HETEROGENEOUS INFORMATION AND THE THEORY OF THE BUSINESS CYCLE

by

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A standard feature of recent business cycle theory is that agents do not have access to complete current period information which, if available, would alter real variables. A useful analytical model for demonstrating this possibility is the "island economy," first proposed by Phelps (1970). Here it is assumed that agents cannot observe commodity prices on other islands at the time supply decisions must be made. This assumption was a key element of Lucas (1972), Barro (1976), and Weiss (1980). However, the assumption appears doubtful both on a priori and empirical grounds. Barro (1980), citing the results of Sargent (1976), Fair (1979) and himself (1979,1980), has argued that price surprises appear to play an empirically insignificant role in generating cycles. Clearly, if cycles stem from incomplete information, the relevant data for agents' optimizing decisions are not summarized in the prevailing vector of commodity prices.

What then is the mechanism by which confusions about real variables leads to variations in output? A mechanism in the spirit of the arguments of Wicksell, Keynes, and Tobin focuses on the determinants of new capital formation by firms. Firms are thought to invest to the point where the supply price of capital is equal to its marginal productivity. An important question is the ability of financial markets to provide adequate signals for this calculation.

In this paper, we present a model of the economy in which financial markets are unable to provide all relevant information for firms' investment decisions. Equilibrium market quoted <u>nominal</u> interest rates may not be sufficient statistics for firms to infer the correct real cost of capital. The inability of firms to infer the correct cost of capital leads to greater variability of aggregate labor input, output, consumption and real interest rates than would occur under complete information.

A major emphasis of the model is the differential effect of aggregate verses relative shocks which affect the technical productivity of new investments. It is shown that a capital market which can facilitate intertemporal exchange induces agents to be more responsive to perceived idiosyncratic shocks than to perceived aggregative shocks. This arises because perceived real interest rate movements stemming from aggregate disturbances retard aggregate investment. However, a key assumption of the model is that agents' own information does not discriminate between relative and economy-wide shocks. In some cases, observation of the two available economy-wide nominal prices -- the money price of the single consumption good and the market clearing nominal interest rate on default free, one-period bonds -- will permit agents to identify the two types of shocks separately. If, however, monetary disturbances are introduced, this feature will no longer hold as agents confuse shocks to aggregate productivity with shocks to the expected inflation rate. The inability to observe the current aggregate shock or, equivalently, the real interest rate, will lead the representative trader to overstate the movement in his relative position. Thus he will vary investment more in response to aggregate disturbances than under full information. This will cause greater cyclical variation in investment, labor supply and consumption than would occur if agents know the true state. This phenomenon may occur even if all agents can observe all current period commodity prices and money supply contemporaneously with investment decisions.

Fluctuations in the perceived productivity of investment is capable of explaining a Phillips curve relationship between unanticipated inflation and aggregate labor supply, even if all agents have the same information. Incomplete (i.e. heterogeneous) information will strengthen this relationship by invoking larger output swings, however.

Emphasis on the informational role of nominal interest rates is a new channel by which disturbances to money demand may be confused with real variables and thereby affect real variables. The model yields different conclusions about the comovements of ex ante expected real rates of return available on financial assets, employment, and monetary disturbances from those suggested in earlier works, e.g. Barro (1980). A complete discussion of these distinctions is deferred until the end of the paper.

The paper is organized as follows. Section I sets up the model under the assumption that agents have complete current period information. In this context, an observed (spurious) Phillips curve relationship is compatible with a model in which the real economy can be described completely independently of monetary variables, i.e. where money is super-neutral. Section II shows that under some circumstances financial signals will not serve to communicate all relevant information. The major result of this modification is that investment fluctuations will be more pronounced than under complete information. The third section incorporates monetary disturbances and shows how this can have real effects even when all current prices are observable. The fourth section discusses the testable conclusions of the model

I. The Model

It is assumed that there are J identical, infinitely long-lived agents with identical preferences:

$$U^j = \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{\alpha} \left(C_t^j\right)^{\alpha} + \frac{1}{a} \left(m_t^j\right)^{a} - L_t^j \right] , \tag{1}$$

where C_t^j = consumption of j at t,

m_t^j = real balances of j at t,

L_t^j = labor input at j at t,

$0 < \beta < 1,\ a,\alpha < 1$.

Agents have access to a private technology which transforms labor input at t into the single perishable consumption good at $t+1$ denoted by X_{t+1}^j, according to the (stochastic) technology

$$(2) \qquad X_{t+1}^j = \left(\tilde{N}_t \tilde{e}_t^j\right) \frac{1}{\sigma} L_t^\sigma \quad , \quad \sigma > 0$$

where $\tilde{N}_t$ = (random) aggregate productivity shock at t (I.I.D),

e_t^j = (random) relative productivity shock at j at t (I.I.D).

It is assumed that each agent can observe his own technology $\left(N_t e_t^j\right)$ before labor supply in t is chosen. However, his private information does not permit him to decompose the two shocks separately. Note that in eq. (2), labor is put in place at time t but produces output at time $t+1$. The labor supply decision for each agent is thus much like an investment decision. Not only is there a time lag between inputs and outputs, but, as will be shown below, the demand for "labor" depends upon the interaction between productivity and interest rates. Using labor as a proxy for investment considerably simplifies the problem by making the utility cost of investment independent of current output. This has the effect of converting an infinite period maximization problem into a sequence of one-period maximizations.

Individuals are permitted to borrow on an economy-wide default-free bond market whereby \$1 in t can be exchanged for \$R in $t+1$. Individuals can trade commodities for money at a price of \$P. Individuals can only work at their own technology, and they cannot observe directly the labor supply decisions of other agents, but can observe aggregate output directly.

