THE RELATIVE EFFICIENCY OF VARIOUS PORTFOLIOS:

SOME FURTHER EVIDENCE

by

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Working Paper No. 26-1979

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I. Introduction

The market portfolio of risky assets has long played a central role in equilibrium theories of the capital markets. In the original one-period, mean-variance world of Sharpe, all investors would hold the market portfolio of risky assets in conjunction with a positive or negative investment in the riskfree asset. Fama [3] and more recently Ross [8] and Roll [7] have shown that a necessary and sufficient condition for the validity of the Sharpe-Lintner capital asset pricing model is that the market portfolio of risky assets be efficient.

Despite this central role of the market portfolio, none of the prior tests of the capital asset pricing model, such as [1], [2], or [4], has really grappled with the potential biases associated with errors in measuring the return on the market portfolio. Recently, Roll [7] has reminded the profession of this omission and has speculated that these prior tests of the capital asset pricing model may be extremely sensitive to such errors.

The purpose of this paper will be to provide some evidence on the relative efficiency of a bond portfolio, a stock portfolio, and various combinations of these two portfolios, including market-weighted combinations. A portfolio will be termed relatively more efficient than an alternative portfolio if, in conjunction with a position in the riskfree asset, it permits an investor to obtain a greater expected utility than would be possible with the alternative. This definition of relative efficiency encompasses two decisions: the selection of the risky portfolio and the determination of the position in the riskfree asset, or, in short, the allocation decision.

In traditional performance studies of efficient portfolios,[1] the relative efficiency of different risky portfolios would be measured by the ratio of the expected risk premium (expected return less the riskfree rate) to its standard deviation--the so-called Sharpe measure. Because of separation, the allocation to the riskfree asset is a mere detail and can be ignored in determining relative efficiency. Implicit in this use of the Sharpe measure is the assumption that the underlying distributions of returns are

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[1] Cf. Sharpe [9].

known. If these distributions were not known, the next section will argue that the Sharpe measure would no longer provide an unambiguous measure of relative efficiency and that the impact of "estimation risk" upon the allocation decision must also be considered. The empirical tests proposed in this paper will incorporate the effect of "estimation risk" in measuring the relative efficiency of different portfolios.

II. The Investor's Decision Problem

To formulate the investor's decision problem, let i index the set of risky portfolios available to the investor and α be the proportion of his wealth placed in the risky portfolio. Further, let R_i denote the random return for risky portfolio i and R_f, the riskfree return. If the investor's initial wealth is taken as the numeraire, his random end-of-period wealth, $W(\alpha,i)$, will be given by

$$W(\alpha,i) = 1 + (1-\alpha)R_f + \alpha R_i \tag{1}$$

If the investor knew the underlying distributions of the risky portfolios, say, $h_i(R)$, and had a utility function of end-of-period wealth $U(W)$, his optimal decision would be that i and α which maximized

$$E[U(W)] = \int_R U[W(\alpha,i)]\ h_i(R)dR \tag{2}$$

In the usual mean-variance world in which separation holds, the maximization of (1) would proceed in two steps: The investor's first step would be to select that risky portfolio which would maximize the Sharpe measure, defined as the ratio of $(E(R_i)-R_F)$ to $\sigma(R_i)$. His second step would be to allocate his initial wealth over this risky portfolio and the riskfree asset. Since the h_i's are the true distributions, the solution to this first step would be the same for all investors, and thus the Sharpe measure itself could be interpreted as a measure of investment performance.

If an investor did not know the h_i's but had to estimate them, it is easy to construct examples in which an investor might rationally select a risky portfolio which he knew to have a lesser Sharpe measure than another. Consider, for instance, an investor who knew for some reason that the Sharpe measure for portfolio A was greater than that for portfolio B. Nonetheless, if he were highly uncertain as to the absolute risk (and, correspondingly, the expected risk premium) of A but quite certain as to this risk for B, he might well choose B over A: with B, he would have more confidence in his decision as to the proportion of his wealth allocated to the riskfree asset. In this case, the Sharpe measure would no longer provide an unambiguous measure of investment performance: estimation risk must somehow be taken into account.

