MAJORITY CHOICE DETERMINATION OF THE LEVEL OF NON-NEUTRAL GOVERNMENT DEBT

by

Simon Benninga*

Working Paper No. 14-80

I wish to thank Yakov Amihud, Elhanan Helpman, Assaf Razin, and members of the Finance Workshop of the Wharton School, University of Pennsylvania, for helpful comments made on an earlier version of this paper. Remaining errors are mine. This research was supported in part by a grant from the Israel Institute of Business Research, Faculty of Management, Tel-Aviv University.

*The University of Pennsylvania and the University of Tel-Aviv.

MAJORITY CHOICE DETERMINATION OF THE LEVEL OF NON-NEUTRAL GOVERNMENT DEBT

by

Simon Benninga

ABSTRACT

With incomplete financial markets government debt may be non-neutral even if individuals perceive that the value of the debt is the discounted value of future taxes and there are no opportunities for individuals to shift the tax burden among themselves. Non-neutrality arises because of the effects of a change in future taxation on individuals' consumption sets. There exists a taxation vector which cannot be defeated by a majority vote of consumers, when these are asked about their preferences as between this taxation vector and any other taxation vector. The equilibrium is non-manipulable and thus stable.

Keywords: government debt, majority choice.

MAJORITY CHOICE DETERMINATION OF THE LEVEL OF NON-NEUTRAL GOVERNMENT DEBT

1. Introduction

The conclusion of a long-lived debate on the real effects of government debt appears to be that as long as individuals perceive that the value of the government's debt is the discounted value of the future taxes used to finance this debt, and as long as there are no opportunities to shift the tax burden from one generation to another (or among individuals), then government debt does not have any real effects.[1]

In this paper we show that with the introduction of financial markets and uncertainty, the two above conditions are not in general sufficient to guarantee the neutrality of government debt. We consider a two-period, non-monetary model in which uncertainty in the second period is represented by the occurrence of one of a number of states.[2] Individuals agree on the number of states and their classification, but not necessarily on their probabilities. They do, however, agree on the values of all market variables, conditional on the occurrence of each state.[3] We represent government as an agent which consumes a fixed amount of the economy's single good in the first period, and which finances this consumption through first-period lump-sum taxes and sales of bonds. Bondholders are repaid out of state-dependent lump-sum taxes collected in the second period. Shares in firms with state-dependent production technologies provide the remaining securities in the model.

Define an exchange-production equilibrium (EPE) as a set of prices for the single commodity and for the firms, a consumption vector and portfolio for each of the economy's consumers, an input of the single commodity for each of the economy's firms, and a lump-sum state-dependent taxation vector for each of the consumers such that there is equilibrium in the commodity and securities markets and such that each firm chooses its inputs by maximizing some well-defined function of its initial shareholders' commodity consumption vectors. Then an alternative lump-sum, state dependent taxation vector will be called neutral with respect to a given EPE if there exists another EPE in which all of the real variables (i.e., consumers' consumption vectors and firms' inputs) are the same.

In a complete market we show that a necessary and sufficient condition for taxation neutrality is that the state-dependent net present value of individual taxation remains the same for each consumer. This net present value is well-defined, since in a complete market each consumer's implicit state prices will be equal and can be determined from the market prices and the return vectors of the securities. This condition is, furthermore, eminently logical: As long as the alternative taxation vector has the property that it leaves every consumer's taxation burden unchanged, it will be neutral with respect to the EPE.

Where markets are incomplete, the situation is considerably more complicated. In Section 8 we show that the constancy of the state-dependent net present value of individual taxation (now using each individual's implicit state prices) remains a necessary condition for taxation neutrality, even in an incomplete market. It need not, however, be a sufficient condition, and this for one of two reasons: First, the market may be incomplete because there are too few securities to span the set of states. Second, even if there are a sufficient number of securities in the market, incompleteness may be caused by market restrictions on short selling which cause consumers to choose their portfolios at a corner solution.

In Section 10 we consider this second case. In order to abstract from the question of the distribution of the burden of taxation, we assume that each consumer pays a fixed proportion of the total lump-sum taxes in the first period and

and in each state of the second period; when full short sales of the government bonds are allowed, this is readily shown to mean that a change in the taxation vector cannot lead to any shifting in the individual tax burdens. Furthermore, the fact that we allow short sales of the government bond results in all individuals accurately perceiving the government debt as being the net present value of second period taxes. In the absence of short sales of firms, we show that individuals have preferences over the vector of total lump-sum (both in this first period and in every state of the second period) taxation.[4] Moreover, we show that these preferences are essentially over a uni-dimensional variable and are single-peaked. so that there exists a lump-sum taxation vector which is favored by a majority of individuals over any alternative proposal preferred by some individuals. Finally, we do not have to ask individuals about their preferences, since the variable which reveals individual preferences is a real choice variable. The equilibrium which we show to exist is thus doubly non-manipulable: 1. There is no need to poll individuals on their preferences, since the individuals' portfolio choices suffice to reveal these; and 2. The only way an individual can change the economy's equilibrium is to make a choice which is both suboptimal for him personally and which will change the equilibrium in a direction not desired by him.

The structure of the paper is as follows: In Sections 2-6 we set out the model of the paper. After defining an exchange-production equilibrium in Section 7, we give necessary and sufficient conditions for taxation neutrality in complete markets in Section 8. Section 9 states the necessary condition for taxation neutrality in an incomplete market, and in Section 10 we discuss consumer preferences and voting over taxation vectors in an incomplete market. The existence of a full equilibrium -- an EPE in which the lump-sum taxation vector is favored by a majority of individuals over any alternative favored by any other individual -- is proved in Section 11. A brief summary follows in the last section.

2. Time-state structure, proces and other preliminaries

We shall consider a two-period model with a single good used for both consumption and production, in which uncertainty in the second period ("tomorrow") will be represented through the artiface of states of nature. We shall assume there to be M such states, and where convenient they will be denoted by $m=1,\ldots,M$. Each of I consumers is assumed to maximize a utility function over a state-dependent consumption vector; to denote this function, we shall write $U_i(x_i) = U_i(x_{io},x_{i1},\ldots,x_{iM})$ for each consumer $i = 1,\ldots,I$; x_{io} denotes consumption in the first period ("today") and x_{im} denotes consumption in state m of the second period. The model allows non-homogeneous (subjective and different) probabilities over the states (indeed, we shall never mention these subjective state probabilities, assuming them to be subsumed in the utility function), but it does require agreement among consumers as to the number of states and their classification.