Formally, in each period, each agent must choose consumption, money holding and labor supply so as to maximize utility in accord with an appropriate budget constraint. Let B_t^j be the net nominal borrowings by person j at time t after period t consumption and income are realized. The sequence of budget constraints is

$$P_o C_o^j + M_o^j + B_o^j \leq P_o X_o + \bar{M}_o$$
$$P_t C_t^j + M_t^j + B_t^j \leq R_{t-1} B_{t-1}^j + M_{t-1}^j + P_t X_t^j \quad .$$

If there is no constraint on borrowings, then the maximization problem in (1) has no solution; the consumer would set $B_j = -\infty$ and $C_t = \infty$. Thus it is necessary to bound borrowing. Let Ω_t be a complete description of the state of the aggregate variables from zero to T. It is the list of prices, interest rates and outputs from zero to T. A convenient and economically plausible way to bound borrowing is to require the agent to choose a plan at time 0 such that

$$\lim_{T\to\infty} E\left[\left(B_T^j \Big/ \prod_{i=0}^{T-1} R_i\right) \Big| \, \Omega_T\right] = 0 \quad ,$$

for all possible realizations of the information, Ω_T. The constraint implies that, for any sequence of per capita incomes, interest rates and nominal prices, the limit of expected borrowings must have a present value , as of time 0, of 0.

In the Appendix, it is shown if a consumption-borrowing policy satisfies the above bound, then the first-order conditions to the maximization of (1) give the best policy for the consumer in the class of all policies where discounted borrowing goes to zero. The first-order conditions to the consumer's problem are, for all j and t

(3) $$\left(C_t^j\right)^{\alpha-1} = \beta E_t\left(r_t \, C_{t+1}^{j\ \alpha-1}\right)$$

(4) $$\left(m_t^j\right)^{a-1} = \beta E_t(r_t - r_{mt})\left(C_{t+1}^j\right)^{\alpha-1}$$

(5) $$1 = \beta E_t\left[N_t e_t^j \left(L_t^j\right)^{\sigma-1}\left(C_{t+1}^j\right)^{\alpha-1}\right] \quad ,$$

where r_{mt} = real return on money holding, equal to P_t/P_{t+1},

r_t = real return on nominal bond holding, equal to R_tP_t/P_{t+1},

where all expectations are with respect to the information possessed by the agent at t, which is described below.

Equation (3) states that the consumer must be indifferent between using a bond to change his consumption today for a change in income tomorrow. Equation (4) can be interpreted as a neoclassical money demand equation by substituting $R_t = r_t/r_{mt}$ to get

$$(4') \qquad \left(m_t^j\right)^{a-1} = \left(M_t^j/P_t\right)^{a-1} = \left(\frac{R_t-1}{R_t}\right)\left(C_t^j\right)^{\alpha-1} ,$$

where M is nominal money.

Equation (5) determines optimal labor supply. Each agent must infer the marginal utility of consumption next period to make this calculation. In market economies, it will be shown that this problem is identical to inferring the real interest rate.

From eq. (5), it is clear that agent j's input, L_t^j, will depend upon how large $N_te_t^j$ is relative to $E(C_{t+1}^j)^{\alpha-1}$. In general, C_{t+1}^j will depend upon the output of agent j at time t+1, X_{t+1}^j, as well as how much was borrowed at time t, B_t. If there was an insurance market, then each consumer could guarantee that his consumption C_{t+1}^j was equal to the economy-wide average $\frac{1}{n}\sum_{j=1}^{n} X_{t+1}^j$, where n is the number of agents. That is, an insurance company could always guarantee the economy-wide average output to each consumer if each consumer promises to deliver his output to the company each period. In this case, a firm's investment would be large at t if he expected the economy-wide average output to be low relative to his own productivity $N_te_t^j$. In the Appendix we show that it is feasible and optimal for each consumer to choose a borrowing policy such that $C_t^j = \frac{1}{n}\sum_{j=1}^{n} X_t^j$ when that consumption level satisfies the first-order conditions (3)-(5). That is, the borrowing policy which permits each consumer to consume the economy-wide average output satisfies the condition that expected discounted borrowing goes to zero. We shall return to the effect of this assumption at the end of this section.

Anticipating later developments, it is useful to write equations (3)-(5) in log linear form. To do so, we assume that r and C are log normally distributed. Further, we now let r, C, m, P denote the logrithim of the previously defined variable with the same level.

$$(6) \qquad (\alpha-1)C_t^j = E_t\left(r_t + (\alpha-1)C_{t+1}^j\right) + \ln \beta + \tfrac{1}{2}\mathrm{Var}\left(r_t + (\alpha-1)C_{t+1}^j\right)$$

$$(7) \qquad (a-1)\left(M_t^j - P_t\right) = \ln\left(\frac{R_t - 1}{R_t}\right) + (\alpha-1)C_t^j$$

$$(8) \qquad (1-\sigma)L_t^j = \left(N_t + e_t^j\right) + E_t\left((\alpha-1)C_{t+1}^j\right) + \ln \beta \quad .$$

For equilibrium, we require that the average of consumption equals the average of output:

$$(9) \qquad \frac{1}{n}\sum_j \exp(X_t^j) = \frac{1}{n}\sum_j \exp(C_t^j)$$

and that money supply equals money demand, where n is the number of agents. Setting $M^s \equiv 0$, this requires

$$(10) \qquad \frac{1}{n}\sum_j \exp(M_t^j) = 1 \quad .$$

We will first solve the model under the assumption that all agents have complete information and thus know N_t at time t. We define equilibrium to be a mapping from the state of the economy to the value of all endogenous variables which satisfy equations (6)-(10), given that agents form rational expectations of future values. The state of the economy may be described by the realization of yesterday's and today's aggregate productivity shocks, N_{t-1} and N_t.

Posit a solution by searching for parameters π_1, π_o such that

$$C_t^j = \pi_1 N_{t-1} + \pi_o \quad . \tag{11a}$$

From (8), $(1-\sigma)L_t^j = N_t + e_j + \pi_1(\alpha-1)N_t + \ln\beta + (\alpha-1)\pi_o$, substituting labor into the log production function

$$\begin{aligned} X_{t+1}^j &= N_t + e_t^j + \sigma L_t^j - \log\sigma \\ &= \frac{1+\pi_1(\alpha-1)\sigma}{1-\sigma} N_t + \frac{1}{1-\sigma} e_t^j + \frac{\sigma}{1-\sigma}(\ln\beta + (\alpha-1)\pi_o) - \log\sigma \end{aligned} \tag{11b}$$

