CORPORATE HEDGING IN FUTURE MARKETS WHEN SHORT SALES ARE RESTRICTED

by

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1. Introduction

If financial markets are complete, there can be no motive for hedging by firms, since corporate hedging will simply result in the creation of securities which were already available to the individual investor. Any financial theory of hedging by publicly-traded firms must, therefore, be a theory of hedging in an incomplete financial market.[1] This paper presents such a theory, by showing that if short selling of securities by consumers is restricted, corporate hedging in futures markets will lead to an increase in the firm's market value. Although the formal argument requires a considerable amount of notation, it is quite simple to motivate the results. In this section we shall do precisely this, reserving the formal model and its results for Sections 2-4.

Consider a market in which income streams are sold. Denote such a stream by the letters a, b; note that such an income stream may be stochastic (state-dependent). Let V(a) denote the market value of the income stream a. In the Section 3 we shall show that it is a general property of a market without short sales (or one in which short sales are limited) that

$$(1) \qquad V(a) + V(b) \geq V(a + b).$$

The reason for this fact is quite simple: If short sales are restricted, not all individuals will agree about the value of income streams. Each income stream will thus be sold to that individual who places the highest value on it; all other individuals will either agree with the purchaser or they will value the income stream at below its market value. (Note that if we were to allow short sales, an individual who valued an income stream at less than its market value would short sell it; this would eventually lead to all individuals placing the same value on every income stream.) Denote the private valuation of individual i by $V_i(a)$. In a market with restricted short sales,

$$(2) \qquad V(a) = \max_i V_i(a).$$

it is now readily seen that

$$(3) \qquad V(a + b) = \max_i V_i(a + b) < \max_i V_i(a) + \max_i V_i(b) = V(a) + V(b).$$

Intuitively, what is happening here is that in splitting up the income stream a + b, we are able to sell off a to its highest bidder and b to its highest bidder. We can do no worse in this case than if we sell off a and b together.[2]

To apply the above argument to futures hedging, suppose that a corporation is currently selling an income stream a, and that it is considering splitting off stream b from a and selling it separately. It follows immediately from (1) that

$$(4) \qquad V(a - b) + V(b) > V(a - b + b) = V(a).$$

Note that in selling a futures contract, a corporation is doing exactly what we have alluded to above: The corporation is splitting off an income stream (that produced by a fixed quantity of the product produced under varying market prices) and selling it to the highest bidder.

The succeeding three sections formalize the above argument. In the next section we outline a two-period model with consumption and production, where uncertainty in the second period is represented by the occurrence of one of a number of states of the world. Section 3 gives first-order conditions for the model when short sales are restricted, and Section 4 discusses the effect of corporate hedging in the futures market on the market value of the corporation. In addition we discuss in Section 4 consumer preferences over corporate hedging. In Section 5 we consider a numerical example.

2. The model

We consider a two-period model. The first period of the model ("today") will be subscripted by 0, whereas states of the world in the second period ("tomorrow") will be subscripted by 1,...,M. Agents in the model (be they consumers or firms) do not know which of the M states of the world will occur tomorrow, but they all agree on the values of all relevant variables (prices, production by firms, consumption by consumers) given the occurrence of each state of the world. In this sense the model is what Radner (1972) has called a "rational expectations" model. Note that we shall not require agreement among consumers about their subjective probabilities as to the occurrence of the various states; we shall in fact make no reference to these probabilities, but shall instead subsume them in the utility function.[3]

Let there be H commodities which are used for both consumption and production. Denote by J the number of firms in the model; each firm shall be assumed to own a stochastic production technology capable of producing one good.[4] We shall thus set $J = H$, and we shall assume that firm j produces good j. A given firm j which buys an input vector today will have the use of these inputs tomorrow for production. Using superscripts to denote the commodities, write the input vector of firm j by

$$(5) \qquad z_j = (z_j^1, \ldots, z_j^H),$$

where z_j^h denotes the physical quantity of commodity h purchased by firm j. If firm j buys an input vector z_j today, we shall assume that its output tomorrow in state m will be determined by a function y_{jm}; the state-dependent output of firm j given its purchases of inputs z_j will be written

