VALUATION OF LOAN GUARANTEES

by

E. Philip Jones*
and
Scott P. Mason**

Working Paper 25-79

RODNEY L. WHITE CENTER
FOR FINANCIAL RESEARCH

The Wharton School

University of Pennsylvania

Philadelphia, PA 19104

*Wharton School, University of Pennsylvania
**Harvard Business School

1. Introduction

Government guarantees of loans made to private corporations are often proposed as integral parts of public policy programs. Examples are the promotion of "essential" economic activity, such as the development of alternative energy sources, or the extension of financial assistance to major corporations. The proposed government guarantee of a loan to the Chrysler Corporation is a specific example of the latter. Thus, lenders and equity investors are confronted with the problem of assessing the value and impact of the government's guarantees. Merton (1977) and Jones and Mason (1978) have addressed this problem using contingent claims analysis. This paper continues the use of the contingent claims method in the evaluation of loan guarantees by considering a number of complexities often encountered in practice but not treated in the earlier work.

Private economic activities, which cost more than the sum of the benefits accruing to private participants but less than their aggregate social benefit, could suggest some form of government financial assistance. Government loan guarantees, as well as direct credit programs and direct subsidies, are examples of financial assistance programs. By guaranteeing a firm's debt, the government has in essence issued an insurance policy at no charge. Just as outstanding policies represent liabilities to insurance companies, outstanding loan guarantees represent liabilities to the government. And, just as insurance policies have value to policyholders, the loan guarantee has value to the firm. The guarantee, in principle, is structured so as to minimize the value of the liability borne by the government but still represent sufficient incremental value so as to attract the participation of private capital suppliers in what would otherwise be an uneconomic activity. Thus, it is important that private investors have some means of evaluating loan guarantee proposals.

Merton (1977) and Jones and Mason (1978) evaluate certain loan guarantees, as well as the associated benefits and incentives accruing to the participants in such loans. This earlier work dealt with the guarantee of a non-callable discount bond (i.e., pays no coupons) issued by a firm paying no dividends. This case was addressed, in part, because the contingent claims formulation of the problem yields an explicit analytic expression for the value of the guarantee. However, as a matter of practical interest, a more relevant problem would be the evaluation of a guarantee on a callable coupon bond issued by a firm paying dividends. The contingent claims formulation of this problem does not result in an explicit analytic solution, but the solution can be approximated by numerical analysis. This paper presents the results of a numerical treatment of the problem as well as an analysis of the issues of partial versus full guarantees, the guaranteeing of junior debt and alternative covenants specifying the value of guaranteed debt given "premature" bankruptcy.

The next section of the paper briefly introduces contingent claims analysis and the formulations of the problems to be treated. The third section presents and discusses the numerical approximations. The last section outlines possible extensions to this paper.

2. Contingent Claims Analysis

The analysis of guaranteed loans in this paper uses the contingent claims valuation model developed by Black and Scholes (1973) and Merton (1973) (1974). This is a general methodology for the valuation of arbitrary contingent claims. Following these authors, assume:

(A1) "Frictionless Markets": There are no transactions costs or differential taxes. Trading takes place continuously in time. Borrowing and lending, at the same interest rate, are unrestricted. Short sales are unrestricted, with full use of proceeds.

(A2) The riskless short-term interest rate, r, is known and constant over time.

(A3) The price history of the firm is always continuous.

(A4) The instantaneous variance of return, σ^2, on asset value, V, is constant over time.

(A5) Total cash payouts, P, to all claimants depend at most on the asset value of the firm.

Under these assumptions, Black and Scholes (1973) demonstrated that any contingent claim whose value can be written as a function of asset value, V, and time is exactly correlated with the underlying asset value over short intervals. Arbitrage considerations require that the ratio of excess expected return to standard deviation of return - the reward to risk ratio - be identical for the contingent claim and the underlying asset value. In a general formulation, Merton (1974) showed that the value of a contingent claim which receives cash payouts over time, such as an issue of unguaranteed debt, $D(V,\tau)$, obeys the partial differential equation:

$$1/2\ \sigma^2 V^2 D_{VV} + (rV-P)D_V - D_\tau - rD + p = 0 \qquad (1)$$

Where p is the cash payout per unit time to the claim, τ is the maturity of the claim and subscripts denote partial derivatives. Similarly, the value of a contingent claim which receives no cash payouts over time, such as a loan guarantee $G(V,\tau)$, obeys the equation;

$$1/2\ \sigma^2 V^2 G_{VV} + (rV-P)G_V - G_\tau - rG = 0 \qquad (2)$$

Note the value of the guaranteed debt, $D^*(V,\tau)$, can be decomposed into the value of the debt without a guarantee and the value of the guarantee, so $D^*=D+G$. Since both the guarantee and debt minus guarantee are contingent claims on the same firm, the parameters for firm value, variance rate, and cash payouts are identical for the valuation equations (1) and (2).

The valuation logic of the contingent claims model is contained in these differential equations, which depend only on the asset value, V, of the firm, the time to maturity, τ, of the claim, the variance rate, σ^2, of asset value, the short-term interest rate, r, and cash payouts to claimants, P and p. The virtue of the model is that all of the above are observable or readily estimated.[1] In particular, the value of the contingent claims does not depend on the expected rate of return on asset value or on market parameters of risk and return.

2.a. Valuing a Fully Guaranteed Issue of Non-Callable Coupon Debt

Differential equations like (1) and (2) require terminal and boundary conditions to give a unique representation to a contingent claim. The terminal condition gives the value of the claim at maturity, $\tau = 0$, as a function of firm asset value. For example, suppose asset value is equal to or greater than the promised principal, B, at maturity. This implies that $V \geq B$ when $\tau = 0$. The debt receives full payment and $D(V,0) = B$. If the asset value is less than the principal, $V < B$, the debt can only be worth as much as the firm, $D(V,0) = V$. Thus, the debt at maturity is worth the minimum of the principal and asset value;[2]

$$D(V,0) = \text{Min } (B,V) \tag{1a}$$

Now consider the value of the guarantee at $\tau = 0$. If the asset value exceeds the promised principal, $V > B$, the guarantee has no value. If the asset value is less than the principal then the guarantee is worth the difference, $G(V,0) = B-V$. Thus the guarantee is worth the maximum of zero and the principal minus the asset value;

$$G(V,0) = \text{Max } (0, B-V) \tag{2a}$$

A lower boundary condition gives the value of the claims if the firm defaults "prematurely," that is to say before $\tau = 0$. If the asset value becomes worthless, $V = 0$, at any time prior to maturity, then the debt becomes worthless;[3]

$$D(0,\tau) = 0 \tag{1b}$$

Under the same circumstances, the value of the guarantee is dependent upon the covenant protecting the bondholder in this situation. Most guarantees would specify that the government is liable for the payment of the promised principal, B, in case of "premature" bankruptcy;