We assume that there are enough agents so that

$$\frac{1}{n}\sum_{j=1}^{n} \exp\left(\frac{1}{1-\sigma} e_t^j\right)$$

can be treated as a constant. With this, we can use market clearing (9) and (11b) to get an identity in N_t.[1/] This identity can hold if and only if

$$\pi_1 = \frac{1+\pi_1(\alpha-1)\sigma}{1-\sigma} \tag{11c}$$

or

$$\pi_1 = \frac{1}{1-\sigma\alpha} \tag{11d}$$

Solving for the other real variables:

$$r_t = \frac{1-\alpha}{1-\sigma\alpha}\left(N_t - N_{t-1}\right) - \ln\beta - \tfrac{1}{2}\mathrm{Var}\Big((\alpha-1)(\pi_1 N_t + \pi_o)\Big) \tag{12}$$

$$L_t^j = \frac{\alpha}{1-\sigma\alpha} N_t + \frac{1}{1-\sigma} e_t^j + (\alpha-1)\pi_o + \ln\beta \quad . \tag{13}$$

Thus, the real variables can be described independently of nominal variables. The major qualitative property of this equilibrium is that an agent's response to relative production shocks is always greater than to aggregate disturbances. This stems from the observation that the proceeds from relative shocks can be consumed over many periods, to that the income effect of depressing the marginal utility of next period consumption is less pronounced. For the case of logarithmic utility of consumption (corresponding to $\alpha = 0$), the income effect associated with aggregate productivity shocks exactly offsets the substitution effect so that each agent's labor supply, and hence aggregate input, is invariant to this disturbance.

An alternative explanation of this phenomenon focuses on the role of interest rates and productivity for determination of current input. The present value of a marginal increase in current period labor is $\left(N_t + e_t^j\right) + (\sigma-1)L - r_t$, which obviously declines with current period interest rates. Since interest rates rise with expectations of greater aggregate consumption, interest rate movements retard the movements in labor supply associated with increased productivity.

To determine the qualitative properties of the price level and nominal interest rates, equation (7) is linearized around the average value of nominal interest rates R:

$$(14) \qquad (1-a)P_t = \lambda \ln R_t + \frac{\alpha-1}{1-\sigma\alpha} N_{t-1} + \text{constants}$$

where

$$\lambda = \frac{d \ln \frac{R-1}{R}}{d \ln R} = \frac{1}{R-1}$$

substituting $\left[\ln R_t = r_t + P_{t+1}^e - P_t\right]$ into (14), yields

$$(15) \qquad (1-a)P_t = \left(\frac{1-\alpha}{1-\sigma\alpha}\left(N_t - N_{t-1}\right) + P^e_{t+1} - P_t\right) + \frac{\alpha-1}{1-\sigma\alpha} N_{t-1} - \lambda \ln \beta$$
$$+ \text{ constants}$$

Positing a solution of the form

$$(16) \qquad P_t = \pi_2 N_{t-1} + \pi_3 N_t$$

and assuming $E_t(N_{t+1}) = 0$, it may be shown that

$$\pi_2 = \frac{(1+\lambda)(\alpha-1)}{(1-a+\lambda)(1-\sigma\alpha)}$$

$$\pi_3 = \frac{-a\lambda(1-\alpha)}{(1-a+\lambda)^2(1-\sigma\alpha)}$$

<u>For the rest of the paper, we ignore additive constants</u>. The expected rate of inflation from t to $t+1$ is $P^e_{t+1} - P_t = (\pi_2-\pi_3)N_t - \pi_2 N_{t-1} =$ $\frac{(\alpha-1)[(1-a)(1+2\lambda) + \lambda^2]}{(1-a+\lambda)^2(1-\sigma\alpha)} N_t + \frac{(1+\lambda)(1-\alpha)}{(1-a+\lambda)(1-\sigma\alpha)} N_{t-1}$, and thus the nominal rate at t

$$R_t = \frac{-a(1-a)(1-\alpha)}{(1-a+\lambda)^2(1-\sigma\alpha)} N_t + \frac{a(1-\alpha)}{(1-a+\lambda)(1-\sigma\alpha)} N_{t-1} .$$

Thus, the relationship between real and nominal interest rates depends on a, the elasticity of the utility of real balances. As is typical, the case of logarithmic utility $(a = 0)$ is the borderline. In this case, the nominal rate is a constant, meaning the approximation made in (14) is exact. Neither the nominal interest nor the current period price level is affected by N_t. The ratio of real balances to consumption is a constant. When the elasticity is greater than 1 $(a < 0)$, the real and nominal rate will move together, and the opposite will occur when the elasticity of the marginal utility of cash balances is less than 1 $(a > 0)$.

This model can demonstrate that a Phillips curve relationship between employment and inflation is compatible with an economic structure in which money is totally passive. The covariance between employment and the current period price level is $-a\lambda(1-\alpha)\alpha/(1-a+\lambda)^2(1-\sigma\alpha)^2$, which is positive for $-a\alpha > 0$. A rise in N_t causes a rise in the real interest rate and a rise in inputs L_t for $\alpha > 0$. If $a < 0$, the real and nominal rates move together and this causes a decrease in the demand for money which raises the current price level. Thus investment productivity shocks which move the nominal and real interest rates together tend to raise current prices. Note that since $\alpha < 1$, a rise in N_t always lowers the expected rate of inflation, $P^e_{t+1} - P_t$. Thus labor input is always negatively correlated with the expected rate of inflation for a given N_{t-1} for $\alpha > 0$.

Throughout this section, we have assumed that consumers insure themselves against idiosyncratic risks, using the bond market. If this was not possible, then (11a) would have to be replaced by a policy which related C^j_t to N_{t-1} and e^j_{t-1}, say $C^j_t = \pi'_1 N_{t-1} + \pi'_2 e^j_{t-1}$. If we put a fixed bound on borrowing, then a solution to the maximum problem would involve a borrowing policy $B(N_{t-1}, e^j_{t-1})$ and a money policy $M(N_{t-1}, e^j_{t-1})$. We would then use the budget constraints and the first-order conditions to solve for these policies. As long as borrowing can cushion some of the effect of idiosyncratic shocks, C^j_t will depend on N_{t-1} as well as $N_{t-1}e^j_t$. When this is the case, our main result that investment depends positively on the ratio of N_{t-1} to $N_{t-1}e^j_t$ will still be true.

II. The Effects of Incomplete Information

The case of logarithmic utility for cash balances ($a = 0$) raises the possibility that financial nominal variables will not permit each agent to distinguish between relative and aggregate productivity shocks. For this case, the nominal interest rate is a constant, and the price level depends only on current period consumption. Thus, each agent can utilize only his own observation of his technological shock $(N_t + e_t^j)$ to infer aggregate disturbances. This will lend to greater variability of labor input, output, consumption than under complete information.

