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A MODEL OF THE PARALLEL TEAM STRATEGY IN PRODUCT DEVELOPMENT

by

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INTRODUCTION

In the late 1950's the idea of employing several teams to attack the development of weaponry systems--now known as the "parallel path strategy" of development--evolved at the RAND Corporation. This emphasis on exploring several technological paths in the development of a product arose in response to the recognition that the expensive part of research and development is not the exploratory work but the latter part of the development process (See [5] and [12]). This proved to be the strategy adopted in the early development stages of the atomic bomb, where five independent teams at various universities pursued the problem of producing large quantities of material capable of producing a self-sustaining reaction. Similarly, the technique was widely applied to the development of aircraft engines during World War II, when it was found that the best way to produce a good engine was to bring several different prototypes to a practical demonstration as quickly as possible, choose that one which exhibited the most favorable characteristics, and work intensively on the selected model to turn it into an effective military device (See [15]). In later periods, the Soviet Union employed this management strategy in developing their MIG fighters, while the United States brought this competition in team research to the development of the Thor and Jupiter missiles. The approach has also proved beneficial in cases where a central authority does not initiate the formation of the several teams. The intial stages in the development of color movies and television were undertaken by several private firms each following its own plan.

While the idea's birth appears to have many fathers (See [12]), only one individual, Richard Nelson [11], can be credited with providing a framework in which the costs and benefits of the parallel path strategy could be analytically studied. The problem considered by Nelson is that of achieving the development of the project at minimum cost. His model assumes that n teams are selected to work on the project up to some review point. When the review date arrives, the performances of the teams are compared, and the team that looks best in terms of successfully completing the project at minimum cost is retained while the remaining n-1 teams are dropped. The basic assumption of Nelson's characterization of the parallel path model is that the cost of using several teams during this initial stage is small relative to the benefits that accrue from the information gathered, however, as development continues into later stages, the cost of employing multiple teams increases disproportionately to the value of information obtained.

The Nelson model does indeed explain many of the instances in which we observe a firm or government agency employing more than one team on a development project. However, the model does not explain those cases where more than one team work until development is completed or failure is conceded--failure ocurring whenever the allocated development funds run out. Nor does it explain why several teams may be employed even if the per team cost during the initial phase is not low relative to the value of information gathered. Stimulated by Nelson's work, we suggest in this paper another model that explains these situations, and we analyze the factors important in determining the optimal number of parallel teams.

We assume that the size of each team is optimal, and the only question is how many teams should be employed in research and development (R&D).

Obviously, when we determine the number of teams to be employed, we determine the total optimal investment in R&D. This issue is crucial in considering the government budget allocated to R&D as well as the amount of money invested in R&D by the private sector. Just to indicate the magnitude of the investment in R&D, note that in 1977, General Motors invested $1,451 million in R&D; and the corresponding investment figures for other leading firms are respectively: Ford Motor Company, $1,170 million; I.B.M., $1,142 million; and AT&T, $718 million.

Finally, we would like to mention that we suggest in this paper a relatively simple model that can be extended in various directions; e.g., flow of information between teams, agreement between competitors working on the discovery of two or more products which either substitute or complement each other, etc. However, we believe that the simple model contains the main message to which we would like to address ourselves.

IA. THE MODEL

The firm is searching for a new product. If the search is successful, the discovered product increases the firm's net worth by its net present value, R. In calculating R all costs to be incurred in the future production of the product being developed are accounted for, except the costs of development, A dollars per team, that are considered separately. Clearly $R>0$, since otherwise no attempt will be made to develop the product.

Given that any team's chances of success are independent of the number of teams used and that the probability of success is $1-q$ while that of failure is q, how many teams should the firm employ during the development stage? If only one team is employed, the expected dollar return, $\Pi(1)$, is

$$\Pi(1) = (1-q)R-A \tag{1}$$

where the development cost of A dollars is incurred with certainty. With two teams the expected return, $\Pi(2)$, is the probability that at least one team will succeed or one minus the probability of both teams failing, q^2, times R, minus the cost of using two teams. Formally,

(2) $$\Pi(2) = (1-q^2)R-2A$$

Suppose that employment of the first team is profitable, should we add a second team? Only if the addition results in an increase in profit. That is, if

