BANK RESERVES AND MACROECONOMIC STABILITY

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Working Paper #18-79

THE RODNEY L. WHITE CENTER FOR FINANCIAL RESEARCH

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I. Introduction

Required reserves on deposits at financial institutions have two major consequences. First, reserves may be regarded as a tax on bank liabilities, if the rate of return on reserves, measured either explicity or implicity, is less than the market rate of interest. Secondly, reserves may be considered an important monetary instrument for controlling the value of monetary aggregates and ultimately such variables as prices, interest rates, and income. Unfortunately, there appears to be little agreement among economists as to the optimal level for such reserves in order to achieve macroeconomic stability. Friedman (1959) had formerly advocated 100 percent reserves on both time and demand deposits, but recently (1978) has claimed that uniformity of reserves on transactions balances is far more important than the level at which they are imposed. At the other end of the spectrum, Carson (1964) has advocated abolishing required reserves on all deposits while Poole and Lieberman (1972) have urged elimination of reserves on time deposits only if M₁ is accepted as the money stock definition.

Recently, the role of the disturbances affecting asset demands and supplies has been explicitly recognized (Kaminow (1977), Laufenberg (1979)) as key to the determination of the optimal level of reserves for the purpose of stabilizing monetary aggregates. This paper extends these models by analyzing how reserve ratios affect the variability of such ultimate targets as the price level. An explicit formula for the optimal reserve ratio in order to minimize price fluctuations is generalized to the case of N assets and competitive returns on money.

Section II of this manuscript describes the structure of the model and the nature of the equilibrium. The level of reserves which minimize price level variability is first derived in the case of two assets where deposit

rates are fixed. The model is then extended to the fixed rate, N-asset case and finally to the case where deposit rates are competitively set.

Section III employs U.S. post-War data to determine the structure of asset shocks and optimal reserve levels. Section IV is devoted to summary remarks.

II. Structure of the Model

A. Asset Equilibrium

The economy is characterized by a simple one good production process with fixed inputs of labor and capital. There are four financial assets: high-powered money (H), currency (C), deposits (D), and equity (E). High-powered money is issued solely by the central bank and is held as reserves by the banking system. Currency and deposits, which will be frequently termed "monetary assets," are issued by the banking system and held by households, while equity is issued by firms against their capital and is held by both banks and households. The households' (h) real demand functions for assets are specified by

$$c_h^d = c^d(r_c, r_d, r_e, K, H/P)$$

(1)
$$D_h^d = D^d(r_c, r_d, r_e, K, H/P)$$

$$E_h^d = E^d(r_c, r_d, r_e, K, H/P),$$

where r_c , r_d , and r_e are the respective yields on currency, deposits, and equity, K is the fixed capital stock, and P is the price of output in terms of high-powered money. It is assumed that the assets are all gross substitutes and non-negatively dependent on capital and real high-powered money.

Banks are internally financed institutions which hold equity and high-powered money as assets against their deposit and currency liabilities. It is assumed that the banking industry is subject to constant returns to scale in reserves and equity so that the competitive supply price of bank liabilities is a function of the exogenous reserve ratio and the rate of return on equity, e.g.,

$$r_c = F(k_c, r_e) = (1 - k_c)r_e$$
(2)
$$r_d = G(k_d, r_e) = (1 - k_d)r_e.^2$$

Under these conditions the real demand of banks for reserves and equity is determined by the demands of the public at the supply price determined in the market. This will also be true if the rates of return on currency and deposits are set by government regulation below the rates shown in (2) above. In this case monopoly profits will accrue to the banking industry but the quantities of monetary asset supplied will still be determined by household demand at the regulated rates. Therefore the banks' (b) real demand for reserves and equity is given by

$$H_{b}^{d} = k_{c}C_{h}^{d} + k_{d}D_{h}^{d}$$
(3)
$$E_{b}^{d} = (1 - k_{c})C_{h}^{d} + (1 - k_{d})D_{h}^{d}.$$

Equilibrium in the economy is determined by summing the households' and banks' aggregate demand for high-powered money and equities and equating these with their respective supply, i.e.,

(4)
$$k_c c_h^d + k_d D_h^d = H/P$$

(5)
$$E_h^d + (1-k_c)C_h^d + (1-k_d)D_h^d = \bar{K}.$$

Since the households' demands are constrained by total wealth, the above system contains only one independent equation. It is assumed that $r_{\rm e}$ is fixed at the marginal product of capital and $r_{\rm c}$ and $r_{\rm d}$ are either determined competitively by equations (2) or fixed by the central bank. Therefore the price level is the endogenous variable which clears the market for both high-powered money and equity.