Since the underlying distributions, the h_i's, are not known, the investor must somehow modify his decision process. The traditional approach has been to assume that the h_i's can be approximated by a family of distributions parameterized by a vector θ_i, say $f(R|\theta_i)$, and to replace θ_i by point estimates $\hat{\theta}_i$. As an example, if f were normal, the $\hat{\theta}_i$'s might consist of point estimates of the means and standard deviations of R. The investor would then select i and α so as to maximize the expected value of U relative to the distribution $f(R|\hat{\theta}_i)$.

Klein and Bawa [6] have argued that this traditional approach does not incorporate estimation risk satisfactorily, and they have suggested a Bayesian approach. Following their approach, an investor would first integrate f over all possible estimates $\hat{\theta}_i$ to obtain a posterior of predictive distribution g_i, defined as

$$g_i(R) = E[f(R|\hat{\theta}_i)] \tag{3}$$

The optimal i and α would be those which maximized

$$\int_R U[W(\alpha,i)] \; g_i(R)dR \tag{4}$$

The notation of (3) and (4) suggests that "estimation risk" has been properly incorporated into the model through the elimination of the conditioning variable $\hat{\theta}_i$ in f. In fact, estimation risk has not been eliminated. What has happened is that the estimation risk has been pushed back one stage. The function g is clearly conditional on the assumed distribution of $\hat{\theta}_i$ used in applying the expected value operator in (3). Moreover, both the traditional and Bayesian approaches are conditional upon the specification of f. Violations of either of these conditioning assumptions may introduce errors in the decision process.

In sum, although an investor would prefer to base his decisions upon the true underlying distributions, he would not in practice know these distributions and would have to base his decisions upon predictive distributions which he knows would only approximate the true distributions. Even so, he would still prefer that decision which maximized his expected utility relative to the true distributions if it were only possible for him to know these distributions. This desire to use the true distributions will provide the key to evaluating the relative efficiency of different portfolios.

III. The Basic Tests

To evaluate the relative efficiency of the various risky portfolios mentioned in the introduction, let us suppose that there was an investor as of December 31, 1950, who had to select one, and only one, of these risky portfolios to hold in conjunction with a position in the riskfree asset. Using historical data on or prior to this date, this investor would first form a predictive distribution for each of the risky portfolios and based upon each of these distributions, would determine the portion of his wealth to allocate to the riskfree asset as well as the corresponding expected utility. With no additional information, he would select that risky portfolio and the corresponding position in the riskfree asset with the greatest expected utility as calculated from his predictive distribution.

Now, although he made the best investment decision possible given the information available to him, he would know that his decision would probably be suboptimal or inferior to that which he would have undertaken had he known the true distributions. The investor would obviously like to know, if he could, how inferior his actual decision really was. An empirically implementable approach to addressing this concern relies on the following observation: The actual return on, say, the bond portfolio for January 1951 could be interpreted as a drawing from the true underlying distribution, and the actual utility associated with this drawing for the investor's strategy determined.

Now, advance the calendar ahead one month to January 31, 1951, and let the investor repeat his decision process using data on or prior to this date to assess his predictive distribution for bonds. Again, the actual return on the bond portfolio for February 1951 could be interpreted as a drawing from the underlying distribution, and the investor's actual utility for this drawing calculated. Repeating this process over and over again through December 1977 would yield a vector of 324 utility values which could be regarded as utility transformations of drawings of returns from the underlying distribution.

Doing the same thing for each of the other risky portfolios would yield similar vectors of utility values. A comparison of the resulting vectors of utility values for two different risky portfolios would indicate whether one portfolio was preferable to another. A straightforward way to do this is to compare the average values of the two vectors and to test the hypothesis that these averages are equal--the procedure to be followed in this paper.

