RATIONAL EXPECTATIONS AND THE ALLOCATION OF RESOURCES UNDER ASYMMETRIC INFORMATION: A SURVEY

by

Sanford J. Grossman*

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RODNEY L. WHITE CENTER
FOR FINANCIAL RESEARCH

University of Pennsylvania

The Wharton School

Philadelphia, PA 19104

*University of Pennsylvania

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1. Introduction

Every good economics textbook contains the cliché that market prices provide signals which fascilitate the allocation of resources to their best use. In a world not subject to random shocks, consumers and producers when faced with competitive prices need look no further than their own preferences or production technology to be able to make a decision. They need give no thought to the tastes, endowments or technology of other agents. However, in a world subject to random shocks, this is no longer the case. Agents are faced with the problem of forecasting future states of nature and more importantly of forecasting the states of nature which affect other agents. Rational Expectations theories provide a model of how agents make those forecasts. Further, recent models of Rational Expectations have shown how even in a world with random shocks and asymmetric information, price can summarize, for each agent, the characteristics and information of other agents.

Expectations are a natural subject for analysis in problems where there is a lag in production. For example, a farmer desires to plant corn "today." The corn is harvested "tomorrow." In order to decide how much to plant the farmer must estimate the price of wheat tomorrow. Nineteenth Century economists attempted to treat this as a problem in capital theory and invoked the idea of

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* University of Pennsylvania.

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a stationary state. In the stationary state prices are always constant so tomorrow's price will be today's. Uncertainty was ignored or treated as if there was certainty.

Marshall (Book V, Ch. III) introduced the idea of temporary equilibrium and what is now known as Marshallian dynamics. Firms expect the "Normal" price and bring to market the supply appropriate to the Normal price. An actual equilibrium prices comes about which may be greater or less than Normal depending upon whether demand and supply determinants are at their normal level. If the market clearing price is higher than normal, say due to unusual demand, then firms will increase production. Unfortunately Marshall is not perfectly clear on how much they increase production. In particular he does not tell us what is the next price which firms expect. Marshall is much clearer about expectations in the long run and actually presents a very simple Rational Expectations model. He wrote about the expectational problems of a cloth manufacturer that "in estimating the wages required to call forth an adequate supply of labour to work a certain class of looms he might take the current wages...or he might argue...that looking forward over several years so as to allow for immigration he might take the normal rate of wages at a rather lower rate than that prevailing at the time... or [the cloth manufacturer] might think that wages of weavers...were abnormally low...in consequence of a too sanguine view having been taken of the prospects of the trade a generation ago." (Book V, Ch. V, sec. 1).

The above passage is extraordinary because the cloth manufacturer is thinking about the determinants of future wages. He is estimating future wages by estimating the determinants of wages. Even more extraordinary, the manufacturer has a very simple Marshallian model for the determinants of future wages, e.g., a large supply of immigrants will drive down wages. As we shall see below, the essence of Rational Expectations (R.E.) is that agents have some economic model of the economy which relates the exogenous variables to the endogenous variables which he is interested in forecasting.[1/]

Though the above passage indicates that Marshall thought about R.E., his short-run analysis was solely concerned with temporary equilibrium. Temporary equilibrium involves the equilibrium of supply and demand brought about at a market clearing price, where the supply function is taken as predetermined by expectations held last period when the production decision was made. If firms held incorrect price expectations, e.g., if the actual price was below the anticipated price, then they increase production and thus next period supply more. Eventually firms have expectations about the next period's price that are fulfilled. Hicks (1939, p.132) suggested the term Perfect Foresight for the situation where producers anticipate a price and then engage in production decisions with the property that the actual price which comes about is the anticipated price. The key point which Hicks emphasized is that the expectations held by firms about future endogenous variables (such as tomorrow's price of corn) actually help determine the true value of the future endogenous variables. Thus perfect foresight is an equilibrium concept rather than a condition of individual rationality.

After Hicks, two streams of literature are identifiable. The literature on multisector growth theory occasionally explicitly used the perfect foresight idea, however we will not have much to say about this stream. The other area of the literature was more applied and concerned with the econometric modeling of expectations. It is from this literature that the term "Rational Expectations" arose. In Section 2 we will review some of the literature which surrouded econometric modeling of expectations. Then the applied economic theories which introduced "Rational Expectations" will be covered along with the simultaneous generalization of Hicks' perfect foresight idea to the case of uncertainty. Section 3 considers the most recent application of the Rational Expectations idea which is to the problem of information aggregation and transmittal. Sections 4 and 5 return to the problem raised in the introductory paragraph: what role do prices play in an environment subject to uncertainty? What is the extent to which competitive prices summarize all the information that traders need for their decisions? Section 6 contains conclusions. This survey is primarily concerned with conceptual issues in the allocation of resources under uncertainty and asymmetric information. No attempt is being made to survey Rational Expectations in macroeconomics or the more technical aspects of the Rational Expectations literature.

2. Rational Expectations

In econometric modeling, R.E. grew out of dissatisfaction with *ad hoc* models of expectation formation. Perhaps one of the first *ad hoc* models of expectation formation to be estimated was by Irving Fisher [1930] in his celebrated work on inflationary expectations and their relationship to nominal interest rates. In what was to become standard practice for many years Fisher assumed that the expected rate of inflation for year T was a weighted average of the actual rate of inflation in years previous to T (i.e., the rate of inflation anticipated for year T is a *distributed lag* of the actual rates in previous years). This *ad hoc* approach to expectations was refined and modified throughout the next two decades. For example, Nerlove [1958] suggested the "adaptive expectations" model where the period t corn price anticipated by a farmer p_t^a satisfies $p_t^a = p_{t-1}^a + \eta(p_{t-1} - p_{t-1}^a)$, where p_{t-1} is the actual spot price at time t and $0 < \eta < 1$. This is, of course, just another distributed lag model:

$$(1) \qquad p_t^a = \eta \sum_{j=0}^{\infty} (1 - \eta)^j \, p_{t-j-1} \quad . \; 2/$$

Since econometric studies rarely had enough data to estimate arbitary distributed lags with any confidence, it became essential to put some a priori restrictions on the form of the lag structure. One restriction, suggested by Muth [1960], was that the distributed lag should be "Rational." By this he meant that if the sequence of actual price $\{p_t\}_{t=-\infty}^{t=+\infty}$ is a particular stochastic process, then the anticipated period t price p_t^a, as of period $t-1$, should be given by $p_t^a = E[p_t | p_{t-1}, p_{t-2}, \ldots]$. That is, if the farmer knows the stochastic structure generating prices then the price he expects (i.e., anticipates) for time t is the conditional expectation of p_t given all past realizations of the stochastic process. The farmer is being rational in that he is correctly computing the true conditional expectation. For example in the adaptive expectations model, the distributed lag which defines p_t^a is given without any reference to the actual stochastic process generating $\{p_t\}$. In particular, suppose that

$p_t = \begin{cases} 1 & \text{for } t \leq 1 \\ 2 & \text{for } t \geq 2 \end{cases}$. Then under adaptive expectations $p_2^a = 1$, $p_3^a = 1(1 - \eta) + 2\eta$, $p_4^a = (1 + \eta)(1 - \eta) + 2\eta$, .., $\lim_{t\to\infty} p_t^a = 2$. While under a rational lag structure $p_2^a = E[p_2 | p_1, p_0, \ldots] = 2$ since $p_2 \equiv 2$. Under adapative expectations it takes time for people to learn that price has changed from 1 to 2 because all they do is look at past prices. Under R.E., people know the price process, so in this simple example they know that price has changed permanently from 1 to 2 after $t = 1$.