The production side of the model will be represented by J firms, each with a state-dependent production function, and each purchasing a quantity of the single good today to serve as an input tomorrow for the production process. Buying shares in the firms today is one way in which consumers may transfer wealth to the second period; another way will be provided by allowing consumers to purchase risky bonds issued by the government. Consumers will be allowed to sell government bonds short (i.e., borrow against future income on the same terms as the government) as long as they run no default risk.[5]

The market price of all of firm j's equity in the first period will be denoted by p_J, and the price of the commodity in the first period will be denoted by p_o. As we shall show (Section 5) we may, with no loss in generality, assume that the market price of the commodity in every state of nature tommorrow is unity. Where convenient, we shall write $p=(p_o,p_1,\ldots,p_J)$.

3. Firms

Each of the J firms in the model will be assumed to purchase a quantity of the single commodity today to serve as inputs tommorrow for the production process.[6] Given purchases of inputs z_j today, firm j will produce outputs $y_{jm}(z_j)$ in state m of the second period. In the second period the firm liquidates, and the total value of the outputs is paid out to consumers who bought shares of the firm in the first period. We shall assume for every firm j and state m that the function y_{jm} is an increasing, concave function and that y_{jm} is strictly concave for at least one state m; thus we shall assume non-increasing returns to scale every state with strictly decreasing returns to scale in at least one state.

Initial ownership of the firms will play an important role in our model. We shall assume that each consumer i possesses an initial portfolio of shares $\bar{s}_i = (\bar{s}_{i1},\ldots,\bar{s}_{iJ})$. $\bar{s}_{ij}$ should be understood to be the proportional ownership of firm j accruing to individual i at the beginning of the first period (i.e., from purchases in the previous period, or from inheritance). We shall assume these initial portfolios to be non-negative and market-clearing:

(1) $\bar{s}_{ij} \geqslant 0$ for every i and j, and $\sum_i \bar{s}_{ij} = 1$ for every j.

In the first period each consumer i will purchase a new portfolio of shares, $s_i = s_{i1}, \ldots, s_{iJ})$. In addition prior ownership of firm j (i.e., possession of a positive initial holding $\bar{s}_{ij} > 0$) will require the individual to participate in the financing of firm inputs z_j. The income of consumer i from his initial shareholdings in the first period will thus be

(2) $\sum_j \bar{s}_{ij}(p_j - p_o z_j)$

and the cost to the consumer i of his new portfolio s_i, purchased in the first period will be

(3) $\sum_j s_{ij}p_j$.

4. Government

We shall employ a highly simplified model of government. Let government be an agent in the economy which consumes a fixed amount of the single commodity in the first period and which has no consumption in the second period.[7] To pay for its consumption, we shall assume that the government imposes real lum-sum commodity taxes on individuals both today and in each state m tomorrow.

Denote the lump-sum commodity vector of taxation for individual i by

(4) $t_i = (t_{io}, t_{i1}, \ldots, t_{iM})$,

and write,

(5) $t_m = \sum_i t_{im}$, $m = 0,\ldots, M$.

t_m is the total commodity taxation imposed on all individuals in the first period (if m = 0) or in state m (m > 0). Suppose that we denote by g the total consumption by the government of the model's single commodity in the first period. Then the deficit of the government in the first period will be given by

(6) $(g - t_o)p_o$.

We shall assume that the proceeds from taxation tomorrow -- i.e., $t_1,\ldots,t_M$ -- will be used to finance today's budget deficit. Suppose individual i decides to purchase proportion b_i of the government's debt in the first period. Then he must pay

(7) $b_i(g - t_o)p_o$

today, and in turn he will be repaid an amount

(8) $b_i t_m$

in each state m = 1,...,M, tomorrow. Throughout this paper we shall assume that the government's deficit is non-negative; that is, we shall assume that t_o never exceeds g. Additionally, while we shall (in Sections 9 and 10) discuss the effect of certain constraints on the s_{ij}'s, the proportional shareholdings of individuals, we shall throughout the paper allow b_i to be either positive or negative. If $b_i < 0$, this means that consumer i is borrowing (i.e., selling short the government's debt) on the same terms as the government itself.

5. Consumers

We have already (in Section 2) discussed the initial portfolio holdings of consumers, as well as the dividends they may expect to get from their new portfolios. We shall require each consumer to choose, in the first period, a share portfolio $s_i = (s_{i1} \ldots, s_{ij})$ and a fraction b_i of the government's debt which he wishes to hold. Given consumer i's taxes $t_i = (t_{io}, t_{i1}, \ldots, t_{iM})$, firm inputs $z = (z_1, \ldots, z_j)$, the first and prices $p = (p_o, p_1, \ldots, p_J)$ consumer i's consumption x_{io} in the first period will then be given by

(9) $$p_o x_{io} = \sum_j (\bar{s}_{ij} p_j - p_o z_j) - \sum_j s_{ij} p_j - t_{io} p_o - b_i(g - t_o)p_o + p_o w_{io} ,$$

where w_{io} is consumer i's commodity endowment in the first period.

In state m of the second period, consumer i's consumption will be given by

$$(10)\quad x_{im} = \sum_j s_{ij} y_{jm}(z_j) + b_i t_m - t_{im} + w_{im} \quad ,$$

where, as before, w_{im} denotes consumer i's commodity endowment.

We note that equation (9) is homogeneous in prices $p = (p_o, p_1, \ldots, p_J)$. so that we may normalize these prices to sum to one. Note further that while we could introduce a commodity price in state m, this would merely multiply each term in equation (10) by a constant, so that we may assume, with no loss in generality, that commodity prices in every state m tomorrow are unity.

We shall assume that each consumer i chooses s_i and b_i in order to maximize a utility function U_i defined over the vector $x_i = (x_{io}, x_{i1}, \ldots, x_{iM})$. We shall assume that U_i is concave, twice differentiable, and has the properties

$$(11)\quad \partial U_i/\partial x_{im} \geqslant 0, \quad \partial U_i/\partial x_{im} \to +\infty \quad \text{as } x_{im} \to 0, \; m = 0, \ldots, M.$$

The first-order conditions for utility maximization, along with the economy's equilibrium conditions, define implicit state prices for each consumer i. These implicit prices will be of considerable importance in the rest of the paper. In the following theorem, we define these prices.