(3) $$\Delta\Pi(1) = \Pi(2) - \Pi(1) = q(1-q)R-A > 0$$

or

(3a) $$q(1-q) > \frac{A}{R}$$

Obviously, the lower the cost-to-present value ratio, A/R, the more likely it is that a second team will be added. The effect of q on the decision to add another team is rather more complicated, for, given A/R, a decrease in q does not necessarily work in favor of increasing the number of teams. The reason is that a decrease in q gives rise to two counterforces. On the one hand, one can always increase the chances of success by adding a team. The higher q is, the more attractive this strategy is bearing in mind that the increase in odds must outweigh the added certain cost of A dollars. On the other hand, the lower q is, the greater the likelihood that a single team will succeed, dictating against the addition of a second team--since the event "two or more teams discover the new product" yields the same return, R, as the event "only one team discovers the new product." For $q > \frac{1}{2}$ the former force dominates in favor of increasing the number of teams, while for $q < \frac{1}{2}$ the latter force prevails.[1]

More generally, if there are n-1 on-going teams, the question of adding an n^{th} team is decided by

(4) $$\Delta\Pi(n-1) = \Pi(n) - \Pi(n-1) > 0$$

where

(5) $$\Pi(n) = (1-q^n)R - nA$$

Substitution of (5) into (4) yields,

(6) $$\Delta\Pi(n-1) = q^{n-1}(1-q)R-A > 0$$

or

(6a) $$q^{n-1}(1-q) > \frac{A}{R}$$

The critical point to focus on is $q = \frac{(n-1)}{n}$, for if q is below this value then a decrease in q works against adding the n^{th} team while if q is above $\frac{(n-1)}{n}$ then a decrease in q tends to favor an increase in teams.[2]

IB. THE OPTIMAL NUMBER OF TEAMS

If n* is the number of teams that produces maximum expected return then it is necessary that $\Pi(n^*)$ satisfy

(7a) $$\Delta\Pi(n^*-1) = \Pi(n^*) - \Pi(n^*-1) > 0$$

(7b) $$\Delta\Pi(n^*) = \Pi(n^*+1) - \Pi(n^*) < 0$$

Employing the relationship given by Equation (6), for the cases n* and n*-1, the following must hold, if indeed n* provides the maximum expected profit,

(8a) $$Rq^{n^*-1}(1-q) - A > 0$$

(8b) $$Rq^{n^*}(1-q) - A < 0$$

Equations (8a) and (8b) imply that,

(9) $$q^{n^*-1} > \frac{A}{R(1-q)} > q^{n^*}$$

Noting that $\ln q < 0$, then n* must satisfy

(10) $$\left(\frac{1}{\ln q}\right)\ln\left[\frac{A}{R(1-q)}\right] < n^* < 1 + \left(\frac{1}{\ln q}\right)\ln\left[\frac{A}{R(1-q)}\right]$$

Condition (10) is sufficient for a global maximum, since the second differential is everywhere negative. That is

(11) $$\begin{aligned}\Delta^2\Pi(n) &= \Delta\Pi(n) - \Delta\Pi(n-1)\\ &= Rq^n(1-q) - A - Rq^{n-1}(1-q) + A\\ &= Rq^{n-1}(1-q)(q-1)\\ &= -Rq^{n-1}(1-q)^2 < 0\end{aligned}$$

for any n.

From condition (10), the importance of q's size in determining the optimal n is evident. We see that as q increases from zero, there is a point above which $\ln[A/(R(1-q))]$ becomes positive,[3] which in turn forces the upper bound on n^* to fall below one.[4] In this event, the firm views the project as unprofitable and no team is assigned to work on it. The above reasoning also tells us that for at least one team to be employed, we should have $\ln[A/(R(1-q))] < 0$, namely $A/R(1-q) < 1$, or $R(1-q) - A > 0$, that is the project must have positive net present value.

Finally, recall that the optimal number of teams n^* is the same under the assumption of certain R and under the assumption that R is a random variable when the firm's goal is to maximize its expected net present value. This identity becomes transparent once we recall that the net present value is $(1-q^n)R - nA$, and the expected net present value is given by $(1-q^n)E(R) - nA$ (see eq. (5) and eq. (21)).

IC. THE CHANGE IN n^* DUE TO PARAMETER CHANGES

Let us turn to the analysis of the economic factors which determine the optimal number of teams, n^*. This analysis relies on the inequality given in (10). Define,

$$B \equiv \left(\frac{1}{\ln q}\right)\ln\left[\frac{A}{R(1-q)}\right] = \left(\frac{1}{\ln q}\right)[\ln A - \ln R - \ln(1-q)] \tag{12}$$

From (10), the higher B, the higher n^*. Therefore to understand how a parameter change affects n^*, one needs only study the effect of that parameter change on B.