Solving (4) for the price level yields

(6)
$$P = \frac{H}{k_c C^d + k_d D^d}.$$

The simplest case to analyze is when the rates of return on currency and deposits are regulated and the demands for currency and deposits are unaffected by the real quantity of high-powered money. 3 In this case P is homogeneous of degree minus one in $k_{\rm c}$ and $k_{\rm d}$. For example, if both currency and deposit reserve level are halved, the price level will double.

B. Stochastic Form of the Model

In order to examine the stochastic behavior of the equilibrium, it is assumed that the demands for currency and demand deposits are subject to $\frac{1}{2} \frac{1}{2} \frac{1}{$

expectations in the economy. Because of the households' wealth constraint, the following condition must hold

(7)
$$c_h^d \tilde{\epsilon}_C + p_h^d \tilde{\epsilon}_D + E_h^d \tilde{\epsilon}_E = 0$$
,

where $\tilde{\epsilon}_E$ is the proportional shock to equity demand. Taking the log of (6) and letting p = log(P) yields

(8)
$$\tilde{p} = \log H - \log (k_c C (1 + \tilde{\epsilon}_C) + k_d D(1 + \tilde{\epsilon}_D)),$$

where C and D represent the demand for currency and deposits when $\tilde{\epsilon}_{C} = \tilde{\epsilon}_{D} = 0. \quad \text{Linearizing (8) around the equilibrium } E(\tilde{\epsilon}_{C}) = E(\tilde{\epsilon}_{D}) = 0$ yields

(9)
$$-\tilde{p} = \frac{k_c C \tilde{\epsilon}_c}{k_c C + k_d D} + \frac{k_d D \tilde{\epsilon}_D}{k_c C + k_d D}$$

where \tilde{p} indicates deviations around the equilibrium. Note that the first term is the fraction of the shock to real high-powered money demand due to shifts in currency demand and the second term is the fraction due to shifts in deposit demand. The variance of (9) is given by the following normalized quadratic form,

$$(10) \quad \sigma_{\rm p}^2 = \frac{k_{\rm c}^2 C^2 \sigma_{\rm c}^2 + k_{\rm d}^2 D^2 \sigma_{\rm D}^2 + 2 k_{\rm d} k_{\rm c} C D \sigma_{\rm cD}}{(k_{\rm c} C + k_{\rm d} D)^2} \ . \label{eq:sigma_problem}$$

C. The Optimal Reserve Ratio for Fixed Deposit Rates

In the case of regulated deposit rates it is immediate that both (9) and (10) are homogeneous of degree zero in $k_{\rm c}$ and $k_{\rm d}$. Hence, the equilibrium and the variability of the equilibrium depends in a non-trivial manner only on the ratio of $k_{\rm d}$ to $k_{\rm c}$. This justifies the procedure of normalizing the reserve level of currency at unity, a condition which is economically equivalent to specifying that high-powered money may circulate as currency, with banks issuing only deposits.

To evaluate the reserve level which minimizes the variance of prices, the derivative of (10) with respect to \mathbf{k}_{d} is set equal to zero which yields, when currency reserves are normalized at unity,

(11)
$$(k_d/k_c)^{*} = k_d^{*} = \frac{C[\sigma_C^2 - \sigma_{CD}]}{D[\sigma_D^2 - \sigma_{CD}]}$$

In words, (11) states that the ratio of reserves behind deposits to reserves behind currency is inversely related to the ratio of the variance of deposits to currency, each corrected by subtracting the covariance between the shocks to the assets. The following observations are made directly from this expression:

- (1) The optimal reserve requirement on deposits is positively related to the size and per dollar variability of currency and negatively related to the size and variability of deposits. An increase in the covariance between shocks to currency and deposits will decrease the optimal reserve ratio if and only if $\sigma_{\text{C}}^2 < \sigma_{\text{D}}^2$.
- (2) The optimal reserve requirement will be zero if and only if the covariance between shocks to currency and deposits is equal to the negative of the variance of currency.

