CORPORATE FINANCIAL STRUCTURE AND MANAGERIAL INCENTIVES

by

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1. Introduction*

In this paper we study the incentive effects of the threat of bankruptcy on the quality of management in a widely-held corporation. In so doing we develop an equilibrium concept which may be useful in studying a wide class of other problems. The equilibrium concept is relevant whenever there is moral hazard in principal-agent relationships. We model the idea that the agent can engage in precommitment or bonding behavior which will indicate to the principal that he will act in the principal's interest.

The starting point of our analysis is the idea that in a corporation which is owned by many small shareholders there is an "incentive problem," i.e., the managers (or directors) have goals of their own, such as the enjoyment of perquisites or the maximization of their own income, which are at variance with the goals of shareholders, which we assume to be profit or market value maximization. There are a number of ways to overcome this incentive problem. First, managers can be given salary incentives schemes (e.g., profit sharing arrangments or stock options) to get their interests to move towards those of shareholders. Secondly, shareholders can write a corporate charter which permits and to some extent encourages takeover bids. As a result, if the corporation is badly run, a raider can make a profit by buying the company at a low price, reorganizing it, and reselling it at a high price. The threat of such a takeover bid will in general lead current management to achieve higher profits.[1]

Although both salary incentive schemes and takeover bids can reduce the seriousness of the "incentive problem," they will not in general eliminate it completely. In this paper we study a third factor which may be important in encouraging managers to pursue the profit motive -- the possibility of bankruptcy. If managers do not seek high profits, then the probability that the corporation will go bankrupt increases. If the benefits which managers receive from the firm are lost in the event of bankruptcy, managers may prefer to maximize profits or come close to it rather than to risk sacrificing their perquisites.

Clearly the efficacy of bankruptcy as a source of discipline for management will depend on the firm's financial structure -- in particular, its debt-equity ratio. As an illustration of this, ignore takeover bids and consider the old argument that, in a perfectly competitive world with free entry, firms must profit maximize if they are not to go out of business. Suppose that the profit maximizing production plan involves an investment outlay of $100 which gives a return stream whose present value is $100 (so that net present value is zero -- this is the free entry condition). Then if a firm raises the $100 investment cost by issuing equity, the firm is under no pressure to maximize profit at all. In particular, once having raised the $100, there is nothing to stop the firm's managers from cancelling the investment project and spending the money on themselves! All that will happen is that the value of the equity will fall to zero, i.e., those who purchased the firm's shares will lose $100. (Of course, if this is anticipated by shareholders, they will not purchase the firm's shares and management will not be able to raise the $100.) Note, however, how the story changes if the $100 investment outlay is raised by debt. Then any choice other than the profit maximizing plan will lead to bankruptcy with certainty. Thus whether or not competition leads to profit maximization depends crucially on the firm's financial structure.[2]

It is clear from the above arguments that it will generally be in the shareholders' interest for a firm to issue debt as well as equity since this raises profit. In a mature corporation, however, the power to determine the firm's financial structure usually rests in the hands of management, not the shareholders. Furthermore, since debt increases the probability of bankruptcy, it would seem never to be in management's interest for there to be debt. In this paper we will develop a model to explain the existence of debt even under the assumption that management controls the firm's financial structure.[3]

The reason that debt may be beneficial to management is the following. If management issues no debt, then it is safe from bankruptcy. Among other things this means that it is in a relatively unconstrained position and therefore has less reason to profit maximize. As a result, the market will put a low valuation on the firm and its cost of capital will be high. Conversely, if management issues debt, then shareholders know that it is personally costly to management not to profit maximize (because managers lose the perquisites of their position when the firm goes bankrupt). Hence, in this case, the market will recognize that profits will be higher and so the firm will have a high market value. Thus, there is a positive relationship between a firm's market value -- i.e., the value of debt plus equity -- and its level of debt. Therefore, to the extent that management would like its firm to have a high market value, it may wish to issue debt.

There are at least three reasons why managers may want to increase the firm's market value, v. (1) Their salaries may depend on v (through a salary incentive scheme). (2) The probability of a takeover bid will be smaller the higher is the price a raider has to pay for the firm, and this price will in general be positively related to v. (3) If managers are issuing new debt and equity, then the higher v is, the more capital the managers will be able to raise. Furthermore, the more capital the managers can raise, the more they can in general increase their perquisites. In Sections 2-4, we analyze a model where the third reason is central to the managers' desire to increase v (this model can be easily modified to analyze the first reason too, however). Section 5 shows that the second reason can also be incorporated into our model.

An alternative interpretation can also be given to the model of Sections

2-4. It applies to the case of an entrepreneur who wishes to set up a firm but does not have sufficient funds to finance it. Other things equal, the entrepreneur would like to finance the firm by issuing equity since he does not then risk bankruptcy. However, in order to achieve a high market value for his project, he may have to issue some debt in order to convince the market that he will pursue profits rather than perquisites. The trade-offs faced by the entrepreneur are the same as those faced by corporate management in the model of this paper.

We think of the issuing of debt as being an example of precommitment or bonding behavior. Consider the shareholders and management as in a principal-agent relationship where shareholders are the principal and management is the agent. Then, by issuing debt, management (the agent) deliberately changes its incentives in such a way as to bring them in line with those of shareholders (the principal) -- because of the resulting effect on market value. In other words, it is as if, by issuing debt, the management bonds itself to act in the shareholders' interest.[4] This relates closely to another example of a bonding equilibrium. Consider the use of bail bonds in criminal proceedings. The accused person offers to put up a sufficiently large bail bond, so that the judge knows that it would be extremely costly (and thus unlikely) for the accused not to return to the court for his trial.

The above example will help to distinguish a bonding equilibrium from a signaling equilibrium. The bail bond is useful for the judge even if he knows the characteristics of the accused person. This is to be distinguished from a signaling or screening situation where the judge does not know the accused's characteristics and bail is a screening device. In particular, suppose that there are two types of accused persons, with one type having an exogenously higher probability of jumping bail (i.e., leaving the country) than the other type. Further suppose that the type with the high probability of jumping bail is less wealthy (or has higher borrowing costs) than the other type. In this case the size of the bail bond is a screening device, and the amount of bail the accused offers to "put up" is a signal about what type of person he is. It will be clear from the formal analysis which follows that a bonding equilibrium is not related to a signaling equilibrium: the former involves agents communicating their endogenous intentions, while the latter involves agents communicating their exogenous characteristics.[5]

The paper is organized as follows. In Sections 2-4, we present the basic model and develop the notion of precommitment or bonding for the case where management wishes to increase v in order to have more funds at its disposal. In Section 5, we generalize the analysis to the case where increases in v are desired in order to reduce the probability of a takeover bid. Finally, in Section 6 we discuss how our explanation of the role of debt differs from others that have been proposed in the literature.

2. The Model

Consider a corporation whose manager (or management) has discovered a new investment opportunity. We assume that the firm must raise all of the funds required for the investment by borrowing or issuing shares in the enterprise.[6] For simplicity, ignore all the old investments of the corporation. Let the investment opportunity be described by the stochastic production function

$$(1) \qquad q = g(I) + s \quad ,$$

where I is the level of investment undertaken by the firm, $g(I)$ is the expected profit from this investment, and s is a random variable with mean zero. We will assume that investors in the firm -- be they creditors or shareholders -- are risk neutral and that the expected return per dollar which can be obtained on investments elsewhere in the economy is given by $R > 0$ (R should be interpreted as one plus the interest rate).[7]

The model is a two-period one. The level of investment I is chosen "today," and profit q is realized "tomorrow." If traders know the investment level I, and they are risk neutral, then the market value of the firm, i.e., the value of its debt and equity, is given by

$$(2) \qquad V = \frac{g(I)}{R} \quad .$$

Note that this is true whatever debt-equity ratio is chosen by the manager. This suggests that the firm's market value will be independent of its debt-equity ratio, i.e., that the Modigliani-Miller theorem will apply. However, this is only the case if the manager's choice of I is independent of the debt-equity ratio. In the model presented below, I will depend crucially on the level of debt, and so V will not be independent of the debt-equity ratio.

Consider first the case where the manager's investment decision is observable to the market. By (2), if the manager chooses the level of investment I, then the net market value of the firm is

$$V - I = \frac{g(I)}{R} - I \quad . \tag{3}$$

If the manager's salary and perquisites are an increasing function of $V - I$, then I will be chosen to maximize the expression in (3). This is of course the classical solution -- the firm's financial decision is irrelevant and the production plan is chosen to maximize net market value.

The assumption that the market observes I is a strong one. In general, monitoring the firm's activities is a costly operation. If a firm's shares and bonds are widely dispersed, it will not be in the interest of individual bondholders and shareholders to incur these costs. In this paper, we investigate the consequences of dropping the assumption that I is observed. We will assume that the market has no direct information about I. However, the market will be assumed to be able to make inferences about I from actions taken by the manager -- in particular his choice of the debt-equity ratio. These inferences are possible because bankruptcy is assumed to be costly for the manager -- this is another difference between the present model and the classical one.

Suppose that the firm raises outside funds equal to V dollars. Let this amount be divided between debt and equity in such a way that the total amount owing to the firm's creditors when "tomorrow" arrives is D dollars. We assume that the firm will go bankrupt if and only if the firm's profits are less than D, i.e., if and only if

$$g(I) + s < D \quad . \tag{4}$$

Note that bankruptcy can occur even if $D = 0$ if s is large and negative (we allow profit to be negative).

Let the manager have a concave von Neumann-Morgenstern utility function $U(C)$ defined over nonnegative consumption C. We assume that, if V dollars are raised and I dollars are invested, the manager will have $(V - I)$ left for consumption and so his utility is $U(V - I)$.[8] It will be assumed, however, that this utility is only realized if the firm does not go bankrupt. If the firm does go bankrupt, we suppose that a receiver is able to recover the $(V - I)$ and so the manager's utility is $U(0)$. Since U is defined only up to positive linear transformation, we will assume without loss of generality that $U(0) = 0$.