<u>Theorem 1</u>: Let prices $p = (p_o, p_1, \ldots, p_J)$, commodity taxes for each individual, $t_i = (t_{io}, t_{i1}, \ldots, t_{iM})$, and firm inputs $z = (z_1, \ldots, z_J)$ be given. Then if (x_i^*, s_i^*, b_i^*) maximizes U_i given constraints (9) and (10), we have

$$(12)\quad p_j = \sum_m q_m^i y_{jm}(z_j) p_o \text{ for every i and j}$$

$$(13)\quad (g - t_o) = \sum_m q_m^i t_m \quad ,$$

where

$$(14)\quad q^i_m = \frac{\partial U_i/\partial X_{im}}{\partial U_i/\partial X_{io}} \quad ,$$

and the partial derivatives are evaluated at x^*_i.

Furthermore, if we constrain s_{ij} to be non-negative, (12) is replaced by

$$(15)\quad p_j \geqslant \sum_m q^i_m y_{jm}(z_j)\, p_o$$

In this case, if $s^*_{ij} > 0$ there will be strict equality in (15), whereas a strict inequality will imply that $s^*_{ij} = 0$.

Proof:

The proof of the Theorem follows readily from the Kuhn-Tucker conditions. For details and a full discussion of the meaning of the implicit prices, see Baron (1979). For a multi-period context, see Benninga (1979).

The implicit prices q^i_m defined in Theorem 1 should be interpreted as personalized present value factors which are state-dependent. If we make no restrictions on the s_{ij}'s, we may obtain the following theorem:

Theorem 2: Suppose the following two conditions hold:

2.1 The J+1 vectors

$$y_j(z_j) = (y_{j1}(z_j), \ldots, y_{jM}(z_j)),\ j = 1, \ldots, J$$

$$(t_1, \ldots, t_M)$$

span an M-dimensional Euclidean space.

2.2 There are no restrictions on individual portfolio holdings s_{ij}.

Then the implicit state prices are equal for all consumers, and we may write

$$q_m = q^i_m = q^h_m \text{ for every } h,\ i, \text{ and } m.$$

Proof:

Given equation (12) the proof of the theorem is an immediate application of elementary linear algebra.

We shall call a market in which the conditions of Theorem 2 hold a complete market.

6. The choice of firm inputs

Before we can define an equilibrium in the economy described in the preceding sections, we must make some assumptions about how firms determine their inputs. The problem of consumer preferences over firm inputs is an extremely complex one, which has not -- despite a great deal of attention in the literature -- been completely solved.[8] While we cannot ignore this problem, it is sufficient for our purposes to assume an extremely general firm objective function. We shall assume that firms choose their inputs by maximizing a concave function of consumers' consumption plans:

$$(16)\quad z_j = \psi_j((x_i)_{i=1}^{i=I}),\ j = 1, \ldots, J.$$

This function is sufficiently general to describe most cases of firm objective functions found in the literature.

7. The definition of an exchange-production equilibrium

In order to discuss consumer preferences over lump-sum taxation vectors, we shall first define an equilibrium for a given set of lum-sum taxation vectors. In succeeding sections we shall then discuss consumer preferences over lump-sum taxation vectors given this exchange-production equilibrium (EPE). A full equilibrium will then be defined as an EPE in which the lump-sum taxation vector chosen by the government accords with consumer preferences.

Let a taxation vector $t_i = (t_{io}, t_{i1}, \ldots, t_{iM})$ be given for each consumer i. We shall call $p^* = (p_o^*, p_1^*, \ldots, p_J^*)$, (x_i^*, s_i^*, b_i^*), $i = 1, \ldots, 1$, and

$z^* = (z_1^*, \ldots, z_J^*)$ an exchange-production equilibrium if the following conditions are met:

(E.1) There is equality of commodity supply and demand in each state tomorrow and in today's market:

$$\sum_i x_{io}^* + \sum_j z_j^* + g = \sum_i w_{io}$$

$$\sum x_{im}^* = \sum_j y_{jm}(z_j^*) + \sum_i w_{im}, \quad m = 1, \ldots, M.$$

(E.2) There is equality of security supply and demand:

$$\sum_i s_{ij}^* = \sum_i b_i^* = 1, \; j = 1, \ldots, J.$$

(E.3) x_i^* maximizes $U_i(x_i)$ given constraints (9) and (10).

(E.4) $z_j^* = \psi_j((x_i^*)_{i=1}^{i=I})$.

8. The neutrality of government taxation policy in complete markets

Let $e^* = \{(x_i^*, s_i^*, b_i^*), (t_i^*), (z_j^*), \overset{*}{p}\}$ fulfill (F.1) - (E.4). Then we shall say that an alternative government taxation policy $(\hat{t}_i)$ is neutral with respect to e^* (or, where no confusion will be caused, we shall simply say that $(\hat{t}_i)$ is neutral) if there exists a vector $\hat{e} = \{(x_i^*, \hat{s}_i, \hat{b}_i), (\hat{t}_i), (\hat{z}_j), \hat{p}\}$ which also fulfills (E.1) - (E.4). Thus, for a given EPE, the neutrality of $(\hat{t}_i)$ implies the existence of another EPE in which all consumer consumption plans remain the same. It follows by (E.4) that neutrality implies that for every firm j, $\hat{z}_j = z_j^*$. Furthermore, it follows from Theorem 1 that if there are no restrictions on short sales, neutrality implies that $\hat{p} = p^*$.

In this section and the following section we wish to discuss the neutrality of government taxation policy in complete and incomplete markets respectively.

