Risk Aversion With Random Initial Wealth

by

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1. INTRODUCTION

How is the economic response to uncertainty influenced by individual risk aversion? This question has been the focus of an important part of the literature on the economics of uncertainty in the past fifteen years. The direct impetus for most, if not all, of this work on risk aversion was the introduction, by Arrow [1971] and Pratt [1964], of measures of the local risk aversion associated with a von Neumann-Morgenstern utility function. They argued that $R^u = -\frac{u''}{u'}$ is an appropriate measure of the local risk of aversion of an individual who maximizes the expected value of the (twice differentiable) von Neumann-Morgenstern utility function u.*

Arrow and Pratt justified this measure by showing that when it rose, economic behavior became more risk averse. Specifically, suppose that there are two individuals, 1 and 2, with individual 1 maximizing the expected value of u_i; suppose also that

$$R^{u_1} = -\frac{u_1''}{u_1'} > -\frac{u_2''}{u_2'} = R^{u_2} \tag{1}$$

holds uniformly on the relevant domain of the u_i's. Arrow and Pratt showed that individual 1's behavior would then be more risk averse.

For example, suppose that the individuals invest their wealth in a safe and a risky asset. Denote the random rate of return of the risky asset by $\tilde{x}$. Individual i with initial wealth y invests B_i in the risky asset and $y - B_i$ in the safe asset. His optimal choice $\hat{B}_i(\tilde{x},y)$ is the value of B_i in $[0,y]$ which maximizes

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*In fact they proposed $-(u''/u')$ as a measure of local absolute risk aversion, as opposed to relative risk aversion. They also proposed a definition for this later term. In this paper, we will be concerned with absolute risk aversion only.

$$Eu_i(y + B_i\tilde{x}) \quad .$$

Arrow and Pratt showed that if (1) holds uniformly, then for any y and non-degenerate random variable, $\tilde{x}$,

$$(2) \qquad \hat{B}_1(\tilde{x},y) < \hat{B}_2(\tilde{x},y) \quad .$$

In other words, individual 2's optimal portfolio includes more of the risky asset.

Arrow and Pratt provided an alternative justification by introducing the risk premium associated with a random variable. If the random variable $\tilde{x}$ is interpreted as a random variation in wealth, the associated risk premium $\pi_i(\tilde{x},y)$ is the reduction in mean wealth that individual i with initial wealth y will accept to eliminate the random variation. Formally, $\pi_i(\tilde{x},y)$ is defined by

$$(3) \qquad Eu_i(y + \tilde{x}) = u_i[y + E\tilde{x} - \pi_i(\tilde{x},y)] \quad .$$

Arrow and Pratt showed that if (1) holds uniformly, then for any y and non-degenerate $\tilde{x}$,

$$(4) \qquad \pi_1(\tilde{x},y) > \pi_2(\tilde{x},y) \quad .$$

Subsequent papers by many other authors have used R^u to study the influence of increases in risk aversion on expected utility maximizing decisions made in the face of economic uncertainty. In the preponderance of cases, increases in R^u imply decisions that represent, in an intuitive sense, more risk averse responses to uncertainty.

In virtually all of this literature, there is assumed to be a single source of uncertainty.* In the portfolio problem, for example, the only risk is that which arises because there is uncertainty about the return to investment in the risky asset. Similarly, the risk premium is defined as the reduction in mean wealth that an individual will accept to completely eliminate random variation in wealth.