When $a = 0$, the utility of real balances is logarithmic. This implies that the loss in the utility of real balances from giving up one dollar is independent of the price level. Recall that

$$\frac{1}{P_t} L'\left(\frac{M_t}{P_t}\right) + \beta E u'(C_{t+1}) \frac{1}{P_{t+1}} = u'(C_t) \frac{1}{P_t}$$

is the first-order condition for real balances, where $L(m)$ is the utility of real balances and all variables are in levels (not logarithims). Therefore, if. for each t, $u'(C_t)/P_t$ is constant when N_{t-1} causes C_t to change, the marginal value of a dollar is left unchanged. Since the nominal interest rate satisfies $L'(M_t/P_t) = [(R_{t-1})/R_t]u'(C_t)$, the assumption that $L(\cdot)$ is logarthmic implies that R_t is constant since $u'(C_t)/P_t$ is constant. Thus economy-wide productivity is always associated with enough deflation to keep the nominal rate constant. This prevents the nominal rate from revealing the real rate (i.e. N_t) to traders.

Since nominal interest rates do not reveal N_t, traders will have asymmetric information. To compute the asymmetric information equilibrium, posit a solution (ignoring additive constants):

$$(17) \qquad C_t^j = \bar{\pi}_1 N_{t-1} \quad .$$

Equilibrium labor supply is given by

$$(1-\sigma)L_t^j = N_t + e_t^j + E[(\alpha-1)\bar{\pi}_1 N_t |\ N_t + e_t^j] \quad . \tag{18}$$

Assuming both N_t and e_t^j are zero mean independent variables, then

$$E[N_t |\ N_t + e_t^j] = \frac{1}{1+y}(N_t + e_t^j) \quad , \tag{19}$$

where $y = \mathrm{Var}(e_t^j)/\mathrm{Var}(N_t)$.

Substituting into the production function yields

$$X_{t+1}^j = \frac{1}{1-\sigma}\left(1 + \frac{\sigma\bar{\pi}_1(\alpha - 1)}{1+y}\right)(N_t + e_t^j)$$

Commodity market clearing implies

$$\bar{\pi}_1 = \frac{1}{1-\sigma}\left(1 + \frac{\sigma\bar{\pi}_1(\alpha - 1)}{1+y}\right) \quad ,$$

so that

$$\bar{\pi}_1 = \frac{1+y}{1-\sigma\alpha + y(1-\sigma)} \tag{20}$$

When there are no relative shocks $(y = 0)$, $\bar{\pi}_1$ is the same as the full information case π_1, see (11d). As y increases, π_1 increases monotonically to a maximum value of $1/(1-\sigma)$.

Equilibrium labor supply is

$$L_t^j = \frac{\alpha + y}{1-\sigma\alpha + y\ (1-\sigma)} \left(N_t + e_t\right)$$

Thus, under incomplete information agents respond to aggregate shocks more, and to relative shocks less, than under full information. This response is similar to the findings of Lucas and Barro who analyzed the impact of increasing monetary uncertainty on the optimal response of labor supplies.

The real rate of interest (i.e. the realized marginal rate of substitution between consumption at $t+1$ and t) is now given by

$$r_t = \frac{(1+y)(1-\alpha)}{1-\sigma\alpha+y(1-\sigma)} \left(N_t - N_{t-1}\right) ,$$

which has a greater variance than under full information. Note that r_t is unobservable at time t because only nominal bonds are traded.

Finally, it may be verified that

$$P_t = \frac{(\alpha-1)\ (1+y)}{1-\sigma\alpha + y\ (1-\sigma)}\ N_{t-1}$$

and that R_t , the nominal interest rate is constant, as before.

Thus, incomplete information leads the representative trader to over state the idiosyncratic part of the shock. He believes he can spread the proceeds from this disturbance over several time periods. However, in the aggregate this is not possible and interest rate movements must be large enough to induce large consumption savings.

III. The Effects of Monetary Shocks

The constancy of the nominal rate is not robust. In general, when there is only a single disturbance to the aggregate economy, the nominal interest rate and price level will be capable of transmitting the value of this disturbance to each agent. In this section, the model will be modified to incorporate random components to money demand which are not directly observable. By influencing the nominal rate of interest, and the representative agent's expectation of the real interest rate, such nominal disturbances may have real effects. This may arise even when all current period commodity prices are directly observable.

Suppose agents' preferences are modified such that the utility of agent j from holding m units of real balances from t to $t+1$ is

$\left(\theta_t\ \rho_t^j\right) \frac{1}{a} \left(m_t^j\right)^a$ where $\theta_t\ \rho_t^j$ is a random variable. We assume that in period t, each agent knows his own shock for period t, $\theta_t\ \rho_t^j$. Individuals do not observe θ and ρ^j separately, so they cannot distinguish relative from aggregate money demand shocks. This modification will alter the equilibrium paths of prices and nominal interest rates. Because interest rates are useful predictors of future consumption, money shocks will effect real consumption and labor supply.

The first order condition for optimal money holdings by agent j in t are

$$(a-1)\ \left[M_t^j - P_t\right] = \ln \frac{R_t-1}{R_t} + (\alpha-1)\ C_t^j - \left[\theta_t + \rho^j\right]$$

Aggregating over agents and setting money supply equal to money demand requires that in each period, ignoring the constants, such as nominal money:

$$(1-a)\ P_t = \ln \frac{R_t-1}{R_t} + (\alpha-1)\ C_t^j - \theta_t \tag{21}$$

Under the stated assumption that output and current consumption are directly observable, this formulation implies that the current period money demand shock, θ_t, is knowable to each agent from his observation of current equilibrium prices and nominal interest rates. In order for money shocks to have real effects, it is sufficient that expectations of future disturbances affect current period prices. We assume that at time t each trader j observes his future money demand shock $\theta_{t+1}\rho_{t+1}^j$. Thus we augment the relevant variables which define the state of the economy to include θ_t and θ_{t+1}, as well as N_{t-1} and N_t. We assume that the productivity shocks are independent of the money demand shocks.