As one suspects,

$$\frac{\partial B}{\partial A} = \frac{1}{A(\ln q)} < 0; \quad \frac{\partial B}{\partial R} = -\frac{1}{R(\ln q)} > 0 \tag{13}$$

An increase in per team cost or a decrease in project profitability tends to reduce the optimal number of teams.

With regard to a change in q

$$\frac{\partial B}{\partial q} = \frac{1}{[\ln q]^2}\left[\frac{\ln q}{(1-q)} - \frac{1}{q}\ln\left[\frac{A}{R(1-q)}\right]\right] \tag{14}$$

So the sign of $\frac{\partial B}{\partial q}$ is determined by the sign of the bracketed terms. Because we assume the project is expected to be profitable for at least one team, i.e., $R(1-q) - A > 0$, then

$$-\frac{1}{q}\ln\left[\frac{A}{R(1-q)}\right] > 0$$

but

$$\frac{\ln q}{(1-q)} < 0$$

Therefore the effect on n* of an increase in q is ambiguous. The best one can do is state the condition under which $\frac{\partial n^*}{\partial q} > 0$. From (14), the condition is

(15) $$q \cdot \ln q - (1-q)\ln\left[\frac{A}{R(1-q)}\right] > 0$$

or

(16) $$q^q\left[\frac{R(1-q)}{A}\right]^{1-q} > 1$$

II RISK ANALYSIS

Even if a team is successful in development, the final product's future income is a random variable, consequently the product's present value, symbolized by R in previous sections, can no longer be assumed known but must be considered a random variable. This uncertainty with respect to the payoff from successful development constitutes one type of risk. The other risk associated with product development is that none of the n* teams will succeed and no new product will be forthcoming. When the above risks are explicitly considered, what constitutes an optimal stragegy? We analyze this issue by employing stochastic dominance criteria and the well-known mean-variance rule.

IIA. STOCHASTIC DOMINANCE ANALYSIS

In this section we make use of two theorems on ordering preferences under risk. Both theorems have been grouped under the heading of stochastic dominance. We begin by defining new notation, state the two theorems, and then proceed with the analysis.

Without loss of generality we assume that the firm faces a decision of employing either n_1 or n_2 teams; where $n_2 > n_1$. We refer to a decision to employ n_1 teams as strategy 1 and to a decision to employ n_2 teams as strategy 2. The net income from an investment in R&D is a random variable which we denote by y. Futhermore, denote by $F_1(y)$ and $F_2(y)$, respectively, the cumulative distribution functions of the net present value of the random variable, y, induced by employing n_1 or n_2 teams.

<u>First Degree Stochastic Dominance</u> (FSD): Given two cumulative probability distributions F_1 and F_2, strategy 1 will be preferred to strategy 2 by every expected utility maximizer, independent of the convexity or concavity of the utility function, if and only if $F_1(y) \leq F_2(y)$ for all values y, and for at least one y value the strict inequality $F_1(y) < F_2(y)$ holds. For proof of FSD see [2], [3], [13].

<u>Second Degree Stochastic Dominance</u> (SSD): Given two cumulative probability distributions F_1 and F_2, strategy 1 will be preferred to strategy 2 by every <u>expected</u> utility maximizer with a concave utility function, if and only if

$$\int_{-\infty}^{y} [F_2(t) - F_1(t)]\, dt \geq 0$$

for all y, and the strict inequality holds for at least one y value. For proof of SSD see [1], [2], [3], [14].

Assume that the probability density function of the project's present value, excluding development costs is given by

$$g(R) = \begin{cases} 0 & R \leq 0 \\ h(R) & R > 0 \end{cases} \tag{17}$$

where

$$\int_{-\infty}^{\infty} g(R)\, dR = \int_{0}^{\infty} h(R)\, dR = 1$$

If a development program using n teams is unsuccessful, that event having probability q^n, then the project's net present value, y, equals -nA; on the other hand if the program is successful, the probability of that event being $1-q^n$, then the y random variable is R-nA. Since the event "success" or "failure" of the development process is independent of the present value random variable, R, the net present value, y, can be described by the following mixed distribution[5]

$$(18) \qquad f(y) = \begin{cases} 0 & y < -nA \\ q^n & y = -nA \\ (1-q^n)h(y+nA) & y > -nA \end{cases}$$

The cumulative distribution of y under the general strategy n is then given by

$$(19) \qquad F(y) = \begin{cases} 0 & y < -nA \\ q^n & y = -nA \\ q^n+(1-q^n)H(y+nA) & y > -nA \end{cases}$$

where

$$H(y+nA) = H(R) = \int_0^R h(t)dt$$

As n increases, -nA decreases and the entire distribution F(y) shifts to the left. Moreover q^n decreases (since $q < 1$) and $1-q^n$ correspondingly increases.