In a complete market $(\hat{t}_i)$ is neutral if and only if the net present value of each individual's tax burden remains unchanged:

<u>Theorem 3</u>: In a complete market $(\hat{t}_i)$ is neutral with respect to e^* if and only if

$$\hat{p}_o\hat{t}_{io} + \sum_m q_m\hat{t}_{im} = p_o^*t_{io}^* + \sum_m q_m t_{im}^* \quad , \quad i = 1, \ldots, 1. \tag{17}$$

<u>Proof</u>:

=>: By neutrality it follows that the state prices q_m remain unchanged. Multiplying x_{im}^* by q_m and summing over m gives

$$\sum_m q_m x_{im}^* = \sum_m q_m \{ \sum_j \hat{s}_{ij}y_{jm}^* + \hat{b}_i\hat{t}_m - \hat{t}_{im} + w_{im}\}$$

$$= \sum_j \hat{s}_{ij} \sum_m q_m y_{jm}^* + \hat{b}_i \sum_m q_m\hat{t}_m - \sum_m q_m\hat{t}_{im} + \sum_m q_m w_{im}.$$

Since by the assumption of neutrality and Theorem 1 all prices are equal, this now becomes

$$(18) \quad \begin{aligned} &= \sum_j \hat{s}_{ij}p_j^* + \hat{b}_i(g - \hat{t}_o)p_o^* - \sum_m q_m\hat{t}_{im} + \sum_m q_m w_{im} \\ &= \sum_j s_{ij}^*p_j^* + b_i^*(g - t_o^*)p_o^* - \sum_m q_m t_{im}^* + \sum_m q_m w_{im} \end{aligned}$$

But

$$p_o^*x_{io}^* = \sum_j (\bar{s}_{ij}p_j^* - p_o^*z_j^*) - \sum_j s_{ij}^*p_j^* - p_o^*t_{io}^* - b_i^*(g - t_o^*)p_o^* + p_o^*w_{io}$$

$$= \sum_j (\bar{s}_{ij}p_j^* - p_o^*z_j^*) - \sum_j \hat{s}_{ij}p_j^* - p_o^*\hat{t}_{io} - \hat{b}_i(g - \hat{t}_o)p_o^* + p_o^*w_{io} \quad .$$

Eliminating like terms from the above equations gives:

$$\sum_j s_{ij}^*p_j^* + p_o^*t_{io}^* + b_i^*(g - t_o^*)p_o^* = \sum_j \hat{s}_{ij}p_o^*\hat{t}_{io} + \hat{b}_i(g - \hat{t}_o)p_o^* \quad .$$

Now write

$$\sum_j s^*_{ij}p^*_j - \sum_j \hat{s}_{ij}p^*_j + b^*_i(g - t^*_o)p^*_o - \hat{b}_i(g - \hat{t}_o)p^*_o = p^*_o\hat{t}_{io} - p^*_o t^*_{io} \quad .$$

We may now use (18) to transform this into

$$\sum_m q_m t^*_{im} - \sum_m q_m \hat{t}_{im} = p^*_o t_{io} - p^*_o t^*_{io} \quad ,$$

which, with an obvious rearrangement, yields the desired result.

<=: Suppose (17) holds. We shall then show that there exists a vector $\hat{e} = \{(x^*_i, \hat{s}_i, \hat{b}_i), (\hat{t}_i), (z^*_j), p^*\}$ which fulfills (E.1) - (E.4). First, by completeness there exists a portfolio $(\hat{s}_i, \hat{b}_i)$ for every consumer i such that

$$\hat{x}_{im} = \sum_j \hat{s}_{ij}y^*_{jm} + \hat{b}_i\hat{t}_m - \hat{t}_{im} + w_{im}$$

By (18) above it follows that

$$\text{(19)} \quad \begin{aligned} \sum_j \hat{s}_{ij}p^*_j + \hat{b}_i(g - t_o)p^*_o &= \sum_m q_m x^*_{im} + \sum_m q_m \hat{t}_{im} - \sum_m q_m w_{im} \\ \sum_j s^*_{ij}p^*_j + b^*_i(g - t^*_o)p^*_o &= \sum_m q_m x^*_{im} + \sum_m q_m t^*_{im} - \sum_m q_m w_{im} \end{aligned}$$

and thus

$$p^*_o x^*_{io} = \sum_j (\bar{s}_{ij}p^*_j - p^*_o z^*_j) - \sum_j s^*_{ij}p^*_j - p^*_o t^*_{io} - b^*_i(g - t^*_o)p^*_o + p^*_o w_{io}$$

$$= \sum_j (\bar{s}_{ij}p^*_j - p^*_o z^*_j) - t^*_{io}p^*_o - \sum_m q_m t^*_{im} - \sum_m q_m x^*_{im} + \sum_m q_m w_{im} + p^*_o w_{io} \quad ,$$

which, by the initial assumption (17)

$$(20)\quad = \sum_j (\bar{s}_{ij}p_j^* - p_o^* z_j^*) - p_o^* \hat{t}_{io} - \sum_m q_m \hat{t}_{im} - \sum_m q_m x_{im}^* + \sum_m q_m w_{im} + p_o^* w_{io} \quad .$$

A suitable transformation of (19) now yields:

$$\sum_m q_m \hat{t}_{im} - \sum_j \hat{s}_{ij}p_j^* + \hat{b}_i(g - \hat{t}_o)p_o^* - \sum_m q_m x_{im}^* + \sum_m q_m w_{im} \quad ,$$

which, when substituted into (20), gives

$$p_o^* x_{io}^* = \sum_j (\bar{s}_{ij}p_j^* - p_o^* z_j^*) - p_o^* \hat{t}_{io} - \sum_j \hat{s}_{ij}p_j^* - \hat{b}_i(g - \hat{t}_o)p_o^* + p_o^* w_{io} \quad ,$$

and thus x_{io}^* is affordable.

The optimality of $(x_i^*, \hat{s}_i, \hat{b}_i)$ follows directly from the completeness assumption; were some other vector (x_i, s_i, b_i) preferred by individual i for taxation vector $\hat{t}_i$, then x_i would have been feasible also given taxes t_i^*, and this would contradict the optimality of (x_i^*, s_i^*, b_i^*).

To show that (E.1) - (E.4) hold, we solve

$$\sum_i x_{im}^* = \sum_i \{\sum_j \hat{s}_{ij}y_{jm} - \hat{b}_i\hat{t}_m - \hat{t}_{im} + w_{im}\}$$

$$(21)\quad = \sum_j \hat{s}_j y_{jm} - \hat{b}\hat{t}_m - \hat{t}_m + w_m \quad ,$$

where

$$\hat{s}_j = \sum_i \hat{s}_{ij}, \; \hat{b} = \sum_i \hat{b}_i \quad .$$

Equation (21) solves for $\hat{s}_j = \hat{b} = 1$. Assuming, without loss in generality, that the vectors $\{y_1, \ldots, y_j, (\hat{t}_1, \ldots, \hat{t}_M)\}$ are independent over R^M, we may conclude that this solution is also unique. This establishes (E.2). (E.1) now follows by summing over consumers' budget equations.