$$(6) \qquad y_j(z_j) = \big(y_{j1}(z_j), \ldots, y_{jm}(z_j)\big).$$

Denote the commodity price vector in the first period by

$$(7) \qquad p_o^c = (p_o^{c1}, \ldots, p_o^{cH}),$$

and denote the comodity price vector in state m of the world tomorrow by

$$(8) \qquad p_m^c = (p_m^{c1}, \ldots, p_m^{cH}).$$

Then the cost of purchasing the inputs of firm j will be $p_o^c z_j$, and the value of firm j's production in state m of the world tomorrow will be $p_m^c y_{jm}(z_j) = p_m^{cj} y_{jm}(z_j)$, where the multiplication will represent the standard vectorial dot product.

<u>Consumers</u>: Let there be I consumers in the model. A typical consumer i states the first period with an endowment of shares in each of the firms and with a commodity vector endowment.[5] Denote the fractional share of firm j

owned by consumer i at the beginning of the first period by $\bar{f}_{ij}$. We shall assume that these fractional shares are non-negative and sum to one (so that initially, every firm is owned totally by the consumers in the model):

$$(9) \qquad \bar{f}_{ij} \geq 0; \ \sum_i \bar{f}_{ij} = 1, \ j = 1, \ldots, J.$$

The initial shareholders of firm j are assumed to choose the inputs of the firm and to pay for these inputs. Denoting the market price of all of firm j's equity in the first period by p_j, the value of consumer i's initial portfolio (given firm j's choice of inputs z_j) will be

$$(10) \qquad \sum_j \bar{f}_{ij}(p_j - p_o^c z_j).$$

In the first period consumer i will be assumed to sell his initial holding in the firms and purchase a new portfolio $f_i = (f_{i1}, \ldots, f_{iJ})$.[6] We shall not allow short sales in the model, and we shall therefore require that $f_{ij} \geq 0$ for every firm j.[7] f_{ij} will be assumed to be the fractional part of firm j's equity purchased by consumer i; purchase of this fraction of firm j's equity will entitle the consumer to a similar fraction of the firm's revenue in each state m of the second period. Thus, having purchased a new portfolio f_i, consumer i will receive

$$(11) \qquad \sum_j f_{ij} p_m^c y_{jm}(z_j)$$

in revenue from this portfolio in the second period in state m. The cost to consumer i of this portfolio in the first period will be $\sum_j f_{ij} p_j$.

Denote consumer i's initial endowment of commodities by $w_i = (w_i^1, \ldots, w_i^H)$, and denote consumer i's consumption of goods in the first period by

$$(12) \qquad x_{io} = (x_{io}^1, \ldots, x_{io}^H);$$

similarly, consumer i's consumption of commodities in state m of the second period will be denoted by

$$(13) \qquad x_{im} = (x_m^1, \ldots, x_{im}^H).$$

Then a consumption vector $x_i = (x_{io}, x_{i1}, \ldots, x_{iM})$ will be feasible for consumer i if there exists a portfolio f_i fulfilling the no-short sales constraint such that

$$(14) \qquad p_o^c x_{io} < \sum_j \bar{f}_{ij}(p_j - p_o^c z_j) - \sum_j f_{ij} p_j + p_o^c w_i$$

$$(15) \qquad p_m^c x_{im} < \sum_j f_{ij} p_m^c y_{jm}(z_j), \; m = 1, \ldots, M.$$

Consumers will be assumed to maximize the utility of their state-dependent consumption x_i. The consumption-portfolio pair (x_i^*, f_i^*) will be said to be <u>optimal</u> for consumer i given market prices for commodities and firms and given his initial endowment of commodities and shares if (x_i^*, f_i^*) is feasible and if

$$(16) \qquad U_i(x_i^*) > U_i(x_i) \text{ for any } (x_i, f_i) \text{ which is feasible.}$$

3. First-order conditions for consumer maximization

From the Kuhn-Tucker theorem we may establish first-order conditions for consumer maximizatino. These first-order conditions give us each consumer's shadow prices for production in each state; we shall call these shadow prices the consumer's implicit state prices.