The implementation of this general approach requires the collection of various portfolio return series, the assessment of predictive distributions, and the adoption of specific utility functions. Each of these components will be described in turn:

The Portfolios: Various series of monthly returns were collected for the period from January 1926 through December 1977. The returns for the stock market were measured by the Standard and Poor's Composite Index adjusted for dividends. The returns for the bond market were derived from the Standard and Poor's High Grade Corporate Bond Index.[2] The riskfree rate was approximated by the monthly equivalent of the three-month Treasury Bill rate as reported in the *Federal Reserve Bulletin*.[3]

Selected statistics for these various return series are presented in Table 1. The autocorrelation function shows, in the case of bonds and stocks, minimal dependency after the first lag; and even for the first lag, the dependencies, though positive, are not large. Although the autocorrelation function for the riskfree asset shows strong positive dependence, the returns on bonds and stocks as measured from the riskfree

[2] The monthly returns were calculated as those which an investor would have realized if he had purchased at the beginning of the month a new twenty-year bond at par and then sold it at the end of the month.

[3] Work is currently underway to assess the sensitivity of the results of the paper to the use of a more realistic borrowing rate.

returns are not strongly correlated over time.

In addition to the bond and stock portfolios, three portfolios formed as combinations of these portfolios were examined. The first two portfolios were market-weighted portfolios. The first variant weighted the stock series in porportion to the market value of all stocks listed on the New York Stock Exchange and weighted the bond series in proportion to the market value of all non-convertible corporate bonds.[4] The second variant weighted the stock series in the same way but weighted the bond series in proportion to the values of these corporate bonds plus all government bonds other than Treasury Bills. The third portfolio did not have prespecified weights, but allowed the weights to vary from month to month so as to maximize the investor's expected utility on the basis of his predictive distribution.

The Predictive Distributions: The amount of prior history used in forming the predictive distributions varied from 60 months to 300 months of immediately prior data in steps of 60 months. The first month for which predictive distribution was assessed was January 1951, and the last was December 1977, for a total of 324 months. The predictive distributions for the risky portfolios were formulated in terms of returns as measured from the riskfree rate or "return differentials," rather than the raw returns themselves. Adding back the current riskfree rate, which would be known at the beginning of the month for which the predictive distribution was formed, produced the actual predictive distributions. The use of return differentials rather than raw returns would be the appropriate procedure if Fisher's hypothesis held, namely, that real returns were independent of the inflation rate.

If the return differentials are normally distributed, Klein and Bawa [6] have shown, under an appropriately defined non-informative prior, that the predictive distribution will be a t-distribution with (T + m) degrees of freedom, where T is the number of periods of prior data and m is the number of portfolios. In this paper, m will be

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[4]The weights were obtained from Table 1 of Stambaugh [10].

for two reasons: First, the investor is ultimately interested in actual utilities. Second, the successive predictive distributions, based upon overlapping data, have induced substantial dependencies in the expected utility series. Attempts were made to adjust for these dependencies, but the resulting statistics were highly unstable, often changing radically with the deletion of just one observation.

The t-statistics for all of the pairwise comparisons of the different strategies are summarized in Table 2 for a coefficient of relative risk aversion of two. Where the rankings differed between the expected and actual utilities, the t-values have been set in italics. The largest absolute t-values are associated with the "optimally weighted" portfolios and indicate that an investor would almost always have preferred the stock portfolio or either variant of the market-weighted portfolios to these "optimally weighted" portfolios despite the fact that the expected utilities of these portfolios were by construction the greatest obtainable. The explanation for this change in rankings is that portfolio optimization procedures may be highly sensitive to estimation errors and can on occasion give results which are far from the true "optimal" results. In view of the apparent inefficiency of the optimal weights, these so-called optimally weighted portfolios will not be discussed further.

The next strongest pattern is exhibited by the bond portfolio, whose average actual, as well as expected, utilities are less than those for the stock market or either variant of the market-weighted portfolios. The t-values of the differences, however, are not significant at the usual levels of significance. Nonetheless, if an investor had only the information in Table 2 and had to pick a risky portfolio to hold in conjunction with an investment in the riskfree asset, he would not choose the bond portfolio. His aversion to the bond portfolio might well increase if his effective tax rate on the returns from bonds were greater than that on stocks.