In the remainder of this section, we analyze the optimal choice of I for the manager, for given values of V and D. We assume that the manager chooses I before s is known. Thus, given V,D, he maximizes expected utility, i.e.,

$$U(V - I)\text{Prob}[\text{No bankruptcy}] = U(V - I)\text{Prob}[s \geq D - g(I)] \quad . \tag{5}$$

We will assume that s is a random variable with distribution function F and density function f. Then we can rewrite the manager's maximization problem as:

$$\max_{I} U(V - I)(1 - F(D - g(I))) \quad \text{subject to} \quad 0 \leq I \leq V \quad . \tag{6}$$

Since $(V - I)$ is decreasing in I and $(1 - F(D - g(I)))$ is increasing in I, the trade-off for the manager, given V and D, is between choosing investment levels which give high consumption but a high probability of bankruptcy, and those which give low consumption and a low probability of bankruptcy.

In order to proceed, we will make some further assumptions about U, F, and g. We will assume that (1) U is thrice differentiable with $U'(0) = \infty$, $U' > 0$, $U'' \leq 0$; (2) F is thrice differentiable with $f(x) > 0$ for all x; (3) g is thrice differentiable with $g(0) = 0$, $g' > 0$, $g'' < 0$ and $g'(0) = \infty$, $\lim_{x\to\infty} g'(x) = 0$. Let

(7) $$h(I,V,D) = U(V - I)(1 - F(D - g(I))) \quad .$$

Then

(8) $$h_I \equiv \frac{\partial h}{\partial I} = -U'(V - I)(1 - F(D - g(I))) + U(V - I)f(D - g(I))g'(I) \quad .$$

Therefore, for I to be a solution to (6), it is necessary that

$$\begin{cases} h_I \leq 0 & \text{if } I = 0 \quad \text{(9a)} \\ h_I = 0 & \text{if } 0 < I < V \quad \text{(9b)} \\ h_I \geq 0 & \text{if } I = V \quad \text{(9c)} \end{cases}$$

However, since $U'(0) = \infty$ and $F(x) < 1$ for all finite x (this follows from the fact that $f(x) > 0$ for all x), $h_I < 0$ if $I = V > 0$. On the other hand, if $I = 0$ and $V > 0$, $h_I > 0$ since $g'(0) = \infty$. Therefore, if $V > 0$, (9a) and (9c) are impossible and

(10) $$h_I = 0$$

must hold at the optimal I, i.e.,

(11) $$\frac{U'(V - I)}{U(V - I)} = \frac{f(D - g(I))}{1 - F(D - g(I))} \cdot g'(I) \quad .$$

On the other hand, if $V = 0$, then the constraint $0 \leq I \leq V$ implies that $I = 0$ is optimal for the manager.

Let us concentrate on the case $V > 0$. Equation (11) is then a necessary condition for an optimuum, but not in general a sufficient one. One case where sufficiency is guaranteed is if $f(x)/(1 - F(x))$ is everywhere an increasing function of x.[9] Let

$$r(x) = \frac{f(x)}{1 - F(x)} \quad .$$

Lemma 1. If $r(x)$ is an increasing function of x, then (11) is a sufficient as well as a necessary condtion for I to be an optimal choice for the manager, given V,D.

Proof. The left-hand side (LHS) of (11) is strictly increasing in I, while, if $r(x)$ is increasing in x, the right-hand side (RHS) of (11) is decreasing in I. Therefore (11) can hold for at most one value of I. Since (11) is a necessary condition for optimality and since an optimum certainly exists by Weierstrass' theorem, this proves that (11) holds if and only if we are at the optimum. Q.E.D.

For most of what follows we will confine our attention to distribution functions $F(\cdot)$ for which $r(x)$ is strictly increasing in x. In Reliability Theory, $r(x)$ is called the "hazard rate." Our assumption of an increasing hazard rate is satisfied for many well-known distributions such as the Exponential, the Gamma and Weibull with degrees of freedom parameter larger than 1, the Normal Distribution, LaPlace, and the Uniform (Barlow and Proschan 1975, p. 79). In our context the hazard rate has the following interpretation. If net liabilities $D - g(I) = y$ then bankruptcy occurs when $s \leq y$. Suppose the manager decreases $g(I)$ by t. Then this increases net liabilities to $y + t$. In the event that $s \leq y$, it doesn't matter that net liabilities are increased to $y + t$ since bankruptcy would have occurred in any case. Therefore the conditional probability of the increase in bankruptcy in going from y to $y + t$, given no bankruptcy at y, is $(F(y + t) - F(y))/(1 - F(y))$. Thus $r(y)$ is the $\lim_{t\to 0} (1/t)(F(y+t) - F(y))/(1 - F(y))$ which is the increase per dollar of net liabilities in the conditional probability of bankruptcy given $s \geq y$. So $r(y)$ is like the marginal cost in the probability of bankruptcy. Thus $r'(y) > 0$ is like an increasing marginal cost condition.

Assumption 1. $r'(x) > 0$ for all x.

We have seen that, if Assumption 1 is satisfied, (11) has a unique solution and so there is a unique optimal choice of I for the entrepreneur. We write the optimal I as $I(V,D)$ (if $V = 0$, $I(V,D) = 0$). It is easy to show by standard arguments that $I(V,D)$ is a continuous function of (V,D). In fact, more can be established.

Lemma 2. If $V > 0$, $I_V \equiv \partial I/\partial V$ and $I_D \equiv \partial I/\partial D$ exist. Furthermore, $0 < I_V < 1$ and $0 \leq I_D < [g'(I(V,D))]^{-1}$.

Proof. Differentiating (8), we obtain

$$I_V = -\frac{h_{IV}}{h_{II}} \quad \text{and} \quad I_D = -\frac{h_{ID}}{h_{II}} \quad \text{if} \quad h_{II} \neq 0 \quad . \tag{12}$$

Now

$$h_{II} = U''(1 - F) - U'fg' - U'fg' + Ufg'' - Uf'g'^2 \tag{13}$$

$$= U''(1 - F) - U'fg' + Ufg'' - (U'fg' + Uf'g'^2)$$

$$= U''(1 - F) - U'fg' + Ufg'' - Ug'^2(f^2/(1 - F) + f') \tag{14}$$

where we are using (11). Therefore

$$h_{II} = U''(1 - F) - U'fg' + Ufg'' - Ug'^2(1 - F)r' < 0 \tag{15}$$

by Assumption 1. On the other hand,

$$h_{IV} = -U''(1 - F) + U'fg' . \tag{16}$$

Putting (12), (15), and (16) together, we get $0 < I_V < 1$. Also

$$h_{ID} = U'f + Uf'g' = Ug'\left(\frac{f^2}{1 - F} + f'\right) = Ug'(1 - F)r' \tag{17}$$

where we use (11) again. Putting (12), (15), and (17) together, we obtain

$0 \leq I_D < [g'(I(V,D))]^{-1}$. Q.E.D.

We see from Lemma 2 that I is increasing in both V and D, i.e., if the manager obtains more funds he will use some of them (but not all) on additional investment, while if the volume of funds stays the same but a greater fraction is in the form of debt, this will also lead him to invest more.[10] Further, $0 \leq I_D < (g')^{-1}$ means that the level of debt is chosen such that an increase of a dollar in the amount to be paid back next period calls forth less investment than is actually required to increase the expected value of output by a dollar.

3. The Determination of Equilibrium

In the last section, we showed how the manager's choice of I depends on V and D. In this section we study how V depends on I. We also analyze the optimal choice of D for the manager.

If I was observable to the market, then as we have noted before V would be given simply by

$$(18) \qquad V = \frac{g(I)}{R} .$$

However, I is not observable. Therefore (18) must be replaced by

$$(19) \qquad V = \frac{g(I^P)}{R}$$

where I^P is the market's perceived value of I. We will assume that the market's perceptions are "rational" in the sense that investors in the market know the utility function U and the distribution function F and so can calculate how much the manager will invest for each value of V and D, i.e., we assume that the market knows the function $I(V,D)$. Hence we may rewrite (19) as

$$(20) \qquad V = \frac{g(I(V,D))}{R} .$$

We see then that, given D, the equilibrium V and I are solutions of equation (20). For each value of V, there is an optimal choice of I for the manager, $I(V,D)$. However, for an arbitrary V, say V_a, $I(V_a,D)$ will not satisfy (20). If $V_a > g(I(V_a,D))/R$ the firm will be overvalued, while if $V_a <$ $g(I(V_a,D))/R$ it will be undervalued. Only if (20) holds will investors be prepared to pay exactly V_a for the firm, and so only in this case will the system be in equilibrium.

<u>Definition</u>. Given D, a bonding or precommitment equilibrium is a nonnega-

tive pair $(I(D),V(D))$ such that

(A) $U(V(D) - I(D))(1 - F(D - g(I(D)))) \geq U(V(D) - I)(1 - F(D - g(I)))$

for all $V \geq I \geq 0$;

(B) $V(D) = g(I(D))/R$.

It is useful to rewrite (20) as

$$g^{-1}(RV) = I(V,D) \quad . \tag{21}$$

The LHS of (21) is a strictly increasing, convex function of V with slope zero at $V = 0$. The RHS of (21), on the other hand, is, by Lemma 2, a strictly increasing function of V with slope between zero and one, satisfying $0 \leq I(V,D) \leq V$. The two functions are drawn in Figure 1. It is clear from Figure 1 that $V = 0$ is a solution to (21) and that there is always at least one other solution $V > 0$, since $g'(0) = \infty$, and $I_V > 0$. Note that the $V = 0$ equilibrium arises because if the market thinks the firm is worthless, then it will be worthless because no capital can be raised.

