

UNEMPLOYMENT AND REAL INTEREST RATES:  
ECONOMETRIC TESTING OF INFLATION NEUTRALITY

by

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Working Paper No. 12-80

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The author gratefully acknowledges advice from members of the Federal Reserve Bank of Philadelphia, colleagues in the Finance Department, and from Mark J. Flannery, Stanley Fischer, and A.M. Santomero.

The contents of this paper is the sole responsibility of the author.

The resurgent neoclassical school in macroeconomics has, in its simpler characterizations, asserted three hypotheses about the economy. First, the short-run expectations-augmented Phillips curve is vertical. Second, the real rate of interest is independent of anticipated inflation. Third, expectations are formed rationally in the sense that people use mathematical expectations conditional on available information. The three propositions together suggest the neutrality of the economy with respect to anticipated inflation. In this paper I propose a set of regression-based econometric tests of these three neutrality propositions. I carry out simple versions of these tests for the postwar American economy.

Such neutrality propositions distinguish the modern neoclassical school from the traditional American Keynes-Hicks school of macroeconomics. It is true that these statements oversimplify the position of the neoclassical school (sometimes called the rational expectations school) and also true that the traditional school today would mostly accept these positions in the "long run". However, the argument between the two schools does not rest on minor disagreements over models, but turns rather on different interpretations of strong gyrations of interest rates and unemployment. By careful testing of simple statements of the neutrality hypothesis it is possible to discover whether empirical evidence is closer to confirming or contradicting the view of the modern neoclassical school.

The arguments develop as follows. First, I explicitly define the null hypothesis so as to state precisely what is to be tested. Second, I derive a proper econometric test and compare it to the classic Fama (1975) tests. Third, I test for neutrality of the postwar American economy. Fourth, I extend both the principles and execution of the tests to consider the case in which unemployment persists due to long-term nominal contracts.

The conclusion from the empirical evidence is the strong rejection of the neutrality hypothesis. The simple characterizations of the neoclassical school

cannot stand in light of the empirical evidence. Throughout the paper I have attempted to separate the principles involved in the econometric tests from questions of data and measurement. It is my hope that the discussions of measurement may indicate what more sophisticated neutrality statements might stand on a firmer empirical footing.

### Statement of the neutrality hypothesis

Three statements taken jointly form the null hypothesis.

1. The expected real interest rate is independent of the expected inflation rate.
2. Deviations of the unemployment rate from the natural rate of unemployment are associated only with unanticipated inflation.
3. Expectations are rational.

"Rational" expectations are taken to mean that the anticipated value of a variable is its mathematical expectation conditional upon available information. Under rational expectations, the actual inflation rate can be divided into a component of expected inflation and an expectational error, these being mutually independent.

If we denote the nominal interest rate and expected real interest rate by  $R$  and  $r$ , and inflation<sup>1</sup> and anticipated inflation by  $\pi$  and  $\pi^e$ , the Fisher equation is:

$$R = r + \pi^e \tag{1}$$

The fundamental hypothesis about financial markets is that the real interest rate is independent of expected inflation. This premise is not a necessary implication of rational behavior, since almost any model recognizes that real

effects may be associated with anticipated inflation, including taxation effects, changes in the opportunity cost of non-interest bearing real balances, changes in real portfolio demand due to shocks to the marginal product of capital, and changes in risk premia due to contemporaneous changes in the level of and uncertainty about the rate of inflation. Nonetheless, it has been argued that the independence of the real interest rate and expected inflation is an empirically reasonable characterization of the economy. Under the null hypothesis we should find (where  $\sigma$  is the covariance operator):

$$\sigma_{r,\pi}^e = 0$$

In real markets, the neutrality hypothesis appears as the expectations-augmented, or accelerationist, Phillips curve<sup>2</sup>. Deviations of unemployment  $u$  from the natural rate  $u^*$ , or deviations of actual from potential GNP, are associated only with unanticipated inflation. Written most simply this hypothesis becomes:

$$\pi = \pi^e + \beta \hat{u}, \text{ where } \hat{u} = u^* - u \quad (2)$$

Implicit in this equation are several restrictions which are not required solely by axioms of rational behavior. The Phillips curve above is linear though most observers suggest that the curve is truly curved. A one sector world is pictured; as footnote 6 discusses, this is not an innocuous assumption. The basic hypothesis on the Phillips curve is that deviations from the natural rate of unemployment result only from unanticipated events. Since both the expected real interest rate and expected inflation are known to agents when contracts are drawn, we should find:

$$\sigma_{u,\pi}^{\hat{}} = 0 \quad \text{and} \quad \sigma_{u,r}^{\hat{}} = 0$$

### Principles of Econometric Testing

In this section I present a new econometric test and prove its validity under the neutrality hypothesis. Before doing so, I review the test proposed in Fama (1975). While there are numerous studies of the Phillips curve and the Fisher effect under assumptions of rational expectations<sup>3</sup>, I concentrate on Fama's test since it provides the intellectual genesis for this paper; understanding Fama's test is a prerequisite to understanding the test proposed below.

Fama examined the Fisher effect and rational expectations part of the neutrality hypothesis under the assumption that the real interest rate is constant. His basic test regressed the inflation rate on the nominal interest rate and a constant. This reversal of the usual regression eliminated the errors-in-variables problem associated with using inflation as a proxy for anticipated inflation. The error term in the regression is the unanticipated component of inflation. Since this is uncorrelated with the nominal interest rate, under the null hypothesis, ordinary least squares is appropriate.

In order to set the scene, I have re-estimated Fama's basic test using my data set. The results are almost identical to those reported by Fama and would appear to confirm neutrality.

$$\pi = -.847 + .98 R \quad R^2 = .98 \quad \text{ser} = 2.35 \quad \text{D.W.} = 1.76$$

(.358)    (.10)

monthly data 1/53 to 7/71.

If the real interest rate varies over time, Fama's test is invalid. By design, it does not take advantage of information from the Phillips curve. However his results do appear to provide strong evidence in favor of the neoclassical view. The

ingenuity of the test lies in using the stochastic assumptions of the null hypothesis to form a regression without using a proxy for anticipated inflation.

Anticipated inflation cannot be directly observed. This presents a major obstacle to estimation of the Phillips curve and confirmation of the Fisher effect. In the past, various sorts of distributed lags have been used to proxy for anticipated inflation. Such a procedure goes clearly against the spirit of the neutrality hypotheses. However, precisely because both the Phillips curve and the Fisher effect rely on anticipated inflation, the stochastic implications of the neutrality hypothesis can be used, in effect, to divide inflation into its anticipated and unanticipated components. This decomposition is used to formulate some fairly straightforward regression tests which are shown to be statistically valid if the neutrality hypothesis is true. These tests then allow for the potential refutation of the neutrality hypothesis. In addition, if the neutrality hypothesis is accepted the test procedure yields a consistent estimate of the Phillips curve tradeoff between unemployment and unanticipated inflation.

By substituting the Phillips curve (2) into the Fisher equation (1) the anticipated inflation term is eliminated. The resulting equation is

$$R = r - \beta \hat{u} + \pi \tag{3}$$

The real interest rate is not observable unless an identifying assumption is imposed similar to Fama's assumption that the real rate is constant. Fortunately, knowledge of the real rate is unnecessary. It turns out that the proper econometric test is an ordinary least squares regression of the nominal interest rate on  $\hat{u}$  and the inflation rate (and a constant). Under the null hypothesis, the probability limit of the coefficient on inflation is unity and the probability limit of the coefficient on  $\hat{u}$  is  $-\beta$ . I give a formal proof of these probability limits here and present a heuristic explanation below.