Economic decisions are rarely taken in such isolated circumstances, however. There are often important multiple sources of uncertainty, and decisions made to avoid, even partially, one source of risk may be affected by the presence of others. In the case of investors making portfolio choices, random income fluctuations may arise from sources other than the random return on risky asset holdings; for example, investors may also receive a wage that varies randomly. It is natural to ask whether Arrow and Pratt's results apply in such situations. Suppose, for example, that each individual received a nonnegative random income $\tilde{y}$ and that he also possesses some initial wealth δ which is nonstochastic and positive. Before knowing $\tilde{y}$ he invests the nonrandom wealth, δ, in a safe and a risky asset. Letting $\tilde{x}$ be as before, we can define $\hat{B}_i(\tilde{x},\tilde{y})$ analogously to $\hat{B}_i(\tilde{x},y)$ as the value of B_i in $[0,\delta]$ which maximizes

$$Eu_i(\tilde{y} + \delta + B_i \tilde{x}) \quad .$$

*There is, of course, a large literature on portfolio choice with many risky assets. We will discuss the relationship of the present analysis to that literature in the concluding section. Problems involving multiple sources of uncertainty have also been treated explicitly by Schlaiffer [1969], Hildreth [1974a,1974b], Hildreth and Tesfatsion [1974,1977], and Ross [1979]. Ross, in particular, proposes a strong measure of risk aversion which is appropriate for use when there are multiple sources of uncertainty which are uncorrelated. We will discuss the relationship of Ross's results to our own in the concluding section. Finally it should be added that, although Pratt does not explicitly treat the case of multiple sources of uncertainty, one of his theorems can be interpreted to obtain a result, specifically Corollary 2 below, that is applicable in these cases.

If (1) holds uniformly, can we then say that

$$(5) \qquad \hat{B}_1(\tilde{x},\tilde{y}) < \hat{B}_2(\tilde{x},\tilde{y})$$

if $\tilde{x}$ and $\tilde{y}$ are nondegenerate random variables?

When the random variable $\tilde{x}$ is added to a random initial wealth $\tilde{y}$, the risk premium $\pi_i(\tilde{x},\tilde{y})$ is defined analogously to $\pi_i(\tilde{x},y)$ by the equality

$$(6) \qquad Eu_i(\tilde{y} + \tilde{x}) = Eu_i(\tilde{y} + E\tilde{x} - \pi_i(\tilde{x},\tilde{y})) \quad .$$

If (1) holds uniformly, will

$$(7) \qquad \pi_1(\tilde{x},\tilde{y}) > \pi_2(\tilde{x},\tilde{y})$$

when $\tilde{x}$ and $\tilde{y}$ are nondegenerate random variables?

The present paper studies these questions. Surprisingly, we find that there are cases where $\tilde{x}$ and $\tilde{y}$ are independent and (1) holds uniformly, but (5) and (7) fail.* Such a case is presented in Section 3. Fortunately, however, we find that fairly painless restrictions allow us to extend the Arrow-Pratt results to the case of multiple sources of uncertainty. The restrictions are of two kinds. First, we must make some assumptions about the independence of the random income variations from different sources. In particular we assume that $\tilde{x}$ and $\tilde{y}$ are independent.** Second, the utility functions must be taken from a restricted class; specifically, we show that it is sufficient that either utility function be non-increasingly risk averse.

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*Cass and Stiglitz [1972] have also dealt with this particular question. Their Example 3 is, in essence, a case in which (1) holds but (5) and (7) fail. They do not have $\tilde{x}$ and $\tilde{y}$ independent, however. The Cass-Stiglitz paper is primarily concerned with multiasset portfolio choice and will be discussed further in the concluding section.

**Ross [1979] considers the related case in which $E[\tilde{x}|y] = 0$. Abstracting from translations of the mean, this assumption is weaker than independence. It does, however, imply that $\tilde{x}$ and $\tilde{y}$ are uncorrelated.

The need for restrictions of the first kind should be obvious. Without some such restrictions the Arrow-Pratt results could not possibly be extended. If, for example, returns to the risky asset were negatively correlated with other sources of uncertainty, increases in holdings of the risky asset could actually reduce risk. Thus more risk averse individuals would invest more in the risky asset. Similarly when $\tilde{x}$ and $\tilde{y}$ are negatively correlated, $\tilde{x} + \tilde{y}$ will be less risky than $\tilde{x}$ plus a certainty equivalent. As a consequence, risk averse individuals would have to be compensated if $\tilde{y}$ were to be replaced by a certain payment; i.e., the risk premium, $\pi(\tilde{x},\tilde{y})$, would be negative. Furthermore, the absolute size of the compensation would have to be higher for more risk averse individuals. Thus $\pi_1(\tilde{x},\tilde{y})$ would be smaller; i.e., more negative; than $\pi_2(\tilde{x},\tilde{y})$ if $R^{u_1} \geq R^{u_2}$.