Theorem 1: Let (x_i^*, f_i^*) be optimal for consumer i. Then there exist implicit state prices $q^i = (q_1^i, \ldots, q_M^i)$ for consumer i such that

$$p_j \geq \sum_m q_m^i p_m^c y_{jm}(z_j), \quad j = 1, \ldots, H. \tag{17}$$

Furthermore, if $f_{ij}^* > 0$, there exists equality in (17), whereas if there is strict inequality in (17), we shall have $f_{ij}^* = 0$. The implicit state price are given by

$$q_m^i = \frac{\partial U_i / \partial x_{im}^1}{\partial U_i / \partial x_{io}^1} \cdot \frac{p_o^{c1}}{p_m^{c1}} \tag{18}$$

and where the partial derivatives are evaluated at x_i^*.

Proof:

We shall only sketch the proof. Dividing equation (14) through by p_o^{c1} and dividing equation (15) through by p_m^{c1}, the result is readily obtainable as an application of the Kuhn-Tucker Theorem. qed

Intuitively, the meaning of equation (17) is clear: Each consumer i evaluates the state-dependent returns of each firm using his own implicit state prices. If the consumer's implicit valuation is less than the value on the market of the firm, then the consumer does not purchase any shares in the firm. In equilibrium it will be found that no consumer's implicit valuation exceeds the market value of any firm; were this the case, additional purchases of the firm's shares would be made until equality were achieved.

Note that it follows from (17) that

$$(18) \qquad p_j = \max_i \sum_m q_m^i p_m^c y_{jm}(z_j), \; j = 1, \ldots, J.$$

This proves that equation (2) holds; the properties of the function may now be used to establish equation (1).

4. Futures contracts

A futures contract is a contract obliging the firm to deliver a fixed physical quantity of its product tomorrow irrespective of the state of the world; the purchaser of the contract promises to pay the producer a fixed (non-state-dependent) price for his product. The price of the contract is set so that the current market value of the contract is zero.

Denote by α_j the quantity of good which firm j has promised to deliver at price β_j. The firm's revenues from this contract in state m may be written

$$(19) \qquad \alpha_j(\beta_j - p_m^{cj}),$$

and the revenues of the purchaser of the contract will be

$$(20) \qquad \alpha_j(p_m^{cj} - \beta_j).$$

it follows from the way we have defined the price β_j and from Theorem 1 that β_j is fixed such that

$$\max_i \sum_m q_m^i(p_m^{cj} - \beta_j) = 0. \tag{21}$$

Thus the contract will be sold to the individual whose evaluation of its proceeds is the highest among all individuals, and the price β_j will be set so that this implicit valuation is zero.

Suppose firm j contracts to sell quantity α_j in a futures contract bearing price β_j. We may calculate the new price of the firm by finding the highest consumer valuation of the new revenues of the firm. Denoting this new price by $p_j(\alpha_j)$ and using Theorem 1, it follows that

$$p_j(\alpha_j) = \max_i \sum_m q_m^i\{p_m^c y_{jm}(z_j) + \alpha_j(\beta_j - p_m^{cj})\}. \tag{22}$$

In the next theorem we shall show that $p_j(\alpha_j)$ will never be less than p_j:

<u>Theorem 2</u>: $p_j(\alpha_j) \geq p_j$ for $\alpha_j \geq 0$.

<u>Proof</u>:

First note that it follows from (21) that β_j is fixed such that

$$\min_i \sum_m q_m^i(\beta_j - p_m^c) = 0. \tag{23}$$

it follows that for any individual consumer e,

$$\sum_m q_m^e(\beta_j - p_m^{cj}) \geq 0. \tag{24}$$

Now suppose that the maximum in (22) is obtained using the implicit prices of individual $\bar{e}$. Then letting e be any consumer for whom $f^*_{ej} > 0$, it follows from the definition of $p_j(\alpha_j)$ that

$$(25) \qquad p_j(\alpha_j) = \sum_m q_m^{\bar{e}}\{p_m^c y_{jm}(z_j) + \alpha_j(\beta_j - p_m^{cj})\}$$

$$\geq \sum_m q_m^e\{p_m^c y_{jm}(z_j) + \alpha_j(\beta_j - p_m^{cj})\}$$

$$= \sum_m q_m^e p_m^c y_{jm}(z_j) + \sum_m q_m^e \alpha_j(\beta_j - p_m^{cj})$$

$$\geq p_j,$$

where the last inequality follows from (24) and from the fact that since $f_{ej} > 0$ there exists equality in equation (17). qed