- (3) The optimal reserve ratio on demand deposits is unity if shocks to the demand for currency are perfectly negatively correlated to those of demand deposits, so that $D\tilde{\epsilon}_D = -C\tilde{\epsilon}_C$. This is the case when currency and deposits are perfect substitutes and total money demand is stable.
- (4) It is possible for the optimal reserve requirement, k_d^* , to be negative or greater than one. A negative reserve ratio would exist if banks are allowed to issue currency as a given fraction of deposits, holding no reserves, and this bank currency is viewed by the public as a perfect substitute for high-powered money. A reserve requirement greater than unity can prevail if banks issue equity to obtain a quantity of reserves greater than the size of their deposits.

By substituting (11) into (10), the minimized variance of the price level, σ_p^{2*} , is

(12)
$$\sigma_{p}^{2*} = \frac{\sigma_{C}^{2}\sigma_{D}^{2} (1 - \rho^{2})}{\sigma_{C}^{2} - 2\rho \sigma_{C}\sigma_{D} + \sigma_{D}^{2}}$$
,

where ρ is the correlation coefficient between currency and deposit disturbances. The minimized variability of prices is independent of the quantity of currency and deposits demanded. The derivative of (12) with respect to ρ is

(13)
$$\frac{\partial \sigma_{p}^{2*}}{\partial \rho} = \frac{-2 \sigma_{C}^{2} \sigma_{D}^{2} \rho}{\sigma_{C}^{2} - 2 \sigma_{CD} + \sigma_{D}^{2}}$$

which implies that the minimized variability of prices decreases as ρ approaches ± 1 .

D. Extension to N assets

It is straightforward to extend the model developed in the previous section to the case of N monetary assets whose rates of return are fixed. Let A_1, \ldots, A_n be the real demand for monetary assets, k_1, \ldots, k_n their respective reserve ratios in terms of high-powered money, and $[\sigma_{ij}]$ the covariance matrix of contemporaneous proportional shocks, ε_i , among the monetary assets. Analogous to (9), proportional shocks to the price level can be expressed.

(14)
$$-\tilde{p} = \frac{\sum_{i=1}^{n} k_i A_i \tilde{\epsilon}_i}{\sum_{i=1}^{n} k_i A_i}$$

and the variance of prices as

(15)
$$\sigma_p^2 = \sum_{i=1}^n \sum_{j=1}^n \frac{A_i A_j k_i k_j \sigma_{ij}}{(\sum A_i k_i)^2}$$
.

It is shown in the appendix that the set of k which minimize (15) can be expressed in the compact form,

(16)
$$k_{i}^{*} = \frac{c \sum_{j=1}^{c} \sigma_{ij}^{-1}}{A_{i}}, i=1,...n,$$

where σ_{ij}^{-1} is the ijth coefficient of the <u>inverse</u> of $[\sigma_{ij}]$, and c is an arbitrary normalization constant. The minimized variance of prices at the optimal set of k_i^* above is simply

(17)
$$\sigma_{p}^{2} (k_{i}^{*}) = \sum_{i,j} \sigma_{ij}^{-1}$$
,

the sum of all the elements of the inverse of the variance-covariance matrix.