Consider the ordinary least squares regression of  $R$  on  $\hat{u}$  and  $\pi$ , yielding coefficients  $\tilde{\beta}$  and  $\tilde{\gamma}$ . (Without loss of generality, consider the variables in deviations from sample means and eliminate the constant from the regression.) In the probability limit the various product moments will achieve their asymptotic covariances.

$$\text{plim} \begin{bmatrix} \tilde{\beta} \\ \tilde{\gamma} \end{bmatrix} = \begin{bmatrix} \sigma_{\hat{u}}^2 & \sigma_{\hat{u},\pi} + \beta\sigma_{\hat{u}}^2 \\ \sigma_{\hat{u},\pi} + \beta\sigma_{\hat{u}}^2 & \sigma_{\pi}^2 + \beta^2\sigma_{\hat{u}}^2 + 2\beta\sigma_{\hat{u},\pi} \end{bmatrix}^{-1} \begin{bmatrix} \sigma_{r,\hat{u}} + \sigma_{\hat{u},\pi} \\ \sigma_{\pi}^2 + \beta\sigma_{\hat{u},\pi} + \sigma_{r,\pi} + \beta\sigma_{r,\hat{u}} \end{bmatrix}$$

$$= \frac{1}{\sigma_{\hat{u},\pi}^2 - (\sigma_{\hat{u}}\sigma_{\pi})^2} \begin{bmatrix} \sigma_{\pi}^2 + \beta^2\sigma_{\hat{u}}^2 + 2\beta\sigma_{\hat{u},\pi} & -[\sigma_{\hat{u},\pi} + \beta\sigma_{\hat{u}}^2] \\ -[\sigma_{\hat{u},\pi} + \beta\sigma_{\hat{u}}^2] & \sigma_{\hat{u}}^2 \end{bmatrix} \begin{bmatrix} \sigma_{r,\hat{u}} + \sigma_{\hat{u},\pi} \\ \sigma_{\pi}^2 + \beta\sigma_{\hat{u},\pi} + \sigma_{r,\pi} + \beta\sigma_{r,\hat{u}} \end{bmatrix}$$

Evaluating the algebra,

$$\text{plim } \tilde{\gamma} = \frac{\sigma_{\hat{u}}\sigma_{\pi} - (\sigma_{\hat{u},\pi})^2 + \sigma_{\hat{u}}\sigma_{r,\pi} - \sigma_{\hat{u},\pi}\sigma_{r,\hat{u}}}{\sigma_{\hat{u}}\sigma_{\pi} - (\sigma_{\hat{u},\pi})^2}$$

If  $\sigma_{\hat{u},\pi} = 0$ ,  $\sigma_{r,\pi} = 0$ ,  $\sigma_{r,\hat{u}} = 0$ , then

$$\text{plim } \tilde{\gamma} = 1 \quad \text{and} \quad \text{plim } \tilde{\beta} = -\beta$$

Therefore  $\tilde{\gamma} = 1$  is an appropriate test<sup>4</sup> of the null hypothesis and  $-\tilde{\beta}$  is a consistent estimate of the slope of the Phillips curve, conditional on the null hypothesis.<sup>5</sup>

It is somewhat surprising that ordinary least squares provides a valid test. The regression of  $R$  on  $\pi$  alone leads to a biased test. Indeed, the insight of Fama's test is that the bias can be avoided by reversing the regression. A regression of  $R$  on  $\hat{u}$  alone should lead (asymptotically) to a zero coefficient, since under neutrality  $\hat{u}$  and  $R$  are uncorrelated. Nonetheless, the regression of  $R$  on both  $\hat{u}$  and  $\pi$  produces a valid test. Two heuristic explanations of the test are worth considering before proceeding to a formal proof.

One way to think about the regression procedure is the following. The nominal interest rate contains only anticipated components of inflation. The inflation rate contains both anticipated and unanticipated components and deviations from the natural rate reflect only unanticipated movements of inflation. Under rational expectations the anticipated and unanticipated components are orthogonal. The regression projects anticipated inflation in the nominal interest rate onto  $\pi$ . The coefficient on  $\hat{u}$  is forced to  $-\beta$  in order to "undo" the projection of the unanticipated component in  $\pi$ .