The implied rankings of the stock portfolio and the market-weighted portfolios are mixed. The stock portfolio is preferred to either of these two market-weighted portfolios on the basis of the 60- or 120-month predictive distributions, but, on the basis one or two and T will be 60 or greater. In these cases, the t-distribution will be approximately normal, so that as a working hypothesis, the predictive distributions will in fact be assumed to be normal.[5]

The Utility Function: The investor's utility will be assumed to be one which exhibits constant relative risk aversion.[6] Three values of the coefficient of relative risk aversion were used: one, two, and five. The qualitative results for any of these three values were similar; and for reasons of space, only those results based upon a coefficient of relative risk aversion of two will be presented in the text. This assumption of constant relative risk aversion is consistent with the evidence reported by Friend and Blume [5], and a coefficient of two is their best estimate of the value for a representative individual.

At first blush, the choice of this particular family of utility functions, or any other for that matter, may seem to limit greatly the generality of this study. However, upon reflection, this choice is not as limiting as it might first appear. The general purpose of the utility function in this paper is to incorporate the effect of estimation risk upon the investor's allocation of his wealth between a risky portfolio and a risk-free asset. Now, it may be that some utility functions are more or less sensitive to certain types of estimation risk than others. For example, a linear utility function would only be sensitive to estimation errors in measuring expected returns, while a quadratic utility function would also be sensitive to errors in measuring the variance. The class of constant proportional risk aversion functions would be sensitive to estimation errors in the measurement of moments of any order. A detailed study of the interrelationship of estimation risk and classes of utility functions would seem to be warranted, but such a study is clearly outside the scope of this paper.

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[5]The empirical results of this paper were replicated for all the risky portfolios except for the optimal combination under the assumption that the risk differentials were log normal. The qualitative results were unchanged.

[6]If γ is the coefficient of relative risk aversion, this family of utility functions is represented by $(1-\gamma)^{-1} W^{1-\gamma}$ for $\gamma \neq 1$ and $\ln W$ for $\gamma = 1$.

The Results: Before examining the overall results, consider for the moment an investor on December 31, 1950, who is considering investing in a stock portfolio using a predictive distribution based upon 300 months of prior data. If his coefficient of relative risk aversion were two, his implied decision for January 1951 would have been to invest 68 per cent of his wealth in the stock portfolio, with the remaining 32 per cent in the riskfree asset. His expected utility would have been -0.9960, while his actual utility would have been -0.9561, reflecting the greater-than-average return on the stock market in that month. In contrast, the investor's optimal decision for that month for the bond portfolio, again using a 300-month predictive distribution, would have been to invest 1519 per cent of his wealth in this portfolio, with the bulk of the investment financed by a short position in the riskfree asset.[7] The expected utility would have been -0.9756, and the actual utility would have been -0.9724. In terms of expected utilities, the bond strategy would have been preferable by 0.0204 utiles. In terms of actual utilities, the stock strategy would have been preferable by 0.0163 utiles.

Over the entire 27 years ending in 1977, the average of the actual utilities for the stock strategy exceeded that for the bond strategy by 0.0110 utiles. The t-value for the difference was 1.03 and is reported in Table 2.[8] The simulation analysis in the next section gives some reason to believe that the significance level implied by this and other t-values in Table 2 may be biased toward zero and thus understate the true significance level.

Over these same 27 years, the average of the expected utilities for the stock strategy happened also to exceed that for the bond strategy. Though various t-statistics were calculated for the expected utilities, these statistics will not be presented

[7] This large short position in the riskfree assets could be reduced to a more plausible level by postulating a greater coefficient of relative risk aversion than 2.0. However, as mentioned, the basic results do not appear terribly sensitive to the assumed value of this coefficient over a wide range of values.

[8] The t-values have been adjusted for any cross-sectional correlation between the two vectors of utility values by estimating the standard deviation of the average directly from the month-by-month differences. Generally, there were no substantial time-series dependencies in these differences except for those involving the optimal portfolios where the first-order serial correlation coefficients were as great as 0.39. In view of this dependency, all the t-values in Table 3 were calculated under the assumption that the differences conform to a first-order autoregressive process.