9. Neutrality in incomplete markets

Recall from Theorem 2 that two conditions are sufficient for the completeness of a market:

2.1) The return vectors of the firms and of the government bond must span R^M.

2.2) There are no restrictions on portfolio holdings (s_{ij}) and (b_i).

It is not possible to duplicate Theorem 3 for incomplete markets, since in general, in an incomplete market a change in one of the return vectors (including the government bond's return vector) will change some consumer's choice of consumption vector. However, with one additional condition, equation (17) is also a necessary condition (though clearly not a sufficient one) for neutrality in incomplete markets:

Theorem 4: Suppose $(\hat{t}_i)$ is neutral with respect to e^* and suppose that the following condition holds:

4.1 For every j there exists i such that $s^*_{ij} > 0$ and $\hat{s}_{ij} > 0$.

Then it follows that

$$(22) \quad p_o\hat{t}_{io} + \sum_m q^i_m\hat{t}_{im} = p_o t^*_{io} + \sum_m q^i_m t^*_{im} \quad .$$

Proof:

Note that by 4.1 we must have that $\hat{p}_j = p^*_j$, $j=1,\ldots, J$, and thus (assuming that prices are normalized to sum to one), $\hat{p}_o = p^*_o$. The proof now follows directly from the proof of necessity in Theorem 3, replacing q_m by q^i_m.

10. Voting over taxation vectors by consumers

In this section we shall assume that condition (2.1) holds, but that condition (2.2) does not necessarily hold. Individuals may thus be at a corner either because short sales are fully restricted or because there are partial restrictions on short sales.[9] We shall make three further assumptions which we shall assume to hold throughout the rest of this paper:

10.1. There are no restrictions on short-selling the government bond.

Assumption (10.1) guarantees that equality will always hold in (13); thus individual implicit valuations of the government bond will always correspond to its market price. An individual who short sells the government bond is effectively borrowing on the same terms as the givernment; (10.1) thus guarantees that there are no restrictions (other than those imposed by the individual on himself through his utility function) on this kind of borrowing.

10.2. The lump-sum taxation vector of each individual i is a proportion α_i of the total lump-sum taxation vector. Thus, given a total taxation vector $t = (t_o, t_1, \ldots, t_M)$, individual i's taxation vector will be given by $t_i = (\alpha_i t_o, \alpha_i t_1, \ldots, \alpha_i t_M)$. We shall assume that $\alpha_i \geq 0$ for every i and that $\sum_i \alpha_i = 1$.

Assumption (10.2) assumes away the question of the division of the tax burden among consumers by supposing that an a-priori decision has been made about the burden to be borne by each consumer. We make this assumption in order not to confuse -- in our discussion of preferences by consumers over taxation vectors -- questions relating to the distribution of the tax burden with those relating to other real effects of changing the taxation vector.

Note that Assumptions (10.1) and (10.2) imply that the present value of each individual's taxes is always $\alpha_i g$;thus there can be no shifting of the burden of taxation by a change in the taxation vector:

$$\alpha_i t_o + \sum_m q^i_m \alpha_i t_m = \alpha_i (t_o + g - t_o) = \alpha_i g, \text{ for every consumer } i.$$

Finally, we shall make a technical assumption without which our voting procedure will not be well-defined:

10.3. Let A be any subset of 1, ..., J . Then there exists an individual i such that

$$p_j = \sum_m q^i_m y_{jm} \quad \text{for all } j \in A.$$

Assumption (10.3) guarantees that for every portfolio combination there is at least one individual whose implicit valuation of the portfolio is equal to its market price.[10]

We may now begin our discussion of consumer preferences over taxation vectors. Suppose that $\hat{t} = (\hat{t}_o, \hat{t}_1, \ldots, \hat{t}_M)$ is an alternative taxation vector to $t = (t_o, t_1, \ldots, t_M)$. Then by (2.1) we have

$$\text{(23)} \qquad \hat{t}_m = t_m + \sum_j \lambda_j y_{jm}, \quad m = 1, \ldots, M.$$

Since the market price of y_j is p_j, the value of the taxation vector will remain unchanged if

$$\text{(24)} \qquad p_o \hat{t}_o = p_o t_o - \sum_j \lambda_j p_j \quad .$$

Thus, given current market prices, $\hat{t}$ can provide alternative financing to t for the government's consumption g. Note that this argument depends on (10.3): There is always to be found an individual who values the vector $\{ \sum_j \lambda_j y_{j1}, \ldots, \sum_j \lambda_j y_{jM} \}$

at $\sum_j \lambda_j p_j$. Given the short sale restrictions, however, not all individuals will necessarily be indifferent between t and $\hat{t}$. In the next theorem we show this to be true.

<u>Theorem 5</u> Let (x_i, s_i, b_i) maximize utility given prices $(p_o, p_j, \ldots, p_j)$, inputs $z = (z_1, \ldots, z_j)$, and taxes $t = (t_o, t_1, \ldots, t_M)$. Now consider an alternative taxation vector $\hat{t}$ of the form given by equations (23) and (24). Then individual i will prefer that any $\lambda_j > 0$ if and only if $\alpha_i - b_i > 0$ and $p_j > \sum_m q_m^i y_{jm}(z_j)p_o$.

<u>Proof</u>:

Taking $\frac{\partial U_i}{\partial \lambda_j}$, we have (substituting (23) and (24) into (9) and (10):

$$\frac{\partial U_i}{\partial \lambda_j} = \frac{\partial U_i}{\partial x_{io}} \frac{\partial x_{io}}{\partial \lambda_j} + \sum_m \frac{\partial U_i}{\partial x_{im}} \frac{\partial x_{im}}{\partial \lambda_j}$$

$$= \frac{\partial U_i}{\partial x_{io}} (\alpha_i \frac{p_j}{p_o} - b_i \frac{p_j}{p_o}) + \sum_m \frac{\partial U_i}{\partial x_{im}} (-\alpha_i y_{jm} + b_i y_{jm}) > 0,$$

which gives

$$(\alpha_i - b_i)p_j > \sum_m q_m^i y_{jm} p_o (\alpha_i - b_i) \quad ,$$

and this holds if and only if $\alpha_i - b_i > 0$ and $p_j > \sum_m q_m^i y_{jm} p_o$.