As before, we posit an equilibrium in which each agent consumes average income and thus implicitly diversifies all idiosyncratic risk

$$C_t^j = \gamma_1 N_{t-1} + \gamma_2 \theta_t$$

For this to be an equilibrium, the necessary first-order condition (3) implies,

$$(\alpha-1)C_t^j = E^j \left[\log R_t + P_t - \tilde{P}_{t+1} + (\alpha-1)\left(\gamma_1 \tilde{N}_t + \gamma_2 \tilde{\theta}_{t+1}\right)\right] + \ln \beta$$

and hence, aggregating over agents

$$\text{(23)} \qquad \log R_t = -P_t + (\alpha-1)\ C_t^j - E^*\left[\ (\alpha-1)\left(\gamma_1 \tilde{N}_t + \gamma_2 \tilde{\theta}_{t+1}\right) - \tilde{P}_{t+1}\right] .$$

where E^* denotes the average over the j agents of the expected value of the expression in the parenthesis, conditional each agent j's information. The equilibrium value of this expression may be computed by the technique in Weiss [1980]. In particular, let

$$Z_{t+1} \equiv (\alpha-1)[\gamma_1 \tilde{N}_t + \gamma_2 \tilde{\theta}_{t+1}] - \tilde{P}_{t+1} \quad ,$$

then we can show that, in a Rational Expectations equilibrium,

$$\text{(24)} \qquad E[Z_{t+1}|\ R_t,\ P_t,\ C_t^j,\ \theta_{t+1}\rho_{t+1}^j,\ N_t e_t^j] = E[Z_{t+1}|\ N_t,\ \theta_{t+1}] \quad .$$

To see this, imagine an (artificial) economy where all traders observe $E[Z_{t+1}|N_t,\theta_{t+1}]$. Let R_t^a, P_t^a denote the equilibrium for this fully-informed (artificial) economy. By showing that P_t^a, R_t^a gives people the same belief that $E[Z_{t+1}|N_t,\theta_{t+1}]$ gives them, we will have shown that P_t^a, R_t^a is a Rational Expectations equilibrium. Note that P_t^a, R_t^a satisfy (23), where E^* is replaced by $E[Z_{t+1}|N_t,\theta_{t+1}]$. Hence R_t^a, P_t^a, C_t^j reveal $E[Z_{t+1}|N_t,\theta_{t+1}]$, so

$$\text{(25)} \qquad E[Z_{t+1}|\ R_t^a,\ P_t^a,\ C_t^j] = E\left[Z_{t+1}|\ E[Z_{t+1}|\ N_t,\ \theta_{t+1}]\right] .$$

But

$$E\left[Z_{t+1} \mid E[Z_{t+1} \mid N_t, \Theta_{t+1}]\right] = E[Z_{t+1} \mid N_t, \Theta_{t+1}]. \tag{26}$$ [2/]

Combining (25) and (26) shows that (24) holds for P_t^a, R_t^a. Note that we ignore $\Theta_{t+1}\rho_{t+1}^j$, $N_t e_t^j$ since they add no information once $E[Z_{t+1}|N_t,\Theta_{t+1}]$ is observed. Further, by construction R_t^a, P_t^a clear markets when everyone observes $E[Z_{t+1}|N_t,\Theta_{t+1}]$. Hence there is a Rational Expectations equilibrium where (24) holds.[3/]

To solve the equilibrium, we note that the money demand equation does not require any expectations of future variables, given current nominal interest rates. Substituting the nominal interest rate into equation (21) and taking a first-order linear approximation around $\bar{R}$, as we did in (14),

$$\begin{aligned}(1-a)\,P_t = \lambda\Big[(\alpha-1)\left(\gamma_1 N_{t-1} + \gamma_2 \theta_t\right) - (\alpha-1)\left(\gamma_1 N_t + \gamma_2 \theta_{t+1}\right) \\ + P_{t+1}^e - P_t\Big] + (\alpha-1)\left(\gamma_1 N_{t-1} + \gamma_2 \theta_t\right) - \theta_t\ ,\end{aligned} \tag{27}$$

where $P_{t+1}^e \equiv E[P_{t+1} \mid N_t, \Theta_{t+1}]$.

Solving this by the familiar method of unknown coefficients, the equilibrium price path is

$$\begin{aligned}(1-a+\lambda)\,P_t = {} & N_{t-1}\left[(\alpha-1)\ (1+\lambda)\gamma_1\right] \\ & + \theta_t\left[(\alpha-1)\ (1+\lambda)\ \gamma_2 - 1\right] \\ & + N_t\left[\gamma_1 a\lambda(\alpha-1)/\ (1-a+\lambda)\right] \\ & + \theta_{t+1}\left[\lambda(\alpha-1)\gamma_2 a - \lambda/(1-a+\lambda)\right].\end{aligned}$$

And the full information expectation of the price level next period (given $E\left(\theta_{t+2}\right) = E\left(N_{t+1}\right) = 0$)

$$\begin{aligned}(1-a+\lambda)\,P_{t+1}^e = {} & N_t\left[(\alpha-1)\ (1+\lambda)\ \gamma_1\right] \\ & + \theta_{t+1}\left[(\alpha-1)\ (1+\lambda)\ \gamma_2 - 1\right] = (\alpha-1)(1+\lambda)C_{t+1} - \theta_{t+1}.\end{aligned} \tag{28}$$

Each agent must decide optimal labor supply. Since he knows his own production technology exactly, he must use market signals to infer the marginal utility of next period's consumption for this calculation.

The information extraction problem faced by each agent has a simple interpretation. From his knowledge of current period output, the price level and the nominal interest rate, each agent knows the full information value of $-P_{t+1} + U'(C_{t+1})$, as we showed just after eq. (23). From this, he must extract the expected value of $U'(C_{t+1})$.

We will compute this equilibrium under the assumption that agents ignore their own private information and use only market signals. It may be shown that this is a limiting case as the variance of idiosyncratic to aggregate disturbance becomes large.