From the above, it may seem as if R.E. involves more than rationality, since it assumes that agents know a lot more about the process generating prices, than does the adaptive expectations model. This is true, but misleading. Agents know something. Whatever it is that they are uncertain about can be modeled from a Bayesian point of view using the R.E. approach. The problem with ad hoc models such as adaptive expectations is that it assumes that agents have too little uncertainty about the structure. Agents act as if they are certain that the stochastic process generating price has some ad hoc form, such as eq. (1).[3/]

Quite naturally, the next question to arise in the literature was: where does the stochastic process $\{p_t\}$ come from? The assumption that agents use a Rational lag structure does not help econometric estimation by adding reasonable a priori restrictions because the econometrician must still make some a priori assumption (which he has too little data to check) about the stochastic process $\{p_t\}$. This problem generated the next contribution, which was a return to, and generalization of, Hicks' perfect foresight idea. In a seminal paper, Muth [1961] presented a perfect foresight model of a stochastic economy. Muth's basic idea can be described by the following simple model. Let a single firm's output q_t in period t have a cost of production $C(q_t)$. Assume that there are many firms. Suppose that at time t the market demand for the firm's product is

(2) $$\tilde{p}_t = D(Q_t,\tilde{\varepsilon}_t) ,$$

where $\tilde{\varepsilon}_t$ is a random variable summarizing the stochastic factors affecting demand at t, while p_t is the price at which the total output Q_t will be demanded. For a given Q_t, $\tilde{p}_t$ is a random variable because $\tilde{\varepsilon}_t$ is stochastic. Muth assumed that firms, acting as price takers, maximize expected profit

(3) $$E\pi_t = Ep_t q_t - C(q_t) .$$

<u>Assume that production takes place with a lag so that</u> q_t <u>must be chosen at time</u> $t - 1$. Then the optimal q_t for the firm will be a function of Ep_t, say $q_t = H(Ep_t)$ where $H(\cdot)$ is the inverse of the marginal cost function. If there are N identical firms then total supply is $Q_t = NH(Ep_t)$, and the actual market price will be given by the random variable

(4) $$\tilde{p}_t = D(NH(Ep_t),\tilde{\varepsilon}_t) .$$

Eq. (4) makes clear the point emphasized by Hicks, namely the expected period t price held by firms at $t - 1$ (i.e., Ep_t) determined their production decision and hence total output at t, which in turn determines the actual spot price at t. Suppose there is a solution to the equation

(5) $$p^* = ED(NH(p^*), \tilde{\varepsilon}_t) ,$$

for the number p^*, where the expectation in (5) is taken with respect to the distribution of $\tilde{\varepsilon}_t$. If p^* solves (5) then it is a perfect foresight equilibrium because firms anticipating a price of p^* make production decisions $NH(p^*)$ so that next period's spot price is a random variable $D(NH(p^*), \tilde{\varepsilon}_t)$ with mean (i.e., expected value of) p^*. Muth called p^* the Rational Expectations equilibrium price. Note that the equilibrium price is a random variable $D(NH(p^*), \tilde{\varepsilon}_t)$ not a number. Muth, by assuming risk neutrality and nonstochastic production, was able to provide a model which used certainty equivalents, and thus avoided the invention of the idea of a stochastic equilibrium.

Note that Rational Expectations is an equilibrium concept. It is a stochastic equilibrium, where the prices are random variables rather than numbers as in a Walrasian equilibrium. To see this more clearly consider a model where certainty equivalent prices will not work. Assume that production is random, i.e., let $C(q_t)$ be the cost of producing $q_t \cdot \tilde{v}_t$, where $\tilde{v}_t$ is a random shock affecting the production of all firms (e.g., the weather). The market-clearing price at time t will be some function of the random shocks affecting supply and demand, say $p(\tilde{\varepsilon}_t, \tilde{v}_t)$. A Rational Expectations equilibrium is a price function $p^o(\tilde{\varepsilon}_t, \tilde{v}_t)$ such that

(6a) $$\text{if } q_t^o \text{ solves } \max_{q_t} E[p^o(\tilde{\varepsilon}_t, \tilde{v}_t)(\tilde{v}_t \cdot q_t) - C(q_t)], \text{ then}$$

(6b) $$p^o(\tilde{\varepsilon}_t, \tilde{v}_t) = D(N(\tilde{v}_t \cdot q_t), \tilde{\varepsilon}_t) \text{ for all } (\varepsilon_t, v_t) \quad .$$ 4/

Condition (6a) requires that firms choose q to maximize expected profit, acting as price takers with respect to the price random variable $p^o(\varepsilon, v)$. Condition

(6a) requires that the anticipated price random variable $p^o(\tilde{\varepsilon}_t, \tilde{v}_t)$ is the actual market-clearing price for every realization of $(\tilde{\varepsilon}_t, \tilde{v}_t)$. Since realized output $\tilde{v}_t \cdot q_t$ is correlated with $p^o(\tilde{\varepsilon}_t, \tilde{v}_t)$, firms need to know more than the average price next period. Firms need to know the whole equilibrium price distribution.[5/]

R.E. is a condition of market equilibrium rather than being only a condition of individual rationality. However, the information requirements it makes on traders is no different than in models with Rational distributed lags. The latter models assume that traders know the stochastic process generating prices and use this knowledge in their decision problem. In a R.E. equilibrium, the information requirements are no greater, traders need only know the stochastic process generating the equilibrium price. Though the theory of market-clearing tells the economist what the underlying structural factors are (such as the forms of the cost and demands functions), in equilibrium traders need not know anything about the structural form of the economy. Of course, the theory does not explain how equilibrium comes about.

Thus far we have emphasized the approach taken to equilibrium under uncertainty by writers interested in applied econometric applications. An alternative approach is to imagine that there are complete state-contingent futures markets.[6/] Thus, imagine that at time $t - 1$ the farmer could sell his output forward at prices contingent on the realizations of $(\tilde{\varepsilon}, \tilde{v})$. If all trade takes place at $t - 1$, the spot markets at t need never open: At time t traders simply carry out the purchases and deliveries contingent on the realization of $(\tilde{\varepsilon}, \tilde{v})$ which they promised they would carry out at $t - 1$. since all trade takes place at $t - 1$, there is no price uncertainty regarding period t and hence no reason to form price expectations. However, even in a complete market, traders must still attempt to learn the probability distribution of the exogenous random shocks. In Section 4 we will be concerned with the ability of state-contingent claims prices to aggregate traders' information about that probability distribution and the allocation of resources in the above kind of markets.[7/]

3. The Role of Prices in Allocating Resources

One interesting aspect of economies subject to uncertainty is that traders are forced to think about their environment. Recall the quote from Marshall presented in the introduction, where each trader was forced to consider the determinants of wages. However, there is a widely held view, obtained from the study of economies not subject to uncertainty, that prices are signals which provide all the information about other traders that an individual trader needs to know.

Though prices are often referred to as signals in nonstochastic economies, they clearly play no formal role in transferring information. No one learns anything from prices. People are constrained by prices (often in just the right way so that individual rationality is transformed into collective rationality); however they are not informed by prices in the classical Walrasian or Marshallian models. It is an old idea that prices contain information. Perhaps the clearest statement appears in Hayek [1945, p. 527]:

> "We must look at the price system as ... a mechanism for communicating information if we want to understand its real function ... The most significant fact about this system is the economy of knowledge with which it operates, or how little the individual participants need to know in order to be able to take the right action ... by a kind of symbol, only the most essential information is passed on...."