As a second heuristic, consider the properties of the error terms under the null hypothesis. Fama's residuals arose from errors in anticipating inflation. Under rational expectations, these are uncorrelated with  $\pi$ . In my regression, residuals arise from variation in the real rate of interest. Under neutrality, these are uncorrelated with both  $\hat{u}$  and  $\pi$ . Consideration of the error terms shows that my test and Fama's are similar in that they provide valid estimators only under their respective null hypotheses. If Fama had rejected the hypothesis that his coefficient  $R$  equaled one, then his constant term would not have been a consistent estimate of the real interest rate. If I reject the coefficient of the inflation rate equaling one, then  $\tilde{\beta}$  is not a consistent estimate of the slope of the Phillips curve.

The test proposed here is more general than Fama's. In particular, it eliminates the very restrictive assumption of a constant real interest rate. The test uses evidence from both the Phillips curve and the Fisher equation. This has the advantage of extending the testable range of the neutrality hypothesis beyond the hypothesis considered by Fama. Pragmatically, however, consideration of the Phillips curve is not an unmixed blessing. The test here may be quite sensitive to measurement and specification error of the Phillips curve.<sup>6</sup> These cause practical difficulties which are not encountered using Fama's test.

### Implementing the Test of Neutrality

Testing neutrality and estimating the slope of the Phillips curve can be done in a straightforward manner if the natural rate of unemployment is known. Most of the work in this section involves the estimation of the natural rate and of the implied values of  $\hat{u}$ . Several stochastic models of the natural rate are considered. Time series representing the natural rate may be developed either through construction of a structural model or by the assertion of identifying statistical restrictions. I use the latter method, since construction of a structural model of unemployment is, in and of itself, a major research project.

I have chosen interest rate and inflation data matching Fama's as nearly as possible, so that differing results are known to arise out of the test rather than the sample.<sup>7</sup> I use the unemployment rate for prime age males in order to minimize the effects of demographic change. All data is quoted as annual percentage points. The sample consists of the 223 monthly observations from 1/53 through 7/71.

As a first statistical model, suppose that the natural rate is constant, but unknown. Let  $\tilde{u}^*$  stand for the estimate of the natural rate,  $\tilde{\alpha}$  for the estimate of the unconditional mean of the real interest rate,  $\tilde{\beta}$  for the estimate of (minus) the

slope of the Phillips curve, and use the t-statistic on the hypothesis  $\tilde{\gamma} = 1$  as a test of neutrality. This gives us equations (4) and (5).

$$u = \tilde{u}^* - \hat{u} \quad (4)$$

$$R = \tilde{\alpha} + \tilde{\beta}(\tilde{u}^* - u) + \tilde{\gamma}\pi + e \quad (5)$$

Equations (4) and (5) can be estimated either by full-information maximum-likelihood or by substituting (4) into (5) and using ordinary least squares. As there are no overidentifying cross-equation restrictions, the two methods are equivalent (except for a slight misstatement of standard errors reported from OLS due to the nonlinear term  $\tilde{\beta}\tilde{u}^*$ ). FIML estimates are presented in the first two lines of Table 1.  $\tilde{\gamma}$  is approximately 30 standard errors away from one and  $\tilde{\beta}$  is 10 standard errors on the wrong side of zero. While this might indicate a rejection of neutrality, it seems more sensible to reject the view that the natural rate is constant; this would also account for the 0.04 Durbin-Watson statistic reported for equation (4).

As a second statistical model I apply a filter rule to the unemployment rate. Under the null hypothesis, deviations of unemployment from the natural rate ought to be white noise. Following Sargent (1973) and many others, I estimate a filter which splits unemployment into a serially correlated series for the natural rate and a set of white noise innovations.<sup>8</sup> Several simple filters were tried, all yielding approximately the same results when used in estimation. A third order autoregressive series is shown in equations (6) and (7) and in Table 1.

$$u = \tilde{\rho}_0 + \tilde{\rho}_1 u_{-1} + \tilde{\rho}_2 u_{-2} + \tilde{\rho}_3 u_{-3} - \hat{u} \quad (6)$$

$$R = \tilde{\alpha} + \tilde{\beta}(\tilde{\rho}_0 + \tilde{\rho}_1 u_{-1} + \tilde{\rho}_2 u_{-2} + \tilde{\rho}_3 u_{-3} - u) + \tilde{\gamma}\pi + e \quad (7)$$

As a further check on the filter rule, I estimated (6) and (7) both separately and jointly and tested equality of the  $\tilde{\rho}$ 's in the two equations. The test statistic, which is distributed (asymptotically)  $F(3,213)$ , equaled 0.97, so the filter rule cannot be rejected at any of the usual levels of statistical significance.