E. Competitive Rates on Monetary Assets.

In the case where deposit and currency rates are determined competitively or influenced by the reserve ratios $k_{\rm c}$ and $k_{\rm d}$, the relationship between the price level and reserve ratios is more complex than the case of fixed deposit rates. In the competitive case, (6) can be written

(18)
$$P = \frac{H}{k_c C^d (k_c, k_d) + k_d D^d (k_c, k_d)}$$
,

where
$$\frac{\partial D^d}{\partial k_c} < 0, \frac{\partial D^d}{\partial k_d} > 0$$

$$\frac{\partial D^{d}}{\partial k_{c}} > 0, \quad \frac{\partial D^{d}}{\partial k_{d}} < 0.$$

Although gross substitutability guarantees that the \underline{sum} of the demand for currency and deposits will fall when either or both reserve ratios rise, the demand for high-powered money may rise if one asset declines in demand and it possess a high reserve ratio. If this extreme case is ruled out, then a proportional increase in reserve ratios will increase the demand for high-powered money, but less than proportionally since deposit rates fall. In this case the price level would be homogeneous of a degree between 0 and -1 in k_c and k_d .

Under the same proportional shock structure analyzed in Section B above, (10) can be rewritten

$$\sigma_{p}^{2} = k_{c}^{2}C^{2}(k_{c}, k_{d}) \sigma_{C}^{2} + k_{d}^{2}D^{2}(k_{c}, k_{d}) + 2k_{d}k_{c}C(k_{c}, k_{d})D(k_{c}, k_{d})\sigma_{CD}$$

$$\frac{(k_{c}C(k_{c}, k_{d}) + k_{d}D(k_{c}, k_{d}))^{2}}{(k_{c}C(k_{c}, k_{d}) + k_{d}D(k_{c}, k_{d}))^{2}}$$

Because (19) is not homogeneous of degree zero in $k_{_{\hbox{\scriptsize C}}},$ and $k_{_{\hbox{\scriptsize d}}},$ there is no natural normalization of reserve ratios. The first order conditions for the minimization of $\sigma_{_{\hbox{\scriptsize D}}}^2$ are

$$(20) \frac{\partial \sigma_{\mathbf{p}}^{2}}{\partial \mathbf{k}_{\mathbf{c}}} = \left[\mathbf{k}_{\mathbf{d}} \mathbf{D} \sigma_{\mathbf{D}}^{2} + \mathbf{k}_{\mathbf{c}} \mathbf{C} \sigma_{\mathbf{C} \mathbf{D}} - \mathbf{k}_{\mathbf{c}} \mathbf{C} \sigma_{\mathbf{C}}^{2} - \mathbf{k}_{\mathbf{d}} \mathbf{D} \sigma_{\mathbf{C} \mathbf{D}} \right] \chi$$

$$\left[\mathbf{C} \mathbf{D} + \mathbf{k}_{\mathbf{c}} \mathbf{D} \frac{\partial \mathbf{C}}{\partial \mathbf{k}_{\mathbf{c}}} - \mathbf{k}_{\mathbf{c}} \mathbf{C} \frac{\partial \mathbf{D}}{\partial \mathbf{k}_{\mathbf{c}}} \right] = 0.$$

(21)
$$\frac{\partial \sigma_{p}^{2}}{\partial k_{c}} = \left[k_{d}D\sigma_{D}^{2} + k_{c}C\sigma_{CD}^{2} - k_{c}C\sigma_{C}^{2} - k_{d}D\sigma_{CD}\right] \chi$$
$$\left[CD + k_{c}C\frac{\partial D}{\partial k_{d}} - k_{c}D\frac{\partial C}{\partial k_{d}}\right] = 0.$$

The first term in the brackets of (20) and (21) is, surprisingly, identical to the first order condition given in (11) above, and can be written

(22)
$$(k_d/k_c)^{\frac{1}{n}} = \frac{C (k_c^{\frac{1}{n}}, k_d^{\frac{1}{n}})}{D (k_c^{\frac{1}{n}}, k_d^{\frac{1}{n}})} \frac{(\sigma_C^2 - \sigma_{CD})}{(\sigma_D^2 - \sigma_{CD})}$$
.

Of course, (22) is an implicit solution of the optimal k_c and k_d and, unless restrictions are placed on the functional forms, (22) may possess no solution or even multiple solutions. If we compare the solution of (22) with the case when $r_c = r_d = 0$, when $k_c^* = 1$, then k_d^* will clearly be lower

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in the competitive regime since the higher deposit rate will reduce the ratio of currency to deposits.