Hayek wrote the above in criticism of the planning literature of the 1940's. That literature, taking the mathematical models of Walras, and the Welfare theorems of Lange literally, assumed that the State could set prices is such a way as to induce an efficient allocation and also the income distribution desired by the leaders of the State. (The Fundamental Theorem of Welfare Economics assures us that any Pareto Optimum can be supported by a competitive price system.) Hayek argued (in a vague way) that such arguments miss the point of price competition and the invisible hand. Each trader knows something about his own customers and neighborhood. Each individual's little piece of information gets aggregated and transmitted to others via trading. The final competitive allocations in some sense are as if an invisible hand with all the economies' information allocated resources. However a planner without all of that information could not have done as well. aa

Grossman, in a series of papers [1975c,1976,1977b,1978], used the Rational Expectations models of Lucas [1972] and Green [1973] to formalize and clarify the above ideas.[8/] Since this survey is avoiding macroeconomic papers we will not fully present Lucas' [1972] remarkable model.[9/] It will suffice to say that he considered an economy with a storable commodity (money). The current price of the commodity p is determined by a variable α which represents permanent increases in the supply of the commodity, and also by the current temporary demand for the commodity, denoted by β; denote this price function by $p(\alpha,\beta)$. Traders do not observe α and β directly. However, a trader would like to know know what α is because it affects the future value of the storable commodity. In our previous discussions of R.E., traders were trying to forecast a future endogenous variable by forecasting its exogenous determinants (e.g., traders went from their knowledge about the probability distribution of $(\tilde{\varepsilon},\tilde{v})$ to knowledge of $p^o(\tilde{\varepsilon},\tilde{v})$). Lucas, in an extraordinary insight, reversed the logic. He argued that if traders observe the current price p and know the relationship between p and α,β then they can use their observation of p to learn about the current realization of α and β.

Green [1973] used the same equilibrium concept but in the context of traders with heterogenous information. In Green's model there is a class of traders (called informed traders) who have some information, say α, about the future value of the commodity. Another group of traders (called uninformed) do not know α. There may be other temporary factors denoted by β affecting the current spot price of the commodity. Hence the current spot price is some function $p(\alpha,\beta)$. As in Lucas, uninformed traders observe the current price p and try to learn something about α.

The above models will be clearer when we show how Grossman used them to study the problem raised by Hayek. Consider the R.E. model of the last section where demand was random because of $\tilde{\varepsilon}$ shocks. Suppose $\tilde{y}_i$ is a random variable correlated with $\tilde{\varepsilon}$ (i.e., defined on the same probabiity space), and further that producer i observes a realization of $\tilde{y}_i$ at $t - 1$ before he has to make his production decision. Thus $\tilde{y}_i$ provides some information to trader i about what realization of $\tilde{\varepsilon}$ to expect. Let

$$\tilde{\underline{y}} = (\tilde{y}_1,\tilde{y}_2,\tilde{y}_3,\ldots)$$

be the vector of all traders' information. One possible definition of a R.E. equilibrium price for period t is a function $p_t^o(\tilde{\varepsilon},\tilde{\underline{y}})$ such that

(7a) $$\text{if } q_{it-1}^o(y_i) \text{ solves } \max_q E[p_t^o(\tilde{\varepsilon},\tilde{\underline{y}})|\tilde{y}_i = y_i]q - C(q), \text{ then}$$

(7b) $$p_t^o(\tilde{\varepsilon},\tilde{\underline{y}}) = D(\sum_i q_{it-1}^o(\tilde{y}_i),\tilde{\varepsilon}) \quad \text{for all} \quad (\tilde{\varepsilon},\tilde{\underline{y}}) .$$

At time $t - 1$ when production decisions are made, producer i observes a realization of y_i. This tells him something about $\tilde{\varepsilon}$ and about other traders' information (since $\tilde{\underline{y}}$, $\tilde{\varepsilon}$ are defined on a common probability space). Thus, his expectation of next period's price is $E[p_t^o(\tilde{\varepsilon},\tilde{\underline{y}})|\tilde{y}_i = y_i]$. When all producers do this the total supply brought to market at t will depend on all their information $\underline{y}$. It is for this reason that the market-clearing price at t depends on $\underline{y}$ as well as $\tilde{\varepsilon}$.

In the above definition, traders have fulfilled expectations about the spot price at t. Further, when they observe the spot price at t, they can learn something about other traders' information. <u>However that information is too late in coming</u>. Production decisions were made at time $t - 1$; it is at that time that each producer would like to learn about the information possessed by the other producers.

What market forces are there which will allow traders to use each others' information at time $t - 1$ when they need it? Grossman [1975c] pointed out that whenever traders have heterogenous beliefs there are incentives to open speculative markets, which as we will see below tends to homogenize their beliefs. In the above example, if $E[\tilde{p}_t^o|\tilde{y}_i]$ is different across the traders, then there is an incentive to open a futures market at time $t - 1$. To analyze this phenomenon we will make the assumption that demand at period t is perfectly elastic, i.e., let the price p_t at which the total supply Q_t is demanded be independent of Q_t and given by $\tilde{D} = \tilde{D}(\tilde{\varepsilon})$. Then the price trader i expects is $E[\tilde{D}|\tilde{y}_i = y_i]$.

Let p_f be the price at $t - 1$ for a unit of the commodity to be delivered at time t. If x_i denotes the amount of sold forward by trader i, then his expected profit is

$$p_f x_i + E[\tilde{D}|y_i](q_i - x_i) - C(q_i) \quad . \tag{8}$$

From (8) it is clear that each trader will want to increase his forward sales as long as $p_f > E[\tilde{D}|y_i]$ and decrease his sales as long as $p_f < E[\tilde{D}|y_i]$. If there is no constraint on short sales or long positions then no equilibrium forward price will exist when $E[\tilde{D}|y_i] \neq E[\tilde{D}|y_j]$. That is, the statement that forward prices reflect people's beliefs about future spot prices is meaningless when people have different beliefs.[10/]

To see the above point more clearly, consider a model of the above economy where a competitive equilibrium does exist. Assume that trader i is risk averse with Von Neumann-Morgenstern utility function

$$U_i(W) = 1 - e^{-a_i W} \qquad a_i > 0 , \tag{9}$$

and further assume that $(\tilde{D},\tilde{\underline{y}})$ are jointly Normally distributed. Then trader i chooses q_i and x_i to maximize $E[U_i(p_f x_i + (q_i - x_i)\tilde{D} - C(q_i)| y_i]$. The solutions satisfy

$$x_i = q_i + \frac{p_f - E[\tilde{D}|y_i]}{a_i \mathrm{Var}(\tilde{D}|y_i)} \qquad C'(q_i) = p_f . \tag{10}$$

Assume that the N firms are the only one operating in the forward market. Then equilibrium in the forward market requires the p_f make $\sum_{i=1}^{N} x_i = 0$. If we assume that $C'(q_i) = cq_i$ for some $c > 0$, then (10) can be used to solve for the market-clearing price p_f^o and writing it as a function of $\underline{y}$:

$$p_f^o(\underline{y}) = \sum_i \frac{E[\tilde{D}|y_i]}{a_i \mathrm{Var}(\tilde{D}|y_i)} \cdot \left[\sum_j (a_j \mathrm{Var}(\tilde{D}|y_j))^{-1} + \frac{N}{c} \right]^{-1} . \tag{11}$$

Equation (11) shows that the market-clearing price p_f^o is a complicated function of $\underline{y}$. It is some kind of weighted average of the expectations of different traders, where the weights are the product of risk aversion (i.e., a_i) and the precision of the information (i.e., $\mathrm{Var}(\tilde{D}|y_i)$). Though some writers have suggested that equations like (11) show the aggregative ability of competitive prices, (11) shows quite the reverse. In particular, to see how bad (11) does, compare the allocations obtainable with what would arise in an imaginary fully-informed <u>artificial</u> economy where <u>each</u> trader i had <u>all</u> of the economy's information $\underline{y}$. Consider the case where having $\underline{y}$ identifies $\tilde{D}$ exactly so that $\text{Var}(\tilde{D}|\underline{y}) = 0$. Thus when each trader possessed $\underline{y}$ he would face no risk and hence the only market-clearing futures price would be

$$(12) \qquad p_f^o(\underline{y}) = E[\tilde{D}|\underline{y}] \ .$$

Equation (12) is clearly different than (11). Further it is easy to show that a planner with all the information $\underline{y}$ could reallocate the x_i and change the production decisions q_i to make everyone better off than the allocations generated by the ordinary competitive equilibrium in (11). If prices and allocations were generated as in the artificial economy where each trader observes $\underline{y}$ then a planner who knows $\underline{y}$ will not be able to Pareto Dominate the competitive allocations.