Market signals permit agents to know $-U'(C_{t+1}) + P^e_{t+1}$, which equals

$$\frac{a(\alpha-1)}{1 - a + \lambda} C_{t+1} - \frac{1}{1 - a + \lambda} \Theta_{t+1} \quad .$$

Equivalently, by performing a linear transformation (assuming $a \neq 0$), agents know

$$C_{t+1} + \frac{\Theta_{t+1}}{a(1-\alpha)} \equiv C_{t+1} + \Theta'_{t+1} \quad .$$

From (8), optimal labor supply is

$$(1-\sigma)L^j_t = (N_t + e^j_t) + (\alpha-1)E[C_{t+1} \mid C_{t+1} + \Theta'_{t+1}] \quad .$$

Thus aggregate output is

$$X_{t+1} = \frac{N_t}{1 - \sigma} + \frac{\sigma}{1 - \sigma} (\alpha-1)E[C_{t+1} \mid C_{t+1} + \Theta'_{t+1}] \quad .$$

The conditional expectation can be evaluated using standard formula for jointly Normal variables

$$X_{t+1} = \frac{N_t}{1 - \sigma} + \frac{\sigma}{1 - \sigma} (\alpha-1)b(C_{t+1} + \Theta'_{t+1}), \tag{29}$$

where

$$(30) \qquad b = \frac{\mathrm{cov}(C_{t+1}, C_{t+1} + \Theta'_{t+1})}{\mathrm{Var}(C_{t+1} + \Theta'_{t+1})} \; .$$

Since $C_{t+1} = \gamma_1 N_t + \gamma_2 \Theta_{t+1}$, (29) can be written as

$$(31) \qquad X_{t+1} = \left(\frac{1}{1-\sigma} + \frac{\sigma}{1-\sigma}(\alpha-1)b\gamma_1\right) N_t + \frac{\sigma}{1-\sigma}(\alpha-1)b\left(\gamma_2 + \frac{1}{a(1-\alpha)}\right)\Theta_{t+1} \; .$$

Commodity market clearing implies $C_{t+1} \equiv X_{t+1}$ can be used to solve for γ_1 and γ_2:

$$(32) \qquad \gamma_1 = \frac{1}{1-\sigma-\sigma(\alpha-1)b}$$

$$(33) \qquad \gamma_2 = \frac{-b\sigma/a}{1-\sigma-\sigma(\alpha-1)b} \; .$$

Substituting $C_{t+1} = \gamma_1 N_t + \gamma_2 \Theta_{t+1}$ into equation (30), and assuming independence of N_t and Θ_{t+1}, it may be seen that

$$(30') \qquad b = \frac{\gamma_1^2 \mathrm{VAR}(N_t) + \gamma_2 a(1-\alpha)(\gamma_2 a(1-\alpha)+1)\mathrm{VAR}\;\Theta'_{t+1}}{\gamma_1^2 \mathrm{VAR}(N_t) + (\gamma_2 a(1-\alpha)+1)^2 \mathrm{VAR}\Theta'_{t+1}}$$

$$= \frac{\mathrm{VAR}(N_t) - b(1-\alpha)(1-\sigma)\sigma \mathrm{VAR}\;\Theta'_{t+1}}{\mathrm{VAR}(N_t) + (1-\sigma)^2 \mathrm{VAR}\;\Theta'_{t+1}} \; .$$

Hence

$$(34) \qquad b = \frac{\mathrm{VAR}(N_t)}{\mathrm{VAR}(N_t) + (1-\sigma)(1-\sigma\alpha)\mathrm{VAR}\;\Theta'_{t+1}} \; .$$

From (34), it may be seen that $0 < b < 1$. This implies that

$$(35) \qquad \frac{1}{1-\sigma\alpha} < \gamma_1 < \frac{1}{1-\sigma} \; .$$

The left-hand side inequality implies that asymmetric information attentuates the impact of aggregate productivity shocks on aggregate labor supply and output, relative to the full information situation considered in the previous section, see e.g. (11d). The right-hand inequality shows that the response to aggregate productivity disturbance when interest rates are noisy signals of future consumption is less pronounced than when market signals provide no information, corresponding to $a = 0$, as considered previously. This is derived by setting $y = \infty$ in equation (20). Making y large corresponds to the assumption in this section that the idiosyncratic shocks are so large that each consumer ignores the information about aggregate disturbances in his own observation of $\theta_{t+1} + \rho^j_{t+1}$ and $N_t + e^j_t$.

Writing γ_2 as $(-b\sigma/a)\gamma_1$, it can be seen that γ_2 has the opposite sign of a. In Section I, it was shown that, in the absence of money demand disturbances, the real and nominal rate move together if $a < 0$, and inversely if $a > 0$. In the present context, increases in $t + 1$ period money demand lowers the expected future price level, and hence, for any current price level and consumption, lowers the nominal interest rate. Substituting equations (28), (32) and (33) into equation (23) permits an explicit solution for the nominal interest rate

$$\text{(36)} \qquad \ln R_t = -P_t + (\alpha-1)C_t + \frac{(\alpha-1)\gamma_1 a N_t}{1-a+\lambda} - \frac{\gamma_1(1-\sigma)}{1-a+\lambda}\theta_{t+1} \quad .$$

Note that (32) and (34) can be used to solve for γ_1, yielding

$$\text{(37)} \qquad \gamma_1 = \frac{\mathrm{VAR}(N_t) + (1-\sigma)(1-\sigma\alpha)\mathrm{VAR}\ \theta'_{t+1}}{(1-\sigma\alpha)\mathrm{VAR}(N_t) + (1-\sigma)^2(1-\sigma\alpha)\mathrm{VAR}\ \theta'_{t+1}} > 0 \quad .$$

Eq. (36) can be used to describe the inferences consumers make regarding N_t and θ_{t+1}, when they observe R_t, P_t and C_t. For each P_t and C_t, R_t and N_t move together when $a < 0$. Thus, when $a < 0$, a positive money demand shock, θ_{t+1}, causes R_t to be low for a given P_t and C_t. Consumers will attribute the low R_t partially to a low N_t. Since this implies a low C_{t+1}, $u'(C_{t+1})$ is high and each agent, believing his relative position favorable, expands output. The increase in output lowers the price level next period. Thus, the money demand shock lowers the ex ante expected return, increases production, but raises the ex post realized return. The last part follows because the increased investment at t leads consumption to be high at $t+1$, causing a low realized real interest rate.

In summary, this section has presented a model where an individual firm invests a lot when its own productivity is high relative to the anticipated real rate of interest in the economy. In the presence of money demand shocks, the nominal interest rate varies both due to variations in the *ex ante* real rate and due to variations in the expected rate of inflation. Thus consumers, when they observe a high nominal rate, attribute part of this to the *ex ante* real rate being high. This decreases investment and output.