The FIML estimates of (6) and (7), imposing the identifying restriction that  $\hat{u}$  and  $e$  are independent are reported in Table 1.  $\tilde{\beta}$  is two standard errors on the wrong side of zero. The test of rationality,  $\tilde{\gamma} = 1$ , fails by about 22 standard errors.

The Durbin-Watson statistic indicates serious autocorrelation in equation (7). While the estimation above is consistent, it is inefficient and reported standard errors are generally incorrect. I corrected equation (7) for first-order serial correlation and re-estimated. The results reported as (6') and (7'), are surprisingly different from the preceding estimate. The slope of the Phillips curve shows that a one point reduction in the unemployment rate is associated with a 20 basis point increase in inflation. This slope coefficient is two standard errors on the correct side of zero. However, the estimate of  $\gamma$  is now 126 standard errors below one.

Above, I use at most the univariate time series properties of unemployment to estimate deviations from the natural rate. As a third statistical model, I purge the estimates of  $\hat{u}$  of any correlation with  $R$ . It is perfectly reasonable that the natural rate may be correlated with the nominal interest rate. Theoretically, changes in the expected real interest rate affect the discounting of future income streams and therefore equilibrium job search time. More practically one might worry that increases in the natural rate of unemployment due to demographic changes have

occurred contemporaneously with a secular increase in expected inflation even absent a causal relation. By projecting the unemployment rate on the nominal interest rate the series for  $\hat{u}$  is forced to be independent of  $R$  in the sample. This procedure, though valid under neutrality, may weaken the test since it partially imposes the null hypothesis.

I forced the independence of  $R$  and  $\hat{u}$  by adding the term  $\eta R$  to the two models used above. The results are reported in rows (5'') and (7''). It is interesting to note that the results in (5'') are now similar to the estimates (7) and that (7'') is little changed from (7). In each case the test  $\tilde{\gamma} = 1$  rejects neutrality by more than 20 standard errors.

All the estimates have used monthly data. Over so short an interval the data may have a fair amount of noise. In particular, the coverage of CPI survey results may not be well synchronized with bond-holding periods. To mitigate such errors I re-estimated equations (4) through (7) using quarterly data. The estimates use the same sample period as for the monthly estimation and use the unemployment rate in the last month of the quarter. The filter is reduced to a second order AR process because a third lag proved to be perfectly multicollinear. The results, labelled (4Q) through (7Q) in Table 1, do not prove to be very different from the monthly estimates. The tests on  $\tilde{\gamma}$  in (5Q) and (7Q) reject neutrality by 8 and 6 standard errors.

The inescapable conclusion of these illustrative tests is that simple statements neoclassical neutrality are contradicted by the empirical evidence.

#### Long-Term Contracts and Rational Expectations

Fischer (1977) modifies neoclassical rational expectations results by recognition of long-term, nominal contracts. Expectations are fully rational. However, expected inflation has real effects when expectations are formed after

contracts are signed. Even though expectations are fully rational, deviations of unemployment from the natural rate persist during the life of the longest outstanding contract. According to long-term contracting theory, the observed serial correlation of the unemployment rate arises from serial correlation in  $\hat{u}$ , where the more purely neoclassical theory explored in the previous sections requires serial correlation in the unemployment rate to arise solely out of serial correlation in the natural rate. In this section of the paper, I extend the econometric principles suggested above into a test of one variant of long-term contracting theory. Specifically, I maintain the hypotheses of rational expectations and the independence of the real interest rate from anticipated inflation, but allow for persistent deviations of unemployment from the natural rate. By combining information from interest rates of different maturities the persistence pattern of  $\hat{u}$  can be replicated and the long-term contracting hypothesis can be tested.