There is something very wrong with the ordinary Walrasian equilibrium when traders have different information. To see this imagine that (11) is O.K. and that the economy described above is replicated many times so that each trader i observes many realizations of $(p_f^o(\tilde{\underline{y}}),\tilde{y}_i,\tilde{D})$. Then each trader will come to know the joint distribution of $(p_f^o,\tilde{y}_i,\tilde{D})$. Hence each trader is able to learn the conditional distribution of $\tilde{D}$ given $\tilde{y}_i,p_f^o$. A trader will in general find that $E[\tilde{D}|y_i] \neq E[\tilde{D}|y_i,p_f^o]$, i.e., observing p_f^o will lead each trader to change his beliefs because it gives him new information. After traders learn this, they will desire to recontract after observing $p_f^o(\underline{y})$. (A smart trader might even say to himself: "Let all the other traders trade naively using their own information. I will wait until the market clears, and after observing the current realization of $p_f^o(\underline{y})$, make my purchases of x_i and q_i to maximize $E[U_i|p_f^o(\underline{y}),y_i]$. Since I am a price taker I will expect to do better than by trading now and maximizing $E[U_i|y_i]$.")

When traders observe $p_f^o(\underline{y})$ and then compute their optimal choices so as to maximize $E[U_i|y_i,p_f^o]$, $p_f^o(\underline{y})$ will no longer, in general, be the market-clearing price which corresponds to $\tilde{\underline{y}} = \underline{y}$. This is because $p_f^o(\underline{y})$ is the market-clearing price when each trader forms his demands naively looking only at his own information, i.e., maximizing $E[U_i|y_i]$. At what price $p_f^e(\underline{y})$ will there be no desire to recontract after traders observe p_f^e? This price must have the property that when traders form their demands by conditioning on p_f^e (i.e., choosing x_i,q_i to max $E[U_i|y_i,p_f^e(\underline{y})]$, then $p_f^e(\underline{y})$ will indeed be the market-clearing price.

Formally $p_f^e(\underline{y})$ is a R.E. equilibrium:

(13a) if $x_i(p_f,y_i),q_i(p_f,y_i)$ solves

$$\max_{x_i,q_i} E[U_i(p_f x_i + (q_i - x_i)\tilde{D} - C(q_i))|\, y_i,p_f^e(\underline{y})] \text{ , and}$$

(13b) $$\sum_{i=1}^{N} x_i(p_f^e(\underline{y}),y_i) = 0 \quad \text{for all } \underline{y} \text{ .}$$

This is the equilibrium concept used by Lucas [1972] and Green [1973]. To see why $p_f^e(\underline{y})$ is called a R.E. equilibrium, consider the view taken earlier. Traders who observe the futures price know that it is in some way a reflection of the information possessed by other traders. They know that price will be high when traders have good news about $\tilde{D}$. Thus traders attempt to invert $p_f(\underline{y})$ and go from an observation of the endogenous variable p_f to learn about the exogenous variable $\underline{y}$. Of course when all traders do this, that affects the price at which the market will clear. $p_f^e(\underline{y})$ is an equilibrium point of the above process. When prices are generated by $p_f^e(\underline{y})$ and when traders use this price function to learn about $\underline{y}$, then the market will clear at $p_f^e(\underline{y})$ when $\tilde{\underline{y}} = \underline{y}$. Again we see that Rational Expectations is an equilibrium concept rather than a condition of individual rationality.

Recall that for the example in (9)-(11), the ordinary competitive equilibrium did not aggregate any information. In particular there was no relationship between it and the equilibrium allocations which occur in the artificial economy where each trader has all the economy's information. Grossman [1976] has shown that the R.E. equilibrium does an extraordinary thing. For the above example (and in a large class of models given below), the R.E. equilibrium is a sufficient statistic, and further the R.E. allocations are exactly the same as in the fully-informed artificial economy. The above remark is particularly easy to prove in the case where $\underline{y}$ provides perfect information about $\tilde{D}$, i.e., where $\mathrm{Var}(\tilde{D}|\underline{y}) = 0$. In this case

$$(14) \qquad p_f^e(\underline{y}) = E[\tilde{D}|\underline{y}]$$

is a R.E. equilibrium. To see this, first note that, under the assumption that $(\tilde{D},\underline{y})$ are jointly Normal, $E[\tilde{D}|\underline{y}]$, thought of as a function of $\underline{y}$, is a sufficient statistic for the conditional distribution of $\underline{y}$ given $\tilde{D} = D$.[11/] Thus a trader who conditions his beliefs on $p_f^e(\underline{y})$ knows as much as a trader who conditions on $\underline{y}$. It remains to show that markets will clear at $p_f^e(\underline{y})$ when $\tilde{\underline{y}} = \underline{y}$. To see this, note that $\mathrm{Var}(\tilde{D}|\underline{y}) = 0 \Rightarrow \mathrm{Var}(\tilde{D}|p_f^e(\underline{y})) = 0$, hence each consumer in (13a) has perfect information about $\tilde{D}$, and thus acts as if he is risk neutral. When all shareholders are risk neutral $p_f = E[\tilde{D}|\underline{y}] = E[\tilde{D}|p_f^e(\underline{y})]$ is a market-clearing futures price.

We made the unnecessarily strong assumption that $\mathrm{Var}(\tilde{D}|\underline{y}) = 0$ in order to show how in a R.E. model the futures price can equal the expected spot price conditional on all the economy's information, even though traders have different beliefs $E[\tilde{D}|y_i]$. Though traders come to the market with different information, they leave the market with allocations which are as if they all had the same information and their common information is the best available, namely $\underline{y}$. From

(13a), the reader may verify that producers set q_i so that $C'(q_i) = p_f^e \equiv E[\tilde{D}|\underline{y}]$. Thus production decisions are guided by all the economy's information (i.e., it is as if each trader had all the economy's information). See Feiger [1978] and Danthine [1978] for further results on R.E. and futures markets.

These results are unobtainable in a standard Walrasian equilibrium; there price is a weighted average of people's information, rather than a sufficient statistic for that information, see (11). The above remarks were derived under the assumption that $Var(\tilde{D}|\underline{y}) \equiv 0$. Suppose we drop that assumption, but maintain the assumption that $(\tilde{D},\tilde{\underline{y}})$ are jointly Normal. Then the following is a R.E. equilibrium

$$(15) \qquad p_f^e(\underline{y}) = \sum_{i=1}^{N} \frac{E[\tilde{D}|\underline{y}]}{a_i Var(\tilde{D}|\underline{y})} \cdot \left[\sum_j (a_j Var(\tilde{D}|\underline{y}))^{-1} + \frac{N}{c} \right]^{-1} .$$

This can be seen by noting that the joint Normality of $(\tilde{D},\tilde{\underline{y}})$ implies that $Var(\tilde{D}|\tilde{\underline{y}} = \underline{y})$ is the same for all realizations of $\tilde{\underline{y}}$. Hence $p_f^e(\underline{y})$ is a linear function of $E[\tilde{D}|\underline{y}]$ and by the previous argument $E[\tilde{D}|\underline{y}]$ is a sufficient statistic. Thus the conditional distribution of $\tilde{D}$ given $\underline{y}$ is the same as the conditional distribution of $\tilde{D}$ given $p_f^e(\underline{y})$. Hence $p_f^e(\underline{y})$ should be the ordinary Walrasian equilibrium for the economy where every trader observes $\underline{y}$, which the reader may verify is true.[12/]

The above argument shows that even if traders are risk averse and the aggregate information $\underline{y}$ is imperfect (i.e., $Var(\tilde{D}|\underline{y}) \neq 0$), then the R.E. price produces allocations which are as if each trader had all the economy's information.[13/] The next section considers a general equilibrium model where welfare propositions can be proved about the Rational Expectations equilibrium.