IV. Conclusions and Extensions

The model presented here shares several features of traditional Keynesian analysis as well as the more recent informationally based equilibrium theories. Like Keynesian theory, the primary determinant of economic activity is the relationship between the productivity of investment (analagous to Wicksell's "natural rate") and the perceived real rate available on financial assets (the "market rate"). As in the *General Theory*, monetary phenomenon can affect the financial rate and thereby influence real activity. There is a presumption that this is an inverse relationship: lower nominal rates imply, for any given productivity of investment, lower expected real rates and thus higher levels of economic activity. This relationship is very different from the Lucas-Barro intertemporal substitution model. In those papers fluctuations are thought to stem from the response of labor to perceived temporary abnormal rates of return. The presumption is that employment responds positively to anticipated real rates.

However, like more recent theories of the Phillips curve, the link from money to the real economy arises only because information is incomplete and agents use nominal interest rates as imperfect predictors of structural variables. However, in our paper we assume that all agents can observe the money supply. Nominal shocks have an effect because of asymmetric information about future money demands (and hence the rate of inflation).

Unlike the Lucas model, demand plays a passive role in our model. All fluctuations are attributable to changes in the perceived technical productivity of investment. A Phillips curve can be generated by these types of disturbances even with complete information. Incomplete information will, however, strengthen this relationship by invoking larger output response to changes in real and nominal variables.

This is not to deny that aggregate demand shocks are important, nor that confusion between relative and absolute demand plays a central role in generating cycles. It would be possible to incorporate such elements into our model. This would not alter our major conclusions that perceived relative shocks cause larger output movements than aggregate shocks because of the perceived opportunity to consume the proceeds over several periods through the capital market. Furthermore, the inability of financial markets to convey aggregate information leads agents to overstate movements in their relative position and thus vary input levels more than under full information.

Emphasis on expectations of the productivity of new investments as generating movements in interest rates and output has a number of potentially testable conclusions. Perhaps the most direct conclusion is that the current nominal interest rate is a sufficient statistic for predicting the product of the future price level times the marginal utility of consumption next period. This concept, suggested in Grossman (1976), implies that no other potentially knowable information can aid in predicting this quantity.

Perhaps the most important prediction of our model is that aggregate investment will be relatively insensitive to predictions of real rates of return based on market generated signals. This arises because of the offsetting income and substitution effects of perceived productivity shocks. That component of investment not related to variations in the expected real rate will, however, be useful in predicting ex post real rates. This effect stems from the fact that investment is related to unperceived aggregate productivity shocks which ultimately produce greater output, lower prices and hence higher real rates.

As is always the case with noisy Rational Expectations models, the introduction of more assets (e.g. an indexed bond) will eliminate the noise and prices will be fully revealing about the underlying aggregate shocks. Of course, more noise will always eliminate the effect of the additional price. (For example, if indexed bonds are introduced, then random risk preferences would be sufficient noise to obscure real interest rate movements.) Thus it is not possible to determine on a priori grounds whether there are "enough" prices. In our context, what matters is whether all current prices are a sufficient statistic for the aggregate of traders' information about capital's productivity.[4/]

APPENDIX

The text used the result that a bond market can substitute for an insurance market in an infinite horizon model. In particular, assume x_t is the per capita output of consumption goods at time t; agent i's income is $x_t + \varepsilon_t^i$; and $\frac{1}{n}\sum_{i=1}^{n} \varepsilon_t^i = 0$. We now show that the solution to the problem

$$\text{(A1)} \qquad \text{Max } E_o \sum_{t=0}^{\infty} \beta^t u(C_t^i)$$

$$\text{subject to} \quad P_oC_o^i + B_o^i \leq P_oX_o$$

$$P_tC_t^i + B_t^i \leq R_{t-1}B_{t-1}^i + P_tX_t^i \qquad t=1,2,...$$

and subject to

$$\lim_{T\to\infty} E_o\left[\left(B_T^i/\prod_{j=0}^{T-1} R_j\right) \Big| \Omega_T\right] = 0 \ ,$$

where $\Omega_T = (X\ ,X_1,...,X_T,P\ ,...,P_T,R\ ,...,R_T)$ is

$$\text{(A2)} \qquad \bar{C}_t^i = X_t \qquad \text{for all } t$$

where we assume that nominal interest rates are given by

$$\text{(A3)} \qquad R_t = \frac{u'(X_t)/P_t}{\beta E_t[u'(X_{t+1})/P_{t+1}]}$$

and hence real rates (realized at $t + 1$) are

$$\text{(A4)} \qquad r_t = \frac{P_tR_t}{P_{t+1}} = \frac{u'(X_t)}{\beta E_t[u'(X_{t+1})/P_{t+1}]P_{t+1}} \ .$$

Note that the policy $\bar{C}_t^i$ satisfies the first-order condition

$$\text{(A5)} \qquad u'(\bar{C}_t^i) = \beta E_t\tilde{r}_t u'(\tilde{\bar{C}}_{t+1}^i) \ .$$

We first show that $\bar{C}_t^i$ satisfies the constraints in (A1). The evolution of borrowings is given by

$$\text{(A6)} \qquad \bar{B}_t^i = P_t \varepsilon_t^i + R_{t-1} \bar{B}_{t-1}^i$$

$$\bar{B}_o^i = 0$$

and thus

$$\text{(A7)} \qquad \frac{\bar{B}_t^i}{\prod_{i=0}^{T-1} R_i} = \sum_{j=0}^{j=T} \left(\prod_{K=0}^{K=j-1} R_K \right)^{-1} (P_j \varepsilon_j^i)$$

Thus, if we assume $E_o(\varepsilon_t^i) = 0$,

$$\text{(A8)} \qquad E_o \left[\frac{\bar{B}_t^i}{\prod_{i=0}^{T-1} R_i} \middle| \Omega_T \right] = \sum_{j=0}^{j=T} \frac{P_j E_o(\varepsilon_j^i)}{\prod_{k=0}^{k=j-1} R_k} = 0. \qquad \text{Q.E.D.}$$

Now, we show that no policy which satisfies the constraint set in (A1) can provide higher expected discounted utility.