FIG. 1 →

Proposition 1. For each D, $V = 0$ is a solution of (21) and there is at least one other solution $V > 0$.

It is worthwhile to consider at this point the role of bankruptcy in the model. In the absence of bankruptcy, the manager's utility would be just $U(V - I)$. Given V, this is maximized by setting $I = 0$. But since, for each V, the manager's optimal response is to set $I = 0$, this means that the only solution of (21) is $I = V = 0$, and thus no capital can be raised. With bankruptcy, however, other equilibria also exist. The reason is that since bankruptcy is undesirable the manager, by taking on debt, can convince the market that he will invest in order precisely to avoid this undesirable outcome. Thus, although bankruptcy is unpleasant for the manager, it permits him to acheive equilibrium situations which make him better off than the no bankruptcy situation $I = V = 0$.

We will assume that the manager selects D and, if there are multiple equilibria resulting, can choose among them. Given D, if there are just two equilibria $V = 0$ and $V > 0$, then it is clear that the manager is better off at the $V > 0$ equilibrium. This is because $I < V$ when $V > 0$ and so $U(V - I)(1 - F(D - g(I))) > 0$. However, if there are two or more equilibria with $V > 0$, then it is not clear which of them is optimal for the manager. If $V_A > V_B > 0$ are two equilibria for some D, then $I_A = g^{-1}(RV_A) > g^{-1}(RV_B) = I_B$ and so the risk of bankruptcy $F(D-g(I))$ is lower at I_A than at I_B; however, $V_A - I_A$ may be less than $V_B - I_B$, in which case consumption benefits are higher at I_B.

For each D, let $V(D)$ denote an optimal equilibrium for the manager, i.e., $V(D)$ maximizes $U(V - I(V,D))(1 - F(D - g(I(V,D))))$ over all V satisfying

$$(22) \qquad V = \frac{g(I(V,D))}{R} .$$

Then, it is optimal for the manager to choose the level of debt D so as to maximize

$$(23) \qquad U(V(D) - I(V(D),D))(1 - F(D - g(I(V(D),D)))) .$$

An analysis of the maximization of (23) is somewhat complicated. First, $V(D)$ may not be uniquely defined for some D, i.e., there may be multiple optima given D. Secondly, even if $V(D)$ is uniquely defined, it may not be continuous in D. Suppose that A is the best equilibrium in Figure 2 for the manager at a particular level of D. Then it is clear that a small change in D can result in the equilibrium A disappearing, with a resulting discontinuous shift in the manager's optimal equilibrium.

FIG. 2 →

Of course, if V is discontinuous in D, we can certainly not apply a calculus argument (at least directly) to determine the optimal D.

It turns out, however, that things become much easier if we regard I rather than D as the manager's choice variable. Let $\hat{I}$ satisfy $g(\hat{I})/R = \hat{I}$. For each $0 < I < \hat{I}$, consider the following equation:

$$(24) \qquad \frac{U'\left(\frac{g(I)}{R} - I\right)}{U\left(\frac{g(I)}{R} - I\right)} \cdot \frac{1}{g'(I)} = r(D - g(I)) \quad .$$

(24) is simply the first-order condition for the manager's problem, equation (11), with V set equal to $g(I)/R$. Let us try to solve (24) for D. By Assumption 1, the RHS is strictly increasing in D. Therefore, if there is a solution at all to (24), it is unique. However, there may be no solution to (24) -- this will be the case if $U'\left(\frac{g(I)}{R} - I\right) \Big/ U\left(\frac{g(I)}{R} - I\right) g'(I) \geq \lim_{x \to \infty} r(x)$.

If there is a solution to (24), denote it by D(I) (we allow D(I) to be negative -- see below). What is the interpretation of D(I)? It is simply the level of debt which sustains the equilibrium (V,I), where $V = g(I)/R$, i.e.,

$$I\left(\frac{g(I)}{R}, D(I)\right) = I \ .$$

The reason is that if we set $V = g(I)/R$, we see from (24) that (11) is satisfied when $D = D(I)$. But by Lemma 1, this means that $I = I(V,D)$, i.e., I is an optimal choice for the manager given V and D. (If $D < 0$, then the equilibrium (V,I) is sustained by the manager lending rather than borrowing.)

Suppose that D(I) is well-defined at $I = I^{o} > 0$, i.e., (24) can be solved at $I = I^{o}$. Then since r(x) is strictly increasing, it is clear that (24) can also be solved in a neighborhood of I^{o}. Furthermore, since the LHS and RHS of (24) are continuous functions, D(I) must be continuous at I^{o}. We will now show that D(I) is also differentiable at I^{o}.

Lemma 3. If $D(I)$ is defined at $I = I^0 > 0$, then it is also defined in a neighborhood of I^0. Furthermore, it is continuous and differentiable at $I = I^0$.

Proof. Only differentiability remains to be established. However, this follows from differentiating (24) with respect to I:

$$[Ug'U''\left(\frac{g'}{R} - 1\right) - U'(Ug'' + U'\left(\frac{g'}{R} - 1\right)g')] \div (Ug')^2$$
$$= r'(x)(D' - g') \quad .$$

Therefore

$$(25) \qquad D' = g' + \left[\frac{g'UU''\left(\frac{g'}{R} - 1\right) - U'Ug'' - U'^2g'\left(\frac{g'}{R} - 1\right)}{(Ug')^2 \; r'(x)}\right] ,$$

which is finite by Assumption 1. Q.E.D.

Note that at $I = 0$, $D(I)$ is completely arbitrary, i.e., any value of D sustains the $I = 0$, $V = 0$ equilibrium.

In order to say something about the sign and magnitude of $D'(I)$, consider (25). Let I^* satisfy

$$(26) \qquad \frac{g'(I^*)}{R} = 1 ,$$

i.e., I^* maximizes $\frac{1}{R}g(I) - I$. Then $g'(I)/R \gtrless 1$ as $I \lessgtr I^*$. Therefore if $I \geq I^*$, the bracketed term in (25) is positive, from which it follows that $D' > g' > 0$. If $I < I^*$, however, the bracketed term in (25) may be positive or negative and so, in particular, we may have $D' < 0$. We have established

Lemma 4. Suppose that $D(I)$ is defined at $I > 0$. If $I \geq I^*$, $D' > g' > 0$. If $I < I^*$, $D' \gtrless g'$ according to whether $\frac{d}{dI}\left\{\left(\frac{U'\left(\frac{g(I)}{R} - 1\right)}{U\left(\frac{g(I)}{R} - 1\right)}\right) \cdot \frac{1}{g'(I)}\right\} \gtrless 0.$

It should be noted that there are some utility functions and production functions for which

$$\text{(27)} \qquad \frac{d}{dI}\left\{\left(\frac{U'\left(\frac{g(I)}{R} - I\right)}{U\left(\frac{g(I)}{R} - I\right)}\right)\cdot \frac{1}{g'(I)}\right\} > 0$$

for all I. An example is $U = c^b$, $0 < b < 1$, and $g(I) = I^{\frac{1}{2}}$, in which case the LHS of (27) is

$$\frac{d}{dI}\left(\frac{2b}{\frac{1}{R} - I^{\frac{1}{2}}}\right) > 0 \quad .$$

In such cases, Lemma 4 tells us that $D'(I) > 0$ everywhere that $D(I)$ is defined. Thus the graph of D against I is as in Figure 3. Since $V = g(I)/R$, D will also be an increasing function of V in such cases.

Fig. 3 →

However, if (27) does not hold everywhere, then the graph of D against I is as in Figure 4. Again the graph of D against V will have a similar shape.

Fig. 4 →

If we invert $D(I)$ to get I as a function at D, we can see again why problems arise when D is regarded as the manager's independent variable. There is no difficulty, of course, in the case of Figure 3, but, in the case of Figure 4, I is multivalued for some values of D, and can also be discontinuous.

Lemma 4 gives us information not only about D' but also about $D' - g'$. If $D' > g'$ (resp. $D' < g'$), this tells us that an increase in I leads to an increase (resp. decrease) in $D(I) - g(I)$ and hence to a higher (resp. lower) chance of bankruptcy. Note also that if we think of D as a function of V rather than I, then

$$\frac{dD}{dV} = \frac{dD}{dI}\frac{dI}{d\left(\frac{g(I)}{R}\right)} = \frac{RD'}{g'} ,$$

and so $D' > g'$ implies that $dD/dV > R$, i.e., an increase in debt repayment of one dollar leads the firm's market value to increase by less than $1/R$ dollars.

Having analyzed the relationship between D and I, let us return to the manager's choice of the optimal debt level D. Regarding I as the independent variable, we may write the manager's problem as follows:

$$\text{(28)} \qquad \underset{I}{\text{Max}}\ U\left(\frac{g(I)}{R} - I\right)(1 - F(D(I) - g(I))) ,$$

where I is restricted to those values for which $D(I)$ is defined.

Lemma 5. The problem in (28) has at least one solution.

Proof. Let $A = \{I \mid D(I) \text{ is defined}\}$. Let $\sup_{I \in A} U\left(\frac{g(I)}{R} - 1\right)(1 - F(D(I) - g(I))) = \alpha > 0$. Then we can find a sequence $I_r \in A$ such that

$$\text{(29a)} \qquad U\left(\frac{g(I_r)}{R} - I_r\right)[1 - F(D(I_r) - g(I_r))] \to \alpha$$

as $r \to \infty$. Clearly $0 \leq I_r \leq \hat{I}$, where $g(\hat{I})/R = \hat{I}$. Therefore we may assume without loss of generality that $I_r \to I'$. Furthermore, $0 < I' < \hat{I}$ since otherwise the LHS of (29a) $\to 0$. Clearly I' is a solution to (28) as long as $I' \in A$. But $I' \notin A$ implies that (24) cannot be satisfied, which in turn implies that

$$\text{(29b)} \qquad \frac{U'\left(\frac{g(I')}{R} - I'\right)}{U\left(\frac{g(I')}{R} - I'\right)} \cdot \frac{1}{g'(I')} \geq \lim_{x\to\infty} \frac{f(x)}{1 - F(x)} .$$

In fact, we can say more than this. Since $I_r \in A$ for all r and $I_r \to I'$, (29b) must hold with equality and $(D(I_r) - g(I_r)) \to \infty$ as $r \to \infty$. But this is impossible since then the LHS of (29a) tends to zero. Q.E.D.