4. General Equilibrium and Welfare Propositions

In this section we consider a pure-exchange economy where traders' endowments depend on a random state of nature. To model this economy let $(\Omega, \mathcal{S}, \mu)$ be a probability space. Let $\tilde{s}$ be a random variable defined on this space which takes on only a finite number of values, say $n > 0$. Assume that in each state of nature there is only one commodity. Assume that there are H consumers. Each consumer h has an endowment $e_{hs} \geq 0$ in state s. Let $\tilde{e}_h \equiv (e_{hs})_{s=1}^n$ be consumer h's endowment vector; $\tilde{e}_h \in R_+^n$. Denote consumer h's consumption bundle by $\tilde{c}_h \in R_+^n$ where $\tilde{c}_h \equiv (c_{hs})_{s=1}^n$ and c_{hs} is his planned consumption in state s. We assume that each consumer h has preferences which can be described by a Von Neumann-Morgenstern utility function $\sum_s \bar{\pi}_{hs} U_h(c_{hs})$, where π_{hs} is the probability h attaches to state s.

We assume that consumers are able to get information about $\tilde{s}$ in the current period before $\tilde{s}$ is realized and before markets open. Assume that consumer h observes the realization of a random variable $\tilde{y}_h$ which is defined on $(\Omega, \mathcal{S}, \mu)$. Let Y_h denote the range of $\tilde{y}_h$. We let $\tilde{y} \equiv (\tilde{y}_h)_{h=1}^H$ denote all of the information possessed by all traders. Note that all traders' information random variables are defined on the same probability space. Let Y denote the range of $\tilde{y}$.

We assume that there is a complete set of Arrow-Debreu contingent commodity markets. That is, traders can exchange promises to deliver (or take delivery) of the commodity contingent on the occurrence of the event $\tilde{s} = s$.

An Arrow-Debreu equilibrium for the above economy (which is just the Walrasian equilibrium for an economy where the "goods" are $\tilde{c}_h$ and endowments are $\tilde{e}_h$), is a price vector $\tilde{p}^w = (p_s^w)_{s=1}^n$ and consumption allocations $\tilde{c}^w = (\tilde{c}_h^w)_{h=1}^H$, such that

(16) $$\tilde{c}_h^w \text{ solves } \max_{\tilde{c}_h} \sum_s U_h(c_{hs})\text{Prob}(\tilde{s} = s|\tilde{y}_h = y_h)$$

$$\text{s.t. } \tilde{p}^w \cdot \tilde{c}_h \le \tilde{p}^w \tilde{e}_h \text{ , and}$$

(17) $$\sum_h \tilde{c}_h^w \le \sum_h \tilde{e}_h \quad .$$

In (16) each trader maximizes his expected utility subject to a budget constraint. Note that each trader's beliefs about $\tilde{s}$ depend on his information y_h, so that his optimal consumption bundle $\tilde{c}_h^w(p,y_h)$ depends on the price and his information. Thus the price at which supply equals demand $\tilde{p}^w(\underline{y})$ will depend on everyone's information.

It is interesting to define the Walrasian equilibrium for the artificial economy where each trader observes all the information $\underline{y}$. Thus let $\tilde{p}^a(\underline{y})$ and $\tilde{c}^a(\underline{y})$ be the Walrasian equilibrium in (16) and (17) when $\text{Prob}(\tilde{s} = s|\tilde{y}_h = y_h)$ is replaced by $\text{Prob}(\tilde{s} = s|\tilde{\underline{y}} = \underline{y})$ in (16). That is, each consumer maximizes his expected utility conditional on <u>all</u> the economy's information. The allocations in the artificial economy correspond to the Walrasian equilibrium where each consumer's preferences over the commodity vector $\tilde{c}_h$ is derived from $\sum_s U_h(c_{hs})\text{Prob}(\tilde{s} = s|\tilde{\underline{y}} = \underline{y})$. Thus by the fundamental theorem of welfare economics a central planner with all the economy's information $\underline{y}$ could <u>not</u> find a feasible allocation $\tilde{c}(\underline{y}) = (\tilde{c}_h(\underline{y}))$ such that

(18) $$E[U_h(\tilde{c}_h(\underline{y}))|\underline{y}] \ge E[U_h(\tilde{c}_h^a(\underline{y}))|\underline{y}] \quad \text{for all } h \text{ and } \underline{y} \; .$$

with strict inequality for some h. This is because the Walrasian equilibrium for the artificial economy induces a Pareto optimal allocation of the commodity vector $(\tilde{c}_h)$ relative to the preferences $\sum_s U_h(c_{hs})\text{Prob}(\tilde{s} = s|\tilde{\underline{y}} = \underline{y})$.

However, the actual Walrasian equilibrium $\tilde{p}^w, \tilde{c}^w$ can, in general, be Pareto dominated by a central planner who knows all the information $\underline{y}$ (i.e., a feasible allocation $\tilde{c}_h(\underline{y})$ will exist so that $E[U_h(\tilde{c}_h(\underline{y}))|\underline{y}] \geq E[U_h(\tilde{c}_h(\tilde{p}^w(\underline{y}),\underline{y}))|\underline{y}]$ for all h and $\underline{y}$ with strict inequality for some h). This is exactly what we illustrated in the example in the previous section. As in that section we can define a R.E. equilibrium. We will prove that as long as markets are complete (in the sense that consumers can buy and sell $\tilde{c}_h$ freely), there exists a R.E. equilibrium for the economy where each trader has his own information, which cannot be Pareto dominated by a central planner who has all the economy's information. This formalizes the idea that the prices have an explicit informational role.

To prove the above theorem, we need only prove that the Walrasian equilibrium to the artificial economy $\tilde{p}^a(\underline{y}),\tilde{c}^a(\underline{y})$ is a R.E. equilibrium. In this context a R.E. equilibrium is a pair $(\tilde{c}^o,\tilde{P}^o(y))$ such that $\tilde{c}^o \equiv (\tilde{c}^o_h(y_h,P))^H_{h=1}$ $\tilde{c}^o_h(y_h,P) \equiv (c^o_{hs}(y_h,P))^n_{s=1}$ $\tilde{P}^o = (P^o_s(y))^n_{s=1}$ where

(19a) $$c^o_h(y_h,P^o) \text{ solves } \max_{c_h} \sum_s U_h(c_{hs})\text{Prob}(\tilde{s} = s|\tilde{y}_h = y_h,\tilde{P}^o = P^o)$$

$$\text{s.t. } P^o \cdot c_h \leq P^o \cdot e_h \quad \text{and}$$

(19b) $$\sum_h \tilde{c}^o_h(y_h,\tilde{P}^o(y)) = \sum_h \tilde{e}_h \quad \forall\, \underline{y} \text{ for all } \underline{y}.$$

If we consider the <u>artificial</u> economy where each consumer h has the information $\tilde{\underline{y}}$ rather than $(\tilde{y}_h,\tilde{P}^o)$, then an equilibrium to the artificial economy which is an invertible function of $\{\text{Prob}(\tilde{s} = s|\tilde{\underline{y}} = \underline{y})\}^n_{s=1} \equiv \pi(\underline{y})$ will be a Rational Expectations Equilibrium. This is because $\pi(\underline{y})$ is a sufficient statistic for the family $\text{Prob}(\tilde{y} = y|\tilde{s} = s)$.[14/] Thus a consumer who conditions on any invertible function of $\pi(\underline{y})$ has the same beliefs as a consumer who conditions on $\underline{y}$. Hence in (19a) when a consumer conditions on y_h and a $\tilde{P}^o$ which is an invertible function of $\pi(\underline{y})$, he will have the same demands as a consumer in the artificial economy who conditions on $\underline{y}$. This is formalized below.