$$G(0,\tau) = B \; . \tag{2b}$$

An alternative covenant would specify that the government is liable for the present value of all future promised payments, calculated at the riskless interest rate. If there is a promised coupon of c per unit time, this amount can be represented by $R(\tau)$, where;[4]

$$R(\tau) = \frac{c}{r}(1-e^{-r\tau}) + Be^{-r\tau}$$

This would then lead to an alternative lower boundary condition for the guarantee;

$$G(0,\tau) = R(\tau) \tag{2b'}$$

An upper boundary condition gives the value of the claims when the asset value becomes large. The value of the debt will approach the value of a riskless bond, $R(\tau)$, as $V \to \infty$;

$$D(\infty,\tau) = R(\tau) \tag{1c}$$

The value of the guarantee would become worthless;

$$G(\infty,\tau) = 0 \tag{2c}$$

Consider a single issue of non-callable coupon debt, with a promised coupon of c per unit time and a promised principal, B, due in τ time periods. Assume that the firm will pay dividends of d per unit time over the life of the debt. The value of the unguaranteed debt will be given by the solution of the partial differential equation (1), with the total cash payout per unit time (P) equal to the sum of the coupon payments (c) and the dividend payments (d), and the cash payout to debt holders (p) equal to the coupon payments (c). This solution is subject to the terminal condition given by (1a) and the lower and upper boundary conditions given by (1b) and (1c) respectively. Similarly, the value of the guarantee will be given by the solution of the partial differential equation (2), again with total cash payouts equal to the sum of the coupon and the divident payments, $P=c+d$. The solution for the value of the guarantee is subject to the terminal condition given by (2a), the appropriate lower boundary condition by either (2b) or (2b'), and the upper boundary condition given by (2c). The value of the guaranteed debt will simply be the sum of the value of the unguaranteed debt and the value of the guarantee.

2.b. Valuing a Partially Guaranteed Issue of Non-Callable Coupon Debt

A closely related problem is that of partial guarantees. Consider a debt issue which has a fraction δ of its principal guaranteed. This leads to different terminal and boundary conditions for the value of the guarantee. The appropriate terminal condition is:

$$G(V,0) = \text{Max }(0, \delta B-V) \tag{2d}$$

which says that if the asset value is greater than δB then the guarantee is worth zero. If the asset value is less than δB then the guarantee is worth the difference between δB and V. The new lower boundary would be;

$$G(0,\tau) = \delta B \tag{2e}$$

The upper boundary condition would be the same as (2c), which says that the value of a partial guarantee goes to zero as the asset value becomes large. Equation (2), appended by conditions (2c), (2d) and (2e) is the contingent claims formulation of the partial guarantee value problem. The value of the unguaranteed debt is still represented by equation (1) and conditions (1a), (1b), and (1c). The value of the partially guaranteed debt is simply the sum of these two values.

2.c. Valuing Junior and Senior Non-Callable Coupon Debt with Guarantees

Now consider a firm with two classes of non-callable coupon debt, junior and senior. The junior debt is promised coupons of c' per unit time and has a promised principal of B'. The senior debt is promised coupons of c per unit time and has a promised principal of B. Assume that both issues have the same maturity date and that the firm will pay dividents of d per unit time. We examine two cases: in the first case, the senior debt is fully guaranteed and the junior debt is unguaranteed; in the second case, the senior debt is unguaranteed and the junior debt is fully guaranteed.

The value of the guaranteed senior debt is simply the sum of the value of unguaranteed senior debt plus the value of the guarantee. The value of the unguaranteed senior debt is represented by equation (1) with $P = c+c'+d$, $p=c$ and conditions (1a), (1b), and (1c). The value of the guarantee is represented by equation (2) with $P = c+c'+d$ and conditions (2a), (2b), and (2c).

The value of guaranteed junior debt is, again, simply the sum of the value of unguaranteed junior debt and the value of the guarantee. The value of unguaranteed junior debt satisfies equation (1) with $P = c+c'+d$ and $p=c'$. The terminal condition says the unguaranteed junior debt receives any residual firm asset value over the senior principal, Max (0,V-B), up to a maximum of the junior principal, B'. This is equivalent to;

$$D(V,0) = \text{Min}\ (B',\ \text{Max}\ (0,V-B)) \tag{1d}$$

Thus the contingent claims formulation of the unguaranteed junior debt problem is represented by equation (1) appended by conditions (1b), (1c) and (1d).

The value of the guarantee on the junior debt satisfies equation (2) with $P = c+c'+d$. The terminal condition says that the guarantor must pay if the asset value is less than the sum of the junior and senior principal payments, Max (0,B'+B-V), up to a maximum of the junior principal, B'. This is equivalent to;

$$G(V,0) = \text{Min}\ (B',\ \text{Max}\ (0,B'+B-V)) \tag{2f}$$

The formulation of the guarantee value problem is therefore equation (2) appended by conditions (2b), (2c), and (2f).

2d. VALUING CALLABLE COUPON DEBT WITH GUARANTEES

In the examples above, the debt was non-callable. however, the value of the unguaranteed debt and the guarantee will generally be affected by a call provision. To explore this, consider a firm with a single issue of callable coupon debt which is promised coupons of c per unit time and a principal payment of B in τ time periods. Assume that the firm will pay dividends of d per unit time and that the indenture specifies a schedule of call prices, $K(\tau)$. The formulation of this problem is identical to that of noncallable debt with the exception of the upper boundary condition. In the case of callable unguaranteed debt, there will exist a time dependent schedule of firm asset values, $\bar{V}(\tau)$, at or above which it is optimal for the equityholders to call the debt at $K(\tau)$. This schedule of asset values is solved for simultaneously with the determination of the debt's value. The new upper boundary condition for the callable debt problem is:

$$D(\bar{V}(\tau), \tau) = K(\tau) \tag{1e}$$

Therefore the formulation of the callable unguaranteed debt problem is equation (1) with $P = c+d$, $p=c$ and conditions (1a), (1b), and (1e).

The moment the debt is called, the guarantee has a zero value. Thus the guarantee value problem has the upper boundary condition;

$$G(\bar{V}(\tau),\tau) = 0 \tag{2g}$$

where $\bar{V}(\tau)$ is the firm asset value schedule determined in the callable debt problem. The guarantee value problem is then represented by equation (2) with $P = c+d$ and conditions (2a), (2b), and (2g). The value of guaranteed callable debt is simply the sum of the value of unguaranteed callable debt and the value of the guarantee.

None of the problems posed in this section have known analytic solutions for finite τ. However, there do exist numerical techniques which can be used to approximate the solutions. The next section presents and discusses the results of the numerical analysis of these problems.