Let I be a solution to the manager's overall maximization problem. It is clear from the proof of Lemma 5 that $0 < I < \hat{I}$. Furthermore, we know that $D(I)$ is defined at I, and hence, by Lemma 3, that it is differentiable at I. Therefore, we may differentiate the expression in (28) to get the following necessary condition for optimality:

$$(30) \qquad U'\left(\frac{g(I)}{R} - I\right)\left(\frac{g'(I)}{R} - 1\right)(1 - F(D(I) - g(I)))$$
$$- U\left(\frac{g(I)}{R} - I\right)f(D(I) - g(I))(D'(I) - g'(I)) = 0 \quad .$$

Rewriting (30), we obtain

$$(31) \qquad \frac{U'\left(\frac{g(I)}{R} - I\right)}{U\left(\frac{g(I)}{R} - I\right)} \cdot \frac{1}{g'(I)} = r(D(I) - g(I)) \cdot \frac{(D'(I) - g'(I))}{\left(\frac{g'(I)}{R} - 1\right)g'(I)} \quad .$$

But we know that (24) must hold at $D = D(I)$. Therefore, putting (24) and (31) together, we get

$$(32) \qquad \frac{D' - g'}{\left(\frac{g'}{R} - 1\right)g'} = 1 ,$$

that is,

$$(33) \qquad D'(I) = \frac{(g'(I))^2}{R} \quad .$$

<u>Proposition 2</u>. A necessary condition for I to be an optimal level of investment for the manager is that (33) holds.

It follows immediately from Proposition 2 that the optimal level of $I < I^*$

where

(34) $$\frac{g'(I^*)}{R} = 1 \ .$$

For $I \geq I^* \Rightarrow \frac{g'(I)}{R} \leq 1 \Rightarrow D' = \frac{g'^2}{R} \leq g'$, which contradicts Lemma 4.

Proposition 3. If I is an optimal level of investment for the manager, then $I < I^*$, where I^* maximizes $\frac{g(I)}{R} - I$. In other words, there is under-investment relative to the "classical" situation where I is observable.

If fact, Proposition 3 is obvious even without appealing to (33). For if $g'/R \leq 1$, then by reducing I the manager increases $U\left(\frac{g(I)}{R} - I\right)$. However, by Lemma 4, $(D - g(I))$ goes down and so the probability of bankruptcy is reduced. Hence the manager is made unambiguously better off.

FIG. 5 → The relationship between $D'(I)$ and $\frac{(g'(I))^2}{R}$ is graphed in Figure 5. In general, there may be more than one I satisfying (33). Thus (33) is not a sufficient condition for optimality.

At this stage, the role of Assumption 1 may become clearer. It enables us to solve for D uniquely in terms of I and hence, by regarding I rather than D as the manager's independent variable, to obtain (33) as a necessary condition for optimality. If Assumption 1 does not hold, then there may be several D's corresponding to a particular I. Furthermore, even if there is a unique optimal choice of D for each I for the manager, the function $D(I)$ may not be continuous. Thus the same problems arise when we regard I as the independent variable as do when we regard D as the independent variable. It would be interesting to know whether some other argument can be used to derive (33) as a necessary condition if Assumption 1 fails.

4. Comparative Statics Results

In the last section, we showed how the optimal (I,V,D) combination is chosen by the manager. We now consider how the manager's choice varies with changes in F and U.

Comparative statics results are particularly difficult in this problem because there are many opposing effects. From (8) it is clear that for a given D, the relevant marginal threat of bankruptcy rises when the hazard rate r rises, and I will increase in response to a rise in r. This can be interpreted as an attempt by the manager to increase investment to lower the threat of bankruptcy. Unfortunately, the situation is far more complex because D will not stay constant. Recall that $D(I)$ is the level of debt necessary to convince the market that I will be the chosen investment level. For a given I, when the threat of bankruptcy changes, the size of debt necessary to convince traders that the manager will invest I changes. From (24) it can be seen that for a given I, an increase in the hazard rate will cause the manager to reduce D at I. This has a further effect on I which may go in the opposite direction to the initial effect.

To analyze the above, let t be a real-valued parameter which represents some aspect of F or U. We want to know how I changes and how $D(\cdot)$ changes as t changes. In what follows we will assume that at the initial value of t there is a unique optimal I for the manager, denoted by $I(t)$. We will sometimes write $D_t(I)$ to emphasize that the function $D(I)$ depends on t. From the analysis of the previous section we know that

$$D_t' = \frac{g'^2}{R}$$

at $I(t)$. In fact, we know more: from the second-order conditions we have

Lemma 6. $D_t'' - \frac{2g'g''}{R} \geq 0$ at $I = I(t)$.[11]

Proof. Suppose that $D_t'' - \frac{2g'g''}{R} < 0$ at $I(t)$. Then for $I > I(t)$ and I close to $I(t)$, $D_t' < \frac{g'^2}{R}$. Therefore, by (24),

$$\frac{U'}{Ug'} = r_t > r_t \cdot \frac{D_t' - g'}{\left(\frac{g'}{R} - 1\right)g'} ,$$

where r_t' is the derivative of the hazard rate for the distribution indexed by t. Hence,

$$U' \cdot \left(\frac{g'}{R} - 1\right)(1 - F) - U \cdot f \cdot (D_t' - g') > 0 .$$

But, from the second-order conditions for the problem in (28), we know that the LHS of (30) cannot be positive for all $I > I(t)$, I close to $I(t)$. Contradiction. Q.E.D.

Definition. We will say that $I(t)$ is a regular maximum if $D_t'' - \frac{2g'g''}{R} > 0$ at $I = I(t)$.

From now on we concentrate on regular maxima. The following proposition is obvious in view of Lemma 6.

Proposition 4. Suppose that at $t = t_o$ $I(t_o)$ is a unique optimal investment level for the manager. Suppose also that $I(t)$ is a regular maximum. Then a sufficient condition for $I(t)$ to be increasing (resp. decreasing) in t in the neighborhood of t_o is that $\frac{\partial}{\partial t}(D_t' - \frac{g'^2}{R}) < 0$ (resp. > 0) at $I = I(t)$.

Of course, if I is increasing in t, then so is $V = \frac{g(I)}{R}$.

Let us use Proposition 4 to calculate how I varies as the distribution F changes. We know from (25) that

$$(35) \qquad D_t' = g' + \left[\frac{g'UU''\left(\frac{g'}{R} - 1\right) - U'Ug'' - U'^2 g'\left(\frac{g'}{R} - 1\right)}{U^2 g'^2 r_t'} \right] .$$

Furthermore, at the optimuum, since $I(t) < I^*$,

$$D_t' = \frac{g'^2}{R} > g' \quad .$$

Therefore, the bracketed term in (35) is positive. It follows that any change in F which causes $r_t'(D_t(I) - g(I))$ to increase will reduce D_t' and hence $(D_t' - \frac{g'^2}{R})$. We have therefore

<u>Proposition 5</u>. Suppose that at $t = t_o$ $I(t_o)$ is a unique optimal investment level for the manager. Suppose also that $I(t)$ is a regular maximum. Then if only the hazard rate $r_t(x) \equiv f_t(x)/(1 - F_t(x))$ depends on t, a sufficient condition for $I(t)$ to be increasing (resp. decreasing) in t in a neighborhood of t_o is that $\frac{d}{dt} r_t'[D_t(I(t_o)) - g(I(t_o))] > 0$ (resp. < 0) at $t = t_o$.

In order to understand the above condition, write $r_t'[D_t(I(t_o)) - g(I(t_o))]$ as $r_t'(x_t)$, where $x_t = D_t(I(t_o)) - g(I(t_o))$. Note that $D_t(I(t_o))$ is just the level of debt the manager must take on in order to sustain $I = I(t_o)$ when t_o changes to t. Therefore, by (24),

$$\frac{U'\left(\frac{g'(I(t_o))}{R} - I\right)}{U\left(\frac{g(I(t_o))}{R} - I\right)} \cdot \frac{1}{g'(I(t_o))} = r_{t_o}(x_{t_o}) = r_t(x_t) \quad .$$

Hence the condition $\frac{d}{dt} r_t'(x_t) > 0$ means that an increase in t leads to an increase in r_t', where r_t' is evaluated at x_t and x_t satisfies $r_t(x_t) = r_{t_o}(x_{t_o})$.

To see how this condition works, consider the uniform distribution which ranges from $-t \leq s \leq t$ (this distribution satisfies Assumption 1 only for

$-t < s < t$). The hazard rate is

$$r_t(x) = \frac{1}{t - x} \quad \text{and} \quad r_t' = \frac{1}{(t - x)^2} = [r_t(x)]^2 \quad .$$

Assume that $-t < D_t(I(t_o)) - g(I(t_o)) < t$. Then

$$\frac{dr_t'}{dt}(D_t(I(t_o)) - g(I(t_o))) = 2r_t \frac{dr_t}{dt} = 0 \; ,$$

where the latter equality follows from the fact that $D_t(I(t_o))$ must change in such a way that r_t is constant. Thus in the uniform case if $I(t)$ is differentiable then $\frac{dI(t)}{dt} = 0$. A complete analysis of the uniform case appears in the Appendix.