QED

The intuition behind Theorem 5 is simple: If $p_j > \sum_m q_m^i y_{jm} p_o$, then consumer i feels that firm j is overvalued at current market prices. Were he given the opportunity, therefore, he would sell the firm's shares short; this opportunity, however, is denied him in our market. If $\alpha_i - b_i > 0$, a change in t in the direction of $\lambda_j > 0$

is equivalent to a short sale by the government of the securities of firm j. To see this, substitute (13) and (24) into equations (9) and (10):

$$(25)\quad p_o x_{io} = \sum_j (\bar{s}_{ij}p_j - p_o z_j) - \sum_j s_{ij}p_j - \alpha_i(t_o p_o - \sum_j \lambda_j p_j) -$$

$$- b_i\{(g - t_o)p_o + \sum_j \lambda_j p_j\}$$

$$(26)\quad x_{im} = \sum_j s_{ij}y_{jm} - \alpha_i(t_m + \sum_j \lambda_j y_{jm}) + b_i(t_m + \sum_j \lambda_j y_{jm})$$

which, collecting terms, gives

$$(27)\quad x_{im} = \sum_j \{s_{ij} - (\alpha_i - b_i)\lambda_j\}y_{jm} - \alpha_i t_m + b_i t_m \quad .$$

Thus, if $(\alpha_i - b_i) > 0$, $\lambda_j > 0$ is equivalent to the government shortselling security j. Since these short sales expand the budget set, and since consumer i, if $p_j > \sum_m q^i_m y_{jm} p_o$, is on the edge of this budget set, the consumer prefers $\lambda_j > 0$. A similar story (with signs of $(\alpha_i - b_i)$ and λ_j reversed) holds in the opposite case. Note that if $p_j = \sum_m q^i_m y_{jm} p_o$, consumer i is indifferent to small changes in λ_j.

It follows from Theorem 2 that consumers may be divided into three groups: All consumers i for whom $\alpha_i - b_i > 0$ prefer (weakly) any alternative taxation plan $\hat{t}$ for which all $\lambda_j > 0$; here we understand by "weak preference" the fact that any such consumer i prefers $\lambda_j > 0$ if $p_j > \sum_m q^i_m y_{jm} p_o$ and is indifferent otherwise. All consumers i for whom $\alpha_i - b_i < 0$ prefer (weakly) any taxation plan for which $\lambda_j < 0$. Finally, consumers i such that $\alpha_i - b_i = 0$ are indifferent among all taxation plans.

Consumer preferences over taxation plans t suggest that a taxation plan is not desirable if a majority of the consumers in the economy would have preferred some other taxation plan $\hat{t}$. Suppose, however, that we could find a taxation vector t such that, when the values $\alpha_i - b_i$ were arranged in ascending order, the median value, which we shall denote by n, were zero. It then follows that neither of the two groups referred to in the previous paragraph which favor a uni-directional change in government taxation policy has a majority. No change in the current taxation vector t which represents the true preferences of either of the two above groups would be preferred by a majority of the economy's consumers.

Note furthermore that the true preferences of consumers may be learned by looking at their revealed preferences; i.e., by looking at their portfolio choices (in this case their holdings of government bonds) and at their tax shares α_i. The usual problem in voting schemes comes from the cheat-proofness (or lack thereof) of such schemes: If we poll individuals, and by this polling establish the group choice, we must first establish that it is not worthwhile for individuals to misrepresent their preferences. In the case we are discussing, no such poll is necessary, since revealed preferences over portfolios also reveal preference over alternative taxation plans. Finally, note (and this is a property which our problem has in common with any scheme in which preferences are single-peaked over a single variable -- see for example Arrow (1951) or Black (1948)) that the only way a single individual can change the group decision in a direction not desired by him. Our scheme, therefore is non-manipulable.[11]

11. <u>General equilibrium with non-neutral government debt: definition</u>

We may now define a full general equilibrium in our model: Prices $p^* = (p_0^*, p_1^*, \ldots, p_J^*)$, inputs z_j^* for every firm j, consumer plans (x_i^*, s_i^*, b_i^*) and a taxation vector $(t_o^*, t_1^*, \ldots, t_M^*)$ will be called an <u>equilibrium</u> if the prices, inputs, and consumer plans constitute an EPE (see section 7) given the taxation vector is preferred by a majority of consumers to $(t_o^*, t_1^*, \ldots, t_M^*)$; i.e., the following condition is fulfilled:

(E.5). $\operatorname{median}_i (\alpha_i - b_i^*) = 0.$

12. The existence of equilibrium

In this section we prove that an equilibrium exists in our economy. We first define compact choice sets for consumers, firms and the government, and then using the Brouwer fixed point theorem show that a fixed point exists which is an equilibrium for the economy.

We start by defining the choice set for firms. Let

$$(28)\quad H = \{z \mid 0 \leqslant z \leqslant \sum_i w_{io}\} .$$

Since no firm can hope to purchase more physical inputs than there exist consumer endowments in the first period, H defines the set from which the equilibrium choices for each firm must be made. Note that this implies that

$$(29)\quad y_{jm}(z_j) \leqslant y_{jm}(\sum_i w_{io})$$

We shall find it useful to define

$$(30)\quad F_m = \sum_j y_{jm}(\sum_i w_{io})$$

In defining the choice set for the government, we encounter a problem: Since the government obviously cannot collect more taxes in the first period than there exist physical inpits, it would seem natural to bound t_o by $\sum_i w_{io}$; similar considerations lead to a bound of t_m by $F_m + \sum_i w_{im}$. Unfortunately, our proposed bound for t_o imposes no bound on individual borrowing b_i; we shall therefore assume that the government's taxation in the first period is bounded by $g - \varepsilon$, where $\varepsilon > 0$.[12] We now define the government choice set by

$$(31)\quad T = \{t = (t_o, t_1, \ldots, t_M) \mid 0 \leq t_o < g - \varepsilon ,\ 0 \leq t_m \leq F_m + \sum_i w_{im}\} .$$

We shall assume that each individual i faces a firm-dependent short sale constraint, thus making individual i's portfolio choice set:

$$(32)\quad G_i = \{(s, b) = (s_1, \ldots, s_J, b) \mid \delta_{ij} \leq s_j \leq 1, |b| \leq (\sum_{ii} w_{io})/\varepsilon, \quad -\infty < \delta_{ij} \leq 0\}$$

Note that the bound on b derives from the fact that government debt will never be less than ε, and consumers may never, in equilibrium, plan to borrow or lend more than the proportion $\sum_i w_{io}/\varepsilon$ of this debt, since otherwise they would be borrowing or lending all of the economy's first period resources.