3. Numerical Results for the Value of Guaranteed Debt Issues

The method of Markov chains is used to approximate solutions to the problems posed in the previous section. Samuelson (1965) proposed a similar technique to test a warrant pricing model. Parkinson (1977) and Mason (1979) use Markov chains to approximate solutions to valuation problems similar to the ones considered in this paper. A single computer algorithm, based on this method, is capable of treating all of the problems posed in this paper plus numerous other contingent claim valuation equations.[5] The numerical results are represented by Tables (1-10). These tables do not represent an exhaustive treatment of the problems but serve to demonstrate an application of contingent claims analysis to various loan guarantee problems given specific parametric assumptions.

3.a. Numerical Results for Non-Callable Coupon Debt with Full and Partial Guarante

The tables have been designed so as to convey as much information as possible and still be easy to interpret. To demonstrate, consider a firm with an asset value of \$100 million and a single issue of guaranteed debt. Assume that the debt is promised a principal payment of \$50 million in 15 years and carries a coupon rate of 12%/year. Let the variance of return on asset value be 20%/year and the riskless short-term interest rate be 10%/year.[6] Finally, assume that the bond indenture specifies that the firm pay no dividends over the life of the debt and that in case of "premature" bankruptcy, the government will pay the bondholders their promised principal. Thus, we have;

$$V = \$100,000,000 \qquad r = .10/\text{yr.}$$

$$B = \$\ 50,000,000 \qquad \sigma^2 = .20/\text{yr.}$$

$$c = \$\ 6,000,000/\text{yr.} \qquad \tau = 15 \text{ yrs.}$$

$$d = 0 \qquad P = \$6,000,000/\text{yr.}$$

Tables (1-3) represent a numerical treatment of this problem. Note that each table assumes specific values for the ratios, r/σ^2, $c/\sigma^2 B$ and $P/\sigma^2 B$. Returning to the example;

$$r/\sigma^2 = 0.5 \qquad c/\sigma^2 B = 0.6 \qquad P/\sigma^2 B = 0.6$$

Thus Table 2 represents the numerical treatment of this example. In order to find the proper table entry, it is necessary to compute the quantities $\sigma^2\tau$ and V/B. Given the example;

$$\sigma^2\tau = 3.0 \qquad V/B = 2.0$$

The table values are presented in units of promised principal, B. The first number, 0.902, represents the value of unguaranteed debt and the second number, 0.232, represents the value of the guarantee. The sum of these two numbers, 1.134, represents the value of guaranteed debt. So, for every $1,000 of promised principal;

D = $ 902; value of unguaranteed bond

G = $ 232; value of guarantee

D* = $1,134; value of guaranteed debt

Note that the bottom row of Table 2 gives the value of a riskless bond, R(T), with the same promised payments. Thus in the above example where the short-term interest rate is a constant 10%/yr., a riskless bond with a promised principal of $1,000 due in 15 years and an annual coupon rate of 12% is worth $1,155.

The virtue of presenting the results in this form is that the same table represents the numeric analysis of any similar loan guarantee problem with the same parametric assumptions. For instance, returning to the example, if r = 8%/yr., σ^2 = 16%/yr. and the debt carried a coupon rate of 9.6%/yr. then Table 2 represents the numeric treatment of this problem. Indeed, it is possible to represent all the information in Tables (1-3) in one table by considering more complex transformations.

Figure 1 depicts the value of unguaranteed debt as a function of firm value for a given time to maturity, τ, and risk free interest rate r. The curves labeled σ_1^2, σ_2^2 and σ_3^2 are based on the data in Tables 1, 2, and 3 respectively. Note the value of unguaranteed debt decreases as risk, σ^2, increases. Figure 2 represents the value of the guarantee as a function of firm value for a given τ and r. Again, the curves labeled σ_1^2, σ_2^2 and σ_3^2 are based on Tables 1, 2, and 3 respectively. Clearly, the value of the guarantee increases as risk, σ^2, increases.

(Insert Fig. 1 and Fig. 2)

Comparing the underlined entries in Tables (1-3), it is clear that the value of the guaranteed debt decreases as σ^2 increases. This is due primarily to the specification of the lower boundary in this problem, which says that if the firm defaults before the maturity date then the bondholders receive the promised principal. The value of guaranteed debt should be invariant to changes in risk. An alternative specification of this covenant, which is more consistent with the notion of "guaranteed" debt, is that if the firm defaults the bondholders receive the present value of all promised future payments. As is evident from Table 4, this will result in the value of a guaranteed bond always being equal