As another application of the proposition, consider the Double Exponential distribution. (this distribution satisfies Assumption 1 only for $x < 0$). The Double Exponential distribution function with mean zero and variance $2t^2$ is given by

$$F_t(x) = \begin{cases} \frac{1}{2}\exp(x/t) & \text{for } x < 0 \\ 1 - \frac{1}{2}\exp(-x/t) & \text{for } x \geq 0 \end{cases} \; .$$

The hazard function is thus given by

$$r_t(x) = \frac{f_t(x)}{1 - F_t(x)} = \begin{cases} \frac{1/2t(\exp(x/t))}{1 - \frac{1}{2}\exp(x/t)} & \text{for } x < 0 \\ 1/t & \text{for } x \geq 0 \end{cases} \; .$$

Consider (24). It is clear from this equation that $D_{t_o}(I(t_o)) - g(I(t_o)) \leq 0$. For if $D_{t_o}(I(t_o)) - g(I(t_o)) > 0$, then, by reducing D, the manager can keep (24) satisfied and at the same time decrease the probability of bankruptcy, hence making himself better off. Suppose that $x = D_{t_o}(I(t_o)) - g(I(t_o)) < 0$. Then

$$r_t'(x) = \frac{\frac{1}{2t^2}\exp(x/t)}{[1 - \frac{1}{2}\exp(x/t)]^2} = \frac{1}{t^2}\frac{z}{(1 - z)^2} = \frac{r_t(x)}{t(1 - z)} \; ,$$

where $z \equiv \frac{1}{2} e^{x/t}$. Consider now a change in t and x which keeps $r_t(x)$ constant. Since $r_t = \frac{z}{t(1-z)}$ stays constant when t increases, it follows that z rises when t rises and hence $t(1-z)$ rises when t rises. Therefore, an increase in t lowers $r_t'(D_t(I(t_o) - I(t_o))$. Thus Proposition 5 implies that an increase in t (i.e., an increase in the variance of s) lowers I.

The above results can be used to explain the role of debt and equity in our model. Consider the market value of the firm's equity, V_e, which in this risk neutral world with limited liability is given by

(36) $$V_e \equiv E \max(0, g(I) - D_t + s) \ .$$

Recall that distribution functions are indexed by t.

Proposition 6. Suppose that the distribution function of s changes in such a way that (a) s becomes more risky (i.e., a mean-preserving spread occurs) and

(b) $$\frac{dr_t'}{dt} (D_t(I(t_o)) - g(I(t_o))) < 0 \ .$$

and (c) the hazard rate increases: $\partial r_t / \partial t > 0$. Then this increases the value of equity V_e, and decreases the value of debt.

Proof. Note that the left-hand side of (24) must be increasing at an optimal I; if it were not, then the manager could reduce D and raise I, and still satisfy the first-order conditions for a maximum and be better off. From Proposition 5, assumption (b) implies that I falls. Thus the left-hand side of (24) falls. However, assumption (c) implies that the right-hand side of (24) rises. Hence equilibrium can be maintained only if $D_t(I) - g(I)$ falls. Thus (b) and (c) imply that $g(I) - D_t(I)$ rises and this tends to raise V_e. Assumption (a) raises V_e for a given $g(I) - D_t$ because $\max(0,s)$ is a convex function of s. Hence all the effects together raise V_e. Finally, since I falls,

$V = g(I)/R$ falls and hence the value of debt $= V - V_e$ falls. Q.E.D.

Note that conditions (a)-(c) are quite strong. For the Double Exponential distribution, a rise in t satisfies (a) and (b) but only satisfies (c) for some values of t. For the uniform distribution, an increase in t always lowers the hazard rate so (c) and (a) can never both be satisfied.

Let us turn now to comparative statics involving changes in U. Suppose that U is replaced by

$$V = U^{\alpha} ,$$

where $\alpha < 1$. Consider the effect on D' in (35). Now

$$\begin{aligned} V' &= \alpha U^{\alpha-1} U' , \\ V'' &= \alpha(\alpha - 1)U^{\alpha-2}U'^2 + \alpha U^{\alpha-1}U'' . \end{aligned} \tag{37}$$

Therefore

$$\begin{aligned} & g'VV''\left(\frac{g'}{R} - 1\right) - V'Vg'' - V'^2 g'\left(\frac{g'}{R} - 1\right) \\ &= g'\left(\frac{g'}{R} - 1\right)[\alpha(\alpha - 1)U^{2\alpha-2}U'^2 + \alpha U^{2\alpha-1}U''] \\ &\quad - \alpha U^{2\alpha-1}U'g'' - g'\left(\frac{g'}{R} - 1\right)\alpha^2 U^{2\alpha-2}U'^2 \\ &= \alpha U^{2\alpha-2}[g'UU''\left(\frac{g'}{R} - 1\right) - U'Ug'' - U'^2 g'\left(\frac{g'}{R} - 1\right)] . \end{aligned} \tag{38}$$

It follows that

$$\begin{aligned} D' &= g' + \left[\frac{\alpha U^{2\alpha-2}[g'UU''\left(\frac{g'}{R} - 1\right) - U'Ug'' - U'^2 g'\left(\frac{g'}{R} - 1\right)]}{V^2 g'^2 r'}\right] \\ &= g' + \alpha\left[\frac{g'UU''\left(\frac{g'}{R} - 1\right) - U'Ug'' - U'^2 g'\left(\frac{g'}{R} - 1\right)}{U^2 g'^2 r'}\right]. \end{aligned}$$

Hence an increase in α raises D' and hence $(D' - \frac{g'^2}{R})$. An application of Proposition 4 therefore yields

Proposition 7. If U is replaced by $V = U^{\alpha}$, where $\alpha < 1$, then I will decrease.

Replacing U by U^{α}, $\alpha < 1$, means, of course, that the manager becomes more risk averse. It might be wondered whether a more general increase in risk aversion -- represented by replacing U by $H(U)$ where H is an arbitrary increasing concave function -- leads to a decrease in I. The answer appears to be no in general.

Comparative statics results can also be obtained for changes in R and $g(\cdot)$. These must be interpreted carefully, however, since changes in R and g will also affect the classical solution I^*. We will not consider such comparative statics in this paper.

5. Comments and Extensions

The previous section examined the relationship between the risk of bankruptcy faced by a manager and his incentive to maximize the value of the firm. To see one implication of the theory, consider the case of no uncertainty. Then the manager looses all his benefits (i.e., goes bankrupt) whenever $g(I) < D$. Thus he will always choose I such that $g(I) = D$, i.e., $I(D) = g^{-1}(D)$. Thus

$$(39) \qquad V(D) = \frac{g(I(D))}{R} = \frac{D}{R} .$$

The manager maximizes his expected utility by maximizing $U(V(D) - I(D)) = U\left(\frac{D}{R} - g^{-1}(D)\right)$ from which it is clear that $g'(I(D)) = R$. Recall that $g'(I^*) = R$, where I^* is the profit maximizing level of invesment. Hence the manager maximizes the net market value of the firm.

It is clear that the equity of the above firm is worthless. In particular, the above firm is financed totally by debt. The firm borrows $g(I^*)/R$ and pays back $D = g(I^*)$. The manager consumes $V(I^*) - I^* = g(I^*)/R - I^*$. Because there is no randomness in production, the manager is able to adjust his investment so that he just pays back the debt with nothing left over for the shareholders. Of course, shareholders take this into account initially, and so it is impossible and unnecessary for the manager to raise capital with equity. Thus we are lead to the conclusion that a firm with no randomness in production will be owned essentially by the manager who receives the residual income $V(I^*) - I^*$ and finances his investment totally by debt!![12]

If s becomes random, however, then equity will in general have a positive value given by $\max(0, g(I) + s - D)$. Thus randomness in production will cause the manager to reduce debt levels to the point where equity finance becomes possible and useful. Equity will have value as long as there are realizations of s where $g(I) + s > D$. When s is random, the only way the manager can reduce his probability of bankruptcy is to create more events where $g(I) + s > D$.

Note that the model can be interpreted as a model of an entrepreneur who wants to raise capital, rather than as a model of corporate directors. In particular, it is a model of an entrepreneur with an investment opportunity but no capital of his own. Suppose that if the entrepreneur raises V dollars by the sale of debt and equity, then he spends $V - I$ on capital equipment useful only for his own consumption and invests I in the firm. If the entrepreneur goes bankrupt, assume that the consumption equipment $V - I$ is taken away from him. (For simplicity, suppose that it is worthless to the creditors who can only get $\min(D, g(I) + s)$.) Under these assumptions, the model applies directly to an entrepreneur who wants to raise capital from investors who cannot monitor how much he invests directly.

In the model which we have presented, the only reason that the manager wants to increase V is because he can thereby increase his consumption benefits $(V - I)$. However, there is another important reason why the manager may desire increases in V: to reduce the probability of a takeover bid. In Grossman and Hart [1979], we analyzed the probability of takeover bids occurring as a function of the differential between the potential worth of the firm to the raider (i.e., the party making the takeover bid) and the price he has to pay to get shareholders to tender their shares. In particular, if a corporation is badly managed because it is widely held and thus each shareholder is too small for it to be in his interest to monitor management, then a raider can become a large shareholder and to some extent internalize externalities which exist across small shareholders. That is, the raider, because he becomes a large shareholder, has an incentive to monitor management.