The restrictions to the class of non-increasingly risk averse utility functions is not too troublesome, since this class has gained acceptance on both theoretical and empirical grounds. See, for example, Arrow [1971] and Stiglitz [1969a, 1969b]. Section 4 shows that non-increasing risk aversion for either individual is sufficient for (1) to imply (5) and (7) when $\tilde{x}$ and $\tilde{y}$ are independent. In fact, we derive these results as Corollary 1 of a more widely useful theorem that will be applicable when there are more than two independent sources of random variation in wealth. Specifically, when $\tilde{x}$ and $\tilde{y}$ are independent, the influence of risk aversion on $\pi(\tilde{x},\tilde{y})$ and $B(\tilde{x},\tilde{y})$ can be studied by investigating the risk aversion of the utility function V defined by

(8) $$V(x) = Eu(\tilde{y} + x) \quad .$$

In fact, it is easily verified that $\pi_2(\tilde{x},\tilde{y})$ is less than $\pi_1(\tilde{x},\tilde{y})$ and

$B_1(\tilde{x},\tilde{y})$ is less than $B_2(\tilde{x},\tilde{y})$ when V_1 is more risk averse than V_2. This result is stated formally in the next section, which serves as a preliminary to the remainder of the paper.

2. PRELIMINARIES

In the analysis to follow, the utility functions u_i; $i = 1,2$; will have as their domain $(\underline{z},\bar{z})$, a non-empty subinterval of real numbers. Each u_i will be assumed to be concave and twice differentiable and R^{u_i} will be defined as in the preceding section. The random variable $\tilde{y}$ will take values in $(\underline{y},\bar{y})$ where $\bar{y} - \underline{y} < \bar{z} - \underline{z}$. The probability measure μ defined on the Borel sets of $(\underline{y},\bar{y})$ by $\tilde{y}$ will be denoted by μ. We now let $\underline{x} = \underline{z} - \underline{y}$ and $\bar{x} = \bar{z} - \bar{y}$. For each x in $(\underline{x},\bar{x})$, $V_i(x)$ is defined by an equation analogous to (8) in which $u_i = u$. We assume that the expectations defining V_i exists and that V_i is twice differentiable on $(\underline{x},\bar{x})$. The Arrow-Pratt risk aversion measure of V_i is R^{V_i}. The comments made at the conclusion of the introductory section are formalized in the following Proposition, which is an immediate Corollary of Pratt's Theorem 1.

<u>Proposition</u>: <u>Let</u> $\tilde{y}$ <u>be a fixed random variable. The inequalities</u>

(9) $$\pi_1(\tilde{x},\tilde{y}) \geq \pi_2(\tilde{x},\tilde{y})$$

<u>and</u>

(10) $$B_1(\tilde{x},\tilde{y}) \leq B_2(\tilde{x},\tilde{y})$$

<u>hold for all</u> $\tilde{x}$ <u>independent of</u> $\tilde{y}$ <u>if and only if</u>

(11) $$R^{V_1}(x) \geq R^{V_2}(x)$$

for all $x \in (\underline{x},\bar{x})$. *If*

(12) $$R^{V_1}(x) > R^{V_2}(x)$$

for all $x \in (\underline{x},\bar{x})$, *then* (5) *and* (7) *will hold for all* $\tilde{x}$ *independent of* $\tilde{y}$.

3. AN EXAMPLE

Given the proposition, it is easy to find a case for which (1) holds uniformly but (5) and (7) fail. In this section we present such a case in which $\tilde{x}$ and $\tilde{y}$ are independent.