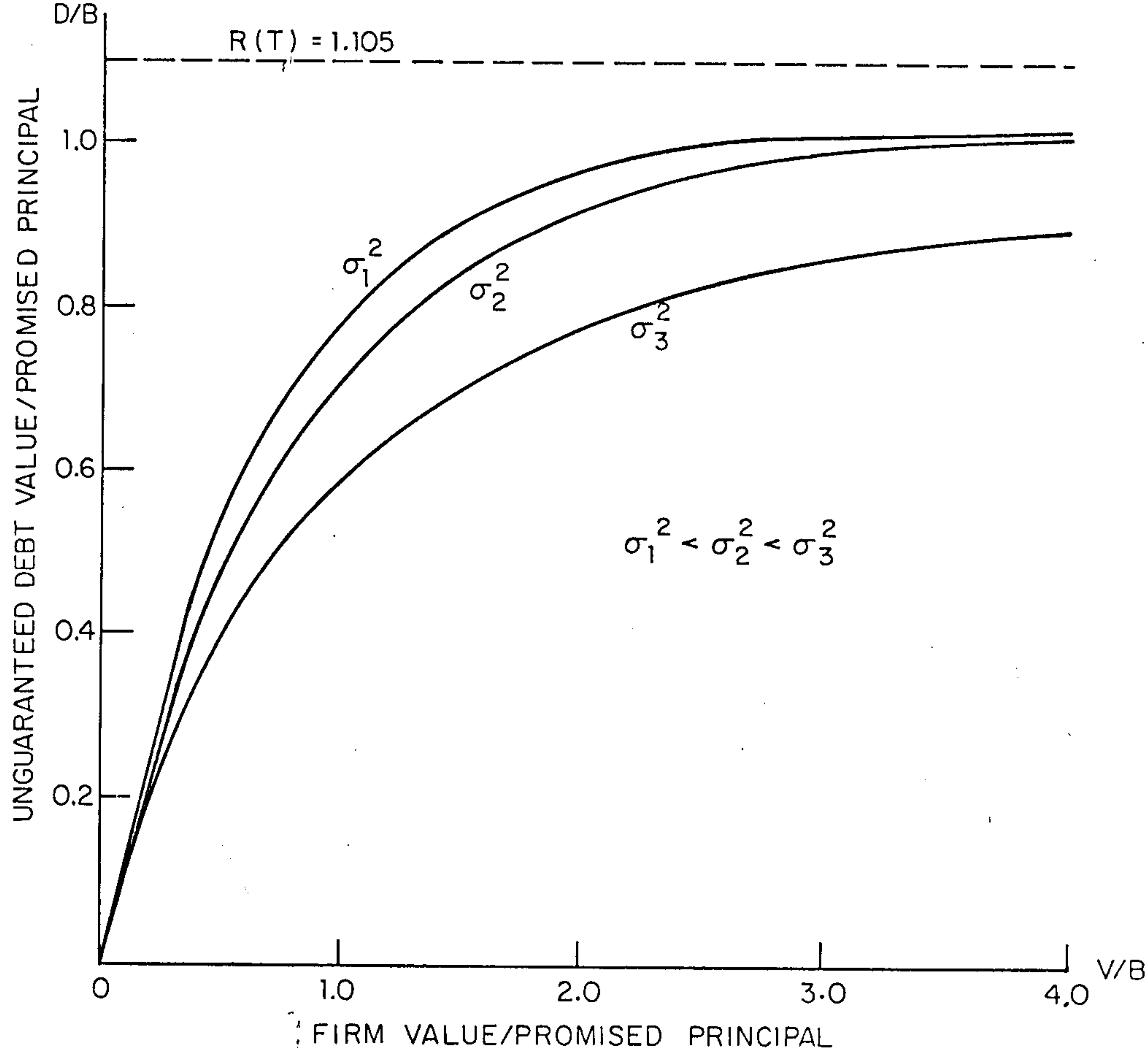

Fig. 1. Unguaranteed debt value as a function of firm value.

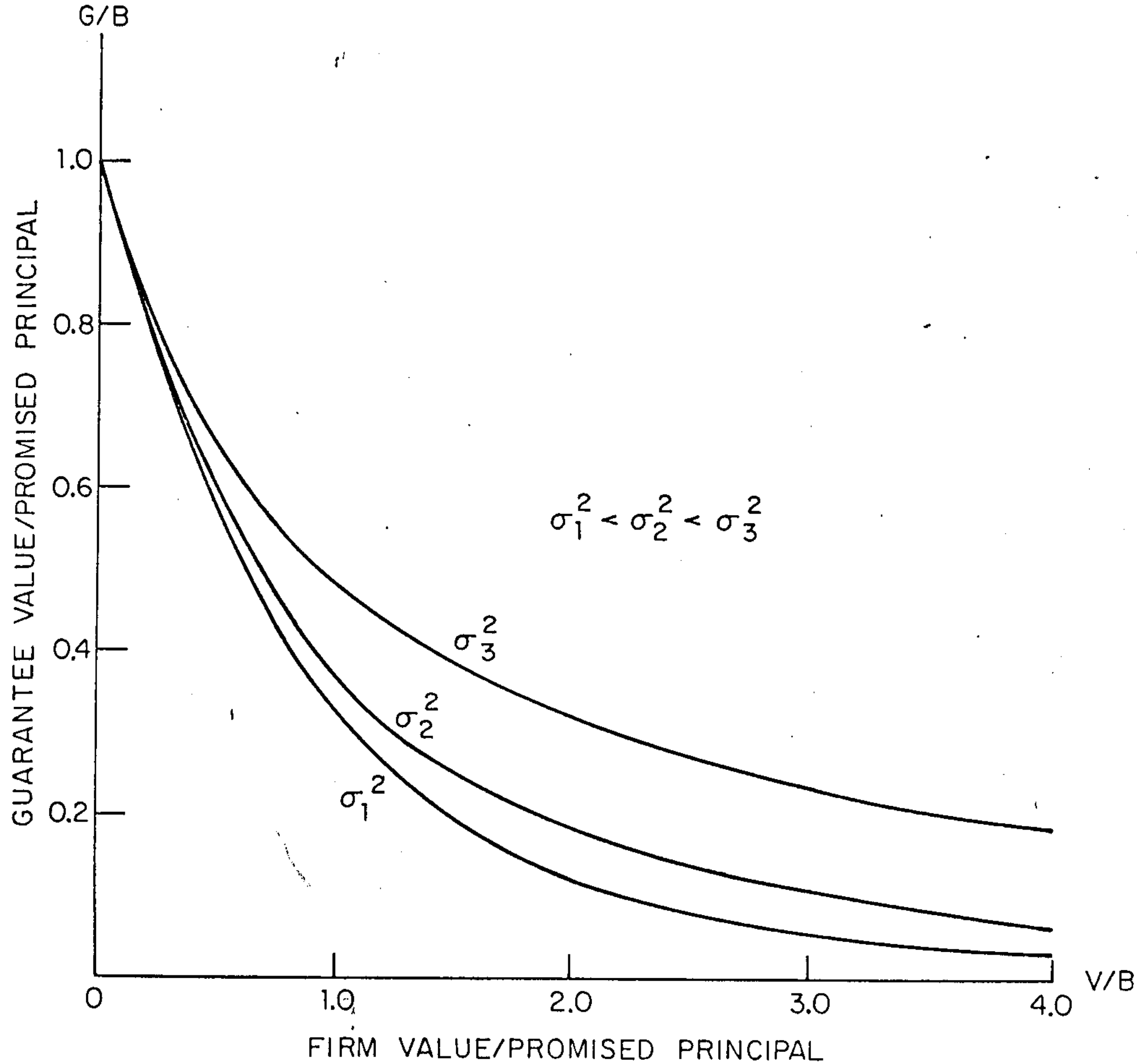

Fig. 2. Guarantee value as a function of firm value.

to the value of its riskless bond equivalent, R(T).

If the firm is making payouts to other claimants, such as dividends or coupon payments to junior debt, then the value of the unguaranteed debt and the value of the guarantee are affected. Table 5 allows for $P > c$, which says that total firm payouts are greater than that being made to the guaranteed debt. Compare Table 5 with Table 2, where $P = c$. The value of the unguaranteed debt decreases and the value of the guarantee increases. The unguaranteed debt value decreases because the payouts have the effect of decreasing the firm value. The guarantee value increases because the presence of payouts increases the probability of bankruptcy.

