliminary results on the estimation of A and β. Estimates of both parameters can be derived using expression (3) for two different assets, which we took as stocks and short-term bonds. Unfortunately, the estimates of A for the more recent subperiods seem implausibly high. This breakdown of the model mirrors the divergence between P^* and P since the early 1950's, as well as the extremely low real return on short-term bond rates in this period. There was an enormous rise in stock prices in that period which cannot be explained by changes in realized dividends or in marginal rates of substitution. Preliminary results show that it cannot be explained by taxes. Irwin Friend and Marshall Blume noticed an extremely high excess return of stocks over bonds in this period relative to all other subperiods from 1890 to date. Their estimated market price of risk was twice as high in the decade 1952-1961 as the highest of any other decade. While the divergence between P_t and P_t^* might be considered an enormous forecast error, we don't have any idea as to why $E(P_t^* | I_t)$ should have changed so much.

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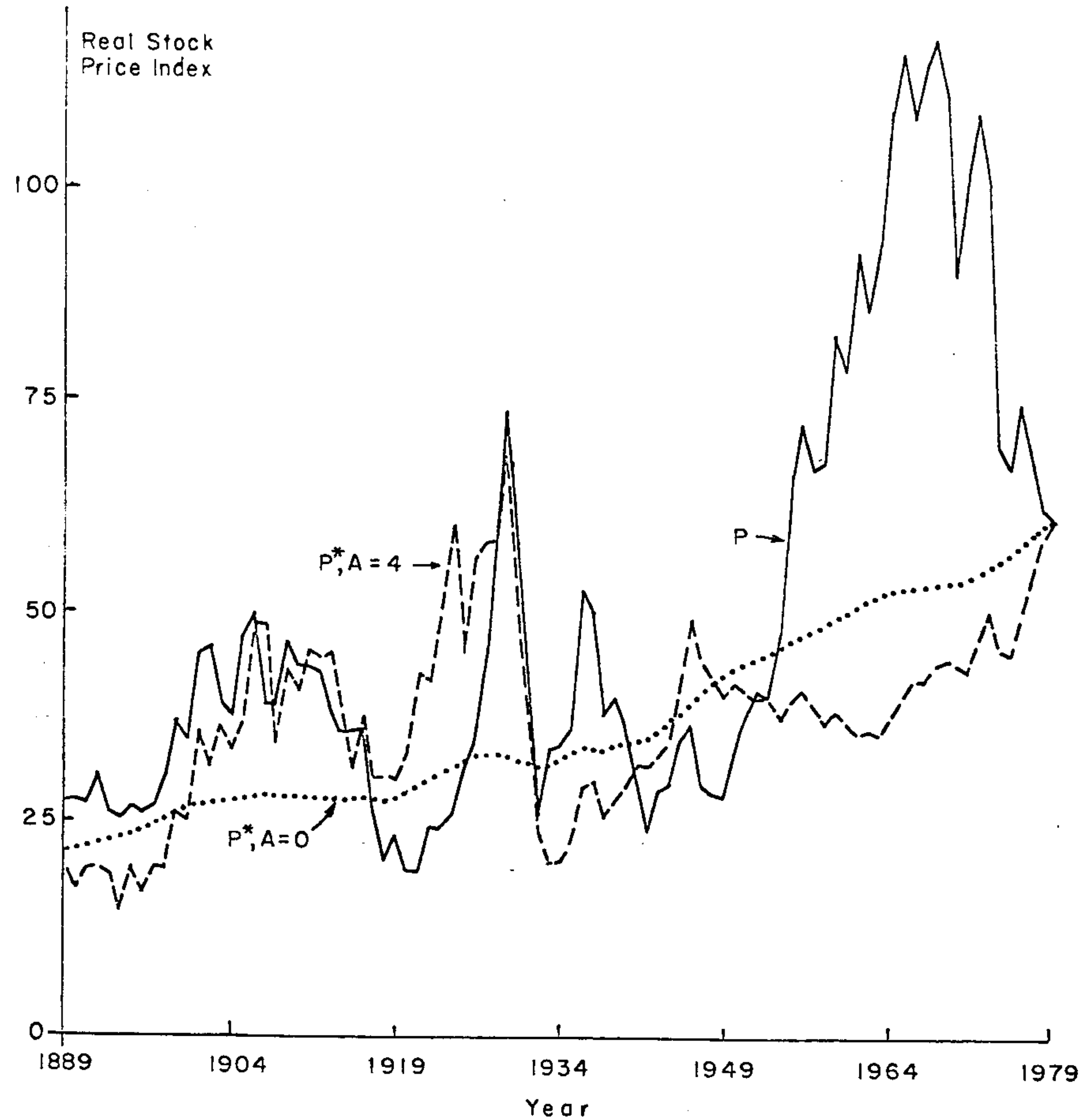
Actual and Perfect Foresight Stock Prices 1889 - 1979



The solid line P_t is the real Standard and Poor Composite Stock Price Average. The other lines are: P_t^* (as defined in expression 5), the present value of actual subsequent real dividends using the actual stock price in 1979 as a terminal value. With A=0 (dotted line) the discount rates are constant, while with A=4 (dashed line) they vary with consumption.