I develop the theory in terms of one and two period contracts and then state the extension to the k-period case. Suppose a fraction  $\theta$  of all contracts start in the current period and that  $(1-\theta)$  carry over from the previous period. Letting  ${}_t P_{t+1}$  be the expectation of the logarithm of the general price level at time  $t+1$ , conditional on information available at time  $t$ , and letting  $\hat{u}_t$  be the deviation from the natural rate of unemployment at the end of period  $t$ , we have

$$\hat{u}_t = \frac{1}{\beta} \{ \theta [P_{t+1} - {}_t P_{t+1}] + (1-\theta) [P_{t+1} - {}_{t-1} P_{t+1}] \} \quad (8)$$

Deviations from the natural rate depend on errors in the one period inflation forecast made at time  $t$  and the two period inflation forecast made at time  $t-1$ . The one period time  $t$  forecast is also built into the one period time  $t$  nominal interest rate and the time  $t-1$  two period forecast is in the time  $t-1$  two period nominal interest rate.

	equation	coefficient estimates and standard errors			SER	R <sup>2</sup>	D.W.
(4)	u* 3.27 (0.079)				1.18	.00	.04
(5)	α 2.73 (0.108)	-β 0.683 (0.065)	γ 0.1897 (0.0275)		1.06	.34	.43
(6)	ρ <sub>0</sub> .1597 (.0461)	ρ <sub>1</sub> 1.032 (0.064)	ρ <sub>2</sub> 0.201 (0.094)	ρ <sub>3</sub> -0.281 (0.064)	.23	.96	1.90
(7)	α 2.48 (0.11)	-β 0.719 (0.367)	γ 0.298 (0.031)		1.28	.33	.62
(6')	ρ <sub>0</sub> .113 (0.046)	ρ <sub>1</sub> 1.054 (0.063)	ρ <sub>2</sub> 0.172 (0.092)	ρ <sub>3</sub> -0.259 (0.063)	.23	.96	1.95
(7')	α 4.22 (0.80)	-β -0.206 (0.095)	γ 0.0097 (0.0079)	(AR1) .966 (.018)	.36	.95	2.33
(5'')	α 2.48 (0.11)	-β -0.035 (0.098)	γ 0.301 (0.031)	η -0.500 (0.039)	1.30	.30	.57
(7'')	α 2.48 (0.11)	-β -0.114 (0.376)	γ 0.301 (0.031)	η - .022 (.014)	1.30	.30	.58
(4Q)	u* 3.26 (0.14)				1.18	.00	.16
(5Q)	α 2.51 (0.21)	-β 0.437 (0.123)	γ 0.405 (0.074)		1.00	.59	.79
(6Q)	ρ <sub>0</sub> 0.45 (0.15)	ρ <sub>1</sub> 1.20 (0.11)	ρ <sub>2</sub> -0.336 (0.105)		0.43	.87	2.09
(7Q)	α 2.18 (0.19)	-β 0.534 (0.289)	γ 0.548 (0.064)		1.06	.53	1.1

Ordinary Least Squares Based Tests

Table 1

At time  $t$ ,  $r_t^1$  and  $R_t^1$  are the  $i$  period expected real rate and nominal rate respectively. The nominal rates matching contracts which expire in the current period are shown in (9) and (10).

$$R_t^1 = r_t^1 + {}_t P_{t+1} - P_t \quad (9)$$

$$2R_{t-1}^2 = 2r_{t-1}^2 + {}_{t-1} P_{t+1} - P_{t-1} \quad (10)$$

Equation (11) shows the substitution of (8) into a weighted average of (9) and (10).

$$\theta R_t^1 + (1-\theta)2R_{t-1}^2 = \{\theta r_t^1 + (1-\theta)2r_{t-1}^2\} - \hat{\beta} u_t + \theta(P_{t+1} - P_t) + (1-\theta)(P_{t+1} - P_{t-1}) \quad (11)$$