Let $\pi_s^\alpha \equiv \text{Prob}[\tilde{s} = s | \underline{y} = \underline{y}^\alpha]$, $\pi_s^\beta \equiv \text{Prob}[\tilde{s} = s | \underline{y} = \underline{y}^\beta]$, and $\pi^\alpha \equiv \{\pi_s^\alpha\}_{s=1}^n$ $\pi^\beta \equiv \{\pi_s^\beta\}_{s=1}^n$. Consider the Arrow-Debreu equilibrium where all traders have probability beliefs π^α, i.e., the Walrasian equilibrium for the artificial economy where we pretend that each trader has all the information $\tilde{\underline{y}} = \underline{y}^\alpha$. Denote this equilibrium by $(\tilde{P}_\alpha^a, \tilde{c}_\alpha^a)$, where $\tilde{P}_\alpha^a = \tilde{P}^a(\underline{y}^\alpha)$ and $\tilde{c}_\alpha^a = \tilde{c}^a(\underline{y}^\alpha)$. Similarly, let $(\tilde{P}_\beta^a, \tilde{c}_\beta^a)$ be the Walrasian equilibrium for the artificial economy when traders have common information $\tilde{\underline{y}} = \underline{y}^\beta$. The following theorem shows that essentially if $\pi^\alpha \neq \pi^\beta$, then $\tilde{P}_\alpha^a \neq \tilde{P}_\beta^a$, i.e., $\tilde{P}^a(\underline{y})$ reveals $\pi(y)$ to all traders.[15/]

<u>Theorem 1</u>: Let $\pi^\alpha >> 0$ and $\pi^\beta >> 0$ (each state always has positive probability). Assume that for all traders h, $U_h(\cdot)$ is strictly concave and differentiable. If $\pi^\alpha \neq \pi^\beta$, and $\tilde{c}_\alpha^a \neq \tilde{c}_\beta^a$, then $\tilde{P}_\alpha^a \neq \tilde{P}_\beta^a$.

<u>Proof</u>: By contradiction: suppose $\tilde{P}_\alpha^a = \tilde{P}_\beta^a$, then for some consumer h, $\tilde{c}_{h\alpha}^a \neq \tilde{c}_{h\beta}^a$. This consumer could afford $\tilde{c}_{h\alpha}^a$ and $\tilde{c}_{h\beta}^a$ in both economies. By revealed preference $\tilde{c}_{h\alpha}^a$ must yield higher utility under beliefs π^α than does $\tilde{c}_{h\beta}^a$. Since utility is strictly concave $\tilde{c}_{h\alpha}^a$ must give strictly more utility than does $\tilde{c}_{h\beta}^a$ under beliefs π^α. Thus for all h

(20) $$\sum_s U_h(c_{hs\alpha}^a)\pi_s^\alpha > \sum_s U_h(c_{hs\beta}^a)\pi_s^\alpha \quad .$$

Rewrite (20) as

(21) $$\sum_s [U_h(c_{hs\alpha}^a) - U_h(c_{hs\beta}^a)]\pi_s^\alpha > 0 \; .$$

Since $U_h(\cdot)$ is a concave function

(22) $$U_h(c_{hs\alpha}^a) - U_h(c_{hs\beta}^a) \leq U_h'(c_{hs\beta}^a)(c_{hs\alpha}^a - c_{hs\beta}^a) \; .$$

Substitute (22) into (21) to get

(23) $$\sum_s U_h'(c_{hs\beta}^a)(c_{hs\alpha}^a - c_{hs\beta}^a)\pi_s^\alpha > 0 \; .$$

From the first-order condition to the consumer's optimization problem in the economy with beliefs π^β, there exists a $\lambda_h^\beta > 0$ such that

(24) $$U_h'(c_{hs\beta}^a)\pi_s^\beta \leq \lambda_h^\beta P_{s\beta}^a \text{ with equality if } c_{hs\beta}^a > 0 .$$

Substitute (24) into (23) to get

(25) $$\lambda_h^\beta \sum_s P_{s\beta}^a \frac{\pi_s^\alpha}{\pi_s^\beta} (c_{hs\alpha}^a - c_{hs\beta}^a) > 0 .$$

Since $\lambda_h^\beta > 0$, (25) implies

(26) $$\sum_s P_{s\beta}^a \frac{\pi_s^\alpha}{\pi_s^\beta} (c_{hs\alpha}^a - c_{hs\beta}^a) > 0 .$$

Note that if for consumer k, $\tilde{c}_{k\alpha}^a = \tilde{c}_{k\beta}^a$, then (26) holds with equality for $h = k$. Sum equation (26) over all consumers h to get

(27) $$\sum_s P_{s\beta}^a \frac{\pi_s^\alpha}{\pi_s^\beta} [\sum_h (c_{hs\alpha}^a - c_{hs\beta}^a)] > 0 .$$

However, aggregate demand must equal aggregate supply for commodities with positive prices, so

(28) $$\sum_h c_{hs\alpha}^a = \sum_h e_{hs} = \sum_h c_{hs\beta}^a .$$

Equations (27) and (28) cannot both be true. Q.E.D.

Theorem 1 almost immediately implies that the equilibrium for the artificial economy $(\tilde{P}^a, \tilde{c}^a)$ is a R.E. equilibrium, i.e., satisfies (19). This is because if consumer h conditions his beliefs on $\tilde{P}^a(\underline{y})$ and $\tilde{y}_h$, then $\tilde{P}^a(\underline{y})$ reveals $\pi(\underline{y})$ and the consumer will have probability beliefs about $\tilde{s}$ given by $\pi(\underline{y})$. $\tilde{P}^a(\underline{y})$ has been constructed to clear markets when all consumers have beliefs $\pi(\underline{y})$, so (19b) holds for $\tilde{P}^a$. Formally,

<u>Theorem 2</u>: Suppose that $\text{Prob}[\tilde{s} = s | \tilde{\underline{y}} = \underline{y}] > 0$ for all $s, \underline{y}$. If $U_h(\cdot)$ is strictly concave and differentiable for all h, then a Rational Expectations Equili-

brium exists with $\tilde{c}^o$ equal to what it would be in an artificial economy where each trader h observed $\tilde{y} = (\tilde{y}_h)_{h=1}^H$.

Proof: If in the artificial economy $\pi^\alpha \neq \pi^\beta$ implies $\tilde{c}^a_\alpha \neq \tilde{c}^a_\beta$, then Theorem 1 is the same as Theorem 2. On the other hand, suppose $\pi^\alpha \neq \pi^\beta$ but $\tilde{c}^a_\alpha = \tilde{c}^a_\beta$. Then, in the rational expectations economy each consumer cannot distinguish between π^α and π^β by observing the price. However, it is irrelevant because each each consumer h would choose $\tilde{c}^a_{h\alpha} = \tilde{c}^a_{h\beta}$ irrespective of whether $\pi(\underline{y})$ is π^α or π^β. Q.E.D.

An immediate consequence of Theorem 2 is that there exists a R.E. equilibrium $(\tilde{p}^o, \tilde{c}^o)$ which cannot be Pareto dominated by a central planner with all the economy's information. Formally,

Theorem 3: There exists a R.E. equilibrium $(\tilde{p}^o(\underline{y}), \tilde{c}^o(\underline{y}))$ such that if $\tilde{c}(\underline{y}) = (\tilde{c}_h(\underline{y}))_h$ is any other feasible allocation (i.e., $\sum_h [c_h(\underline{y}) - e_h(\underline{y})] \leq 0$ for all $\underline{y}$), then for each $\underline{y}$, it is impossible to have:

$$\sum_s U_h(c_{hs}(\underline{y})) \operatorname{Prob}(s|\underline{y}) \geq \sum_s U_h(c^o_{hs}(\underline{y})) \operatorname{Prob}(s|\underline{y}) \quad \text{for all } h \tag{29}$$

and strict inequality for some h.

Proof: Choose the R.E. equilibrium in Theorem 2, $(\tilde{p}^o, \tilde{c}^o)$. Then, by Theorem 2, for each $\underline{y}$, $\tilde{c}^o_h(\underline{y})$ is the ordinary Walrasian equilibrium for the economy where traders have preferences on $(c_{hs})_s$ induced by the von Neumann-Morgenstein utility function $\sum_s U_h(c_{hs}) \operatorname{Prob}(s|\underline{y})$. Thus (29) is impossible because the Walrasian equilibrium is, for each $\underline{y}$, Pareto optimal. Q.E.D.