Theorem 2 thus shows that any firm which sells a futures contract will not lower (and will, in most cases, raise) its market value. It is by now well known that in imperfect capital markets the maximization of a firm's market value is not necessarily an objective desired by all of the firm's shareholders.[8] Thus, even though we have now proved that a firm will raise its market value (or not lower it) by engaging in futures hedging, we cannot yet say unequivocally whether its shareholders will desire to do so. To derive the conditions under which a firm's shareholders will desire it to engage in the futures market, we shall take the derivative of a typical individual i's utility with respect to α_j.

Theorem 3: If $\bar{f}_{ij} \geq f^*_{ij}$, then individual i will want firm j to engage in futures hedging.

Proof: We wish to determine when

$$(26)\qquad \frac{\partial U_i}{\partial \alpha_j} = \frac{\partial U_i}{\partial x^1_{io}}\frac{\partial x^1_{io}}{\partial \alpha_j} + \sum_m \frac{\partial U_i}{\partial x^1_{im}}\frac{\partial x^1_{im}}{\partial \alpha_j}$$

$$= \frac{\partial U_i}{\partial x^1_{io}}\left\{\frac{1}{p_o^{c1}}(\bar{f}_{ij} - f^*_{ij})\frac{\partial p_j(\alpha_j)}{\partial \alpha_j}\right\}$$

$$+ \sum_m \frac{\partial U_i}{\partial x^1_{im}}\left\{\frac{1}{p_m^{c1}} f^*_{ij}(\beta_j - p_m^{c1})\right\} > 0.$$

Dividing through by $\partial U_i/\partial x^1_{io}$, and using (25), we find that the inequality in (26) holds if and only if

$$(27)\qquad (\bar{f}_{ij} - f^*_{ij})\sum_m q_m^{\bar{e}}(\beta_j - p_m^{cj}) + f^*_{ij}\sum_m q_m^i(\beta_j - p_m^{cj}) > 0.$$

Since $f^*_{ij} \geq 0$, the second term in (27) is always non-negative. By the proof of Theorem 2 (equation (24)), it follows that

$$\sum_m q_m^{\bar{e}}(\beta_j - p_m^{cj}) \geq 0.$$

It thus follows that if $(\bar{f}_{ij} - f^*_{ij}) \geq 0$, equation (27) holds. qed

Two remarks are in order about Theorem 3:

Remark 1: The increase (non-decrease) in the price of the firm as a result of its hedging operations established in Theorem 2 cuts two ways. On the one hand, it increases the wealth of initial shareholders; on the other hand, it makes new purchases of the firm's shares more expensive. Call the first of these effects the <u>wealth effect</u> and the second the <u>consumption effect</u> of the hedging operation. Then Theorem 3 may be interpreted as saying that corporate hedging is preferred by the firm's shareholder if the wealth effect outweighs the consumption effect. If we conceive of the first period as representing a typical period in a multi-period model, and if we assume that most shareholders of the firm will make no change in their portfolios, then the conditions of Theorem 3 will always hold, and hedging will be preferred by the firm's shareholders. Another condition under which hedging will be preferred is if there are many firms which are similar to firm j. We may then assume that initial shareholders of firm j will set $f^*_{ij} = 0$; this may be done, since they can always purchase shares in come other firm whose production function is the same as that of firm j and which does not engage in hedging operations.[9]

Remark 2: Theorem 3 gives only sufficient conditions for shareholders to prefer hedging. By manipulating equation (27), it follows that

$$(28) \qquad f^*_{ij} < \frac{\bar{f}_{ij} \sum_m q^{\bar{e}}_m(\beta_j - p^{cj}_m)}{\sum_m (q^{\bar{e}}_m - q^i_m)(\beta_j - p^{cj}_m)}$$

is both a necessary and sufficient condition for consumer i to prefer corporate hedging. I have, however, been unable to find a satisfactory operational meaning for this equation.