We shall write consumer i's choice set for consumption vectors as

$$(33)\quad X_i = \{x = (x_o, x_1, \ldots, x_M) \mid 0 \leq x_o \leq \sum_i w_{io}, \ 0 \leq x_m \leq F_m + \sum_i w_{im}\} \quad .$$

Finally, as noted in Section 5 prices may be normalized to sum to one. Define, therefore, the economy's price set by

$$(34)\quad S = \{p = (p_o, p_1, \ldots, p_J) \mid \sum_j p_j + p_o = 1, \ p \geq 0\} \quad .$$

We now define the function

$$(35)\quad \Phi: X^I \times \prod_i G_i \times H^J \times T \times S \to X^I \times \prod_i G_i \times H^J \times T \times S$$

where

$$(36)\quad \Phi((x_i, s_i, b_i)_{i=1}^{i=I}, (z_j)_{j=1}^{j=J}, t, p) = ((\hat{x}, \hat{s}_i, \hat{b}_i)_{i=1}^{i=I}, (\hat{z}_j)_{j=1}^{j=J}, \hat{t}, \hat{p})$$

is defined as follows:

$\hat{x}_i$ maximizes $U_i(x_i)$ given (z, t, p) and such that $(\hat{x}_i, \hat{s}_i, \hat{b}_i)$ fulfills the budget constraints (9) and (10) and $(\hat{s}_i, \hat{b}_i) \in G_i$.

$$(37) \quad \hat{z}_j = \begin{cases} \psi_j((x_i)_{i=1}^{i=I}) & \text{if this is} \leqslant \sum_i w_{io} \\ \sum_i w_{io} & \text{otherwise} \end{cases}$$

We define p by writing

$$\eta = \underset{i}{\text{median}} \, (\alpha_i - b_i)$$

$$E_o = \sum_i \hat{b}_i - 1$$

$$E_j = \sum_i \hat{s}_{ij} - 1, \; j = 1, \ldots, J.$$

Writing

$$E = ((x_i, s_i, b_i)_{i=1}^{i=I}, (z_j)_{j=1}^{j=J}, p, t) = \text{Max } (|E_o|, |E_1|, \ldots, |E_J|, \; |\eta| \;)$$

We may now define

$$(38) \quad \hat{p}_o = \frac{p_o + E}{1 + (J+1)E}$$

$$(39) \quad \hat{p}_j = \frac{p_j + E}{1 + (J+1)E} \; , \quad j = 1, \ldots, J.$$

Finally, we write

$$(40) \quad \hat{t} = t \quad ,$$

We may now prove:

Theorem 6: Let $(w_{io}, \ldots, w_{iM}) \gg 0$ for every i. Let $\alpha = (\alpha_1, \ldots, \alpha_I)$ be defined so that $0 \leq \alpha_i g \leq w_{io}$, $i = 1, \ldots, I$. Then there exists an equilibrium fulfilling (E.1) - (E.5).

Proof:

All of the components of the function are continuous (for details, see Debreu 1959). Therefore by the Brouwer fixed point theorem, there exists

$$e^* = \{(x_i^*, s_i^*, b_i^*)_{i=1}^{i=I}, (z_j^*)_{j=1}^{j=J}, t^*, p^*\}$$

such that

$$e^* = \Phi(e^*) \quad .$$

(E.2) - (E.5) now follow immediately. To see (E.1), we sum (9) and (10) over all i. The fact that securities' markets clear will now give us that the commodity markets also clear.

13. Conclusions and summary

Government debt may be non-neutral if financial markets are incomplete (as they usually are). If the incompleteness is caused by short sales restrictions, individuals have preferences over the level of government debt, even if individual tax burdens are fixed, so that changes in taxation do not alter the proportion of government consumption financed by each consumer. A natural optimality criterion for a government taxation vector in such a case is to choose the taxation vector so that any alternative proposal of taxation would not be supported by a majority of the economy's consumers. We have shown that an equilibrium exists in which the taxation vector chosen by the government has such a property; moreover, we have shown that consumers need not be asked about their preferences, since these can be inferred from their portfolio choices. The optimality criterion revolves about the proportion of taxes paid by the individual consumer minus the proportion of the government's debt purchased by him; if the median value of the difference of these two variables is zero, we have shown that fewer than half of all consumers would prefer that second period taxes be raised, with first period taxes lowered, and similarly, less than half of all consumers hold the opposite view. Were either of these groups to present their true preferences for a vote by all consumers, fewer than a majority would support the alternative taxation vector; the current taxation vector may thus be said to be the majority choice.

FOOTNOTES

1. The view that government debt is neutral under the circumstances mentioned was first expounded by Ricardo (1951); Ricardo did not, however, believe this to be the case (see O'Driscoll (1977) and Shoup (1962)). The Ricardian view has been revived by Barro (1974); for the debate which followed this article, see Buchanan (1976), Feldstein (1976) and Barro (1976). Using a framework similar to Barro's, Drazin (1978) has shown that government debt may have a net wealth effect even with an effective mechanism for intergenerational bequests, if the bequests consist of human capital. An earlier debate, which dealt largely with the shifting of the tax burden, involved Bowen, Davis and Kopf (1960), Lerner (1961), and Shoup (1962). Excerpts from these earlier articles may be found in Houghton (1970).
2. The models of Barro (1974) and Drazen (1978) are also non-monetary, although they employ the overlapping-generations model and not the state-preference model used in this paper. This latter framework allows for focus on the question of uncertainty, which is not dealt with by Barro or Drazen.
3. Radner (1972) calls an equilibrium in this type of model a "rational expectations" equilibrium. I have avoided using this term because of its different meaning in macroeconomic models, where rational expectations are generally taken to mean that individuals base their actions on expectations of the future and that these expectations are "the true mathematical expectations of future variables, conditional on all variables in the model which are known to the public at time t" (Schiller (1978)).
4. The assumption that absolutely no short sales of any security except government bonds are allowed is made for simplicity only; our results hold even where short sales in some securities are allowed. Note that Barro (1974) also discusses non-neutrality in imperfect capital markets, but his results refer explicitly to the case where government debt represents net wealth. This is not the case in our model;

all individuals correctly perceive the debt as being the net present value of future taxes and non-neutrality arises because of the debt's effect on the feasible consumption sets of individuals.