In the above economy, there are n commodities $(c_{hs})_{s=1}^n$, and thus n prices which consumers observe to get information about $\underline{y}$ $(p^o_s(\underline{y}))_{s=1}^n$. Note that the information in $\underline{y}$ is summarized by the n vector $\pi(\underline{y}) = (\pi_s(\underline{y}))_{s=1}^n =$

$(\text{Prob}(\tilde{s} = s|\tilde{y} = y))_{s=1}^{n}$. From Theorem 1, it is clear that $P^a(y) \equiv \bar{P}^a(\pi(y))$, i.e., $P^a(y)$ depends on y only through $\pi(y)$. Theorem 1 showed that $\bar{P}^a(\pi)$ is essentially an invertible function of π. The n-vector $\pi(y)$ is a sufficient statistic for y, and there are n traded speculative commodities.[16/] That is, there are enough prices around so that all the information relevant to consumers can be extracted from $P^a(y)$. If there were fewer markets, this would not, in general, be true.[17/] The next section discusses how the number of speculative markets is determined by incentives for information collection.

5. The Informational Role of Markets

The previous section showed that if there is a complete set of markets, then the allocations in an economy where each individual possesses only a small piece of information are as if each individual possessed all the economy's information. Note that Rational Expectations creates a positive externality in that many consumers benefit from the information possessed by a single consumer. Formally, this externality can be seen to exist because an individual's preferences over the commodiities $\tilde{c}_h$ are derived from $E[U_h(\tilde{c}_h)|y_h, \tilde{P}^o(y) = P^o]$. The latter depends on the whole economy's information vector y. This section discusses some consequences of the externality.

If information is costly to each individual, then the existence of a R.E. equilibrium price which is a sufficient statistic provides strong disincentives for any individual to collect information. If $\tilde{P}^o$ is a sufficient statistic, then $E[U_h|\tilde{y}_h, \tilde{P}^o] = E[U_h|\tilde{P}^o]$, so each individual finds his own information irrelevant. In this case no individual will be willing to devote resources to information collection. The Theorems of the previous section guarantee that even if some traders stop collecting information (so that $\tilde{y}_h \equiv$ constant for many traders), then there is a R.E. price which is a sufficient statistic for the remaining information. This is because no restrictions were put on the information vector $\underline{y}$ in proving the theorems. Thus, whenever anyone collects information, he finds his information redundant. Therefore, it is not a competitive equilibrium for anyone to collect information in an Arrow-Debreu complete market. On the other hand, suppose no one collects information, then the $\tilde{\underline{y}}$ which $\tilde{P}^o$ reveals is degenerate and so $\tilde{P}^o$ reveals nothing to traders. Assume that the information $\tilde{y}_h$ has some private value in the situation where trader h has access to no information other than $\tilde{y}_h$. Under this assumption it is not a competitive equilibrium for no one to collect information. This is because each trader, acting as a price taker, will find it valuable to collect information. Thus no competitive equilibrium exists when information collection is costly and when markets are complete.[18/]

Since the economy is presumed to exist, we must drop one of the following three assumptions: (A) that information is costly, or (B) that markets are complete, or (C) that there is perfect competition. Consider first the case where information is free. There are two senses in which information may be free. First any individual h may be able to acquire the whole economy's information $\underline{y}$ for free. In this case there are no disincentive problems with having a R.E. equilibrium which is a sufficient statistic. However, the competitive price system is not doing very much by revealing information which is freely obtainable to anyone even without prices.

There is another sense in which free information is compatible with the existence of a R.E. equilibrium which is far from degenerate. Suppose that each trader collects information $\tilde{y}_h$ in the course of his ordinary business. A shoe seller learns a little about the demand for shoes, for free, from his customers. That is, information is often produced complementary with the production, and distribution of other commodities. Each trader can for free learn something about his own demanders and suppliers. This is the situation where $\tilde{y}_h$ is free to trader h, but $\tilde{y}$ is costly or impossible to acquire. In this case, when $\tilde{P}^o$ is a sufficient statistic, no disincentives to acquire $\tilde{y}_h$ are generated (since it is free), but yet every trader is being given information by $\tilde{P}^o$ which he could not obtain otherwise, except at great cost. This is the sense in which a R.E. equilibrium in a complete market captures Hayek's idea that a fundamental role of competitive prices is the aggregation of information.

One problem with models of complete speculative markets is that the price system is so refined that differences in beliefs disappear, even though they are necessary for trade to take place. Any functioning speculative market must support a large enough daily trading volume so that traders can cover the fixed costs of participating in the market. Trade can occur for many reasons other than the differences in beliefs (e.g., differences in risk preferences, differences in endowments, etc.), however most speculative markets in existence seem to support large trading volumes because of frequent changes in information. Another similar effect can be seen from the model in Section 3, where traders are risk averse (see (9)-(15)), and hedge their stocks of commodities. Each trader faces risk because he has imperfect information about the future spot price. However, we showed that if the economy's information is perfect (i.e., if a trader who knows <u>all</u> other traders' information would face no risk), then there is a R.E. equilibrium futures price with the property that each trader knows all other traders' information and faces no risk. That is, trade in the futures market takes place because of differences in information, but when traders extract the information of other traders from the futures price no differences in information can persist. This leads us to believe that if information is costly then markets will not be complete. That is, the nonexistence of the competitive equilibrium which we mentioned earlier will express itself by particular markets closing down due to a lack of trade.

When markets are incomplete, the R.E. price will not in general be a sufficient statistic. For example, suppose that there are 1500 states of nature so that $\pi(\underline{y})$ is a 1500-vector. Instead of assuming that every commodity can be brought forward in every state of nature, assume that say only 5 forward markets (i.e., contingent claims markets) exist. Then the vector of current contingent claims prices $P(\underline{y})$ under R.E. will not be invertible in $\pi(\underline{y})$. When there are fewer prices than states of nature, the R.E. price becomes a noisy signal of the information that a given trader wants to know. For example, consider a totally uninformed trader for whom $\tilde{y}_h \equiv$ constant. This trader may be interested in the price of wheat next period in the event there is a lot of rain. The ordinary (uncontingent) futures price of wheat may be high because many traders believe that there will be very little wheat around next period due to a lack of rainfall (which is what the uninformed trader is interested in) or the ordinary futures price may be high because traders have current information indicating that there will be a shortage of beef, and beef is a substitute for wheat (this is information which is totally irrelevant to the uninformed trader and is called noise). If there is a complete set of contingent markets then the above never happens because all traders equalize their marginal rates of substitution for wheat in all contingencies. Thus in a complete market each trader is interested in all the information possessed by other traders. In an incomplete market some of the information may be noise to an uninformed trader which prevents him from discovering the information relevant to him.[19/]

When the R.E. equilibrium leaves traders with differences in beliefs (i.e., when the price is not a sufficient statistic due to the incompleteness of markets), there is an incentive to open new markets -- "differences in beliefs make for a good horse race." If too many speculative markets are created, then traders will lose their incentives to collect information. There is some equilibrium number of markets where the R.E. price is just noisy enough so that traders can earn a return on information collection, and there is enough trade to cover the fixed costs of operating the market.

6. Conclusions

Rational Expectations (or Perfect Foresight) models began as an attempt to generalize the competitive equilibrium concept to economies subject to uncertainty. In the ordinary Walrasian model of an economy not subject to uncertainty, there is a price P^e, such that if all traders believe the market will clear at P^e, then it will clear at P^e, where P^e is the competitive equilibrium price. Hicks extended this idea to models with lags in production: if producers make input decisions which are optimal for them when they anticipate next period's price to be P^e, then their input decisions will cause a supply next period such that the market does clear at P^e. Muth, Radner, Lucas, and Prescott generalized this to uncertainty; if traders make decisions today anticipating that the market price tomorrow is some function $P^e(\tilde{\varepsilon})$ of the exogenous random disturbances $\tilde{\varepsilon}$, then those decisions will lead the market clearing price to be $P^e(\varepsilon)$ when $\tilde{\varepsilon} = \varepsilon$. This is not a radical departure from the competitive equilibrium under certainty paradigm. The quote in Section 1 from Marshall indicates that he thought of traders forming their expectations about endogenous variables via a model of how these variables are determined by exogenous forces. These models give one the idea that uncertainty doesn't make much difference.