The model of Section 2-4 can easily be modified to take into account the possibility of takeover bids. In order to concentrate on the implications of the takeover threat, let us drop the assumption that the manager's consumption benefits depend directly on the firm's market value, V. In particular, assume that the manager has a fixed amount of capital, K, at his disposal which he can spend either on investment I or consumption $(K - I)$ (we imagine that K was raised sometime in the past). Thus the manager's utility is given by $U(K - I)$. We assume, however, that this utility is realized only if the firm is not taken over and does not go bankrupt. As in Grossman and Hart [1979], we assume that the probability of a takeover bid is a decreasing function of the market value of the firm V: $\bar{q}(V)$, where $0 \leq \bar{q}(V) < 1$.[13]

Stiglitz [1972] has emphasized the fact that raiders, by purchasing equity, can often change production plans to hurt bondholders. Bondholders can write restrictive covenants to try to prevent this from occurring. To avoid analyzing this issue, we assume that the bondholders are perfectly protected, in that we require the raider to buy up the equity plus the bonds of the firm. It is for this reason that we take the probability of a takeover bid to be a function $\bar{q}(V)$ of the firm's total market value.

It is straightforward to compute the optimal action for the manager under the threat of a takeover bid. Given V and D, the manager chooses I to maximize

$$(40) \qquad U(K - I)(1 - F(D - g(I)))q(V) ,$$

where $q(V) = 1 - \bar{q}(V)$. It is clear from (40) that V has no effect on the manager's choice of I since $q(V)$ appears multiplicatively. This occurs because we have assumed that the investment policy of current management is reversible by the raider, so that the probability of a raid depends on the price the raider must pay for the firm, which is V, and the potential worth of the firm -- which is independent of what current management is doing.

Note that V is the value of the firm under current management in the event that there is no raid. Thus, as in Sections 2-4, V is just the discounted expected output of current management.

Although V does not affect the manager's choice of I, it does affect his utility. Let $I(D)$ be the solution to (40). Then the manager's expected utility is

$$U(K - I(D))(1 - F(D - g(I(D))))q(g(I(D))/R) , \tag{41}$$

and the manager will choose D to maximize this expression. An analysis of this problem can be carried out along the same lines as in Sections 2-4. Although there are some mathematical differences, the basic conclusions are the same. In particular, it will be in the interest of management to issue debt because this increases I (through the $I(D)$ function) and hence $V = g(I)/R$, and therefore reduces the probability of a takeover bid. Note that, as in the case of the analysis of Sections 2-4, the existence of bankruptcy penalties is crucial to this argument. In the absence of such penalties, I does not depend on D and hence issuing debt will affect neither V nor the probability of a takeover.

6. Conclusions

In this paper, we have developed a theory to explain the use of debt as a financial instrument. The theory is based on the idea that the managers of a firm which is mainly equity-financed do not have a strong incentive to maximize profit -- in particular, since without debt bankruptcy does not occur, bad managers are not penalized in the event of low profit. Thus such a firm will have a low value on the stock market. We have argued that management can use debt to precommit itself in such a way that managers can only avoid losing their positions by being more productive. Thus debt increases the firm's market value. We have also analyzed the determinants of the optimal level of debt for management.

It is interesting to compare our theory of the role of debt with other theories which have been advanced. The most celebrated result on a firm's financial structure is the Modigliani-Miller theorem, which states that the owners of a firm will be indifferent about its debt-equity ratio -- in particular, the firm's market value will be independent of the debt-equity ratio. Since this result was established, numerous efforts have been made to relax the assumptions underlying the theorem in order to find a role for debt. Early attempts focused on the assumptions of no bankruptcy and/or no taxes. It was argued that if the probability of bankruptcy is positive, then, as long as investors cannot borrow on the same terms as the firm, i.e., go bankrupt in the same states of the world, then, by issuing debt, the firm is issuing a new security, and this will increase its market value. Also, if debt payments are tax deductible -- as they are in the USA or the UK, for example -- then the debt-equity ratio will again affect market value.

More recent attempts to explain the importance of debt have focused on a different assumption of the Modigliani-Miller theorem -- that the firm's production plan is independent of its financial structure. Stiglitz (1974) and Jensen and Meckling (1976) consider the situation of an entrepreneur who has access to an investment project, but does not have the funds to finance it. If the entrepreneur raises the funds by issuing equity, then since he will have a less than 100% interest in the project, he will not manage it as carefully as he should from the point of view of all the owners, i.e., in the language of this paper, he will take too many perquisites. On the other hand, if the entrepreneur issues debt, then his incentive to work will be reduced much less since, except in bankrupt states, he gets the full benefit of any increase in profits. Thus, to Stiglitz and Jensen and Meckling, debt is a way of permitting expansion without sacrificing incentives. The trade-off for the entrepreneur is between issuing equity, which permits the sharing of risks, and issuing debt, which leads to a high market value for the project through the incentive effect.

The model developed in this paper also relies on the idea that, for incentive reasons, a firm's production plan will depend on the firm's financial structure. However, we have assumed that management has a zero (or close to zero) shareholding in the firm. As a result, a switch from debt finance to equity finance does not change management's marginal benefit from an increase in profit directly. Rather the incentive effect comes from the desire to avoid bankruptcy. Thus, in particular, whereas a change in bankruptcy penalties would not have a significant effect in the Jensen-Meckling, Stiglitz analysis, it is of crucial importance in our model.

In recent papers, Ross (1977,1978) develops a signaling model to explain the debt-equity ratio. He considers a situation in which there are firms of exogenously given different qualities, where these qualities are known to management but not to the market. Ross shows that under these circumstances debt can be a signal of firm quality. In particular, in equilibrium, there will be a positive relationship between a firm's debt-equity ratio and its market value. The point is that it will not pay a firm of low quality to try to signal that it is of high quality by issuing a lot of debt since the costs to management -- in the form of a high probability of bankruptcy -- are too high.

We have already noted in the introduction the difference between a signaling equilibrium and the bonding or precommitment equilibrium developed in this paper. As another illustration of the difference, note that, in contrast to Ross's analysis, our analysis, without further assumptions, says nothing about the cross-section relationship between market value and the debt-equity ratio. This is because we explain the equilibrium level of debt of a firm as a function of the firm's stochastic production function, the rate of interest, and managerial tastes. The cross-section relationship will therefore depend entirely on how these parameters are distinguished in the population of firms. For example, if different firms are identical except with respect to the riskiness of s, then Proposition 6 gives sufficient conditions for the debt-equity ratio to be an increasing function of market value. However, if the conditions of Proposition 6 are not satisfied, or if firms differ in other ways, then the debt-equity ratio could easily be a decreasing function of market value.

Appendix

In this appendix, we give a complete analysis of the case where s is uniformly distributed on $[-\bar{s},\bar{s}]$ and U,g take the following special forms:

$$U = c^b, \; 0 < b < 1$$
$$g = I^{\frac{1}{2}}.$$

To simplify matters, we will assume that $R = 1$. Note that $I^* = \frac{1}{4}$ in this example.

Note that the uniform distribution satisfies Assumption 1 only for $-\bar{s} < x < \bar{s}$. For this reason we must modify slightly the analysis of previous sections.

Suppose that the manager chooses the investment level I, where $\hat{I} > I > 0$. See page 18 for the definition of $\hat{I}$. Then in equilibrium $V = g(I) = I^{\frac{1}{2}}$. As in Section 3, we can ask what is the smallest level of debt which will sustain this equilibrium. Let $D(I)$ represent this. Then, if $-\bar{s} < D - I^{\frac{1}{2}} < \bar{s}$, we know that (24) must hold, i.e.,

(A1)
$$\frac{b}{(I^{\frac{1}{2}} - I)} \cdot \frac{2}{I^{-\frac{1}{2}}} = \frac{1/2\bar{s}}{1 - \left(\frac{D - I^{\frac{1}{2}} + \bar{s}}{2\bar{s}}\right)} = \frac{1}{\bar{s} - D + I^{\frac{1}{2}}},$$

Hence

(A2)
$$I^{\frac{1}{2}} = \frac{1 - 2b\bar{s} + 2bD}{2b + 1},$$

or

(A3)
$$D = \frac{(2b + 1)I^{\frac{1}{2}} - 1 + 2b\bar{s}}{2b}.$$

However, it is also possible that $D - I^{\frac{1}{2}} = \bar{s}$ or $D - I^{\frac{1}{2}} = -\bar{s}$ (it is easy to see that $D > I^{\frac{1}{2}} + \bar{s}$ or $D < I^{\frac{1}{2}} - \bar{s}$ cannot be solutions since small reductions in D leave the manager's optimuum unchanged and increase his utility). The first case is impossible since the manager goes bankrupt for sure. He can always do better by raising I slightly. Hence we need only consider the second case $D - I^{1/2} = -\bar{s}$. Since $f(x)/(1 - F(x))$ is differentiable to the right at $x = -\bar{s}$, a necessary condtion for this case is

(A4) $$\frac{b}{(I^{1/2} - I)} \cdot \frac{2}{I^{-1/2}} \leq \frac{1}{\bar{s} - D + I^{1/2}} ,$$

which gives

(A5) $$I^{1/2} \leq \frac{1 - 2b\bar{s} + 2bD}{2b + 1}$$

or

(A6) $$D \geq \frac{(2b + 1)I^{1/2} - 1 + 2b\bar{s}}{2b} .$$

We are now in a position to establish the following:

Result. $D(I) = \max\left\{\frac{(2b + 1)I^{1/2} - 1 + 2b\bar{s}}{2b} , I^{1/2} - \bar{s}\right\}$.

The proof of this is simple. We have already established that D(I) equals either $\frac{(2b + 1)I^{1/2} - 1 + 2b\bar{s}}{2b}$ or $I^{1/2} - \bar{s}$. Suppose that

(A7) $$\frac{(2b + 1)I^{1/2} - 1 + 2b\bar{s}}{2b} > I^{1/2} - \bar{s} .$$

Then D cannot equal $I^{1/2} - \bar{s}$ since (A6) is contradicted. On the other hand, if

(A8) $$\frac{(2b + 1)I^{1/2} - 1 + 2b\bar{s}}{2b} < I^{1/2} - \bar{s} ,$$

then setting $D(I) = \frac{(2b + 1)I^{1/2} - 1 + 2b\bar{s}}{2b}$, we get from (A8) that

(A9) $$D(I) - I^{1/2} < -\bar{s} ,$$

which, as we have argued previously, cannot be a solution (a small reduction in D leaves the manager's optimum unchanged and increases his utility).