Restrict $\tilde{y} + \tilde{x}$ to the interval $(0,1)$, and let

$$u_1(y) = y - \tfrac{1}{2}y^2, \text{ and } u_2(y) = y - \frac{1}{22}y^{11} .$$

Then

$$u_1'(y) = 1 - y ,$$
$$u_1''(y) = -1 ,$$
$$R^{u_1}(y) = \frac{1}{1-y} ,$$
$$u_2'(y) = 1 - \tfrac{1}{2}y^{10} ,$$
$$u_2''(y) = -5y^9 ,$$
$$R^{u_2}(y) = \frac{5y^9}{1 - \tfrac{1}{2}y^{10}} .$$

It can be shown that for all $y \in (0,1)$, $R^{u_1}(y) > R^{u_2}(y)$. Hence (1) holds uniformly on the relevant interval.

Now let $\tilde{y}$ be a random variable that takes on the values .01 and .99 each with probability ½, and let $x = 0$. Simple calculations establish that

$$R^{V_1}(x) = \frac{-Eu_1''(\tilde{y} + x)}{Eu_1'(\tilde{y} + x)} = 2.00 , \quad R^{V_2}(x) \cong 2.95 .$$

Since we are dealing with polynomial utility functions, there of course exists a neighborhood of $x = 0$ for which

$$R^{V_2}(x) > R^{V_1}(x)$$

holds throughout. By the argument presented at the end of Section 1, it follows that we can find $\tilde{x}$'s for which (5) and (7) are violated.

4. THE CASE OF NON-INCREASING ABSOLUTE RISK AVERSION

The example of the previous section involved independent $\tilde{x}$ and $\tilde{y}$ variables and two utility functions which exhibited increasing absolute risk aversion. We will show in this section that if either of the utility functions in question exhibit non-increasing risk aversion and if $\tilde{x}$ and $\tilde{y}$ are independent, then (1) implies (12), (5), and (7). In fact, we will work with the weak form of these inequalities throughout. The reader can easily verify, however, that only slight modifications in the analysis are required to obtain the results with strong inequalities.

Theorem: If

$$R^{u_1}(z) \geq R^{u_2}(z) \tag{13}$$

for all $z \in (\underline{z},\bar{z})$ and either R^{u_1} or R^{u_2} is a non-increasing function of z on $(\underline{z},\bar{z})$, then (11) holds on $(\underline{x},\bar{x})$.

Before giving the proof, we state and prove the following.

Lemma: For any z_a, z_b in $(\underline{z},\bar{z})$, we define r by

$$r = \left[\frac{u_1'(z_a)}{u_1'(z_b)}\right] \Bigg/ \left[\frac{u_2'(z_a)}{u_2'(z_b)}\right] \quad . \tag{14}$$

<u>If, for all</u> $z_a \geq z_b$ <u>in</u> $(\underline{z},\bar{z})$,

$$\{[R^{u_1}(z_a) - R^{u_2}(z_a)] + [R^{u_1}(z_b) - R^{u_2}(z_b)]\}r + (1 - r)[R^{u_1}(z_b) - R^{u_2}(z_a)] \geq 0 \quad (15)$$

<u>then</u> (12) <u>holds on</u> $(\underline{x},\bar{x})$.

Proof:

Straight forward algebraic manipulations demonstrate that (15) is equivalent to

$$-[u_1''(z_a)u_2'(z_b) + u_1''(z_b)u_2'(z_a)] \geq -[u_2''(z_a)u_1'(z_b) + u_2''(z_b)u_1'(z_a)] \quad (16)$$

Note that (16) is symmetric in z_a and z_b. If $z_\alpha \geq z_\beta$, we can now let $z_\alpha = z_a$ and $z_\beta = z_b$ and obtain

$$-[u_1''(z_\alpha)u_2'(z_\beta) + u_1''(z_\beta)u_2'(z_\alpha)] \geq -[u_2''(z_\alpha)u_1'(z_\beta) + u_2''(z_\beta)u_1'(z_\alpha)] \quad (17)$$

from (16). If $z_\beta \geq z_\alpha$, then (17) is obtained from (16) by letting $z_\beta = z_a$ and $z_\alpha = z_b$. Thus (17) holds for all z_α, z_β in $(\underline{z},\bar{z})$ if and only if (15) holds for all $z_a \geq z_b$ in $(\underline{z},\bar{z})$.