Some debt is partially guaranteed, for example, the government guarantees that the bondholder will receive at least X% of the promised principal in case of default. Table 6 considers a 75% guarantee where the bondholders are assured of receiving 75 cents for every dollar of promised principal. Compare Table 6 with Table 2, which is, of course, a 100% guarantee. The value of the unguaranteed debt is unaffected by the presence of a partial guarantee. The value of a partial guarantee is, as would be expected, worth less than the value of a full guarantee. However, note than an X% guarantee is worth less than X% of a full guarantee.[7]

3.b. Numerical Results for Guaranteed Junior Coupon Debt

Tables (7-9) are concerned with the problem of guaranteed junior debt. These tables assume that the junior principal and the senior principal are equal, $B = B'$. The response of the value of unguaranteed junior debt and the value of the guarantee to changes in risk, σ^2, is ambiguous. Note the underlined entries in Table 9. Table 8 represents the same problem as Table 9 except σ^2 has been increased by 50%. For high asset values the value of unguaranteed junior debt decreases and the value of the guarantee increases. This is similar to the behavior of guaranteed senior debt. However, for low asset values, the reverse can occur. The value of the unguaranteed junior debt can increase and the value of the guarantee can decrease. The same results obtain in comparing Table 8 with Table 7, which represents a 100% increase in σ^2 .

Figure 3 depicts the response of the value of unguaranteed junior debt and the guarantee to changes in risk, σ^2, for high firm values. Figure 4 represents the response of the value of unguaranteed junior debt and the guarantee to changes in σ^2 for low firm values. The value of the unguaranteed junior debt will initially increase and the value of the guarantee will initially decrease for increases in σ^2. The reason for this is that the junior debtholders and the guarantor have "nothing to lose" from increases in risk for sufficiently low firm values. However, if the variance becomes too large, the value of the unguaranteed junior debt will begin to decrease and the value of the guarantee will start to increase.[8]

(Insert Fig. 3 and Fig. 4)

This phenomenon has an interesting implication for the structure of loan guarantee programs. Assume that a firm has a single class of unguaranteed debt and the government has agreed to fully guarantee a new issue of debt. Further, assume that the guarantee specifies that in the event of default, the guaranteed debt will receive R(T), the riskless bond equivalent. This means that the guaranteed debt will always trade like a riskless bond, independent of the risk level of the firm and therefore the guaranteed debtholders will have no incentive to monitor the actions of the firm. However, as has been shown, the value of the guarantee will in most cases increase if the risk of the firm increases. Since the guarantee is a liability, the government has an incentive to monitor the firm's behavior. This monitoring function would represent an additional expense to the gov-

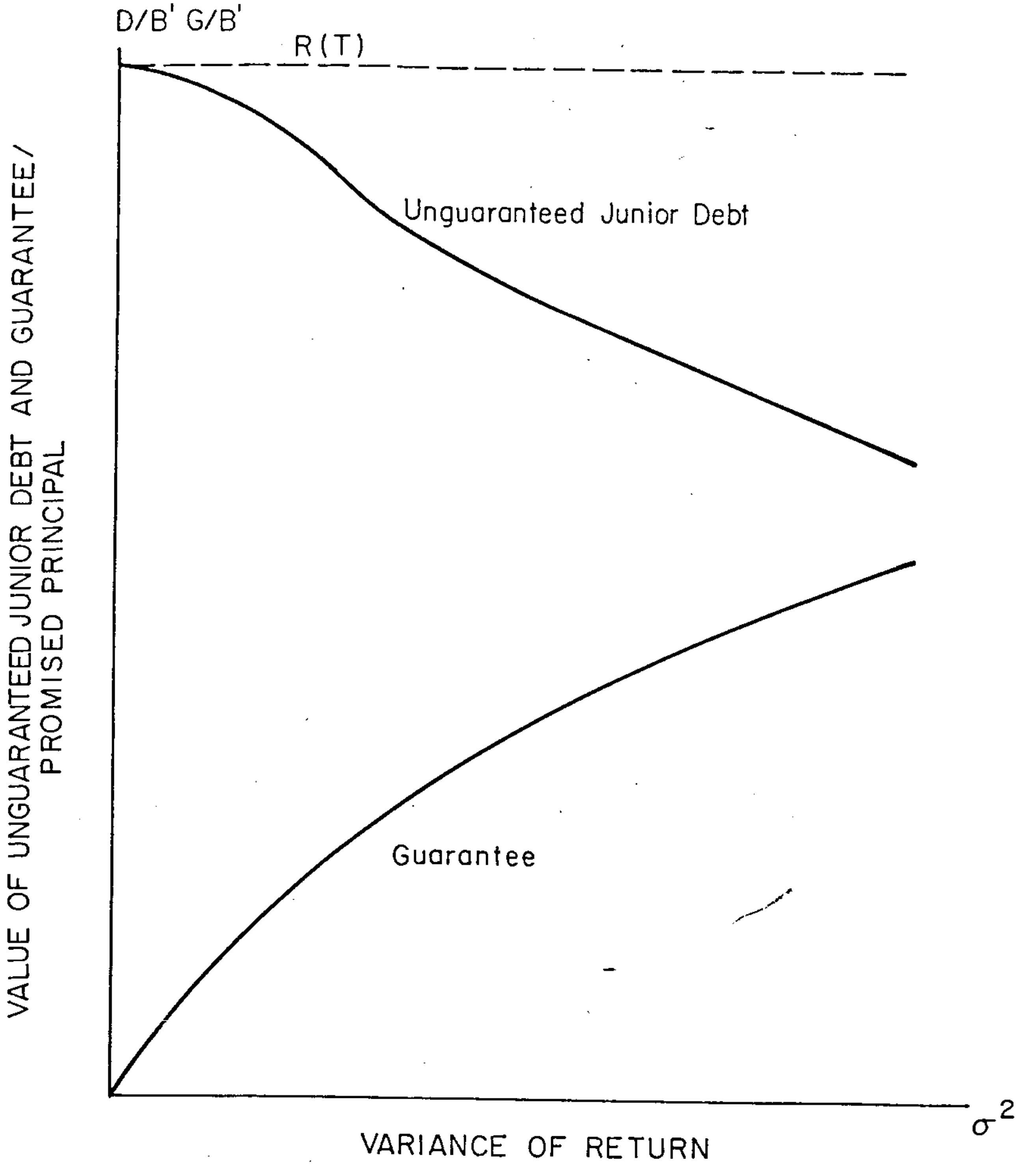

Fig. 3. Value of unguaranteed junior debt and guarantee, as a function of the variance, for large firm values.