FIG. A →

We illustrate the above result in Figure A. The heavy black line gives D as a function of $I^{\frac{1}{2}}$. Note that if

$$\text{(A10)} \qquad \frac{-1 + 2b\bar{s}}{2b} > -\bar{s} \ ,$$

then $\frac{(2b+1)I^{\frac{1}{2}} - 1 + 2b\bar{s}}{2b}$ and $I^{\frac{1}{2}} - \bar{s}$ do not intersect at any $I > 0$ and $D(I)$ is given by $\frac{(2b+1)I^{\frac{1}{2}} - 1 + 2b\bar{s}}{2b}$ for all I.

Define I_0 to be the positive value of I at which the two lines intersect, if such an I exists, and to be zero otherwise. By simple calculation,

$$\text{(A11)} \qquad I_0^{\frac{1}{2}} = \max(1 - 4b\bar{s}, 0) \quad .$$

We consider now the manager's choice of the optimal I. The manager maximizes

$$U(g(I) - I)(1 - F(D(I) - g(I))) \ .$$

Three cases can be distinguished:

(1) The optimal $I > I_0$. Then D is differentiable at I and so, by the results of Section 3,

$$D'(I) = g'(I)^2 \quad .$$

Hence

$$\left[\tfrac{1}{2}\,\frac{(2b+1)}{2b} \cdot I^{-\frac{1}{2}}\right] = \frac{1}{4I} \ ,$$

which yields

$$I = \left(\frac{b}{2b+1}\right)^2 \quad .$$

(2) The optimal $I < I_o$. Again D is differentiable and so

$$D'(I) = g'(I)^2 .$$

This time we get

$$\tfrac{1}{2} I^{-\frac{1}{2}} = \frac{1}{4I} ,$$

i.e.,

$$I = \tfrac{1}{4} .$$

(3) The optimal $I = I_o$. Now D is no longer differentiable, but it is differentiable to the right and to the left. Since I_o is optimal, we know therefore that the expression in (30) is nonpositive if $D'(I)$ is evaluated according to (A3) and is nonnegative if $D'(I)$ is evaluated according to $D = I^{\frac{1}{2}} - \bar{s}$. This yields a modified version of (33):

$$D'(I) \begin{array}{ll} \geq g'(I)^2 & \text{if } D'(I) \text{ is evaluated according to (A3),} \\ \leq g'(I)^2 & \text{if } D'(I) \text{ is evaluated according to } D = I^{\frac{1}{2}} - \bar{s} . \end{array}$$

Hence we get

$$\left(\frac{b}{2b+1}\right)^2 \leq I_o \leq \tfrac{1}{4} .$$

Cases (1)-(3) establish the following result.

Result. If $I_o > \frac{1}{4}$, then it is optimal to set $I = \frac{1}{4}$. If $\left(\frac{b}{2b+1}\right)^2 \leq I_o \leq \frac{1}{4}$, it is optimal to set $I = I_o$. Finally, if $I_o < \left(\frac{b}{2b+1}\right)^2$, it is optimal to set $I = \left(\frac{b}{2b+1}\right)^2$.

The optimal debt levels can also be easily computed. If $I = \frac{1}{4}$, $D = I^{\frac{1}{2}} - \bar{s} = \frac{1}{2} - \bar{s}$. If $I = I_o$, $D = I_o^{\frac{1}{2}} - \bar{s}$. Finally, if $I = \left(\frac{b}{2b+1}\right)^2$, $D = \frac{1}{2} - 1/2b + \bar{s}$.

FIG. B →

Using (A11), we may graph the optimal I and D as functions of $\bar{s}$. We see that I decreases as the variance of s rises. Further, $I \to I^*$, $D \to D^* = g(I^*)$ as variance $s \to 0$. It is also worth noting that D is not monotonic in $\bar{s}$.

Footnotes

* We would like to thank David Champernowne and David Kreps for helpful suggestions.

1 See Grossman and Hart (1979). In that paper it is shown that the takeover threat will only be effective if free riders are excluded to some extent from sharing in the improvements in the firm brought about by the raider.

2 See Winter (1971) for a discussion of other factors concerning the impact of competition on profit maximization.

3 Note that the initial shareholders may want to write a corporate charter which constrains the debt-equity ratio to be at the level which maximizes managerial effort. However, suppose that the best ratio depends on variables which are not known at the time the charter is written, and may only be observable at great cost in the future. It may then be preferable for the initial shareholders to give management some flexibility in the choice of financial structure because this will allow management to base its actions on private information. We will consider the extreme case where management has complete freedom about the choice of financial structure.

4 Jensen and Meckling (1976) have studied another example of precommitment or bonding behavior. They consider a situation where an entrepreneur must decide what share to retain in his firm. If the entrepreneur runs the firm, his share in the firm will determine to what extent he pursues profits rather than perquisites. Therefore, his shareholding (assumed to be publicly known) is an example of <u>precommitment</u> in the same way that debt is in our model.

5 The work by Ross (1977) and Leland and Pyle (1977) derives signaling equilibrium debt-equity ratios. In both cases, the debt-equity ratio is a signal to

the financial market about the exogenous quality of the firm. The assumption that bankruptcy costs are correlated with the exogenous quality of firms drives their signaling equilibrium.

6 This assumption is made for simplicity only. The analysis can easily be generalized to the case where the corporation has some initial capital and needs to raise only a fraction of the required funds from outside sources.

7 The assumption of risk neutrality is justified if investors hold well diversified portfolios and the firm's return is uncorrelated with the returns of other firms in the economy.

8 In order to concentrate on the role of debt, we are ignoring any incentive schemes which are designed to raise the value of equity when a new investment project is undertaken. That is, for simplicity we assume that the manager gets all the surplus (V-I) from the firm's investment opportunity and that none of it accrues to initial shareholders. It is easy, however, to generalize our analysis to the case where the manager gets only some part of an increase in V, i.e., the manager's utility is $U(\phi(V) - I)$, where $0 \leq \phi(V) \leq V$. Note that it is essential that the manager get some surplus from working for the firm for the threat of bankruptcy to be an effective incentive scheme. See Calvo (1978) on a related topic.

9 We say that $h(x)$ is increasing if $x' > x \Longrightarrow h(x') \geq h(x)$. If $x' > x \Longrightarrow h(x') > h(x)$, we say that h is strictly increasing.

10 A "revealed preference" proof can be used to show that Assumption 1 is unnecessary for the result that I is increasing in V.

11 Note that D_t'' is well-defined -- simply differentiate (25) in the proof of Lemma 3.

12 This result should be treated with some caution. In this paper, we have ignored incentive schemes which make the manager's remuneration a function of ex post profit, $g(I) + s$. If such schemes are possible, then in the case $s \equiv 0$, the investment level $I = I^*$ can also be sustained by having 100% equity and an incentive scheme of the form: the manager is fired unless ex post profit = $g(I^*)$. Clearly in the case of certainty this accomplishes exactly what debt does in our model, namely the manager is removed if his profit falls below some critical level.

13 For example, if the corporate charter is such that the raider can buy the firm for V, then the raider's profit is $g(K)/R - V$ since once he takes over he can set $I = K$, if the actions of the previous manager are reversible.