5. A Numerical Example

Consider an economy with two future states and three firms, each producing, with a stochastic production function (i.e., state-dependent) one good. Suppose that firm 1 produces 4 units of its good in state 1 and 24 units in state 2, and that the price the good produced by the firm is .75 in state 1 and .25 in state 2. The revenue of the firm is thus given by the vector

$$(4 \times .75,\ 24 \times .25) = (3,\ 6).$$

Without going into the same kind of detail, we shall assume that the revenue of the other two firms is given by the vectors

firm 2: $(1, 1)$

firm 3: $(7, 2)$.

Now suppose that the firms allocate all of their revenues to their shareholders, and that these shareholders maximize state-dependent utility functions of these revenues. The purchase by a consumer of the equity of a firm j is thus equivalent to purchasing a proportion of that firm's (state-dependent) revenue vector. Suppose that the price of the whole revenue vector of firm 1 is 4, the price of firm 2 is .75, and the price of firm 3 is 3. Then a consumer who maximizes a state-dependent utility function will typically be faced with the following maximization problem:

$$\max U(x_1, x_2)$$

s.t.

(29) $$x_1 = 3f_1 + f_2 + 7f_3$$

(30) $$x_2 = 6f_1 + f_2 + 2f_3$$

(31) $$4f_1 + f_2 + 3f_3 = W$$

(32) $$f_1, f_2, f_3 > 0.$$

In the above equations, f_1, f_2, f_3 represent the proportion of each firm's equity purchased by the consumer, and equation (31) represents the consumer's wealth constraint. The last equation (32) gives the no-short sales constraint.[10]

Figure 1 gives a graphical representation of the (x_1, x_2) possibilities afforded the consumer by the equations (29) - (32) above. The rays extending from the axis are the revenue vectors of the firms, and the solid lines -- AB and BC -- represent the optimal (x_1, x_2) opportunities. A consumer such as consumer 1 will maximize his utility by investing half his initial wealth W in the equity of firm 1 and half in firm 2's equity. Consumer 2, on the other