Consider three alternative strategies n_1, n_2 and n_3 where $n_3 > n_2 > n_1$. Using (19) we plot the corresponding cumulative distributions in Figure 1. As we see from Figure 1, no strategy can dominate a strategy with a lower n either by FSD (since $F_1(-n_2A) < F_2(-n_2A)$ and $F_2(-n_3A) < F_3(-n_3A)$) or by SSD (since $\int_{-\infty}^{-n_1A} [F_1(t) - F_2(t)]\, dt < 0$ and $\int_{-\infty}^{-n_2A} [F_2(t) - F_3(t)]dt < 0$). However, a given strategy n may dominate a strategy with a higher n. Figure 1 illustrates a case where strategy n_1 dominates n_3 by FSD since $F_1(y) \leq F_3(y)$ for all y and a strict inequality obtains for $y > -n_3A$. Strategy n_1 may also dominate n_2 by SSD but not by FSD (since the two cumulative distributions F_1 and F_2 intersect.) The only way to determine whether strategy n_1 dominates n_2 by SSD is to calculate the (algebraic) area between the two curves, i.e., to apply the SSD rule directly.

In order to calculate the relevant area one has first to find all the intersection points between the two cumulative distributions under consideration. In Figure 1, we demonstrate a case where F_2 and F_1 intersect only twice. In this case, if the negative area which is between $-n_1A$ and y_0 is smaller than the positive area between the two cases over the range $(-n_2A, -n_1A)$, we can conclude that $F_1(y)$ dominates $F_2(y)$ by SSD. Though for some distributions of R (e.g., a uniform distribution) it can readily be seen that either there is no intersection at all (e.g., $F_3(y)$ and $F_1(y)$ in Figure 1), or if the cumulative distributions cross, then there must be exactly two intersection points (as for $F_2(y)$ and $F_1(y)$ in Figure 1), such assertion is not true in general, and two cumulative distributions may intersect any number of times. Thus, in general, in order to establish dominance one has to analyze the specific distribution of R in the relevant case. Finally if R is given such that there is only one intersection between the two distributions under consideration, then $F_1(y)$ dominates $F_2(y)$ by SSD if and only if the expected net profit by $F_1(y)$ is greater than the expected net profit by $F_2(y)$. $F_1(y)$ dominates $F_2(y)$ for all concave utility functions since SSD is equivalent in this case to the simple rule: the mean of $F_1(y)$ is greater than the mean of $F_2(y)$. In general, given two distributions F_1, F_2 with mean values μ_1, μ_2 respectively, such that for some $y_0 < \infty$, $F_1 \leq F_2$ for $y < y_0$ (and $F_1 < F_2$ for some $y_1 < y_0$) and $F_1 \geq F_2$ for $y \geq y_0$, then F_1 dominates F_2 (for all concave utility functions, i.e, SSD) if and only if $\mu_1 \geq \mu_2$. (The proof of this statement is presented in [3], p. 371.)