III. Empirical Results

In order to calculate the optimal reserve ratio on demand deposits shown in Eq. (11), it is necessary to determine the variance-covariance matrix of unanticipated shocks to currency and deposit demands. The following equations for real demand deposits (D) and real currency (C) are estimated from quarterly data from 1952:02 to 1973:04.

(23)
$$\log(D) = 0.087185 + 0.10327 \log GNP - 0.0045529 CPR$$

(.43) (3.49) (-5.00)
- 0.0094301 TDEP + 0.85822 $\log(D)$
(-2.19) (13.22)

 \bar{R}^2 = .9627 SEE = .0081262, Durbin h = 1.1467.

(24)
$$\log(C) = -.09821 + .025859 \log GNP - 0.0015135 CPR$$
 (-2.51) (4.49) (-2.72)

$$+ 1.2815 \log (C)_{-1} - 0.30053 (C)_{-2}$$
 (12.76) (-3.02)

$$\bar{R}^2$$
 = .9969, SEE = .004961, Durbin h = -.1.99,

where GNP = real gross national product, CPR = rate on 90-day prime commercial paper, and TDEP = average rate on time deposits. t-statistics are noted in parentheses under the coefficient. The Durbin h statistic indicates no autocorrelation of the residuals in the demand deposit equation, but border-line rejection (at the 5 percent level) of the hypothesis of no autocorrelation in the residuals in the currency equations. These equations and estimates are similar to those estimated by Goldfeld (1973).

The variance-covariance estimates of the residuals are

(25)
$$\sigma_{C}^{2} = 2.3185 \times 10^{-5}$$
 $\sigma_{D}^{2} = 6.2239 \times 10^{-5}$ $\sigma_{CD}^{2} = 0.9722 \times 10^{-5}$.

The ratio of currency to deposits in the latest quarter is .27429.

Substituting these values into (11) yields

(26)
$$k_{\hat{d}}^{\star} = (.27429) \times (.2564) = 7.0311\%.$$

Therefore, the optimal reserve ratio for stabilizing prices is approximately 7 percent, below the nationwide average of 11.5 percent and approximately one-half the ratio found for Federal Reserve member banks. The positive covariance between deposits and currency decreases the optimal ratio from 10.22 percent which would be the value of $k_{\tilde{d}}^*$ if σ_{CD} were zero. It is clear that the rather low value of $k_{\tilde{d}}^*$ is due to the fact that (1) the per-dollar variability of deposits is approximately $2\frac{1}{2}$ times the per dollar variability of currency, and (2) the value of deposits is $3\frac{1}{2}$ times that of currency.

Substituting (25) into (12) indicates that the minimized variance of the price level is

(27)
$$\sigma_{\rm p}^{*2} = 2.0438 \times 10^{-5}$$
.

This indicates that the standard deviation of the price level is 0.452 percent per quarter. If reserves on demand deposits were zero, it can be seen from Eq. (10) that the variance of the price level would be identical to the variance of currency, equalling 2.3185×10^{-5} , or 13.5 percent higher than (27). As k_d approaches infinity, the variance of prices approaches σ_D^2 or 6.2239×10^{-5} . The variance of the price level at the current level of reserves is 2.1×10^{-5} , or 2.8 percent higher then (27). At a reserve level of 100 percent, the variance is more than double the minimized value. For a reserve ratio of less than -.274 (-C/D), the system is unstable, since an increase in the demand for high-powered money will cause a rise in the price level. Figure 1 displays the variance of the price level for differing levels of reserves on demand deposits. As can be seen from the asymmetry of the curve, reserve levels below the optimal level cause a greater increase in price variation than the equivalent reserve ratios

The above empirical results are subject to a number of qualifications. First, no attempt has been made to correct for the potential simultaneity problem involved in equations (23) and (24). If the price level does not move quickly enough to continually equilibrate the demand and supply of high-powered money, then the effects of asset supplies should be taken into account. However, Goldfeld (1973, 1976) has not found a significant simultaneity bias in similar demand equations. Secondly, time deposits are ignored in the empirical work, although the optimal theoretical ratio on such deposits has been derived in Section II.D. In the United States, nearly 90 percent of all high-powered money is held behind currency and demand deposits, so that any recommendation calling for a substantial increase in reserves on time deposits would clearly amount to a major institutional change.