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Figure 1

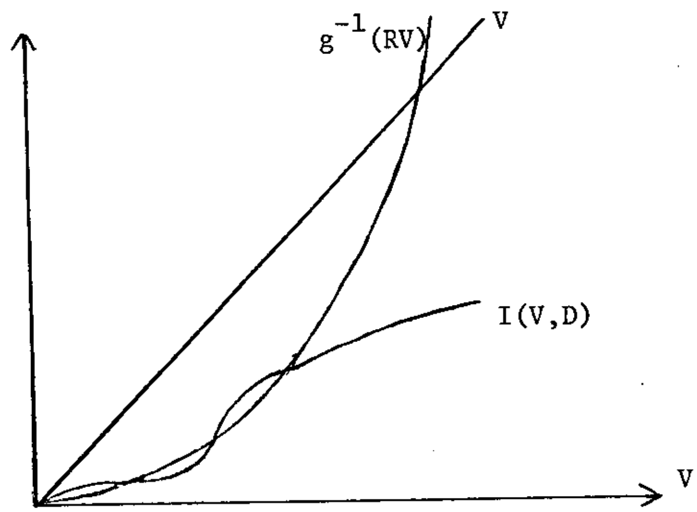


Figure 2

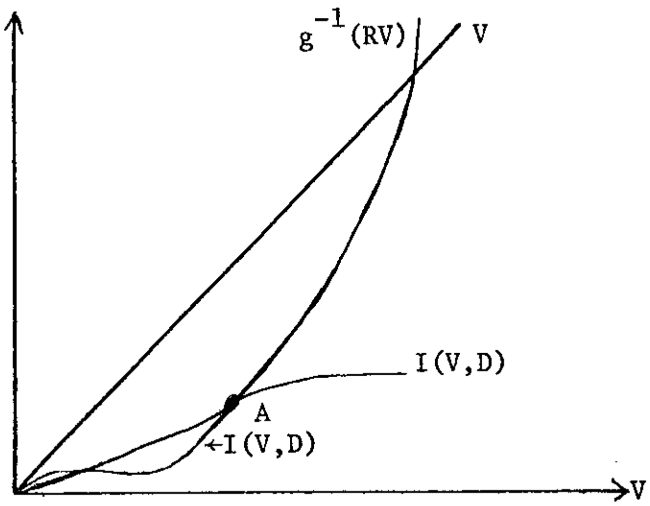


Figure 3

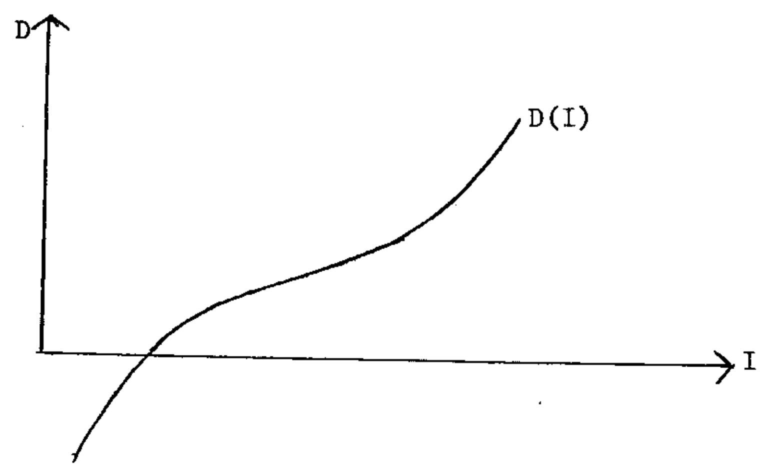


Figure 4

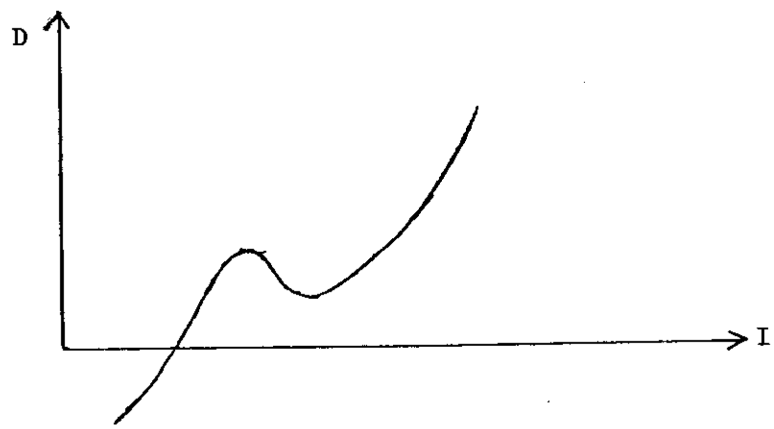


Figure 5

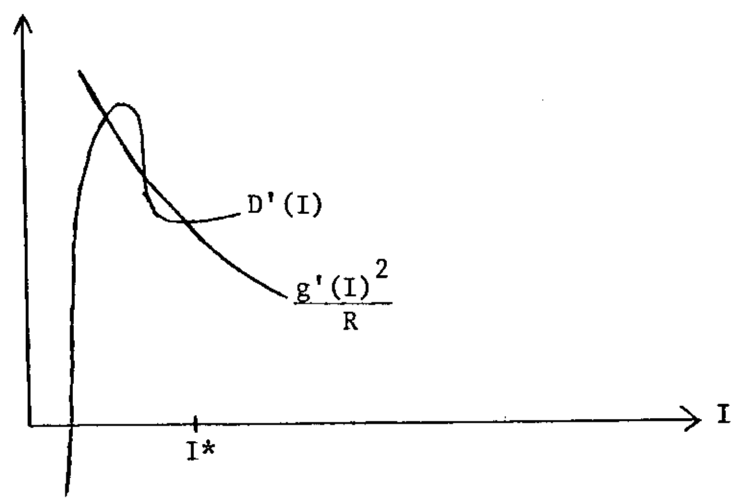


Figure A

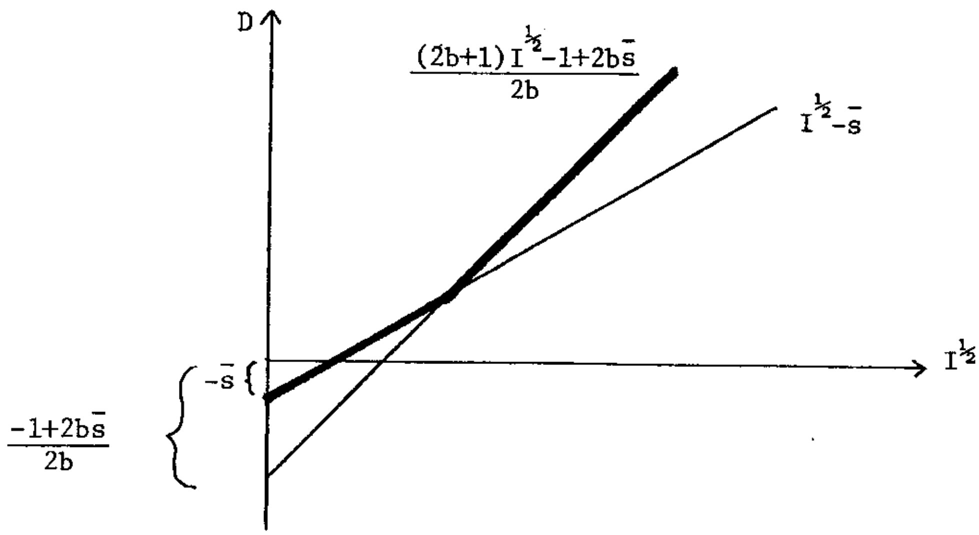
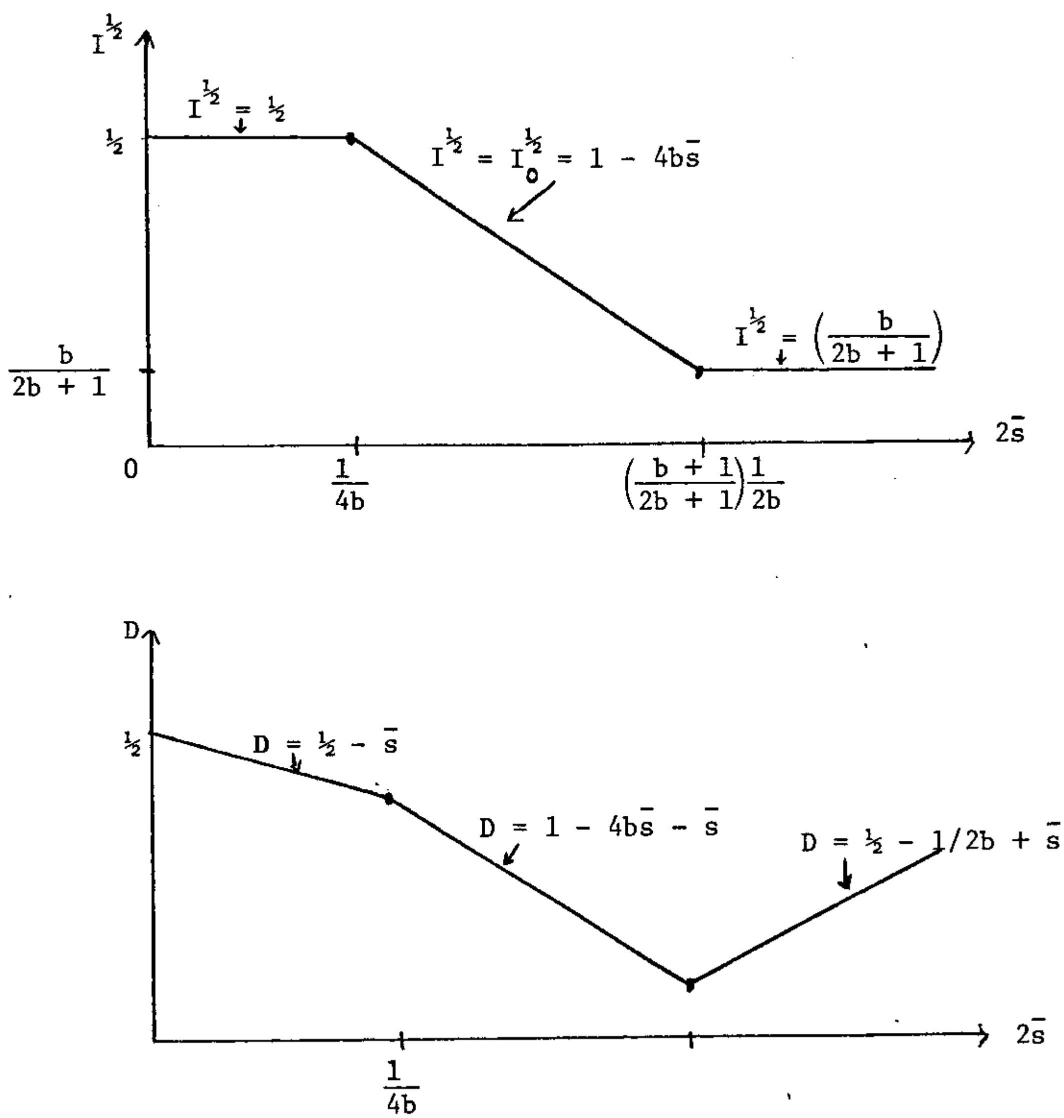

D
$\frac{(2b+1)I^{\frac{1}{2}}-1+2b\bar{s}}{2b}$
$I^{\frac{1}{2}}-\bar{s}$
$I^{\frac{1}{2}}$
$-\bar{s}$
$\frac{-1+2b\bar{s}}{2b}$

Figure B



I^½
I^½ = ½
½
I^½ = I_0^½ = 1 - 4bs̄
I^½ = (b/(2b + 1))
b/(2b + 1)
2s̄
0
1/4b
((b + 1)/(2b + 1)) 1/2b
D
½
D = ½ - s̄
D = 1 - 4bs̄ - s̄
D = ½ - 1/2b + s̄
2s̄
1/4b