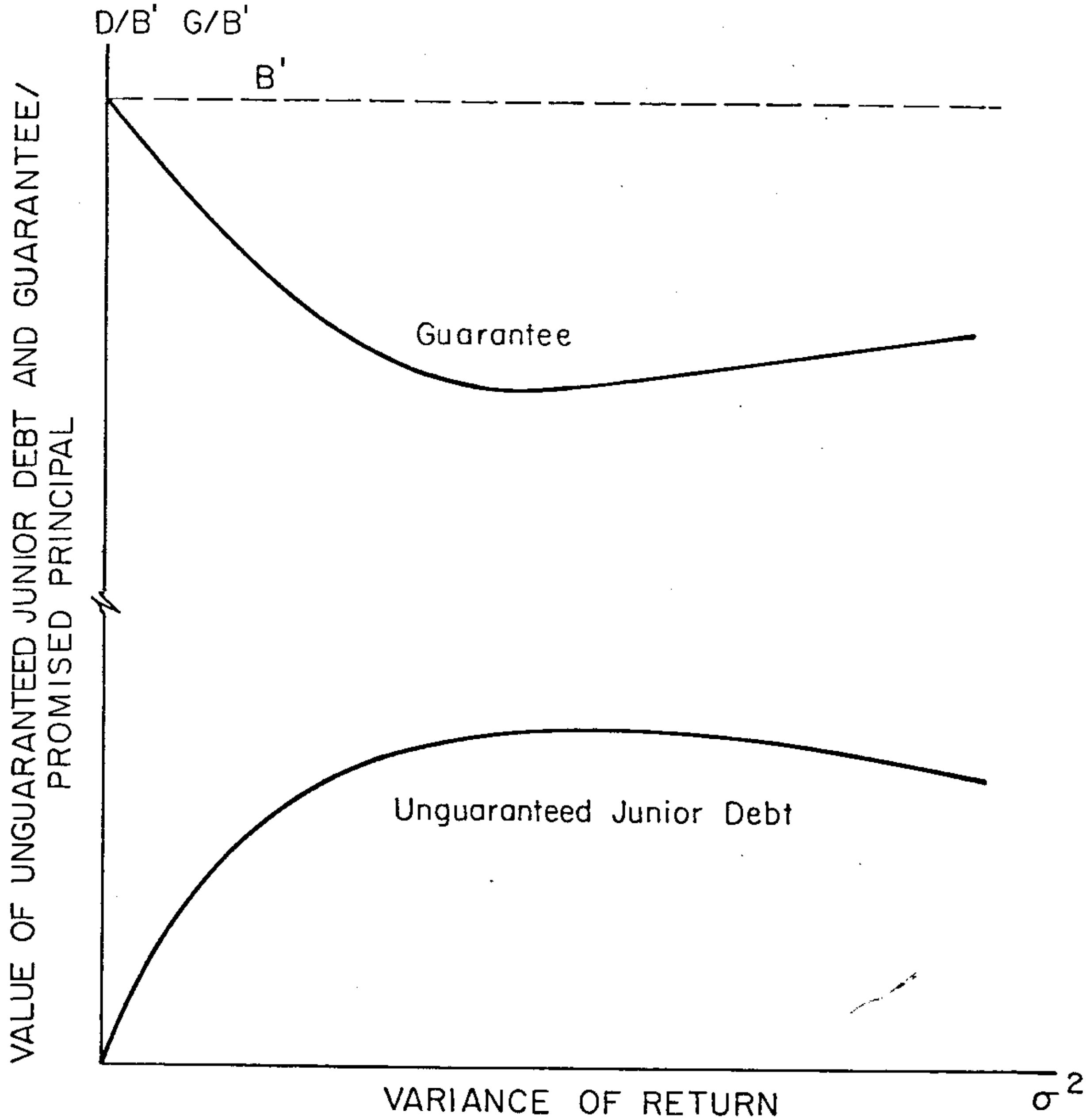

Fig. 4. Value of unguaranteed junior debt and guarantee, as a function of the variance, for small firm values.

ernment, thus it is of interest to ask if it is possible for the guarantee to be structured such that the incentives of the existing unguaranteed debt are consistent with those of the guarantor. For instance, consider positioning the guaranteed debt senior to the existing unguaranteed debt. Will the unguaranteed debt consistently guard against increases in firm risk and therefore relieve the government of the task of monitoring the firm? Clearly the answer is no, since it was earlier demonstrated that unguaranteed junior debt will at times benefit from increases in firm risk. What if the guaranteed debt is placed junior to the existing debt? Then it is true that the unguaranteed senior debt will always have the incentive to guard against increases in firm risk. Of course, given a fixed amount of debt to guarantee, it will cost more (the value of the guarantee is larger) to guarantee the junior debt. Table 5 and Table 8 demonstrate this point.

3.c. Numerical Results for Guaranteed Callable Coupon Debt

The last problem to be treated is that of callable guaranteed debt. Consider the following call schedule;

$$K(\tau) = \gamma(R(\tau)-B) + B$$

where $0 \leq \gamma \leq 1$. Table 10 represents the value of unguaranteed callable debt and the value of the guarantee when $\gamma = .25$. Table 2 is the non-callable counterpart to Table 10. As is well known, and as is verified by comparing the tables, non-callable unguaranteed debt is more valuable than callable unguaranteed debt. And, as would be expected, the guarantee is less valuable in the case of the callable debt, since the call feature has the effect of taking the guarantor "off the hook."

4. Extensions

This paper has not fully exploited contingent claims analysis or the Markov chains approximation algorithm in analyzing guaranteed loan problems. There are a number of interesting extensions which can be readily treated. For instance, in this paper it was assumed that the government is the guarantor and therefore there is no risk associated with the payment of the guaranteed amount. A possible extension is to allow for a risky guarantor such as another firm. Callable convertible debt, as well as certain tax effects, may be incorporated into this analysis. This paper also assumed a constant riskless short-term interest rate. It is possible to allow for stochastic interest rates but this results in valuation models which require additional assumptions. The work of Merton (1973) suggests that stochastic interest rates could be treated as simply an increase in total risk, σ^2. In this sense, the results of this paper are low estimates of the true value of loan guarantees.

Footnotes

[1]The short-term interest rate is a known market parameter. The time to maturity of the claims and cash payouts can be deduced from the indentures on claims. For an existing firm, asset value equals the sum of market values of securities of the firm minus the value of the guarantee. So there is a consistent set of values for the assets and guarantee which sum to the market value of all securities. Suppose the expected rate of return on asset value is constant. Then the wealth relative of asset value, including reinvestment of cash payouts, will be lognormally distributed over any interval. So the logarithm of this wealth relative will be normally distributed, with a variance of σ^2 multiplied by the interval length. For a given sequence of past intervals, the sample variance per unit time is an unbiased and efficient estimate of σ^2 . For a new firm, without a price history, standard valuation models must be used to determine asset value. Also, the variance rate can be estimated from existing firms with similar risk characteristics.

[2]The asset value is defined to be net of (the market value of) any costs of financial distress, including bankruptcy costs. Note this is consistent with the observation that the market value of all securities of the firm equals the asset value plus the value of the guarantee.

[3]In the case we are considering, lenders can force bankruptcy only in the case of default on a promised payment. Thus "premature" bankruptcy can only occur when asset value is insufficient to meet the coupon payment. An alternative indenture would provide for bankruptcy if asset value falls below a predetermined level, which might vary with time and maturity. Black and Cox (1976) examine the effect of such safety covenants on the value of a debt issue with no coupons. We could incorporate a safety covenant in our analysis by rewriting lower boundary conditions in terms of the critical asset value level for a given time to maturity.