A third caveat is the recent evidence of a substantial shift in the demand for money, particularly that of demand deposits (see Goldfeld (1976) and Garcia and Pak (1979)). When the currency and deposit equations are estimated through 1977, the optimal reserve ratio declines by approximately $2\frac{1}{2}$ percentage points. This is due primarily to the increased instability of the demand for deposits relative to currency. It was judged that the earlier time period, during which money demand was relatively stable, is more appropriate for estimating asset disturbances until more is understood about the sources of the recent shift in deposit demand.

IV. Summary

The result of this research is the development of a stochastic financial model from which one can derive the reserve levels on financial assets which minimize price level fluctuations. It is shown that the optimal

reserve ratio on demand deposits in the two-asset case is a inversely related to (1) the ratio of deposits to currency, (2) the ratio of the unanticipated variability of deposits to that of currency, and (3) the covariance of unanticipated shocks between these assets (if currency demand is relatively more stable than deposit demand, as is empirically the case). The general solution for the N-asset case as well and the case where deposits rates are competively determined are also solved.

Empirically, it is found that the optimal reserve ratio on demand deposits is approximately 7 percent in the United States, almost half the current average. However, because price level variance is more sensitive to reserve ratios below their optimal level than above, the current, above-optimal level of reserves does not significantly increase price fluctuations.

APPENDIX

The formal problem of finding the optimal reserve ratios in the N-asset case is

(A1)
$$\min_{k} f(K) = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{A_{i} A_{j} k_{i} k_{j} \alpha_{ij}}{(\sum_{i=1}^{n} A_{i} k_{i})^{2}}$$

$$i=1$$

where $K = (k_1, k_2, \ldots, k_n)$ and, due to the homogeneity property, K is defined up to an arbitrary constant. In matrix form, (A1) can be written

(A2)
$$f(K) = \frac{Q^{\prime} \Sigma Q}{Q^{\prime} UQ}$$

where

(A3)
$$Q = (Q_1, Q_2, \dots Q_n), Q_i = A_i k_i$$

$$\Sigma = [\sigma_{ij}]_{nxn}$$

$$U = \begin{bmatrix} 1 & \dots & 1 \\ \vdots & \vdots & \vdots \\ 1 & \dots & 1 \end{bmatrix} = \text{ the unity matrix}$$

Minimizing f(k) is equivalent to maximizing $\lambda = 1/f(K)$, or,

(A4)
$$\max_{\mathbf{k}} \lambda = \frac{Q'UQ}{Q'\Sigma Q}.$$

This problem is formally equivalent to the simultaneous diagonalization of the two matrices U and Σ encountered in discriminant analysis. The first order condition for maximization is the vector condition

=

(A5)
$$\frac{\partial \lambda}{\partial Q} = \frac{2 \left[UQ(Q^{\prime}\Sigma Q) - \Sigma Q(Q^{\prime}UQ) \right]}{(Q^{1}\Sigma Q)^{2}} = 0.$$

This can be simplified by substituting $\lambda = (Q'UQ)/(Q'\Sigma Q)$ to obtain

$$.(A6) UQ - \lambda \Sigma Q = 0.$$

Assuming ∑ is nonsingular, (A6) can be written

(A7)
$$(\Sigma^{-1} U - \lambda I) Q = 0.$$

The solution to this problem is to find the largest eigenvalue of Σ^{-1} and its associated eigenvector. Let Σ^{-1} be indicated by