Using some of the properties of SSD efficiency we can determine the efficient set of strategies. A strategy n_i is included in the efficient set if there is no other strategy which dominates it. First a necessary condition for FSD and SSD is that the dominated strategy have a lower expected return.[6] Therefore, the strategy that provides the highest expected return, symbolized by n^*, is undominated. Does strategy n^* dominate any other strategy? Recalling the reasoning used in previous paragraphs to prove that strategy $n_2 (>n_1)$ cannot dominate n_1, we conclude by the same reasoning that n^* cannot dominate (either by FSD or SSD) any strategy n where $n < n^*$. Thus, all the strategies $n_i \leq n^*$ are FSD as well as SSD efficient, in the sense that there is no strategy which dominates n_i either by FSD or by SSD.[7] What about strategies $n_j > n^*$? It is obvious that there is no strategy n_j which dominates any of the strategies $n_i \leq n^*$, since the cumulative distribution of n_j starts further to the left than any cumulative distribution of n_i. However, n^* (or any other n_i) may dominate one or all distributions of n_j either by FSD or by SSD. Such dominance is possible since on the one hand n^* is the strategy with the highest expected net present value (this stems from eq. (10), since the analysis in the certainty case is equivalent to an analysis involving maximation of the expected net present value), and on the other hand the cumulative distribution of n^* starts farther to the right than the cumulative distribution of n_j. In order to see if such dominance exists, one has to apply directly either the FSD or the SSD rule and compare n^* with all other strategies n_j. However, while all strategies $n_i \leq n^*$ are FSD — as well as SSD-- efficient, for $n_j > n^*$ the efficient sets of strategies derived by FSD and SSD rules do not coincide. Figure 2 illustrates such a possiblity when the SSD efficient set is smaller than the FSD efficient set.

IIB. MEAN-VARIANCE ANALYSIS

Almost all of the risk-return efficiency analysis of economic decision making has been done under more restrictive assumptions: 1) the attributes of any risky strategy are completely defined by the strategy's expected return and variance of return; and 2) the decision maker views expected return favorably and variance unfavorably.[8] We now investigate what can be said about the n-efficient set in mean-variance space.

Since the return y from a strategy n is R-nA with probability $(1-q^n)$ and -nA with probability q^n, the expected dollar return y is given by

(20) $$E(y) = (1-q^n) \int_0^\infty (R-nA)h(R)dR - q^n \cdot (nA)$$

or

(21) $$E(y) = (1-q^n)\ [E(R)-nA] - q^n(nA) = (1-q^n)E(R) - nA$$

The variance of $\sigma^2(y)$ is derived as follows:

(22) $$\begin{aligned}\sigma^2(y) &= \int_0^\infty [R-nA-(1-q^n)E(R) + nA]^2(1-q^n)h(R)dR \\ &+ [-nA - (1-q^n)E(R) + nA]^2 q^n \\ &= (1-q^n) \int_0^\infty [(R - E(R)) + q^n E(R)]^2 h(R)dR \\ &+ (1-q^n)^2 [E(R)]^2 q^n \\ &= (1-q^n)\sigma^2(R) + [E(R)]^2[(1-q^n)q^{2n} + q^n - 2q^{2n} + q^{3n}] \\ &= (1-q^n)[\sigma^2(R) + q^n[E(R)]^2].\end{aligned}$$

From the analysis in section II we know that E(y) reaches a maximum at some n, labelled n*. What about $\sigma^2(y)$? Partially differentiating $\sigma^2(y)$ with respect to n, we obtain

(23) $$\begin{aligned}\frac{\partial\sigma^2(y)}{\partial n} &= -q^n(\ln q)[\sigma^2(R)+q^n\ E(R)]^2 + (1-q^n)q^n(\ln q)[E(R)]^2 \\ &= [-q^n \ln q]\,\sigma^2(R)+q^n(\ln q)[E(R)]^2[1-q^n-q^n]\end{aligned}$$

or

(24) $$\frac{\partial\sigma^2(y)}{\partial n} = [\ -q^n \ln q]\,\sigma^2(R)+q^n(\ln q)[E(R)]^2[1-2q^n]$$

Since $\ln q < 0$ and $q^n < 1$, the first term on the right hand side of (24) is positive while the second term is ambiguous; consequently the sign of $\frac{\partial\sigma^2(y)}{\partial n}$, in general, is ambiguous at any n. However, if $[1-2q^n]$ which appears on the right hand side of (24) is negative then one can determine that $\frac{\partial\sigma^2(y)}{\partial n} > 0$. This implies that $\frac{\partial\sigma^2(y)}{\partial n} > 0$ if $1 < 2q^n$ or $n \ln q > \ln \frac{1}{2}$. Recalling that lnq is negative, the condition for a positive sign in (24) is $n < (\ln\frac{1}{2})/(\ln q)$. Thus, what one has to do is to calculate the positive number $\ln(\frac{1}{2})/\ln(q)$ which we denote by n^{**}. Compare n^{**} to n^*, which is the strategy with the higher expected return. If $n^{**} > n^*$ one can conclude that in the range $n^* < n < n^{**}$, $E(y)$ decreases and $\sigma^2(y)$ increases with an increase in n, and hence all the strategies of n such that $n^* < n \leq n^{**}$ are dominated by stragegy n^*.