[4]For risky bonds, the coupon rate $\frac{c}{B}$ exceeds the riskless interest rate r, so $R(\tau)$ is greater than B.

[5]The method of finite differences has been used by Brennan and Schwartz (1975), (1976), (1977) to treat contingent claims equations. The methods of Markov chains and finite differences are very similar, as demonstrated in Brennan and Schwartz (1976a) and Mason (1978).

[6]The tables assume that r/σ^2 = .25, .50 or .75. Given that r = .10, this implies that σ^2 = .40, .20 or .13. Rosenfeld (1979), in his study of common stock returns, has identified unlevered firms which have a variance of return in excess of .40. The use of this upper bound is justified given that a selection bias will exist in terms of the inherent risk of those firms/projects which request loan guarantees.

[7] Consider two firms with the same value and risk characteristics. Both firms have a coupon bond outstanding with a coupon rate of c per unit time and a promised principle, B, due in τ time periods. The first firm's debt is fully guaranteed and the second firm's debt is δ% guaranteed where $\delta < 1$. Now consider two securities positions: position I holds δ% of the first firm's debt and position II holds all of the second firm's debt. The question is in which of these two positions is the guarantee worth more. The guarantee will only pay in cases of premature bankruptcy or at $\tau = 0$. In the case of premature bankruptcy it is clear that both positions will receive the payment of δB from the guarantor. Now consider the payoffs at $\tau = 0$. If the value of the firm is greater than or equal to the promised principal, $V \geq B$, the guarantor will pay nothing to either position. If $B > V \geq \delta B$ then the payoffs from the guarantor to each position are;

I: $\delta(B-V) > 0$

II: 0

and if $V < \delta B$ the payoffs are;

I: $\delta(B-V)$

II: $\delta B-V$

It is clear that the payoffs from the guarantor to the first position are always equal to or greater than those to the second position. Thus δ% of a full guarantee is worth more than a δ% guarantee.

[8] A heuristic explanation of this behavior is that as σ^2 becomes very large, the probability of any future state obtaining becomes very small. Since the payoffs to junior debt are finite in all states, the expected payoff becomes smaller and the value of the debt decreases.

References

Black, F. and J. C. Cox, 1976, "Valuing Corporate Securities: Some Effects of Bond Indenture Provisions," Journal of Finance, 31, 351-402.

Black, F. and M. Scholes, 1973, "The Pricing of Options and Corporation Liabilities," Journal of Political Economy, 81, 637-659.

Brennan, M. J. and E. S. Schwartz, 1975, "Convertible Bonds: Valuation and Optimal Strategies for Call and Conversion," University of British Columbia Working Paper No. 336.

Brennan, M. J. and E. S. Schwartz, 1976, "The Valuation of American Put Options," University of British Columbia Working Paper.

Brennan, M. J. and E. S. Schwartz, 1976a, "Finite Difference Methods and Jump Processes Arising in the Pricing of Contingent Claims: A Synthesis," University of British Columbia Working Paper No. 432.

Brennan, M. J. and E. S. Schwartz, 1977, "Savings Bonds, Retractable Bonds and Callable Bonds," Journal of Financial Economics, 5, 67-88.

Jones, E. P. and S. P. Mason, 1978, "Contingent Claims Analysis of Loan Guarantees," Harvard Business School Working Paper, No. 78-51.

Mason, S. P., 1978, "The Numerical Analysis of Certain Free Boundary Problems Arising in Financial Economics," Harvard Business School Working Paper, No. 78-52.

Mason, S. P., 1979, The Numerical Analysis of Risky Coupon Debt Contracts," Harvard Business School Working Paper, No. 79-35.

Merton, R. C., 1974, "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," Journal of Finance, 29, 449-470.

Merton, R. C., 1977, "An Analytic Derivation of the Cost of Deposit Insurance and Loan Guarantees: An Application of Modern Option Pricing Theory," Journal of Banking and Finance, 1, 3-11.

Merton, R. C., 1973, "A Rational Theory of Option Pricing," Bell Journal of Economics and Management Science, 4, 141-183.

Parkinson, M., 1977, "Option Pricing: The American Put," Journal of Business, 5-, 21-36.

Rosenfeld, E., 1979, "Stochastic Processes of Common Stock Returns: An Empirical Examination," M.I.T. Dissertation.

Samuelson, P. A., 1965, "Rational Theory of Warrant Pricing," Industrial Management Review, 6, 13-31.

Table 1

Unguaranteed Debt Values and Guarantee Values

$r/\sigma^2 = 0.75 \quad c/\sigma^2 B = 0.90 \quad P/\sigma^2 B = 0.90$

$T = \sigma^2 \tau$

V/B	3.0	1.5	1.0	0.0
4.00	1.101 0.070	1.086 0.047	1.076 0.028	1.000 0.000
2.00	0.982 0.173	0.979 0.149	0.984 0.120	1.000 0.000
1.00	0.760 0.361	0.762 0.348	0.770 0.326	1.000 0.000
0.50	0.476 0.600	0.477 0.597	0.479 0.592	0.500 0.500
0.25	0.250 0.787	0.250 0.787	0.250 0.787	0.250 0.750
0.00	0.000 1.000	0.000 1.000	0.000 1.000	0.000 1.000
R(T)	1.178	1.135	1.105	1.000

Table 2

Unguaranteed Debt Values and Guarantee Values

$r/\sigma^2 = 0.50 \quad c/\sigma^2 B = 0.60 \quad P/\sigma^2 B = 0.60$

$T = \sigma^2 \tau$

V/B	3.0	1.5	1.0	0.0
4.00	1.032 0.115	1.036 0.068	1.039 0.039	1.000 0.000
2.00	0.902 0.232	0.918 0.182	0.938 0.140	1.000 0.000
1.00	0.700 0.408	0.713 0.378	0.731 0.344	1.000 0.000
0.50	0.455 0.616	0.459 0.608	0.465 0.596	0.500 0.500
0.25	0.248 0.789	0.248 0.788	0.249 0.787	0.250 0.750
0.00	0.000 1.000	0.000 1.000	0.000 1.000	0.000 1.000
R(T)	1.155	1.105	1.078	1.000

Table 3

Unguaranteed Debt Values and Guarantee Values

$r/\sigma^2 = 0.25 \quad c/\sigma^2 B = 0.30 \quad P/\sigma^2 B = 0.30$

$T = \sigma^2 \tau$

V/B	3.0	1.5	1.0	0.0
4.00	0.905 0.195	0.964 0.098	0.992 0.052	1.000 0.000
2.00	0.769 0.324	0.835 0.226	0.880 0.163	1.000 0.000
1.00	0.595 0.484	0.642 0.416	0.681 0.362	1.000 0.000
0.50	0.406 0.652	0.425 0.624	0.442 0.598	0.500 0.500
0.25	0.238 0.796	0.242 0.791	0.245 0.787	0.250 0.750
0.00	0.000 1.000	0.000 1.000	0.000 1.000	0.000 1.000
R(T)	1.105	1.062	1.044	1.000