It is macroeconomic phenomena such as unemployment which are the most difficult to reconcile with the "competitive equilibrium under certainty" paradigm. Thus it is in models in this area where we should expect the most radical departure from that paradigm. Lucas [1972] in attempting to model unemployment implicitly made the innocuous-looking remark that if traders could have a model $P^e(\varepsilon)$ which allowed them to forecast future prices, why not assume they have a model of the determination of the current prices as a function of its exogenous determinants say $P^o(\varepsilon)$. Each trader can then learn something about ε by inverting $P^o(\varepsilon)$ after he observes the current price.

This is a radical departure because it allows us to model the role of prices as creating an externality by which a given individual's information gets transmitted to all other traders. That is, one of the determinants of current price is the information which traders possess about future states of nature. An uninformed trader can invert the current price to learn something about the informed traders' information. Thus the R.E. paradigm can be used to model the idea that a fundamental role of competitive markets (especially speculative markets) is to provide a mechanism by which traders can earn a return on information collection, while the information gets transmitted and aggregated along with other traders' information.

Footnotes

1/ It is interesting to note that Marshall did not apply R.E. to his short-run dynamics. In the discussion of his dynamic process (Book V, Chapter III), he seems totally unaware of the fact that if the economic agents knew the process, then the market-clearing price would come about immediately (i.e., Marshallian dynamics would not be observed if agents anticipated Marshallian dynamics). See Arrow [1959].

2/ See Nerlove [1972] for a survey of distributed lag models.

3/ See, e.g., Grossman [1975a,b] or Cyert and DeGroot [1974].

4/ Since this paper is conceptual and expository we will not be careful about measure theoretical statements, e.g., statements like "for all (ε_t, v_t)" mean "for all realizations of $(\tilde{v}_t, \tilde{\varepsilon}_t)$ except possibly for a set of probability zero."

5/ In this example, firms actually need to know only $EP^o(\tilde{\varepsilon}, \tilde{v}) \cdot \tilde{v}$.

6/ See Debreu [1959], Arrow [1964], and Radner [1968].

7/ Muth's Rational EXpectations idea has been generalized to economies which do not have a complete set of markets, but have many commodities and/or periods. See Radner [1972], Brock [1972], Lucas and Prescott [1971]. The only new conceptual issue to arise in these generalizations is what decisions do firms make when the firm has more than one shareholder. In a world of certainty, all shareholders desire the firm to maximize profit. In a world of uncertainty, profit is a random variable. If different shareholders have different risk preferences, **or endowments**, then it is not clear what objective function the firm maximizes. Since this is not a problem caused by expectations formation, but rather by the incompleteness of markets, we will not cover this literature here, but see Grossman and Hart [1979] and Bell Journal Symposium [1974] for a survey of this problem.

Another topic studied by writers on R.E. is the welfare consequences of incomplete markets. We will not survey that work, as the issues raised relate to the incompleteness of markets rather then Rational Expectations; see Hart [1975] and Grossman [1977a].

8/ Akerlof [1970] in the context of adverse selection presented one of the first models in which prices explicitly convey information. In his case, price revealed the quality of the item being sold. Akerlof's model has the complexity of adverse selection in addition to the informational role of prices, and the latter was not emphasized much by him or others writing about his work.

9/ See Shiller [1978] for a survey of Macroeconomic and empirical work on R.E.

10/ A similar statement can be made about the idea of "efficient capital markets" as surveyed in Fama [1970].

11/ A statistic $t(\underline{y})$ is sufficient if the conditional density of $\underline{y}$ given $\tilde{D}$, i.e., $\text{Prob}(\underline{y}|\tilde{D} = D)$ factors as follows $\text{Prob}(\underline{y}|\tilde{D} = D) = H_1(\underline{y})H_2(D, t(\underline{y}))$. If $t(\underline{y})$ is a sufficient statistic, then $\text{Prob}(\tilde{D}|\underline{y}) = \text{Prob}(\tilde{D}|t(\underline{y}))$. See the Appendix in Grossman [1978] for some other results and references on sufficient statistics.

12/ This argument is given in more detail in Grossman [1978].

13/ Note that a R.E. price has two functions: (A) to clear markets, and (B) to transmit information. Kreps [1977] and Green [1977] have produced examples where standard sorts of assumption, like convexity, are satisfied but no R.E. equilibrium exists. See Allen [1979a] and Radner [1979] for some general results on when a R.E. equilibrium which is a sufficient statistic exists.

14/ The reader can verify that $\text{Prob}(\tilde{s} = s|\underline{y}) = \text{Prob}(\tilde{s} = s|\pi(\underline{y}))$. To see this, let $F(s)$ be any function of s and let $\bar{Q}(\underline{y}) = E[F(s)|\underline{y}] = \sum_s F(s)\text{Prob}(s|\underline{y}) \equiv Q(\pi(\underline{y}))$. Hence $E[F(s)|\underline{y}]$ depends on $\underline{y}$ only through $\pi(\underline{y})$. But $E[F(s)|\pi(\underline{y})]$ $E\{E[F(s)|\underline{y}]|\pi(\underline{y})\} = E[Q(\pi(\underline{y}))|\pi(\underline{y})] = Q(\pi(\underline{y})) \equiv E[F(s)|\underline{y}]$.

15/ This is an original method of proof. The theorem is proved in Green [1973] for the case where there are only two types of traders, using a totally different method of proof involving much calculus. Theorem 1 below and some generalizations are proved in Handel [1979] using a different method of proof. His work stimulated the proof in the text. See Kihlstrom and Mirman [1975] and Jordon [1977] for more theorems about when prices will be fully revealing of trader information.

16/ Note that $\sum_s \pi_s(y) \equiv 1$ so that $\pi(\underline{y})$ is really an element of an $n-1$ dimensional space. Similary when there are n commodities, there are only $n-1$ relative (i.e., real) prices. Douglas Gale has pointed out to me that if the size of the numeraire is a function of $\underline{y}$ (i.e., if there are $n-1$ relative prices that depend on $\underline{y}$ <u>and</u> 1 additional price whech depends on $\underline{y}$), then the absolute price of the numeraire can always be chosen so that it is an invertible function of $\underline{y}$, irrespective of whether there is a complete set of markets. This is because the absolute size of the numeraire does not affect demands other than through an information affect. To avoid this absurdity, it is important in R.E. models to set the numeraire to be a constant independent of $\underline{y}$.

17/ The most general theorem presently available is due to Allen [1979b]. Therein it is shown that if there are $n-1$ relative prices and the lowest dimensional sufficient statistic has dimension $n-2$, then for almost all economies there will exist a R.E. equilibrium which is a sufficient statistic. The above theorem does <u>not</u> assume that there is a complete set of markets. See also Allen [1979a].

18/ See Grossman [1975c] and Grossman and Stiglitz [1980] for a further analysis of this problem.

19/ For a further discussion of "noise" see Green [1973], Grossman [1975c,1977b], and Grossman and Stiglitz [1976]. In those papers noise refers to randomness other than information which makes the current price vary. For example, the futures price of wheat may be high because some well-informed traders have information $\tilde{y}_I$ that there will be a small crop next period. However the current futures price may be high because of a temporary rise in current demand for wheat; denote this demand shift factor by $\tilde{x}$. So the futures price is a function $P_f(\tilde{y}_I,\tilde{x})$. Uninformed **firms** which have to decide how much wheat to store are interested in $\tilde{y}_I$ but not in $\tilde{x}$. The randomness in $\tilde{x}$ prevents them from attributing variations in P_f to variations in $\tilde{y}_I$. In uninformed firms do not know y_I <u>and</u> x then they will not be able to extract y_I from P_f. This may seem different from the model in the text but is actually the same. Just define $\tilde{\underline{y}} = (\tilde{y}_I,\tilde{x})$.

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