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TABLE 1

Summary Statistics of the Monthly Returns Series

Statistic	Period or Lag	Returns: Riskfree	Returns: Bonds	Returns: Stocks	Returns less the Riskfree Rate: Bonds	Returns less the Riskfree Rate: Stocks
Average	1926-77	0.00211	0.00310	0.00891	0.00099	0.00680
	1926-38	0.00136	0.00517	0.00921	0.00381	0.00785
	1939-51	0.00048	0.00233	0.01007	0.00184	0.00960
	1952-64	0.00207	0.00199	0.01174	-0.00009	0.00967
	1965-77	0.00453	0.00294	0.00462	-0.00159	0.00009
Standard Deviation	1926-77	0.00178	0.01094	0.06129	0.01120	0.06143
	1926-38	0.00135	0.01205	0.09806	0.01234	0.09808
	1939-51	0.00040	0.00716	0.04838	0.00725	0.04834
	1952-64	0.00071	0.00982	0.03518	0.00983	0.03536
	1965-77	0.00108	0.01339	0.04245	0.01355	0.04274
Autocorrelation Function						
1926-77	1	0.99	0.21	0.10	0.25	0.10
	2	0.98	0.08	-0.02	0.12	-0.02
	3	0.97	-0.03	-0.14	0.01	-0.13
	4	0.96	0.05	0.03	0.09	0.03
	5	0.95	0.02	0.06	0.06	0.06
	6	0.93	-0.02	-0.04	0.02	-0.04
	7	0.92	-0.03	0.01	0.01	0.01
	8	0.92	0.06	0.07	0.09	0.07
	9	0.91	0.05	0.12	0.07	0.12
	10	0.90	0.02	-0.01	0.05	-0.01
	11	0.89	0.03	-0.03	0.06	-0.02
	12	0.88	0.13	0.00	0.15	0.00
	13	0.87	-0.02	-0.04	0.00	-0.04
	14	0.86	0.04	-0.09	0.06	-0.09
	15	0.85	-0.06	-0.01	-0.03	-0.01
	16	0.84	-0.02	-0.06	0.01	-0.06
	17	0.83	0.00	0.10	0.02	0.10
	18	0.82	0.03	0.06	0.05	0.06
	24	0.79	0.04	0.02	0.05	0.03

TABLE 2

The t-Statistics for the Pairwise Comparison
of Different Strategies Based Upon Actual Utilities
1952-1977

Predictive Distributions Based Upon	Portfolio	Portfolio: Bonds	Stocks	Market-1	Market-2	Opt. Wgt.
		(positive value indicates row preference)				
60 Months	Bonds		-1.52	-1.50	-1.27	-1.36
	Stocks	1.52		0.19	1.35	*0.83*
	Market-1	1.50	-0.19		1.83	*0.80*
	Market-2	1.27	-1.35	-1.83		-0.11
	Opt. Wgt.	1.36	*-0.83*	*-0.80*	0.11	
120 Months	Bonds		-1.37	-1.37	-1.28	-1.24
	Stocks	1.37		*0.06*	0.95	*1.24*
	Market-1	1.37	*-0.06*		1.12	*1.25*
	Market-2	1.28	-0.95	-1.12		*1.13*
	Opt. Wgt.	1.24	*-1.24*	*-1.25*	*-1.13*	
180 Months	Bonds		-1.37	-1.37	-0.95	-0.43
	Stocks	1.37		*-0.07*	*0.92*	*1.79*
	Market-1	1.37	0.07		1.09	*1.85*
	Market-2	0.95	*-0.92*	-1.09		*1.22*
	Opt. Wgt.	0.43	*-1.79*	*-1.85*	*-1.22*	
240 Months	Bonds		-0.96	-1.03	-0.78	*0.01*
	Stocks	0.96		-0.96	*0.32*	*1.68*
	Market-1	1.03	0.96		0.71	*1.87*
	Market-2	0.78	*-0.32*	-0.71		*1.50*
	Opt. Wgt.	*-0.01*	*-1.68*	*-1.87*	*-1.50*	
300 Months	Bonds		-1.03	-1.13	-0.91	*0.03*
	Stocks	1.03		-1.06	*0.16*	*1.69*
	Market-1	1.13	1.06		0.87	*1.87*
	Market-2	0.91	*-0.16*	-0.87		*1.51*
	Opt. Wgt.	*-0.03*	*-1.69*	*-1.87*	*-1.51*	

Note: When the preferences differed as between expected and actual utilities, the t-value has been set in italics.

of the longer assessment periods, the first variant of the market-weighted portfolio seems preferable. It will be recalled that the first variant weighted bonds by the market value of corporate issues.