(A8)
$$\Sigma^{-1} = \begin{bmatrix} \sigma_{11}^{-1} & \sigma_{1n}^{-1} \\ & & \\ \sigma_{n1}^{-1} & \sigma_{nn}^{-1} \end{bmatrix}$$

where σ_{ij}^{-1} is the ij^{th} element of Σ^{-1} (not $1/\sigma_{ij}$). Note that Σ^{-1} is not symmetrical even though Σ is. Σ^{-1} U can then be written

$$\Sigma^{-1}U = \begin{bmatrix} a_1, a_1, \dots & a_1 \\ a_i, a_i, \dots & a_i \\ a_n, a_n, \dots & a_n \end{bmatrix}$$

where
$$a_i = \sum_{j=1}^{n} \sigma_{ij}^{-1}$$

=

The rank of Σ^{-1} U is obviously unity, which means that the matrix has n-1 eigenvalues equal to zero and one non-zero value. Since the sum of all the eigenvalues is equal to the sum of the diagonal elements, and n-1 are null, the largest eigenvalue is therefore

(A10)
$$\lambda_1 = \sum_{i=1}^{n} a_i = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij}^{-1},$$

Thus the eigenvalue is equal to the sum of all the elements of Σ^{-1} . The associated eigenvector is found by solving

(A11)
$$(\Sigma^{-1}U - \lambda_{1}I) Q = 0$$

whose solution is

$$(A12) Qi = c ai,$$

where c is an arbitrary constant.

Finally the vector K is found as

(A13)
$$k_{i} = \frac{c \sum_{j=1}^{n} \sigma_{ij}^{-1}}{A_{i}} \quad i = 1, ..., n,$$

the constant determined by the normalization constraint.

FOOTNOTES

I would like to thank William Milne for providing computational assistance for this paper. Marcel Genet help provided the mathematical structure for solving the N-asset case.

¹This also appears to be the opinion of Alan Metzler (1969) and Deane Carson (1963, 1964).

²These rates could be modified so that constant per unit costs for the production of money services can be included without changing any qualitative results in the paper.

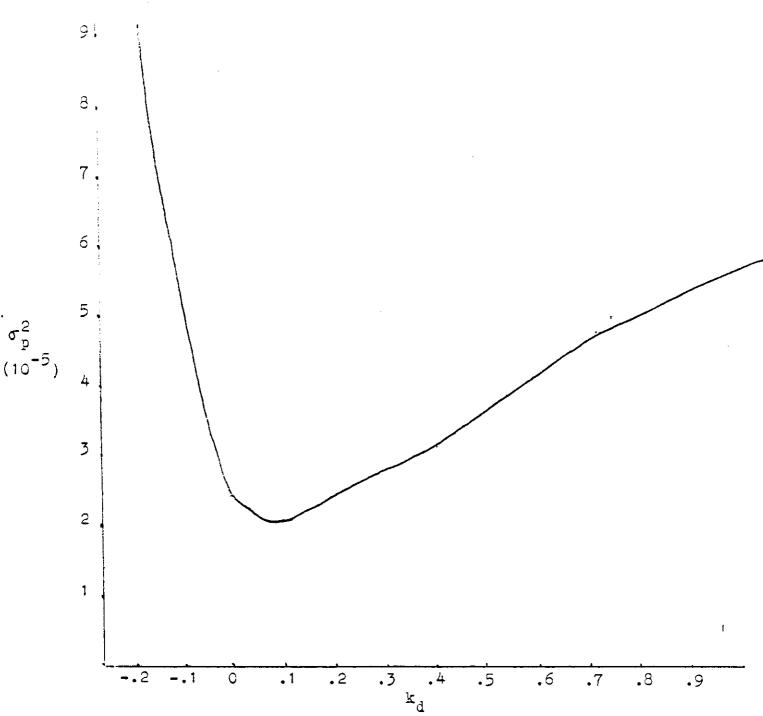
³Since real capital is approximately fifty times high-powered money, this simplification does not have much empirical effect.

 ^4The rate of return on deposits may also affect the variances and particularly the covariances between currency and deposits. It might not be unreasonable to assume that higher deposit rates would reduce σ_{CD} and increase σ_D^2 . This case is not considered here.

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This Figure plots the relationship between the variability of prices and the reserve ratio on demand deposits. The minimized price variability is $2.04 \times 10(-5)$ achieved at a reserve ratio of 7.03%.