Table 4

Unguaranteed Debt Values and Guarantee Values

$r/\sigma^2 = 0.50 \quad c/\sigma^2 B = 0.60 \quad P/\sigma^2 B = 0.60 \quad LB = R(T)$

$T = \sigma^2 \tau$

V/B	3.0	1.5	1.0	0.0
4.00	1.032 0.123	1.036 0.069	1.039 0.039	1.000 0.000
2.00	0.902 0.253	0.918 0.186	0.938 0.140	1.000 0.000
1.00	0.700 0.455	0.713 0.392	0.731 0.347	1.000 0.000
0.50	0.455 0.699	0.459 0.646	0.465 0.613	0.500 0.500
0.25	0.248 0.906	0.248 0.856	0.249 0.829	0.250 0.750
0.00	0.000 1.155	0.000 1.105	0.000 1.078	0.000 1.000
R(T)	1.155	1.105	1.078	1.000

Table 5

Unguaranteed Debt Values and Guarantee Values

$r/\sigma^2 = 0.50 \quad c/\sigma^2B = 0.60 \quad P/\sigma^2B = 1.40$

$T = \sigma^2\tau$

V/B	3.0	1.5	1.0	0.0
4.00	0.876	0.929	0.978	1.000
	0.254	0.172	0.099	0.000
2.00	0.658	0.697	0.759	1.000
	0.444	0.390	0.314	0.000
1.00	0.414	0.425	0.449	1.000
	0.650	0.636	0.608	0.000
0.50	0.227	0.227	0.229	0.500
	0.806	0.806	0.804	0.500
0.25	0.122	0.122	0.122	0.250
	0.894	0.894	0.894	0.750
0.00	0.000	0.000	0.000	0.000
	1.000	1.000	1.000	1.000
R(T)	1.155	1.105	1.078	1.000

Table 6

Unguaranteed Debt Values and Partial Guarantee Values

$r/\sigma^2 = 0.50 \quad c/\sigma^2 B = 0.60 \quad P/\sigma^2 B = 0.60 \quad \delta = 0.75$

$T = \sigma^2 \tau$

V/B	3.0	1.5	1.0	0.0
4.00	1.032	1.036	1.039	1.000
	0.083	0.042	0.020	0.000
2.00	0.902	0.918	0.938	1.000
	0.170	0.123	0.083	0.000
1.00	0.700	0.713	0.731	1.000
	0.302	0.271	0.232	0.000
0.50	0.455	0.459	0.465	0.500
	0.458	0.448	0.433	0.250
0.25	0.248	0.248	0.249	0.250
	0.587	0.586	0.584	0.500
0.00	0.000	0.000	0.000	0.000
	0.750	0.750	0.750	0.750
R(T)	1.155	1.105	1.078	1.000

Table 7

Unguaranteed Junior Debt Values and Guarantee Values

$r/\sigma^2 = 0.25 \quad c'/\sigma^2 B' = 0.40 \quad P/\sigma^2 B' = 0.70$

$T = \sigma^2 \tau$

V/B'	3.0	1.5	1.0	0.0
4.00	0.896 0.381	0.866 0.318	0.870 0.262	1.000 0.000
2.00	0.691 0.540	0.653 0.520	0.623 0.507	1.000 0.000
1.00	0.465 0.701	0.441 0.702	0.402 0.717	0.000 1.000
0.50	0.264 0.833	0.259 0.835	0.246 0.842	0.000 1.000
0.25	0.129 0.918	0.129 0.918	0.129 0.919	0.000 1.000
0.00	0.000 1.000	0.000 1.000	0.000 1.000	0.000 1.000
R(T)	1.316	1.187	1.132	1.000

Table 8

Unguaranteed Junior Debt Values and Guarantee Values

$r/\sigma^2 = 0.50 \quad c'/\sigma^2 B' = 0.80 \quad P/\sigma^2 B' = 1.40$

$T = \sigma^2 \tau$

V/B'	3.0	1.5	1.0	0.0
4.00	1.116 0.275	1.055 0.248	1.015 0.218	1.000 0.000
2.00	0.843 0.463	0.809 0.455	0.768 0.453	1.000 0.000
1.00	0.524 0.670	0.516 0.670	0.498 0.675	0.000 1.000
0.50	0.271 0.830	0.270 0.830	0.269 0.831	0.000 1.000
0.25	0.127 0.920	0.127 0.920	0.127 0.920	0.000 1.000
0.00	0.000 1.000	0.000 1.000	0.000 1.000	0.000 1.000
R(T)	1.466	1.316	1.236	1.000

Table 9

Unguaranteed Junior Debt Values and Guarantee Values

$r/\sigma^2 = 0.75 \quad c'/\sigma^2 B' = 1.20 \quad P/\sigma^2 B' = 2.10$

$T = \sigma^2 \tau$

V/B'	3.0	1.5	1.0	0.0
4.00	1.237 0.213	1.180 0.200	1.127 0.184	1.000 0.000
2.00	0.918 0.421	0.896 0.418	0.862 0.417	1.000 0.000
1.00	0.541 0.661	0.538 0.660	0.531 0.662	0.000 1.000
0.50	0.268 0.832	0.268 0.832	0.268 0.832	0.000 1.000
0.25	0.123 0.922	0.123 0.922	0.123 0.922	0.000 1.000
0.00	0.000 1.000	0.000 1.000	0.000 1.000	0.000 1.000
R(T)	1.536	1.405	1.316	1.000

Table 10

Unguaranteed Callable Debt Values and Guarantee Values

$r/\sigma^2 = 0.50$ $c/\sigma^2 B = 0.60$ $P/\sigma^2 B = 0.60$ $\gamma = 0.25$

$T = \sigma^2 \tau$

V/B	3.0	1.5	1.0	0.0
4.00	1.005 0.093	1.009 0.052	1.013 0.025	1.000 0.000
2.00	0.892 0.224	0.909 0.178	0.929 0.136	1.000 0.000
1.00	0.697 0.406	0.710 0.377	0.729 0.343	1.000 0.000
0.50	0.454 0.616	0.458 0.608	0.464 0.596	0.500 0.500
0.25	0.248 0.789	0.248 0.788	0.246 0.787	0.250 0.750
0.00	0.000 1.000	0.000 1.000	0.000 1.000	0.000 1.000
K(T) R(T)	1.038 1.155	1.026 1.105	1.019 1.078	1.000 1.000