Although the relationships in Table 2 are quite similar regardless of the number of months used in assessing the predictive distributions, more confidence should probably be placed in the results based upon the longer assessment periods. For the shorter assessment periods, the expected values of the predictive distribution are frequently less than the riskfree rate, implying short positions in the risky portfolios, with the proceeds invested in the riskfree asset (Table 3). For the 240- and 300-month assessment periods, the proportions invested in the stock portfolio or either variant of the market-weighted portfolios are always positive. Only for the bond portfolio would a short position be indicated in some months, but even here the average proportion is positive. The investment strategies for these longer assessment periods are probably more consistent with most people's view of appropriate investment strategies in a risk-averse world than those based upon the shorter periods.[9]

IV. Two Further Analyses

In this section, two further analyses are reported. The first is a study of the rankings of the various risky portfolios according to the usual Sharpe's measure of performance. The second is a Monte Carlo simulation designed to explore the sampling properties of the statistics used in this paper.

Sharpe Measure: The Sharpe measure was estimated for each risky portfolio over the 1951-1977 period as follows: First, the monthly return differential was calculated and divided by an estimate of the standard deviation of the return differential.[10]

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[9] The large number of short positions associated with the shorter assessment periods could, on a theoretical basis, be rationalized as consistent with market equilibrium by noting that bonds, stocks, or market-weighted combinations do not constitute all risky assets and then by hypothesizing that an appropriate market-weighted portfolio of all risky assets would not involve such short positions in risky assets.

[10] Estimating the standard deviation of returns from the return differentials rather than the raw returns would be the preferred procedure if Fisher's hypothesis held.

TABLE 3

Summary Statistics for
the Proportion in the Risky Portfolio, α
1951-1977

Predictive Distribution Based Upon	Statistic	α			
		Bonds	Stocks	Market-1	Market-2
60	Minimum	-17.02	-1.89	- 2.08	- 2.09
	Average	- 1.81	2.66	3.13	3.99
	Maximum	27.85	8.69	10.36	12.94
120	Minimum	-11.30	-1.06	- 1.23	- 1.32
	Average	- 0.50	2.83	3.38	4.81
	Maximum	43.84	6.04	7.14	13.17
180	Minimum	- 9.44	0.10	0.17	0.21
	Average	1.55	2.75	3.38	4.69
	Maximum	28.43	4.67	5.71	11.55
240	Minimum	- 7.80	0.64	0.97	1.02
	Average	3.33	2.52	3.23	4.38
	Maximum	22.79	4.43	5.49	7.97
300	Minimum	- 5.81	0.56	1.26	1.65
	Average	4.19	2.12	2.87	3.79
	Maximum	15.19	4.02	4.83	6.94

Second, these monthly ratios, 324 in all, were averaged to obtain an estimate of the Sharpe measure. Two estimates of the standard deviation were used. The first was a predictive estimate derived from the 300 months of data immediately preceding the month of the return differential and therefore varied from month to month. The second was the standard deviation as estimated from the same 324 months for which the return differentials were calculated and therefore did not vary from month to month. In addition, the Sharpe measure for each portfolio was calculated over the longer 1926-1977 period, but using only one estimate of the standard deviation, namely, standard deviation as estimated over these years.

Pairwise differences of the Sharpe measure along with their t-values[11] are shown in Table 4. Over the 1951-77 period, the Sharpe measure ranks the bond portfolio behind the stock portfolio or either variant of the market portfolio; the t-value is highly significant. A similar relationship was found in the previous section, but the relationship was not as significant. The difference between the stock portfolio and either variant of the market is insignificant, in conformity with the prior results. The significant difference between the two market portfolios is an unexpected result, but it should be noted that the absolute size of the difference is not great.