In (12), (11) is re-arranged in a form suitable for estimation.

$$R_t^1 = \left\{ r_t^1 + \frac{1-\theta}{\theta} 2r_{t-1}^2 \right\} - \frac{\beta}{\theta} \hat{u}_t + (P_{t+1} - P_t) - \frac{1-\theta}{\theta} \{2R_{t-1}^2 - (P_{t+1} - P_{t-1})\} \quad (12)$$

Just as in the previous section, the coefficient on the inflation rate equals one under the null hypothesis. The contract weights can be retrieved from the coefficient on the last term in (12). Notice that if  $\theta$  equals one, equation (12) reduces to the model of the previous section.

The generalization of (12) to the  $k$ -period contract case is shown in (13).

The  $\theta_j$  must sum to one.

$$R_t^1 = \left\{ \sum_{j=1}^k \frac{\theta_j}{\theta_1} (j r_{t-j+1}^j) \right\} - \frac{\beta}{\theta_1} \hat{u}_t + (P_{t+1} - P_t) - \left\{ \sum_{j=2}^k \frac{\theta_j}{\theta_1} (j R_{t-j+1}^j - (P_{t+1} - P_{t-j+1})) \right\} \quad (13)$$

The last term, the long-term, ex post real interest rate, is highly correlated with the unobserved long-term expected real interest rate. Ordinary least squares is inconsistent even under the null hypothesis.

For consistent estimation, it is necessary to purge the last term of correlation with the expected real rate while retaining correlation with unanticipated inflation. The  $j$ -period inflation rate,  $P_{t+1} - P_{t-j+1}$  is the perfect instrument for this requirement. Equation (13) can be consistently estimated, under the null hypothesis, using as instruments  $k-1$  multi-period inflation rates, a constant,  $\hat{u}$ , and  $\pi_t$ .

The first four lines of Table 2 present results for 1-3, 1-6, 1-9, and 1-12 period contracts, based again on a constant natural rate.<sup>9</sup> The test of the null hypothesis,  $\tilde{\gamma} = 1$ , can be rejected by  $t$ -statistics of 15, 29, 7.7, and 4.6 respectively. In the last two lines of Table 2 I have re-estimated using (6) to form  $\hat{u}$ . The null hypothesis is still decisively rejected. The empirical evidence lends no support to this particular variant of long-term contracting theory.

### Conclusion

This paper has two parts. First, econometric tests are developed in order to test some of the classical neutrality propositions. Second, these tests are illustrated on simple formulations of the neutrality propositions. The tests repeatedly reject the null hypothesis.

In light of the strong rejection, it seems valuable to review possible extensions to the way the testing principles are implemented. First, it would be desirable to find deviations from the natural rate of unemployment through structural models rather than statistical means. Second, the identification of real market shocks other than those correlated with unemployment would eliminate a possible source of asymptotic bias. Third, correction for tax effects might be very important. However, the most straightforward corrections suggest we ought to have observed  $\tilde{\gamma}$  greater than, not less than, one. Fourth, correction for risk aversion should be considered since there is at least casual evidence that periods of high

expected inflation are associated with substantial inflation uncertainty. Fifth, the coverage of the long-term contract tests ought to be extended to a longer period.

These caveats notwithstanding, the tests are done on much the same terms as previous tests that have tended to support the neutrality propositions. The strong rejections of the null hypothesis cast serious doubt on the simpler representations of the neoclassical school of macroeconomic thought.

Periods	$-\beta$	$\gamma$	$\theta_1$	Dates	SER	D.W.
1-3	1.0 (0.15)	-0.0013 (0.0667)	0.838 (0.041)	1/53-7/71	1.52	0.28
1-6	0.481 (0.67)	0.0010 (0.0343)	0.933 (0.024)	6/59-7/71	0.558	0.71
1-9	-1.25 (1.41)	0.0311 (0.1246)	0.956 (0.203)	7/64-7/71	1.342	0.61
1-12	-1.52 (2.86)	-0.038 (0.226)	0.898 (0.148)	7/64-7/71	2.37	0.97
1-3 [using(6)]	-1.45 (0.91)	0.0088 (0.0960)	0.745 (0.056)	1/53-7/71	2.21	0.34
1-12 [using(6)]	3.04 (5.34)	0.0019 (0.2399)	0.998 (0.192)	7/64-7/71	2.55	0.94

Instrumental Variable Estimates for Long-Term Contracts

Table 2

FOOTNOTES

<sup>1</sup>As Fama points out, we are actually interested in the change in the purchasing power of money. The inflation rate is actually measured as the negative of the deflation rate, throughout.