If on the other hand we find that $n^{**} < n^*$, no further general progress can be made toward the derivation of the n-team efficient set in $E(y) - \sigma^2(y)$ space. Nevertheless given estimates of q, μ_R, and σ_R one can obtain the efficient (μ_y, σ_y) set by a numerical analysis; the efficient set generated will, of course, vary with the values of the q, μ_R, and σ_R parameters.

III. CONCLUDING REMARKS

Working on research and development can be carried out by using many alternative production functions. However, in many cases the firm estimates the number of people and the required resources needed by one investigating team for a predetermined time period. Looking at the costs and the potential revenues that may result from discovery, the firm may decide to go ahead with the project. If this is the case, should the firm add another investigating team? Assuming certainty or that the firm's objective function is to maximize expected net present value, we solve for the optimal number of teams that should be employed and analyze the impact of changes in the relevant parameters on the optimal number of teams to employ. Obviously, the higher the potential net present value of the new product, apart from development costs, and the lower the investigating cost per team, the higher the optimal number of teams that should be employed. The impact of a change in the probability of discovery of a new product by each team on the optimal solution is somewhat ambiguous: because of counterforces, given two projects, identical in all respects except in the probability of success, it is not clear *a priori* which project should employ the larger number of teams.

Extending the analysis to incorporate uncertainty yields a range of efficient strategies from which the firm should select (subjectively) the optimal strategy. In this analysis we assume either risk aversion and use stochastic dominance criteria, or that the firm works in a mean-variance framework. The first screening of inefficient strategies under the above rules is function of the estimated present value, the investigating cost per team as well as each team's discovery probability.

Given a subjective function of the potential profit from the new discovery (i.e., f(R)), one can use it to derive more explicitly the SSD as well as the mean-variance efficient sets of strategies, from which the firm's management should select its optimal strategy and thus determine the optimal investment in R&D.

Footnotes

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1. Note that the marginal profit induced by adding a second team is given by $\Delta(\Pi_1) = q(1-q)R-A$. Thus, $\frac{\partial\Delta(\Pi_1)}{\partial q} = R(1-2q)$, which is always a decreasing function of q. However, for all $q < \frac{1}{2}$, $\frac{\partial(\Delta\Pi_1)}{\partial q}$ is positive and for all $q > \frac{1}{2}$ $\frac{\partial(\Delta\Pi_1)}{\partial q}$ is negative, which explains the above two counterforces. In technical terms these two counterforces stem from the fact that $q(1-q)$ is a parabola, reaching its maximum at the value $q = \frac{1}{2}$.

2. This is true since the function $q^{n-1}(1-q)$ reaches its maximum at the value $q = \frac{n-1}{n}$, (see also footnote 1).

3. This critical point is given by $q = 1 - (\frac{A}{R})$, where $\frac{A}{R} < 1$--otherwise the project is not worth considering.

4. Recall that $\ln q < 0$.

5. Obviously,

$$\int_{-\infty}^{+\infty} f(y)dy = q^n + (1-q^n)\int_0^{\infty} h(R)dR = q^n + (1-q^n) = 1.$$

Recall that the net present value is $y = R-nA$, hence $R = y+nA$ as stated in eq. (18).

6. Let $u(y) = y$, where $u(\cdot)$ denotes utility function, be a member of the class of utility functions for which the FSD and the SSD theorems hold. If F_1 dominates F_2 by FSD or SSD, then $\int_{-\infty}^{+\infty} y\,dF_1 \geq \int_{-\infty}^{+\infty} y dF_2$. Thus $\mu_1 \geq \mu_2$ is a necessary condition for FSD and SSD.

7. Since expected profit monotonically increases with n up to n* then by the same argument used to prove that n* cannot be dominated by n < n* we know that any two strategies n_3 and n_4, $n_3 < n_4 < n^*$, cannot dominate each other.

8. The mean-variance rule is quite convenient to apply. However, one can safely apply this rule if one either assumes normal probability distributions with risk aversion or quadratic utility functions. (See [8], [9], [16], [11]). It is worth mentioning that in the quadratic utility case the mean-variance criterion is only a sufficient rule. (See [4]). In a recent dispute with regard to the theoretical validity of the mean-variance rule see [6], [18] and on its empirical approximation to a maximization of a direct expected utility, see [7].

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