Over the longer 1926-77 period, the bond portfolio still ranks lower than the other portfolios, but the margin is no longer significant. In contrast to 1951-77, either variant of the market-weighted portfolio now ranks ahead of the stock portfolio, and by a significant margin. The anomalous superiority of one variant of the market portfolio over the other observed in the 1951-77 period disappears.

The net impression from this analysis and that of the last section is that an investor would find it very difficult to be certain that the stock portfolio or either variant of the market portfolio was more efficient. There is some evidence that the bond portfolio is less efficient than any of the other three portfolios, but the signi-

[11] With the exception of the first-order autoregressive adjustment, which did not appear necessary, the t-values were calculated in the same way as in footnote 7.

TABLE 4

Differences of Sharpe's Performance Measures
and Associated t-Values

Time Period	Standard Deviation Estimate	Portfolio	Portfolio			
			Bonds	Stocks	Market-1	Market-2
			(positive value indicates row preference)			
1951-1977	Prediction	Bonds		-0.193 (-2.78)	-0.210 (-3.09)	-0.226 (-3.20)
		Stocks	0.193 (2.78)		-0.017 (-2.65)	-0.033 (-2.03)
		Market-1	0.211 (3.09)	0.017 (2.65)		-0.016 (-2.39)
		Market-2	0.226 (3.20)	0.033 (2.03)	0.016 (1.39)	
1951-1977	Concurrent	Bonds		-0.219 (-3.27)	-0.220 (-3.40)	-0.206 (-3.30)
		Stocks	0.219 (3.27)		-0.000 (-0.11)	0.014 (1.62)
		Market-1	0.220 (3.40)	0.000 (0.11)		0.014 (2.63)
		Market-2	0.206 (3.30)	-0.014 (-1.62)	-0.014 (-2.63)	
1926-1977	Concurrent	Bonds		-0.020 (-0.39)	-0.049 (-1.00)	-0.048 (-1.03)
		Stocks	0.020 (0.39)		-0.079 (-4.04)	-0.028 (-2.44)
		Market-1	0.049 (1.00)	0.029 (4.04)		0.001 (0.10)
		Market-2	0.048 (1.03)	0.028 (2.44)	-0.001 (-0.10)	

ficance of this conclusion hinges upon the time period and how estimation risk is incorporated into the analysis.

<u>Monte Carlo Simulation</u>: To examine the sampling properties of the previously used statistics, a Monte Carlo analysis was undertaken as follows: 1) Pick at random two vectors of 624 unit normal variates apiece. 2) Multiply each element of the first vector by σ_1 and add μ_1, and then multiply each element of the second vector by σ_2 and add μ_2; the resulting numbers can be interpreted as random drawings from normal distributions with respective means μ_1 and μ_2 and standard deviations σ_1 and σ_2. 3) Interpret each vector as a return series for a portfolio and replicate the prior analysis. In these replications, it was assumed that the riskfree return was 0.005 or 0.5 per cent per month, that the coefficient of relative risk aversion was 2.0, and that the investor based his decisions upon 300-month predictive distributions.

Using the <u>same</u> two vectors of unit normal variants, steps two and three were repeated 25 times for all combinations of μ_1 and μ_2 ranging from 0.006 to 0.014 in steps of 0.002; both σ_1 and σ_2 were kept constant at 0.04. The range of values for μ is sufficiently great that it probably encompasses most reasonable estimates of expected monthly returns for stocks or market-weighted portfolios of bonds and stocks. The standard deviation is approximately that for stocks in the post-World War II period. The use of the same pair of unit normal vectors for each of the 25 simulations will, of course, introduce a common bias across the simulations, but this common bias is really an advantage in that changes in the simulated results as the μ's change can more confidently be attributed to real effects.