<sup>2</sup>For a review of this literature, see Santomero and Seater (1978).

<sup>3</sup>See notably McCallum (1976) and Sargent (1976a). For a discussion of some of the problems encountered in a number of studies see Levi and Makin (1978, 1979). For further discussion of Fama's test see, Carlson (1977), Joines (1977), Nelson and Schwert (1977), and the rejoinder by Fama (1977).

<sup>4</sup>Unlike the original Fama specification, here serial correlation is not inconsistent with rationality, as the expected real interest rate may be serially correlated. Therefore, the consistent estimation of standard errors may require serial correlation correction.

<sup>5</sup>Notice that the test on  $\tilde{\gamma}$  has zero power against the alternative hypothesis,  $\sigma_{r,u}^{\wedge} = 0$ ,  $\sigma_{r,\pi^e} = 0$ ,  $\sigma_{u,\pi^e}^{\wedge} \neq 0$ . In this special case,  $\tilde{\beta}$  remains a consistent estimator of  $-\beta$ .

<sup>6</sup>The execution of the tests assumes a one sector world. The principles of econometric testing extend directly to the more general case in which unanticipated inflation is due to shocks in any of several markets. If  $\hat{\varepsilon}$  represents deviations from equilibrium in, say, the energy markets, then the full expression for the "Phillips curve" ought to be  $\pi = \pi^e + \beta\hat{u} + \delta\hat{\varepsilon}$ . If covariance conditions for  $\hat{\varepsilon}$  analogous to those stated above for  $\hat{u}$  hold, then an ordinary least squares regression of  $R$  on  $\hat{u}$ ,  $\hat{\varepsilon}$ , and  $\pi$  produces consistent results. Of course, if the world cannot be described by a one sector model, then the reported regressions are misspecified due to an omitted variable. Nor would the problem be resolved if  $\hat{u}$  and  $\hat{\varepsilon}$  happened to be uncorrelated. Indeed, as either a little tedious algebra or a moment's thought will show, the probability limit of  $\tilde{\gamma}$  goes to one under the null hypothesis only if  $\hat{u}$  and  $\hat{\varepsilon}$  are perfectly correlated. In such a case  $-\tilde{\beta}$  would be a consistent estimator of  $\beta + \delta$ .

<sup>7</sup>Estimation in the paper uses monthly data from 1/53 to 7/71, the same sample period as in Fama(1975). The consumer price index is used to measure changes in purchasing power. The unemployment rate for male workers between ages 25 to 54 is used to eliminate the changes in the natural rate due to the sweeping changes in labor force composition which have occurred in the latter part of the post-war period. The interest rate data used is for treasury bills, quoted on a discount basis, developed in Bildersee (1976).

<sup>8</sup>Even though this is a very popular technique - being a somewhat simpler version of that used by Barro in a series of articles on "unanticipated money", Barro (1977), Barro (1978), for example - one probably ought to be less than sanguine about its use. While statistical properties can be used to reject the hypothesis that a particular series represents unanticipated deviations from the natural rate, no purely statistical test can be applied to the series which allegedly represents the natural rate itself. Of course, this filter is valid only under the null  $\hat{u}$  hypothesis. If rationality is rejected we cannot make use of the estimates of  $u$  for other purposes. The illustrative tests are conditional on the assumption that the estimated model adequately captures the time series properties of the natural rate. For further discussion, see Sargent (1976) and Germany and Srivasta (1979).

<sup>9</sup>Due to data limitations the regressions for longer term contracts cover considerably shorter periods.

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