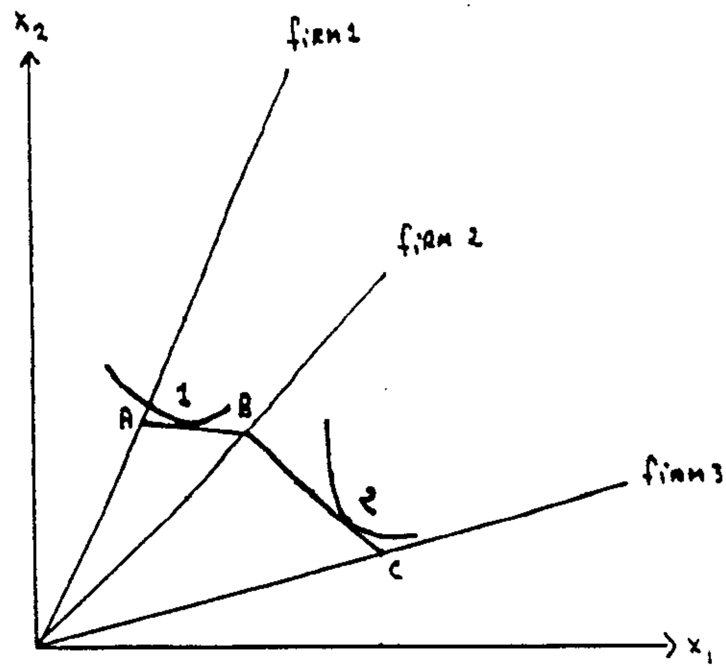

Figure 1

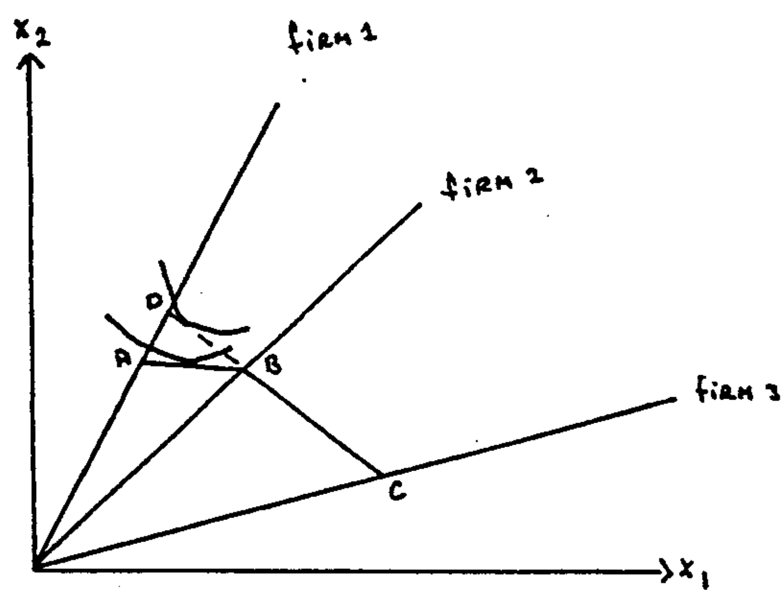

Figure 2

hand, will invest in a portolio which has $\frac{1}{4}$ of his wealth invested in firm 2 and $\frac{3}{4}$ in firm 3. (Note that the line AC is feasible but dominated by AB and BC).

Each of the line segments AB and BC corresponds to a unique set of Kuhn-Tucker shadow prices for a unit of revenue in states 1 and 2. To derive such prices for segment AB we solve the equations

(33) $\quad 3q_1 + 6q_2 = 4.$

(34) $\quad q_1 + q_2 = .75.$

The solution to these equations yields

(35) $\quad q_1 = .1667, \; q_2 = .5833.$

Similarly we may derive implicit state prices for the segment BC by solving

(36) $\quad r_1 + r_2 = .75$

(37) $\quad 7r_1 + 2r_2 = 3,$

yielding

(38) $\quad r_1 = .3, \quad r_2 = .45$

Thus all consumers of type 1 will price securities' return vectors by shadow prices q_1 and q_2, and all consumers of type 2 will price return vectors by using their shadow prices r_1 and r_2.

Note that the short sales restriction means that consumers such as consumer 1 will view securities such as those of firm 3 to be overpriced. To see this, we value firm 3 at the implicit state prices used by consumer 1,

(39) $\quad 7q_1 + 2q_2 = 2.3335 < 3.$

A similar calculation will show that consumer 2 considers firm 1 to be overvalued. Were short sales to be allowed, this situation would be corrected. To see this consider the dotted segment BD in Figure 2. Points on BD represent portfolios in which firm 2 is held in positive amounts and firm 3 is shorted. Clearly, if consumer 1 could short unlimited amounts of firm 3, he could move to a higher utility indifference curve (show in Figure 2).

Now let firm 1 hedge some of its production. Letting β represent the futures price, each unit of production hedged gives the firm the vector (β,β) instead of the commodity price vector (.75, .25). The firm thus nets the vector $(\beta - .75, \beta - .25)$ from the hedging operation. The purchaser of the hedged goods will correspondingly receive $(.75 - \beta, .25 - \beta)$[11].

An individual of type 1 will value this contract at his implicit prices q_1 and q_2, and since futures prices are set so that the net present value of the contract equals zero, individuals of type 1 will determine the amount they are willing to pay for the contract by solving equation

$$(40) \qquad q_1(.75 - \beta) + q_2(.25 - \beta) = 0$$

Thus individuals of type 1 will bid $\beta = .3611$ to purchase the hedged commodity.

Solving the similar equations for individuals of type 2, we solve for $\beta = .4333$. Since consumers of type 2 are willing to offer a higher futures price for the contract, it will be sold to them.