5. In section 5 we shall introduce conditions on individual utility functions which will guarantee that in equilibrium no consumer will desire to be in default in any state of nature.

6. It would be possible to add into the model production by firms in the first period from inputs purchased yesterday; this feature would not significantly enrich the model, however, and would add to the notation. We have chosen, therefore, not to include it.

7. We could also introduce government consumption tomorrow but this would add needless notation without achieving additional results.

8. Arrow (1964) showed that in a complete market all consumers will be unanimous in desiring that each firm j maximize the net present value of the firm's production with respect to the state prices; i.e., all consumers will want each firm to maximize

$$\sum_m q_m y_{jm}(z_j) - z_j,$$

where the state prices are treated as constants for the purposed of the maximization. The extension of this unanimity proposition to incomplete markets has been the subject of considerable discussion; cf. Ekern and Wilson (1974), Leland (1974), and Radner (1974). Benninga and Muller (1979) have shown that even in incomplete markets it may be possible to find a choice of firm inputs which is desired by the majority of the firm's initial shareholders. In a recent article Hart (1979) shows that in large markets individuals will be unanimous in wishing firms to maximize their market value, though they will in general be in disagreement about the choice of inputs for which this maximization will be attained.

9. In this section we shall assume that the short sale constraint is absolute; i.e.,

and assume that short sales are constrained by $s_{ij} \geq \delta_{ij}$, where $-\infty < \delta_{ij} \leq 0$. The criterion for a taxation vector to constitute the majority choice remains the same, however.

10. This assumption is fulfilled, for example, of there exists one individual who holds a portfolio of all stocks in the market.

11. Note that if we needed to poll individuals for their preferences over vectors $\lambda = (\lambda_1, \ldots, \lambda_J)$, our results would not hold, since in general voting schemes over multiple criteria are manipulable (see Kalai, Muller, and Satterthwaite (1979);for a special case in which a multi-criteria voting scheme is not manipulable, see Kramer (1972)).In our model, however, there is no need to poll individuals, since, as mentioned in the paper, portfolio choices indicate preferences. Furthermore, examination of a single portfolio choice, that of b_i, would indicate whether or not expressed preferences over some vector λ were truthful or not, so that our result follows from the Black (1948) theorem on uni-variate voting.

12. For a similar problem involving a bound on borrowing, see Radner (1972). Note that by limiting $t_o \leq g - \varepsilon$, we are requiring the government to keep the bond market open.

BIBLIOGRAPHY

Arrow, K. J. Social Choice and Individual Values. John Wiley & Sons, New York, 1951.

----------. "The Role of Securities in the Optimal Allocation of Risk-Bearing." Review of Economic Studies 31 (1964), 91-96.

Baron, D. P. "On the Relationship between Complete and Incomplete Financial Market Models." International Economic Review 1979.

Barro, R. J. "Are Government Bonds Net Wealth?" Journal of Political Economy 82 (1974), 1095-1117.

----------. "Reply to Feldstein and Buchanan." Journal of Political Economy 84 (1976), 343-50.

Benninga, S. "General Equilibrium with Bankruptcy in a Sequence of Markets." International Economic Review 1979.

---------- and E. Muller. "Majority Choice and the Objective Function of the Firm under Uncertainty." Bell Journal of Economics 1979 (Fall).

Black, D. "On the Rationale of Group Decision Making." Journal of Political Economy 56 (1948), 23-34.

Bowen, W. G., R. G. Davis, and D. H.Kopf. "The Public Debt: A Burden on Future Generations?" American Economic Review 50 (1960), 701-706.

Buchanan, J. M. "Barro on the Ricardian Equivalence Theorem." Journal of Political Economy 84 (1976) 337-342.

Debreu, G. Theory of Value. Joh Wiley & Sons, New York, 1959.

Drazen, A. "Government Debt, Human Capital, and Bequests in a Life-Cycle Economy." Journal of Political Economy 86 (1978), 505-517.

Ekern, S. and R. Wilson. "On the Theory of the Firm in an Economy with Incomplete Markets." Bell Journal of Economics 5 (1974), 171-180.

Feldstein, M. "Perceived Wealth in Bonds and Social Security: A Comment." Journal of Political Economy 84 (1976), 331-336.

Hart, O. "On Shareholder Unanimity in Large Stock Market Economies." Econometrica 47 (1979), 1057-1084.

Houghton, R. W., Public Finance: Selected Readings. Penguin Press, Baltimore, 1970.

Kalai, E., E. Muller, and M. Satterthwaite. "Social Welfare Functions When Preferences are Convex, Monotonic, and Continuous." Public Choice 1979.

Kramer, G. "Sphisticated Voting over Multidimensional Choice Spaces." Journal of Mathematical Sociology 2 (1979), 165-180.

Leland, H. E. "Production Theory and the Stock Market." Bell Journal of Economics 5 (1974), 125-144.

Lerner, A. P. "Burden of Debt." Review of Economics and Statistics 43 (1961), 139-142.

O'Driscoll, G. P. "The Ricardian Nonequivalence Theorem." Journal of Political Economics 85 (1977), 207-210.

Radner, R. "Existence of Equilibrium of Plans, Prices, and Price Expectations in a Sequence of Markets." Econometrica 40 (1972), 289-304.

---------. "A Note on Unanimity of Stockholders' Preferences among Alternative Production Plans: A Reformulation of the Ekern-Wilson Model." Bell Journal of Economics 5 (1974), 181-184.

Ricardo, D. Funding System. in P. Sraffa (ed.), Works and Correspondence, Vol. Cambridge Press, 1951.

Schiller, R. J. "Rational Expectations and the Dynamic Structure of Macroeconomic Models." Journal of Monetary Theory 4 (1978), 1-44.

Shoup, C. S. "Debt Financing and Future Generation." Economic Journal 72 (1962), 887-898.