After completing the first set of simulations, another two vectors of unit normal variates were drawn and steps two and three repeated twenty-five more times. In all, 100 sets of 25 simulations were run and are summarized in terms of the number of t-values which were greater than 1.282 or less than -1.282--the .90 and .10 fractiles of the t-distribution as approximated by the normal distribution. The Sharpe measures were based upon the last 324 elements of each vector and would correspond in Table 4 to the 1951-77 results using the standard deviation as estimated from the

To interpret the numbers, consider the Sharpe measure with μ_1 and μ_2 both equal to 0.006. Since the null hypothesis is true and the Sharpe measure is a t-statistic, one would expect on the basis of a one-tail test at the 10 per cent level that 10 of the hundred t-values would be greater than 1.282 and 10 less than -1.282. The actual numbers, 8 and 7, are less than expected, but well within normal sampling variability. The probability of observing 8 or less would be 0.321 and 7 or less, 0.206. Even so, that there are fewer than expected suggests that the numbers in Table 5 for all the μ's would exhibit a slight downward bias.

In contrast, in the comparison of the actual utilities with μ_1 and μ_2 both equal to 0.006, only 6 of the t-values are greater than 1.282 and only 4 are less than -1.282--considerably fewer than expected. The reason may be that the utility transformation of the return vectors is non-linear, and, though the test statistics would eventually approach normality, the test statistics based upon only 324 numbers may still be far from normality. If so, the significance levels of the t-statistics in Table 2 would be understated.

When μ_1 is not equal to μ_2, the numbers in Table 5 can be interpreted as the probabilities that the null hypothesis would be correctly rejected.[12] These probabilities are not large unless the differences in the μ's are great. For example, if μ_1 were 0.006 and μ_2 were 0.010, the probability of correctly ranking the two portfolios based upon the Sharpe measure is just slightly greater than 50 per cent, and upon the actual utilities, slightly greater than 25 per cent. In sum, unless the differences in expected returns were unrealistically large, there is a substantial probability that an investor with 27 years of monthly data would be unable to rank correctly the relative inefficiencies of different risky portfolios on the basis of the usual Sharpe measure or the new measures used in this paper.

V. Conclusion

This paper has presented some weak evidence that a bond portfolio may be less efficient than a stock portfolio or a market portfolio of bonds and stocks. In com-

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[12] These numbers subtracted from 100 would be estimates of the Type II error.

TABLE 5

Summary of Monte Carlo Analysis for 100 Replications

Statistic	μ_1	$t < -1.282$					$t > 1.282$				
		μ_2					μ_2				
		0.006	0.008	0.010	0.012	0.014	0.006	0.008	0.010	0.012	0.014
Differences	0.006	4	2	0	0	0	6	9	26	54	77
of Average	0.008	12	5	2	0	0	0	7	16	44	75
Utilities	0.010	27	18	7	1	0	1	2	7	22	47
	0.012	52	37	16	7	1	1	2	4	7	24
	0.014	75	55	37	16	6	1	1	2	4	9
Difference of	0.006	7	4	0	0	0	8	26	54	83	94
Sharpe Measure	0.008	16	8	4	0	0	5	8	26	53	83
Using Actual	0.010	46	16	6	4	0	4	5	9	26	52
Standard	0.012	76	47	16	6	4	1	4	5	8	25
Deviation	0.014	90	74	57	15	6	1	1	4	5	10

paring the relative efficiency of a stock portfolio to two variants of a market-weighted portfolio of bonds and stocks, there was no clear dominance, with the specific results depending upon the time period and type of analysis. With more accurate predictive distributions, the results might well have been different, but it should be noted that the procedures used in this paper to assess the predictive distributions were in the same spirit as those used in prior tests of the Sharpe-Lintner model.

A Monte Carlo analysis of the techniques used in this paper helped to put these results in perspective by showing that an investor may find it extremely difficult to ascertain differences in the relative efficiency of different risky portfolios with only 27 years of monthly data--the amount of data used in most of the tests in the paper. What all this suggests, though it does not prove it, is that prior tests of the Sharpe-Lintner model may not be as sensitive to the correct specification of the "market" portfolio as some have suggested.

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