What will be the value of firm 1 to consumers after the hedge? Denoting by α the quantity of the good hedged, we get (since firm 1 is priced at implicit price q_1 and q_2) that the value of firm becomes (using vector product notation):

$$(41) \qquad (q_1, q_2) \begin{vmatrix} .75(\ 4 - \alpha) \\ .25(24 - \alpha) \end{vmatrix} + (q_1, q_2) \begin{vmatrix} \alpha\beta \\ \alpha\beta \end{vmatrix}$$

which, substituting $q_1 = .1667$, $q_2 = .5833$, $\beta = .4333$ becomes

$$(42) \qquad (.1667,\ .5833)\begin{vmatrix}3 - .3167\alpha \\ 6 + .1833\alpha\end{vmatrix} = 4 + .0541\alpha > 4.$$

A value-maximizing hedge at current implicit prices would imply that the firm set α as high as possible. If the firm is constrained to hedge no more than it can supply to the purchaser of the hedge with certainty, then it should set $\alpha = 4$.

6. Summary and conclusions

By hedging some if its future production in a firm can raise its market value if short selling of securities is restricted. This is true even if the shares of the firm are publicly traded. The change in the value of the firm occurs because all individuals need not agree about the valuation of return vectors. It is thus worthwhile splitting off an income stream from the firm and selling it to that individual who places the highest value on it. A value-maximizing firm should always hedge its lowest anticipated level of future production.

Footnotes

1. Authors who have dealt with hedging in a portfolio context, such as Johnson (1960) and Peck (1975), and Stoll (1979) implicitly assume that markets are incomplete.

2. Litzenberger and Sosin (1977, footnote 5) make an argument similar to the above in their discussion of dual-purpose funds: It follows from the above argument that a closed-end fund should never be valued at more than its net-asset value.

3. Suppose we denote consumer i's commodity consumption in the first period by x_{io} and his consumption in state m tomorrow by x_{im}. Then it might well be that

$$U(x_{io}, x_{i1}, \ldots, x_{im}) = V(x_{io}) + \sum_m \gamma\pi_m^i V(x_{im}),$$

where the π_m^i are consumer i's subjective state probabilities and γ is some constant indicating time preference. Our model is open to such an interpretation, but we shall always write the utility functions of consumers in their most general form (the left-hand-side of the above equation). Thus, while our consumers might be expected utility maximizers using their own subjective probabilities, the model allows other kinds of maximizing behavior.

4. This assumption is not strictly necessary to the analysis, but it allows for some notational simplification.

5. Note that we have assumed that the firms do not produce in the first period. The consumers' commodity endowments replace first-period firm production. As in the above footnote, this assumption is made for simplicity only.

6. As long as we assume the absence of transactions' costs, the assumption that consumers first sell their share endowments and then purchase new portfolios does not impair the model's generality.

7. The assumption that no short sales are allowed comes closer to representing current economic reality than the assumption that consumers may short any stock they wish. The theoretical short sales mechanism invoked in many financial models calls for the short seller to have the use of the money raised at the time of the short sale. However, common brokerage practice calls for the money to be deposited with the broker (thus giving him its use) until the short seller has replaced the shares. In addition, due to the difficulty of borrowing large numbers of most stocks (all but the most widely traded), it is exceedingly difficult to sell short shares of small firms which the seller feels to be overvalued. This latter remark applies especially to firms in the economy which are most likely to engage in hedging (smaller firms, commodity producers, farmers). In countries with less developed financial markets, short sales are practically impossible.

8. See Leland (1974), Ekern and Wilson (1974), and Radner (1974). Under certain conditions, shareholders may not agree on what constitutes market value, but voting solution may be possible; see Benninga and Muller (1979). Hart (1979) has shown that in large markets all shareholders will agree that market value is to be maximized, even though they may disagree what determines market value.

9. In a general equilibrium sense this strategy may be contradictory, since the shareholders of the firms which have the same production function as firm j may also decide to engage in hedging. In a sufficiently large economy, however, this would probably not be the case.

10. In the example we ignore first period consumption and the firm's choice of its inputs. These could be added, but the graphical demonstration would then have to be in three dimensions.

11. Note that the purchaser of the hedged commodity is not buying equity in firm 1; rather he is buying the price risk of the commodity produced by the firm, leaving the firm's shareholders with the quantity (production) risk.

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