Let $(\hat{C}_t^i, \hat{B}_t^i)$ be feasible. Assume further that $u(\cdot)$ is strictly concave so that

$$\text{(A9)} \qquad u(y) - u(x) > u'(y)(y-x) \quad \text{for} \quad x \neq y \quad .$$

Then

$$\Delta_T = E_o \left(\sum_{t=0}^{T} \beta^t \, u(\bar{C}_t^i) \right) - E_o \left(\sum_{t=0}^{T} \beta^t \, u(\hat{C}_t^i) \right)$$

$$= E_o \sum_{t=0}^{T} \beta^t \, [u(\bar{C}_t^i) - u(\hat{C}_t^i)]$$

$$> E_o \sum_{t=0}^{T} \beta^t \, u'(\bar{C}_t^i)(\bar{C}_t^i - \hat{C}_t^i)$$

if $\langle \hat{C}_t^i \rangle \neq \langle \bar{C}_t^i \rangle$, so

$$\text{(A10)} \qquad \Delta_T > E_o \sum_{t=1}^{T} \beta^t u'(\bar{C}_t) \left[-\frac{\bar{B}^i_t}{P_t} + \frac{\hat{B}^i_t}{P_t} + r_{t-1}\left(\frac{\bar{B}^i_{t-1}}{P_{t-1}} - \frac{\hat{B}^i_t}{P_{t-1}}\right)\right] + u'(\bar{C}^i_o)(\bar{C}^i_o - \hat{C}_o)$$

where the last step follows from the fact that $(\bar{C}^i_t, \bar{B}^i_t)$ and $(\hat{C}^i_t, \hat{B}^i_t)$ satisfy the budget constraint.

Using (A5),

$$\text{(A11)} \qquad E_o \sum_{t=1}^{T} \beta^t u'(\bar{C}^i_t) r_{t-1} \left(\frac{\bar{B}^i_{t-1}}{P_{t-1}} - \frac{\hat{B}^i_{t-1}}{P_{t-1}}\right)$$

$$= E_o \sum \beta^t E_{t-1} u'(\bar{C}^i_t) r_{t-1} \left(\frac{\bar{B}^i_{t-1}}{P_{t-1}} - \frac{\hat{B}^i_{t-1}}{P_{t-1}}\right) \quad ,$$

since the information set at time 0 is coarser than at $t-1$.

$$= E_o \sum_{t=1}^{T} \beta^t \frac{1}{\beta} u'(\bar{C}^i_t) \left(\frac{\bar{B}^i_{t-1}}{P_{t-1}} - \frac{\hat{B}^i_{t-1}}{P_{t-1}}\right) \quad .$$

Substituting (A11) into (A10) to get

$$\text{(A12)} \qquad \Delta_T > u'(\bar{C}^i_o) \left[(\bar{C}^i_o - \hat{C}^i_o) + \left(\frac{B^{-i}_o}{P_o} - \frac{\hat{B}^i_o}{P_o}\right)\right] - E_o \beta^T u'(\bar{C}^i_T) \left(\frac{\bar{B}^i_T}{P_T} - \frac{\hat{B}^i}{P_T}\right)$$

so

$$\text{(A13)} \qquad \Delta_T > - E_o \beta^T u'(\bar{C}^i_t) \left(\frac{\bar{B}^i_T - \hat{B}^i_T}{P_T}\right) = - E_o E \left[\beta^T u'(\bar{C}^i_t) \left(\frac{\bar{B}^i_T - \hat{B}^i_T}{P_T}\right) \middle| \Omega_T \right] \quad .$$

Note that

$$\frac{\bar{B}^i_T - \hat{B}^i_T}{\prod_{j=\theta}^{T-1} R_j} = \beta^T (\bar{B}^{i}_T - \hat{B}^i_T) \frac{u'(X_T)/P_T}{u'(X_o)/P_o} \times \prod_{j=1}^{T} \frac{E_{j-1}[u'(X_j)/P_j]}{u'(X_j)/P_j} \quad .$$

By the borrowing constraint in (A1),

$$0 = \lim_{T\to\infty} E \left[\frac{\bar{B}_T - \hat{B}^i_T}{\prod_{j=0}^{T-1} R_j} \middle| \Omega_T \right] = \lim_{T\to\infty} \left[\prod_{j=1}^{T} \frac{E_{j-1}[u'(X_j)/P_j]}{u'(X_j)/P_j} \right] \frac{P_o}{u'(X_o)}$$

$$\times E_o \left[\beta^T u'(\bar{C}^i_T) \left(\frac{\bar{B}^i_T - \hat{B}^i_T}{P_T}\right) \middle| \Omega_T \right]$$

which implies

$$\lim_{T\to\infty} \Delta_T > 0 \quad ,$$

which proves $(\bar{C}_t^i, \bar{B}_t^i)$ is optimal since $\lim_{T\to\infty} \Delta_T$ is the limit of the difference in expected utility between the two policies. Q.E.D.

This shows that it is a competitive equilibrium for interest rates to be given by (A4), and each consumer's consumption to be X_t in period t. All idiosyncratic risks ε_t^i are diversified away by banks via loan markets.

Note that we could not have proved this result if X_o had been different for different agents. This is because an agent with a relatively high X_o is better off over his whole life than an agent with a low X_o. Since loan markets can only be used to insure against low lifetime discounted income, this insurance is impossible <u>after</u> each trader already knows his relative position. (This is similar to the difficulty of getting cancer insurance in a world where everyone knows exactly who is going to get cancer.) If there were complete insurance markets for each state and date, then traders could insure against a low income in a particular date and state even if one of them has a higher expected lifetime wealth than others.

FOOTNOTES

1/ $$\exp\left(\frac{1+\pi_1(\alpha-1)\sigma}{1-\sigma}\cdot N_t\right)\frac{1}{n}\sum_{j=1}^{n}\exp\left(\frac{1}{1-\sigma}e_t^j+\frac{\sigma}{1-\sigma}(\ln\beta+(\alpha-1)\pi_o)\right)\equiv$$

$$\exp(\pi_1 N_t)\exp(\pi_o) \; .$$

2/ For any two random variables, X,Y:

$$E[X|E(X|Y)] = E[E(X|Y)|E(X|Y)] = E[X|Y] \; .$$

3/ See Grossman [1978] for more details about how the artificial equilibrium economy equilibrium can be used to find a Rational Expectations equilibrium.

4/ See Grossman (1977) for an argument which shows that when information about asset returns is privately costly to acquire, then there cannot be enough prices for this information to be revealed. For if prices did reveal all the information, traders would not earn a return on its acquisition costs.

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