The proof can now be completed by demonstrating that if (17) holds for all z_α, z_β in $(\underline{z},\bar{z})$, then (11) holds on $(\underline{x},\bar{x})$. To begin, we assume that $x \in (\underline{x},\bar{x})$ and that $\tilde{y}_\alpha$ and $\tilde{y}_\beta$ are two independent random variables, each of which has the same distribution as $\tilde{y}$. We then define the random variables $\tilde{z}_\alpha$ and $\tilde{z}_\beta$ by letting

$$\tilde{z}_\alpha = x + \tilde{y}_\alpha, \quad \text{and} \quad \tilde{z}_\beta = x + \tilde{y}_\beta \quad . \quad (18)$$

If we now substitute (18) in (17), take expectations of both sides of the resulting inequality and make use of the independence of $\tilde{y}_\alpha$ and $\tilde{y}_\beta$ we obtain

$$(19)\qquad \begin{aligned} &-[Eu_1''(x+\tilde{y}_\alpha)Eu_2'(x+\tilde{y}_\beta) + Eu_1''(x+\tilde{y}_\beta)Eu_2'(x+\tilde{y}_\alpha)] \\ &\quad \geq \; -[Eu_2''(x+\tilde{y}_\alpha)Eu_1'(x+\tilde{y}_\beta) + Eu_2''(x+\tilde{v}_\beta)Eu_1'(x+\tilde{y}_\alpha)] \; . \end{aligned}$$

Since $\tilde{y}_\alpha$ and $\tilde{y}_\beta$ each have the same distribution as $\tilde{y}$, (19) is equivalent to

$$-2Eu_1''(x+\tilde{y})Eu_2'(x+\tilde{y}) \;\geq\; -2Eu_2''(x+\tilde{y})Eu_1'(x+\tilde{y})$$

which is in turn equivalent to (11). ||

Proof of the theorem:

Pratt has shown that $r \leq 1$ for $z_a \geq z_b$ if $R^{u_1}(z) \geq R^{u_2}(z)$ on $(\underline{z},\bar{z})$. If $R^{u_1}(z) \geq R^{u_2}(z)$ on $(\underline{z},\bar{z})$, then the first term in (15) is nonnegative. If R^{u_1} is a non-increasing function of z, then we write

$$(20)\qquad R^{u_1}(z_b) - R^{u_2}(z_a) = [R^{u_1}(z_b) - R^{u_1}(z_a)] + [R^{u_1}(z_a) - R^{u_2}(z_a)] \; .$$

If $R^{u_1}(z) \geq R^{u_2}(z)$ on $(\underline{z},\bar{z})$, then the second term in (20) is nonnegative. The fact that R^{u_1} is non-increasing implies that the first term in (20) is nonnegative. Since $r \leq 1$, the second term in (15) is therefore nonnegative.

If now R^{u_2} is non-increasing in x, we write

$$(21)\qquad R^{u_1}(z_b) - R^{u_2}(z_a) = [R^{u_1}(z_b) - R^{u_2}(z_b)] + [R^{u_2}(z_b) - R^{u_2}(z_a)] \; .$$

Then $R^{u_1}(z) \geq R^{u_2}(z)$ on $(\underline{z},\bar{z})$, implies nonnegativity of the first term in (21) and R^{u_2} non-increasing implies nonnegativity of the second term. Again $r \leq 1$ implies that the second term in (15) is nonnegative.

We can now apply the lemma to establish the theorem. ||

Corollary 1: *If* (13) *holds on* $(\underline{z},\bar{z})$ *and either* R^{u_1} *or* R^{u_2} *is a non-increasing function of* z *on* $(\underline{z},\bar{z})$, *then*

$$(22) \qquad \hat{B}_1(\tilde{x},\tilde{y}) \leq \hat{B}_2(\tilde{x},\tilde{y})$$

and

$$(23) \qquad \pi_1(\tilde{x},\tilde{y}) \geq \pi_2(\tilde{x},\tilde{y})$$

if $\tilde{x}$ *and* $\tilde{y}$ *are indepenedent and range over values such that wealth always lies in* $(\underline{z},\bar{z})$.

Corollary 1 follows immediately from the Proposition and from the preceding theorem. The follow corollary of Corollary 1 can also be obtained as a trivial corollary of Theorem 5 in Pratt [1964].

Corollary 2: *If* $R^u(z)$ *is a non-increasing (decreasing) function of* z, *then* $B(\tilde{x},\tilde{y} + z)$ *is a nondecreasing (increasing) function of* z *and* $\pi(\tilde{x},\tilde{y} + z)$ *is a non-increasing (decreasing) function of* z.

Proof:

If $z_1 < z_2$, let

$$u_i(z) \;\equiv\; u_i(z + z_i) \quad .$$

Now apply Corollary 1. ||

5. CONCLUDING REMARKS

In this section, we briefly discuss the relationship of our results to the literature on multiasset portfolio choice. We also compare the results described in this paper to those obtained by Ross [1979].

Much of the literature on multiasset portfolios, which began with the work of Markowitz [1959] and Tobin [1958], deals with the case of "separability." In this case, the choice of a multiasset portfolio can be reduced to the choice of a portfolio containing one safe asset and shares of one mutual fund of risky assets. In this literature, the return to the mutual fund is the only risky asset. Thus when there is separation, the analysis of risk aversion's influence on investor behavior can be carried out using the techniques developed for the case of a single source of risk. Cass and Stiglitz [1970] have characterized the class of cases in which separation holds. Their work has demonstrated the restrictiveness of this assumption. In a subsequent paper [1972], they have shown that, in general, it is impossible to extend the above mentioned Arrow-Pratt theorem on the influence of risk aversion on portfolio behavior to a multiasset context in which there is no separation. Oliver Hart [1975] strengthened this negative result by showing "that the separation property is necessary for the generalization of Arrow's results"* to the case of many risky assets. He, in fact, was able to show that "given a utility function which does not possess the separation property, only trivial wealth effect, comparative statics properties can hold for all probability distributions of security returns."** From this result Hart concluded "that the only hope of obtaining wealth effect comparative statics properties for a larger class of utility functions than the separation class is to restrict the probability distributions of security returns in some way."***

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*Hart [1975], p.615.

**Ibid, p. 616.

***Ibid, p. 616.

It is possible to interpret our theorem as yielding a comparative statics property that holds for the class of non-increasing absolute risk averse utility functions when the returns to different risky assets are restricted to be independent. The non-increasing absolute risk aversion class includes many utility functions not in the separation class, but it does not include the quadratic utility functions for which the separation property holds.

For the purpose of making this interpretation, suppose that there are two risky assets and one safe asset in which wealth can be invested. Assume, furthermore, that the amount invested in the first risky asset is fixed in advance; i.e., suppose that all investors are restricted to holding portfolios which include exactly this predetermined amount of the first risky asset. Such a situation can be identified with the formal model of the present paper if we let δ represent the wealth which remains available for investment in a portfolio containing the safe asset and the second risky asset and if we let $\tilde{y}$ be the random income yielded by the amount invested in the first risky asset. In this interpretation, $\tilde{x}$ is the random return on the second risky asset and $B(\tilde{x},\tilde{y})$ is that part of δ which is invested in this asset. The theorem now can be interpreted to assert that if there are one safe and two risky assets with independent returns and if the amount invested in one of the risky assets is fixed in advance, then a more risk averse investor will invest less of his remaining wealth in the other risky asset. This result will, of course, only apply if one of the utility functions under consideration exhibits non-increasing absolute risk aversion. When this condition fails, the example of Section 2 can be interpreted to show that, for more risk averse investors, the amount of wealth invested in the second risky asset is not necessarily a smaller part of the wealth that is uncommitted to investment in the first risky asset.

As mentioned in the footnotes to the preceding discussion, Ross [1979] treats the questions dealt with here in a slightly different context. He assumes that $\tilde{x}$ and $\tilde{y}$ are uncorrelated in the strong sense that $E[\tilde{x}|y] = 0$, for all y. If we had restricted our attention to cases in which $E\tilde{x} = 0$, any $\tilde{x}$ satisfying our hypotheses would have satisfied those of Ross. Our analysis can thus be interpreted as applying to a smaller class of pairs of random variables $(\tilde{x},\tilde{y})$ than the class to which Ross' analysis applies. Ross' results parallel those presented here but they do not overlap. In particular, he presents an example, analogous to the one in Section 3 of this paper, in which u_1 is more risk averse than u_2 but $\pi_1(\tilde{x},\tilde{y}) < \pi_2(\tilde{x},\tilde{y})$ when $E[\tilde{x}|y] = 0$ for all y. His example cannot serve, however, as an illustration of the point made by our example; viz., that even if $\tilde{x}$ and $\tilde{y}$ are independent and u_1 is more risk averse than u_2, $\pi_1(\tilde{x},\tilde{y})$ may be less than $\pi_2(\tilde{x},\tilde{y})$. Ross' example is inappropriate for that purpose because it is not a case in which $\tilde{x}$ and $\tilde{y}$ are independent. Since the utility functions in Ross' example are from the constant risk aversion class, his example does serve, however, to illustrate the fact that our Corollary 1 cannot be extended to apply when $E[\tilde{x}|y] = 0$ for all y, even if u_1 and/or u_2 exhibit non-increasing absolute risk aversion.

Ross' main theorem gives conditions under which u_1 more risk averse than u_2 implies $\pi_1(\tilde{x},\tilde{y}) > \pi_2(\tilde{x},\tilde{y})$ when $E[\tilde{x}|y] = 0$ for all y. In fact, he characterizes the relationship which must hold between u_1 and u_2 if $\pi_1(\tilde{x},\tilde{y})$ is to exceed $\pi_2(\tilde{x},\tilde{y})$ whenever $\tilde{x}$ and $\tilde{y}$ are such that $E[\tilde{x}|y] = 0$ for all y. Because of the constant risk aversion of the utility functions in his example, the conditions imposed on the utility functions are necessarily stronger than those imposed in our theorem. The necessity for stronger restrictions on the utility functions arises in Ross' analysis because his theorem applies to the class of random variables $\tilde{x},\tilde{y}$ for which $E[\tilde{x}|y] = 0$ for all y. As already mentioned, this is a wider class than those which are independent and for which $E\tilde{x} = 0$.

The contrast between Ross' results and our own is somewhat surprising in light of the apparently small difference in the statistical relationship between $\tilde{x}$ and $\tilde{y}$ permitted in the two analyses. In this connection, it may be useful to emphasize that, in this paper, the function V, defined by eqution (8), plays a crucial role. This is possible because $\tilde{x}$ and $\tilde{y}$ are independent. When $\tilde{x}$ and $\tilde{y}$ are not independent, however, the Proposition of Section2 above will no longer hold even if $\tilde{x}$ and $\tilde{y}$ are uncorrelated in the strong sense that $E[\tilde{x}|y] = 0$ for all y. As a result, the function V is irrelevant if $\tilde{x}$ and $\tilde{y}$ are not independent even if they are uncorrelated in the strong sense used by Ross.

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